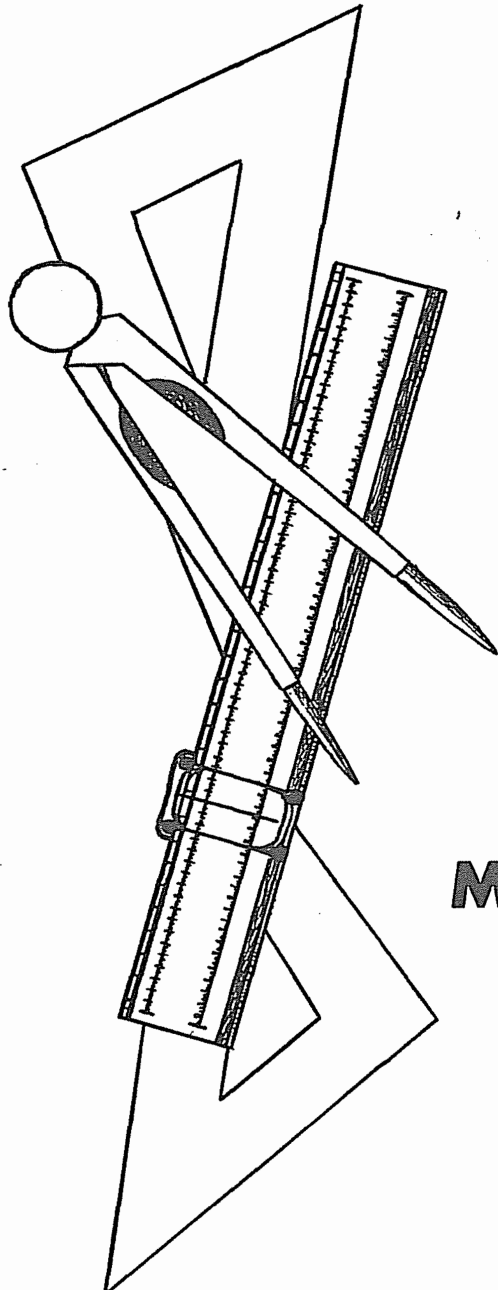


VOLUME 5
NUMBER 3
MAY 1967

ONTARIO MATHEMATICS GAZETTE

sponsored by
THE SAMUEL BEATTY FUND
THE ONTARIO MATHEMATICS COMMISSION
THE ONTARIO ASSOCIATION OF TEACHERS OF MATHEMATICS

C O N T E N T S

****

EDITORIAL

Keats called autumn a "season of mists and mellow fruitfulness". Spring is traditionally the season of new beginnings, and this is so for the Gazette as far as the office of editor-in-chief is concerned. We welcome Dr. Norman Rice to the board and hope, with your help, that we can maintain and improve the quality and relevance of the Gazette content to the teaching of mathematics in Ontario. To this end we are glad to note that several of the contributors to this issue are practising teachers. We hope that we may soon expand the Editorial Board to have a stronger teacher representation in it, and so ensure that the magazine may reflect the teacher's interests, needs and problems as fully as possible.

If this issue has a central theme, it is geometry. In a scholarly, yet delightful, article, that unique gentleman Prof. H.S.M. Coxeter gives his personal interpretation of the long-awaited and now published K-13 Geometry Report. An interesting contrast is provided by the article by Bill Abercrombie from B.C. We hope these articles will stimulate you to try a little experimental teaching or at least to let us hear what you think! There is certainly much material here to generate lively discussion.

In his Presidential Report, Professor Arn Harris points both to the solid contributions of the O.M.C. in the past, and to the need to examine its function and purpose anew, while in the concluding article, Don Attridge gives an excellent account of the recent C.T.F. Conference in Ottawa, at which foundations were laid for a Canadian Association of Mathematics Teachers. Perhaps we are finally beginning to learn that there is no true independence without interdependence. Let us learn by learning more from others.

Finally, we are sure you will enjoy the graceful femininity of the OATM convention report. There must be many capable ladies out there teaching. Let's hear from you, and banish the idea that mathematics teaching is a man's world!

D.H.C.

PRESIDENT'S REPORT

by

A.W. Harris

Althouse College of Education

The past history of the Ontario Mathematics Commission since its inception in 1960 is probably well known to most of you. My purpose in this article is to give you a fairly detailed report of our activities during the past year or so, some of our plans for the coming year and some thoughts on the future of O.M.C.

