RESEARCH SUMMARY – REPRESENTATIONS OF FRACTIONS

Meaning of Fractions

According to Small (2009a), "a fraction is a number that describes a relationship between a part (represented by the numerator) and a whole (represented by the denominator)." Key ideas include:

- a fraction is not meaningful without knowing what the whole is
- there are always two fractions involved in any single fraction situation: the part you are considering and the rest of the whole (e.g., whenever there is 3/4, there has to be a 1/4)
- fractions can be used to describe any partitions of a whole
- fractions can represent parts of regions, parts of sets, parts of measures, or ratio. These meanings are equivalent (e.g., 1/4 of a region is 1 whole divided into 4 equal parts).

Fractions as Division

A divided quantity or partitioned quantity usually refers to a whole quantity being divided into the same number of equal-sized parts. According to Fosnot and Dolk (2003), fractions can be represented as partitive and quotative division.

Cookie Sharing Problem A:

12 cookies are shared among 4 children. How many cookies does each child get?

12/4 = 12 ÷ 4 = 3 -> partitive division where you think about fair sharing 12 cookies with 4 children

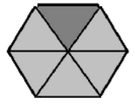
Cookie Grouping Problem B:

12 cookies are divided up so there are 4 cookies to a bag. How many bags are needed?

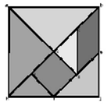
12/4 = 12 ÷ 4 = 3 -> quotative division where you think about how many groups of 4 there are in 12

Fractions as a Part of a Whole or as Part of a Set of Objects

"Equal parts into which the whole is divided are equal, but do not have to be identical. For parts of a set, this means that the members of the set do not have to be identical" (Small, 2009b, p. 44).



1/6 of 1 hexagon is 1 triangle (congruent)



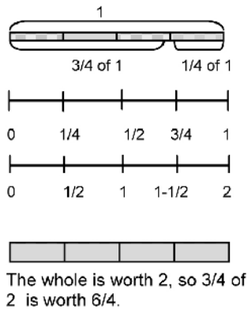
1/7 of 1 tangram is any 1 tan (some tans are not congruent)

Fractions as a Ratio and Rate

When a quotient of a divided quantity refers to a multiplicative relationship between two quantities (e.g., how many times larger or smaller is one than the other), the term, ratio is often used. According to Fosnot (2007), Small (2009), and Smith (2002), fractions are also relations. Small (2009) adds that a ratio is a comparison between two numbers with the same unit, while a rate compares quantities with different units. So, for the cookie problem, a ratio shows part of a whole (12 cookies: 3 cookies or 12:3), while a rate could be 12 cookies per 4 children (12/4) or a unit rate be 3 cookies per 1 child (3/1).

Fractions as Part of a Measure

Watanabe (2006) explains that the use of fractions in measurement contexts, such as linear or liquid (litre) measures helps students to understand fractions as quantities. Understanding fractions as quantities is a common challenge for students when their knowledge of fractions is focused on fractions as part of a whole. Thompson and Saldanha (2003) report that students who are primarily using the part-whole representation often have difficulty making sense of fractions greater than 1 (e.g., improper fractions, mixed numbers). So, Watanabe (2006) explains that with an emphasis on measurement, the idea that non-unit fractions (e.g., 2/3, 3/4, 4/5,) are a collection of unit fractions (e.g., 1/3, 1/4, 1/5) emerges.



The whole is worth 2, so 3/4 of 2 is worth 6/4.

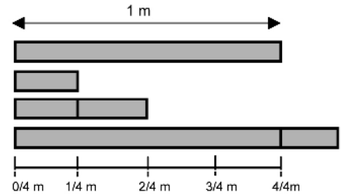
What does 3/4 mean? How do you know?

- 3 parts of a whole that is partitioned into 4 equal parts
- representation of measured quantities such as 3/4 L (litres) or 3/4 m (metres)
- 3 times of the unit obtained by partitioning 1 into 4 equal parts
- a quotient fraction (3 ÷ 4)
- A is 3/4 of B; if we consider B as 1 (a unit), then the relative size of A is 3/4

Paper Ribbon Problem:

A satin ribbon that is 1 metre long is partitioned into 4 equal pieces.

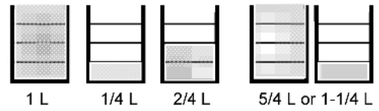
a) How much of one metre is 1 piece of ribbon?
b) How much of one metre are 2 pieces of ribbon?
c) How much of one metre are 5 pieces of ribbon?



Water Bucket Problem:

A water bucket that holds 1 litre of water is partitioned into 4 equal parts.

a) How much of one litre is 1 part of the bucket.
b) How much of one litre is 2 parts of the bucket.
c) How much of one litre is 5 parts of the bucket.



