▲ Assessment Corner: Assessing Mathematical Processes - A Complex Process

CHRIS SUURTAMM
E-MAIL: suurtammc@uottawa.ca

Chris Suurtamm is an Assistant Professor of Mathematics Education at the University of Ottawa. She is an experienced mathematics teacher who is committed to helping teachers with the implementation of the new curriculum and assessment practices.

Mathematics is more than content. Doing mathematics is a complex process and proficiency in mathematics means a certain level of expertise, competence, knowledge, and facility in mathematics. This article discusses the importance of recognizing mathematical processes and the challenge of examining mathematical processes in the assessment process.

In an analysis of a variety of curricula from several jurisdictions (Suurtamm & Vézina, 2003) mathematics content strands such as number sense, geometry, measurement, algebra, statistics, and probability can be easily matched across curricula from most countries and provinces. However, mathematical processes are often not as easily matched. International assessments such as TIMSS and PISA have prompted international standards for curriculum. Hence, curricula from different jurisdictions share similar characteristics such as:

- Curriculum objectives are usually divided into content strands
- Content strands are similar across jurisdictions
- Many curricula have process strands

However, different jurisdictions define and discuss mathematical process strands in different ways. Some countries and provinces have additional frameworks to support the processes of mathematics. Occasionally one or more of the processes may be included as a content

area, or more frequently, a process is seen as cutting across several content areas. One or several of:

- applying mathematics,
- developing mathematical thinking,
- logic,
- problem solving, and
- mathematical inquiry

are process components that appear in some of the curricula (Ruddock, 1998). Denmark uses mathematical competencies as a basis for the structuring of math education, includes eight mathematical competencies:

- Mathematical thinking
- Problem tackling
- Modelling
- Reasoning
- Representing
- Symbol and formalism
- Communicating
- Using aids and tools

and considers the mastering of mathematics to be the mastering of these competencies (Blomhoej & Hoeigaard, 2003).

The processes of mathematics are highly emphasized in several jurisdictions such as Quebec, New Zealand and Singapore. For instance, in Quebec, there are three particular competencies:

- problem solving
- reasoning through concepts and mathematical processes
- communication with language of mathematics

that are thread throughout the curriculum. Singapore's curriculum (Ministry of Education, Singapore) focuses on mathematical processes and includes this diagram of the processes (figure 1) in every level of their curriculum:

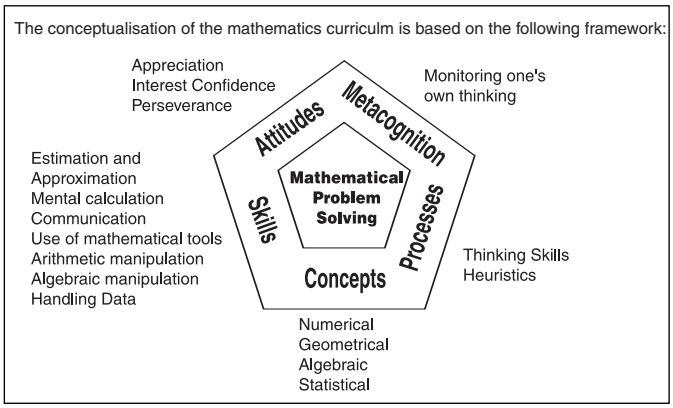

Figure 1. Singapore Framework of the Mathematics Curriculum

This framework encompasses the whole mathematics curriculum from primary to secondary school. The primary aim of the Singapore mathematics curriculum is to enable pupils to develop their ability in mathematical problem solving. Mathematical problem solving includes using and applying mathematics in practical tasks, in real life problems and within mathematics itself. In this context, a problem covers a wide range of situations from routine mathematical problems to problems in unfamiliar contexts and open-ended investigations that make use of the relevant mathematics and thinking processes. As seen in the diagram, the attainment of problem solving ability is dependent on five inter-related components; attitudes, skills, concepts, processes and metacognition, that all form a part of problem solving. Concepts refer to the basic mathematical knowledge needed for solving mathematical problems, covering numerical, geometrical, algebraic, and statistical concepts. Skills refers to manipulative skills that pupils are expected to perform when solving problems. They include estimation and approximation, mental calculation, communication, use of mathematical tools, arithmetic and algebraic manipulation, and handling data. Processes refer to the thinking and heuristics involved in mathematical problem solving such as comparing, identifying patterns and relationships, spatial visualization, using a diagram/model, working backwards, using guess and check, or simplifying the problem. Metacognition refers to the ability to monitor one's own thinking processes in problem solving and attitudes refers to the affective aspects of mathematics learning such as enjoying mathematics, showing confidence in using mathematics, and persevering in solving a problem.

As recognized by many, mathematics is a complex task. Kilpatrick, Swafford, & Findell (2001) define mathematical proficiency as having five components:

- *conceptual understanding*—comprehension of mathematical concepts, operations, and relations
- *procedural fluency*—skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- *strategic competence*—ability to formulate, represent, and solve mathematical problems
- *adaptive reasoning*—capacity for logical thought, reflection, explanation, and justification
- *productive disposition*—habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

These components represent different aspects of a complex whole and are not independent. Rather, they are interwoven and interdependent and cannot be easily separated.

This complexity of mathematics and the interconnectedness of the processes of mathematics have strong implications for assessment. An assessment program that merely uses paper and pencil tests will overlook many of the mathematical processes. An assessment program that uses a variety of strategies and tools will be more suitable to capturing the complexity of mathematics. Rich mathematical tasks allow students to develop and show their competencies in problem solving, communication skills, mathematical thinking and perseverance. Further, the timely use of self-assessment helps students foster metacognitive skills such as a positive attitude towards mathematics and responsibility towards one's own learning, that are strongly recognized in the Singapore curriculum as being essential to learning mathematics. Assessment that includes observation and conferencing with students allows teachers to have a window into students' mathematical thinking and strategies for solving problems. Comments and anecdotal feedback to students helps students to determine the processes that they are mastering well and those that need further development. Such an assessment program is a complex task for the teachers. However, in order to develop mathematical proficiency in students, the full range of content and mathematical processes must be a foundation for curriculum, instruction, and assessment.

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