The problem of financing our activities has been a difficult one for the Executive since the beginning. We continue to receive our main support from the Ontario Teachers' Federation and the Department of Education. During this school year we have received $9,000.00 from O.T.F., $4,000.00 from the Department and several smaller amounts from other sources mainly in support of the publication of the Gazette. The donations have in themselves been generous; however, the main difficulty for the Executive is uncertainty over long range plans due to the interim nature of our financial support.

Since the beginning, the main work of the Commission has been done through two large committees, the Advisory Committees on Elementary Education and Secondary Education. These committees continue to be active, although their roles have changed considerably since the early days. The Commission is best known to teachers for the preparation of proposals for Grades 9-13, by the Secondary Committee. These materials were transmitted to the Department of Education for its consideration. This resulted in new courses for Grades 11-13 and experimental textbooks for Grades 9 and 10 which have been in use in the schools for several years now.

Liaison with O.I.S.E.

Perhaps the most significant recent development has been the formation of a liaison with the Ontario Curriculum Institute which has evolved into a link with the Ontario Institute for Studies in Education. The credit for growth of this relationship is largely due to Dr. Andy Elliott and Mr.Norman Sharp, two of our Past Presidents. It grew out of two studies conceived by our committees. Our Secondary Committee initiated a Study on the Four Year Programme, which resulted in a report published by Ontario Curriculum Institute and a trial of experimental materials in 40 Grade 10 classrooms this year controlled and financed by O.I.S.E. Our Elementary Committee formed a study group to recommend curriculum changes for Kindergarten to Grade 6. A report was published in 1965 by O.C.I.; this was followed by the setting up of an elaborate experiment which has been carried on by O.I.S.E. with a budget of approximately $40,000.00 for 1967-68. Our Elementary Committee still provides some of the personnel of the writing team and offers advice on the details of the experiment through appropriate channels.

The Commission has now formed a Liaison Committee with O.I.S.E., consisting on our side of Messrs. Sharp, Elliott, Dr. George Duff, our vice-president, and the President. This Committee along with two or three O.I.S.E. personnel, forms the Mathematics Committee of that organization. We work with Dr. Ken Prueter who is in charge of Curriculum Projects for O.I.S.E. We meet periodically to offer advice on current projects and recommend new ones for joint action. A listing of O.I.S.E. projects in Mathematics is given below.

K-6: 1966-67 Experiment with geometric solids, relations and graphs; July 1967 Summer course for participating teachers 1967-68 extension of experiments to the following topics:

Grades 1-3 measurement and graphing
4-6 Geometry
2 and 5 methodology - laboratory approach

K-13 Geometry: 1966-67 Publication of report of Committee
July 1967 Writing group to prepare experimental materials for grades 7-10, Chairman-David Boyle
1967-68: Trial of materials in selected schools.

Intermediate Division:	July 1967 Study group to consider impact of new teaching methods in the elementary school on the secondary school-Chairman-Norman Sharp
Four Year Program:	Grade 10: 1966-67 trial of experimental materials in 40 classrooms 1967-68 continued trial of materials in some schools; evaluation of trial.
	Grade 11: 1966-67 preparation of experimental materials by writing group, Chairman-Ronald Dunkley 1967-68 trial of materials in selected schools.
	Grade 12: As above in succeeding years.

Advisory Committee on Colleges of Applied Arts and Technology

Following a strong recommendation from the Secondary Committee, the Constitution was revised at the semi-annual meeting on February 18 to permit the setting up of a 15 man committee to conduct a thorough study of the mathematics programmes of the new Colleges and the relationship of the new institutions to the secondary schools and universities. The Executive is seeking additional funds to allow for an effective study. The organization meeting will be held on April 8th.

Ontario Mathematics Gazette

For the past two years the Gazette has been under the editorship of Dr. Jacke Hogarth of Queen's, who has done an excellent job of maintaining the high standard set by his predecessor. Perhaps the outstanding success was the publication of a Special Elementary Edition in September of 1966. That issue was in charge of Dr. Doug Crawford, also of Queen's, one of the associate editors, who, as usual, gave outstanding leadership and produced an edition which has been widely praised and quoted. We are delighted that Dr. Crawford has agreed to take over the editorship of the Gazette for the next year and know that under his guidance our magazine will continue to develop. Another Special Elementary Edition is planned for next September, with Eldon Pipher as Guest editor.

Teacher Education

For some time the Commission has wrestled with the problem of the pre-service and in-service training of teachers. A sub-committee of the Elementary Committee, chaired by Dr. Crawford, prepared a brief which was submitted to the Department of Education. Recently we have explored the possibility of studies sponsored by O.I.S.E. and/or the Colleges of Education in which we could participate. We are particularly concerned about the preparation of teachers for curriculum changes.