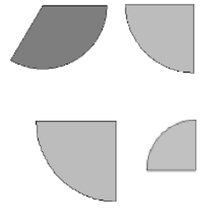
In addition Randall (2005, p. 13) outlines other key mathematical understandings of fractions:

- A fraction describes division.(a/b = a ÷ b, a & b are integers & b ≠ 0), and it can be interpreted on the number line in two ways. For example, 2/3 = 2 ÷ 3. On the number line, 2 ÷ 3 can be interpreted as 2 segments where each is 1/3 of a unit (2 x 1/3) or 1/3 of 2 whole units (1/3 x 2); each is associated with the same point on the number line.
- Each fraction can be associated with a unique point on the number line, but not all of the points between integers can be named by fractions.
- There are an infinite number of fractions between any two fractions on the number line.
- A decimal is another name for a fraction and thus can be associated with the corresponding point on the number line.
- Whole numbers and integers can be written as fractions (e.g., 4 = 4/1).
- A percent is another way to write a decimal that compares part to a whole where the whole is 100 and thus can be associated with the corresponding point on the number line.

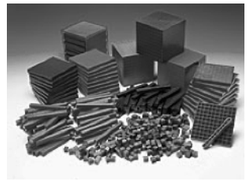
LINKS TO MANIPULATIVES:

A fraction is not meaningful without knowing what the whole is.

- What might this idea look like using fraction circles?
- What is the fractional name for each of the following pieces.
- Did you have to visualize the whole circle in order to be able to name the fraction circle piece?



How would you name these fraction circle pieces? Are they both one fourth? Why?

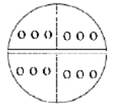


When we identify base ten blocks as units, rods, and flats, to specify its numerical value, we need to know the value of the whole.

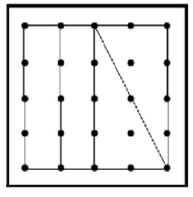
For example, a rod can have a value of 10, 1, 0.1 (1/10), or 0.01 (1/100) depending on which block has been identified as the whole.

Manipulatives make explicit the connections between fractions as part of a whole or part of a set as well as the idea of fractions as division.

- How can we share 12 in 4 equal groups?
- What is 1/4 of the set of 12?



Place 12 counters on a fraction circle divided into fourths to divide them into 4 equal groups.



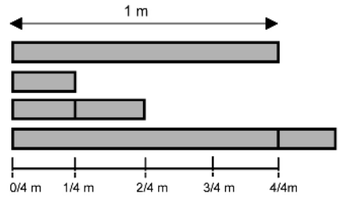
We can use manipulatives to see that equal parts do not have to have identical shapes.

- Is this geoboard divided into fourths? How do you know?
- Which model of representation of fractions are you using?

Paper Ribbon Problem:

A satin ribbon that is 1 metre long is partitioned into 4 equal pieces.

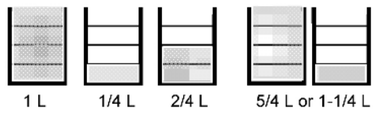
a) How much of one metre is 1 piece of ribbon?
b) How much of one metre are 2 pieces of ribbon?
c) How much of one metre are 5 pieces of ribbon?



Water Bucket Problem:

A water bucket that holds 1 litre of water is partitioned into 4 equal parts.

a) How much of one litre is 1 part of the bucket.
b) How much of one litre is 2 parts of the bucket.
c) How much of one litre is 5 parts of the bucket.



In addition Randall (2005, p. 13) outlines other key mathematical understandings of fractions:

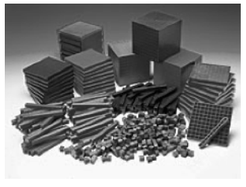
- A fraction describes division.(a/b = a ÷ b, a & b are integers & b ≠ 0), and it can be interpreted on the number line in two ways. For example, 2/3 = 2 ÷ 3. On the number line, 2 ÷ 3 can be interpreted as 2 segments where each is 1/3 of a unit (2 x 1/3) or 1/3 of 2 whole units (1/3 x 2); each is associated with the same point on the number line.
- Each fraction can be associated with a unique point on the number line, but not all of the points between integers can be named by fractions.
- There are an infinite number of fractions between any two fractions on the number line.
- A decimal is another name for a fraction and thus can be associated with the corresponding point on the number line.
- Whole numbers and integers can be written as fractions (e.g., 4 = 4/1).
- A percent is another way to write a decimal that compares part to a whole where the whole is 100 and thus can be associated with the corresponding point on the number line.

LINKS TO MANIPULATIVES:

A fraction is not meaningful without knowing what the whole is.

- What might this idea look like using fraction circles?
- What is the fractional name for each of the following pieces.
- Did you have to visualize the whole circle in order to be able to name the fraction circle piece?

How would you name these fraction circle pieces? Are they both one fourth? Why?



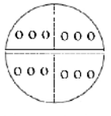
When we identify base ten blocks as units, rods, and flats, to specify its numerical value, we need to know the value of the whole.

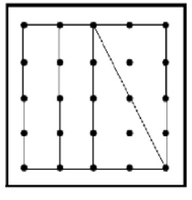
For example, a rod can have a value of 10, 1, 0.1 (1/10), or 0.01 (1/100) depending on which block has been identified as the whole.

Manipulatives make explicit the connections between fractions as part of a whole or part of a set as well as the idea of fractions as division.

- How can we share 12 in 4 equal groups?
- What is 1/4 of the set of 12?

Place 12 counters on a fraction circle divided into fourths to divide them into 4 equal groups.





We can use manipulatives to see that equal parts do not have to have identical shapes.

- Is this geoboard divided into fourths? How do you know?
- Which model of representation of fractions are you using?