Intermediate Division

The Commission is turning its attention to an area of the curriculum for which it has never made any detailed proposals to the Department, except in connection with the K-13 Geometry report and as a "spin-off" of the original work in the early sixties. Now that the new courses for Grades 7 and 8 have been introduced in practically all of the schools of the province a close look at the results is in order. The Executive, on the advice of the Elementary Committee, is considering setting up a joint sub-committee of the Elementary and Secondary Committees to study the Intermediate Division, with considerable emphasis being placed on the method of teaching to be used.

Secretary

Last May our Secretary Mr.Henry Campbell resigned and was replaced by Mr. W.J. Morrison, who gave excellent service in the past few months. On March 12th Mr. Morrison moved to Newfoundland for a year, where he will be doing work in Adult Education at Stephensville. Mr. Campbell has been substituting in his usual fine fashion. We expect to have a replacement by the time this edition goes to press.

Other Activities

The Elementary Committee is preparing a questionnaire to K-6 teachers on their reaction to the new Curriculum P1, J1.

The Secondary Committee met and forwarded suggestions to the Department about future changes in the Grade 11 and 12 Courses of Study.

The Scholarship Committee has prepared suggestions for student research projects, which have been distributed to the schools.

The Future

There are several organizations which compete for the attention of mathematics teachers. Before O.M.C. moves forward it is well for us to consider our purposes, whether they are unique and whether we can achieve them better than some other agency. I believe we have a role to play.

The Commission is an independent body of some 60 members; it has representatives from all areas of education connected with mathematics teaching.

Our members have a duty to represent the views of those who have appointed them and report back to them. For example, each of our universities is represented; O.M.C. provides an excellent forum for familiarization of the universities with developments in the schools.

The Commission can act in a consultative capacity for the Department and O.I.S.E., providing a "grass roots" reaction to new courses and methods. Perhaps in the future, similar assistance may be provided to the Colleges of Education and Teachers' Colleges. O.M.C. gives teachers, through O.T.F., a voice in curriculum reform they can be proud of. It gives the Department and O.I.S.E. a reservoir of practising teachers who have been thinking about the general problems of curriculum reform, when the time comes to set up revision committees or field-test new concepts. These are only a few of the services of the Commission to mathematics education which I hope we can continue to give for many years.

Finally, the Commission, with the help of O.A.T.M. and the Beatty Fund supervises the publication of the Gazette, a project

I am sure all would agree is desirable. We need a flourishing, independent <u>Ontario</u> magazine for the exchange of information and ideas relevant to the local scene.

The present activities of the Commission and the possibilities for the future I have outlined above lead me to believe that O.M.C. will continue to play an important part in the mathematics education in the future.

****

ONTARIO MATHEMATICS COMMISSION - EXECUTIVE AND COMMITTEE CHAIRMEN

Past President:	Mr. J.N.C. Sharp, Etobicoke Board of Education
President:	Prof. A.W. Harris, Althouse College of Education
Vice-President:	Dr. George Duff, University of Toronto
Executive:	Miss Audrey Baird, Hawthorn Public School, Cooksville
	Mr. Eugene Durrant, Etobicoke Board of Education
	Mr. James MacLean, Department of Education
	Mr. David Skoyles, J.L. Forster Collegiate Inst. Windsor
Acting Secretary:	Mr. Henry J. Campbell, c/o Ontario Mathematics Commission, 1260 Bay Street, Toronto.
Committee Chairmen:	
Secondary Advisory:	Mr. Wilfred Baxter, Grand River C.I. Kitchener, Ont.
Elementary:	Mr. James MacLean
Scholarship:	Prof. D.L. Mumford, The College of Education, University of Toronto, Toronto.
Research and Publications:	Dr. George Duff
Colleges of Applied Arts $ Technology:	to be elected on April 8th.

PROGRESS TO DATE IN THE K-6 MATHEMATICS EXPERIMENT

by

E. Pipher

Field Consultant, Development Division, O.T.S.E.

[Editor's Note: Two experiments in elementary school mathematics are in progress in approximately thirty-five centres in the province of Ontario. The projects are administered by the Division of Development of the Ontario Institute for Studies in Education with Dr. H.A. Elliott as project director. The topics, Geometry (Grades 1-3) and Relations, Mappings and Graphs (Grades 4-6) were recommended by the elementary advisory committee of the Ontario Mathematics Commission, and are based on the Report of the Committee Considering the Mathematics Programme (K to 6) 1965. The following article is adapted from the Interim Progress Report.]

GENERAL COMMENTS

There seems to be a general feeling among the participating teachers and their inspectors that the project has been worthwhile. It appears to have created new interest in some pupils and revived latent enthusiasm in others as well as providing a challenge for those children who have a natural interest in mathematics.

In general, the teachers have been enthused about their work. As an indication of this, most of the teachers in the Fall experiment planned to continue the project after the ten-week period was completed. They have worked very hard, perhaps too hard, spending long hours preparing lessons and in providing materials. In a number of cases, the teachers have been asked to give talks on the experiment to meetings of home and school associations and at in-service courses for teachers.

There is general agreement among the teachers that the pupils are enjoying the work in the experimental classes. They show outward signs of pleasant anticipation as the mathematics period is about to begin. In talking to the children at the beginning of the class, one often gets the impression that they would prefer to get down to work than to listen to a visitor.

SOME TENTATIVE EVALUATIONS

It would appear that significant learning of mathematical ideas is taking place. The primary pupils are becoming aware of shapes in mathematics and in the world around them. Some parents reported that their children frequently made reference to things in the home, making such remarks as "That is like a rectangular prism," or "This reminds me of a sphere."

The pupils in Grades 1-3 are learning to classify solids in different ways noticing that things that don't look alike may be "in the same family."

In the junior grades the pupils are beginning to see relationships between things and later, between sets of numbers. They seem to be beginning to see how graphical methods can help them convey mathematical information to others and can sometimes help them see relationships they hadn't noticed before. They are beginning to learn how graphical methods can help them predict facts that are not shown directly on the graph.

In both experiments other phases of mathematics are being related to the topics at hand. For example, some computational skills are being developed as the pupils determine the number of edges on solids or as they calculate the weights of different materials.

Perhaps the most significant aspect of the experiment is the way in which the learning takes place. The learning in mathematics is arising from activities in which the pupils are directly involved, either individually or in small groups.

However, some large group instruction is also provided to complement the laboratory approach.

The mathematics experiments are taking into account individual differences of maturation, innate ability, and interests. Some pupils have been observed working on a relatively advanced problem at the same time that other pupils are involved with more basic concepts. It is not uncommon for pupils to do additional work in both geometry and graphing voluntarily outside of regular class time.

Most of the teachers report a significant social development takes place as the work proceeds. Children are learning to share, to get along with others, sometimes to be a leader, sometimes to be a follower.

Correlation with English is markedly apparent. Grade 1 pupils are developing reading skills as they study the assignment cards that explain the mathematical task at hand. Both oral and written reporting is a fundamental part of the programme, and skills in the use of precise language are being developed.

The project seems to have been good for the teachers as well as for the pupils. One teacher reported this experiment "has got her out of a rut." Others report that it has made them take a new look at methods, at their role in the classroom, and at how the activity-centred approach can be used in other subjects in the curriculum.

SOME CAUTIONS

While many aspects of the programme are most encouraging, a few words of caution might be noted. The laboratory approach to mathematics involving a new-found freedom for many pupils requires an adaptation of the traditional view of classroom behaviour. There tends to be considerably more noise but in many cases it appears as "busy-noise" and in no way objectionable. In other situations, pupils have not learned a reasonable noise level and are distracting others trying to work and nearby classes. It seems quite apparent that when good work habits have been established prior to the laboratory approach, the teacher can maintain an acceptable standard without much difficulty.

The experiment has brought to light certain problems of a psychological nature. It is difficult to know how much of what Piaget calls conservation can be expected from young children studying geometry. Problems are encountered, too, by many pupils as they move back and forth between two-dimensional and three-dimensional shapes.

It is most gratifying to know that the Division of Applied Psychology of OISE is interested in these and other psychological problems inherent in our work and is giving us assistance.

CONCLUSION

Many more things could be said about the experiment but for an interim progress report, perhaps this is sufficient. Without doubt, it can be said that this is a most exciting project and it has been most rewarding to observe children as they experienced the pleasure of discovering what was for them a new mathematical idea.

THE ONTARIO K-13 GEOMETRY REPORT

by

H.S.M. COXETER
University of Toronto

In the prevalent desire for single courses in mathematics rather than separate courses in its various branches, there has been a tendency for geometry to be squeezed out. This tendency is not only regrettable but unreasonable. Geometry has interactions with other branches of mathematics, and should be taught alongside them, not before or after; but it should by no means be neglected. The purpose of the Ontario K-13 Geometry Report is to outline a program whereby this down-grading of geometry can be avoided.

Synthetic geometry develops and refines spatial intuition. In the physics of crystals or the chemistry of complicated organic compounds, many significant geometric relationships are revealed. There are also increasingly complicated problems of architecture and the subtle intricacies of the space-time continuum.

We believe that visual and intuitive work is indispensable at every level of mathematics and science, both as an aid to clarification of particular problems and as a source of inspiration. Great care should be taken to encourage the mental constructions of the student, whether rudimentary or advanced. Self-reliance goes hand in hand with the cultivation of intuitive judgement and artistic taste.

By dealing with geometry informally, by plausible reasoning rather than by strict proof, it is possible to reach interesting and surprising results much more quickly; the student does not spend a whole hour on a proof of something he regards as obvious. Thus several theorems can be covered in one lesson, and students can be given a bird's-eye view of the subject as a whole.

The good and bad features of our geometrical tradition must be carefully disentangled. In the hands of a good teacher, who does not take the textbook too seriously, a geometry lesson can be a stimulating experience. One Ontario student said, in an essay, that geometry lessons were a revelation to him, because in arithmetic and algebra he was told what to do, but in geometry there was discussion and reasons were given.

In geometry, perhaps more than in the other subjects, a student can exercise originality and ingenuity in devising a construction or seeking a proof.

In former times, the wholesale adoption of Euclid's axiomatic method as an authority, and as a model to be emulated, presented an enormously difficult program for most students. Instead of the axiomatic approach, with rules and definitions, we recommend the intuitive "interest" approach through problems significant to the student. Certain properties of simple figures are assumed. These lead to short chains of easy deductions. Later a more ambitious use of assumptions can be made, so that a wider range of problems is accessible, and some of the old tentative assumptions become theorems. This method minimizes the laying down of authority and the making of apparently arbitrary rules at the outset.

It has been well remarked that, in general, a definition sums up an experience and should not precede it.

The systematic use of axioms in geometry is admissible only after the students have already had several years of experience with simple deductions. Actually, for exercises in deductive reasoning, algebra is probably more suitable than geometry. Geometry should be taught rather for its interesting results and as an exercise in informal reasoning. After all, the work that culminated in the discovery of non-Euclidean geometry occurred before the logical gaps in Euclid had been noticed. Neither Bolyai nor Lobachevsky lived to see a proof of the relative consistency of hyperbolic geometry. Incidentally, as the discovery of the non-Euclidean geometries is the most significant development in the whole field of geometry since Euclid, neglect of this development would hardly be compatible with the position of geometry in contemporary liberal education. As Felix Klein once remarked: Non-Euclidean geometry "forms one of the few parts of mathematics which is talked about in wide circles, so that any teacher may be asked about it at any moment."

In the Primary School, children should become familiar with simple objects that illustrate the ideas of shape, size and measurement. Solids such as spheres, cylinders, cones, pyramids, prisms, antiprisms and other polyhedrons can be appreciated at an earlier age than their two-dimensional counterparts: circles, triangles, squares, rectangles, parallelograms, pentagons, hexagons and other polygons. The square first arises as a face of a cube!

As soon as a child has become familiar with two- and three-dimensional figures, he should begin to make patterns. He will soon see that some shapes, such as triangles, regular hexagons and cubes, can be repeated to fill and cover the plane or space, whereas other shapes, such as regular pentagons, circles and spheres, cannot. When a child is ready to handle a straight-edge and a pair of compasses, he should be encouraged to invent and color patterns of his own liking, and to construct models by such means as straws and pipe cleaners, soft wire, sticks with glue, plasticene, cardboard and ready-made polygons. Pairs of plane mirrors can be used to study reflection, rotation, translation, and the simplest notions of symmetry (as in the Minnemath film Dihedral Kaleidoscopes). The teacher should draw attention to the fact that the rim of a lampshade may cast shadows that are circles, ellipses, parabolas, and hyperbolas.

The comparison of size of similar figures and of angles can be considered at an early age, in preparation for the idea of measuring volume, area, length and angle, and for the use of instruments such as set square, parallel-ruler, compasses and protractor, for making scale models and maps.

A good informal treatment of mensuration is illustrated by the problem of finding the volume of a pyramid of height z based on a rectangle $2x \times 2y$, where x,y,z can have any convenient values, such as $3,4,5$ (inches or centimetres). A cuboid x,y,z is dissected into six pieces ("orthoschemes") by planes joining one pair of opposite vertices to each of the other three pairs in turn. It is very plausible that these six pieces all have the same volume $xyz/6$. (Those pairs of pieces which are congurent are not directly congruent but oppositely congruent, like a pair of shoes or an arbitrary solid and its mirror image.) The whole construction is then repeated so as to produce twelve pieces. (These can be mixed up. One child is asked to choose a piece, and another to find a mate for it, either directly congruent or oppositely congruent.) Finally, eight of the twelve pieces are reassembled to form the desired pyramid, whose volume is thus seen to be $4xyz/3$, that is, one-third the base times the height.

The remaining four pieces, with four of the eight, make another pyramid (with base $2y \times 2z$ and height x, or base $2z \times 2x$ and height y).

We follow the British and Russians in recommending the introduction of geometric transformations (or "motion geometry") as early as possible, not only as a tricky way to prove theorems but as a means of inculcating a feeling for space. This idea is closely related to symmetry, and thus appeals to the artistic side of children. A child is aware of the symmetry of a butterfly before the concept of distance has become fully clarified. He will enjoy making his own "butterfly" by folding a sheet of paper with a wet spot of ink near the crease. The classification of frieze patterns according to their seven symmetry groups can be appreciated by children in Grades 4 through 6, and still holds interest for much older students.

Motion geometry includes the concept of translation, another name for which is vector, and for the first time children meet the plus sign in a non-arithmetical context. In expressing a translation as the "sum" of two half-turns, they obtain a first taste of a non-commutative algebra.

Cartesian coordinates can easily be introduced in Grade 6 or 7, so as to provide a stimulating synthesis of geometry, arithmetic, algebra, and trigonometry. Vectors, having previously appeared as translations, can be represented by pairs or triples of numbers, and then by forces or velocities, thus linking pure and applied mathematics.

The algebraic aspect of motion geometry can be developed by using matrices (of two rows and two columns) to represent translations, half-turns, quarter-turns, reflections (in the coordinate axes or their angle bisectors) and dilatations (from the origin). Trigonometry, which many children used to dislike, can be enlivened by judicious use of vectors and of polar coordinates. The dull routine of numerical solution of triangles should be replaced by a taste of the most elegant trigonometric identities.

Any child old enough to handle a pair of compasses can appreciate the idea of coaxal circles, leading naturally to inversion: the simplest example of a transformation that changes shape (more precisely, a conformal transformation that is not merely a similarity). From this it is an easy step to continuous transformations and an informal introduction to topology: the sphere, torus and Möbius strip, maps and colouring problems, topological networks or "graphs", the unsolved problem of classifying knots, and Euler's formula connecting the numbers of vertices, edges, and faces of a polyhedron.

Exceptionally able students should be encouraged to study quadric surfaces, the geometry of complex numbers, and the possibility of using various sets of axioms so as to replace Euclidean geometry by other geometries: affine, inversive, projective, absolute, spherical, hyperbolic. Such excursions will give the correct impression of geometry as a subject that is still developing in a lively manner.

GEOMETRY - AN INTERLUDE OR AN ESSENTIAL

(Summary of a speech delivered in Calgry, August 1966)

by

W.E. Abercrombie
British Columbia Association of Mathematics Teachers

The purpose of this paper is to discuss the place of geometry in the modern high school curriculum. This is becoming a topic of major concern in British Columbia, and, from what I have been told, it should soon become one here in Alberta.

Before beginning a discussion on geometry, it would be of value to observe the entire British Columbia system. By 1960 a few experiments were being conducted using SMSG or Illinois materials. But at a conference held at UBC in December 1960 it was apparent that there was a considerable degree of hostility to this new approach. Nonetheless, it was decided in 1961 systematically to convert the entire secondary mathematics program to modern mathematics.

In 1962 the first text was introduced into Grade 8. At this time, there was nowhere near the variety of books that is available today, and it was decided to use Introduction to Mathematics, by Brumfiel, Eicholz and Shanks. In 1963, Modern Elementary Algebra by Nichols and Collins became the Grade 9 text. In 1964, Geometry by Moise and Downs became the Grade 10 text. In 1965, it was announced that Modern Algebra with Trigonometry would be the Grade 11 course. In the spring of 1966 it was announced that the remainder of Modern Algebra (minus the chapter on probability) plus a unit on calculus would be the Grade 12 course.

Parallel adoptions were occurring in the elementary system from Grades 3 to 7.

It is not my purpose here to criticize the entire system, but I think you should be aware of one or two flaws in the method of adoption. At no time was more than one year's course announced. In other words, we began to teach the Grade 10 course with no idea of what was going to be expected in Grade 11. These texts all represent different approaches to modern mathematics. For example, the Grade 9 text is based on Illinois material, while the Grade 10 text is based on SMSG material. It is quite apparent to me that there was a lack of philosophical over-view in the selection of this curriculum.

You might be asking yourself - is it not possible to adapt a curriculum after it has been started? We have a lovely little device in our province which locks a curriculum for a number of years. This is the textbook rental scheme. Once a book is adopted, it must be kept in use for at least five years. A new revised edition of Modern Elementary Algebra has been published correcting many of the errors of the original. This would mean a new textbook adoption with all the problems consequent to such an adoption.

This last comment points to a second major flaw. There was an almost complete lack of experimentation. I must be a pretty poor reader for I find it literally impossible to read a textbook and recognize all the problems that are going to develop when I try to teach from it.

But this is enough negative thinking. Let us now observe the geometry course in isolation. Previously we had combined algebra and geometry in the Grade 10 and 11 courses on a half-and-half basis. Now geometry comprises the entire Grade 10 course. (As an aside - I have heard it suggested that the motivation was that they did not want to have to pick two courses at once.)

Is it reasonable to take an entire year of secondary school mathematics for the study of geometry? Is not algebra the basis of modern mathematics? Consider the traditional arguments for the study of geometry. (Not being a traditionalist, my interpretation is probably going to be open to argument.) (a) The problems of elementary geometry require an active type of thinking. (b) Skill at mathematics (at all levels) consists largely of recognizing pattersn. (c) Geometry develops intellectual confidence. (d) Geometry, when properly taught, has great aesthetic appeal. (e) Geometry provides an unrivaled field for the exercise of elementary deductive argument. In other words, geometry should be considered as a discipline designed to develop clear thinking.

But this does not really justify the taking of an entire year. There must be some stronger reason. The Moise-Downs text is based upon material from the School Mathematics Study Group. SMSG is convinced that the geometry is a logical extension of the algebra presented in Grade 9. Let me present their arguments for the inclusion (again I am giving a personal interpretation):

The major aim of mathematics through teachers should be to acquaint students with the facts of mathematics through working with them. This utilitarian approach is often ignored or derided by many so-called modern mathematicians. A second aim should be to develop appreciation of clear, logical reasoning and ability to transfer to other situations. A third aim - and here is perhaps the key to an understanding of the place of geometry - is to develop an understanding of the structure of mathematical systems.

Theory is an element of study that has been largely ignored in the past. The acts of naming, classifying, and generalizing are conceptual in nature. A child has no understanding of the real world until he has constructed a conceptual world to correspond to it. I strongly recommend that you read Jean Piaget's The Child's Conception of Geometry.

Do not leap to the conclusion that I am advocating a pure approach to geometry. This is not what I mean by theory. I like Herbert Fiegl's definition of theory - that it is a set of assumptions from which can be derived by purely logico-mathematical procedures a larger set of empirical laws. An isomorphism must exist between the real world and the theoretical world.

I think Irving Adler has indicated the direction in which our teaching of geometry must go. He indicates the major concern of geometry is the study of congruent figures, involving the traditional group of Euclidean transformations. But it is almost as important to study similar figures, which obviously involves a different set of transformations. To use Adler's words: "a geometry is now defined as the study of figures that can be mapped into each other by a group of transformations, and of the properties of figures that remain unchanged in the group when the transformations in the group are applied. We do not study geometry, but some geometries."

Despite all my arguments, I am still not convinced that geometry should occupy such an important place in the curriculum. So instead of dealing in generalities, let us look at the specific text. I must admit a very serious bias - I am firmly convinced that SMSG material represents the soundest mathematical and psychological work that any group has accomplished. Begle's picture is displayed prominently in my classroom.

Such a prejudiced attitude is likely to minimize the flaws in the text, so let me refer to an article by Morris Kline in the April 1966 edition of The Mathematics Teacher.

a. Mathematics is and should be presented as a set of deductive structures.

I think one of the outstanding sections of this book is the sequence beginning with the theorem: "Two parallel lines lie in exactly one plane." It continues through the parallel postulate and finally arrives at the theorem "For every triangle, the sum of the measures of the angles is 180." This is infinitely superior to presenting this concept inductively. Here is a perfect example where it is possible to discover the bridge between the theoretical world and the real world.

b. Mathematics is to be presented rigorously.

This text is certainly rigorous. I cannot really find a single omission. Such a criticism implies that a teacher is incapable of selecting material from a text. Rather than present an endless list of postulates, the authors present their first theorem "Let AB be a ray, and let x be a positive number. Then there is exactly one point P of AB such that AP = x ." There is no place in mathematics for sloppy thinking.

c. Mathematics is built up as an isolated, self-sufficient, pure body of knowledge.

This is absolute nonsense. Because this text does not use examples from the real world does not mean it has no application in the real world. It is becoming quite apparent that many projects in the future will involve such diverse sciences as mathematics, psychology, sociology, political science, etc. Again, let me suggest that the first task is to develop a child's ability to conceptualize. He will then be able to understand the real world and apply his knowledge to it.

d. Mathematics is self-generating.

This is one of the real powers of this text. Using only the undefined terms "point", "line," and "plane," and 23 postulates, a scientific model of reality is constructed. There is not a theorem in the book that could not be substantiated intuitively, but this is not necessary.

e. Students are expected to learn abstract concepts in the expectation that if they learn these, the concrete realizations will be automatically understood.

This can be valid criticism, if we lose sight of the utilitarian aim of mathematics which I presented earlier. The average Grade 10 student has a rather extensive history. Is this theorem too abstract; "Two nonvertical lines are perpendicular if and only if their slopes are negative reciprocals of each other"?

f. The more terminology, the better.

This criticism depends upon your philosophy of mathematics. Piaget has suggested that a child cannot be considered to possess a concept until he is capable of verbalizing it. The vague ambiguity of some teachers who say that a child will learn a term when he needs it, is unacceptable to me. The definitions in this text are outstanding. Again, it is the teacher who must select how they are to be used.

g. Never use words where symbols can be substituted.

A lot of criticism is offered concerning this book's insistence on differentiating between congruence and equality. Its justification depends upon the degree of rigor you feel is necessary.

I think enough has been said to clearly delineate the problem. It is not really possible to evaluate the role of geometry in isolation. Its importance will depend upon the philosophy with which the curriculum is developed.

Let us now consider the ways in which geometry can be presented.

a. A full year of geometry following a full year of algebra.

This is the method we use. It allows geometry to develop an analogous set of postulates and theorems that will enlarge and advance the concepts learned in algebra. An aware teacher will constantly relate that which is being taught in geometry to that which was learned in algebra. I could spend considerable time illustrating ways in which this text utilizes algebra. The chapter on co-ordinate geometry, in which algebra and geometry are completely integrated, is enough evidence. The major criticism of this approach is that it causes a break in algebraic development. The student may not know enough algebra to be able to relate it to geometry.

b. Integrated courses of geometry and algebra in Grades 10 and 11.

This is the technique used in Ontario. It allows the two streams to be developed in a parallel manner. Algebra and geometry will seem more similar. Almost the only criticism I have been able to discover is that it can present a real headache in timetabling.

c. Geometry moved from Grade 10 to Grade 11.

Students will have a firmer foundation in algebra and will be much more mathematically mature. It does present the administrative problem of having students fufill the requirements for university algebra courses.

d. Geometry should be taught only as required for algebra. The chief proponents of this have been the Organization for European Economic Co-operation. Professor Jean Dieudonne has suggested that the mature student should be capable of mastering Euclid in a matter of hours. "The old quarrel of pure versus analytic geometry becomes meaningless, both being mere translations of the language of vectors."

e. Several courses could be created so that a student could take algebra and geometry in the same year.

While this seems to be an admirable solution, it is perhaps unreasonable in the light of demands of other subject areas.

I am quite aware that I have not presented an answer to the question posed at the beginning of this talk. At present, I am convinced there is no answer. It might be suggested that all the developments in mathematics, despite their revolutionary aspects, have been nothing more than a preamble to the real revolution. For many years we have been intensely concerned with what to teach. The Cambridge Report makes rather staggering reading. Perhaps it is time to consider the "can" and "ought" of teaching. We must learn how to teach and we must learn how children learn.

The secondary school is a unique institution of learning. Lawrence Downey has suggested that the secondary school represents a phase of strategized inquiry between the systematized inquiry of the primary years and the specialized inquiry of higher education. I propose that geometry serves an essential function in the secondary mathematics program. But before it can be of real value, a complete and radical change will have to take place in our philosophy of education.

TWO QUOTATIONS ON THE TEACHING OF SCHOOL GEOMETRY

Excerpt from "Comments by Marshall H. Stone, University of Chicago".*

"In my opinion one of the most difficult problems in designing a good modern mathematics program is encountered in the field of geometry. A satisfactory solution to the problem of teaching school geometry demands the introduction of much more physical or intuitive geometry at the elementary level and the elaboration at the secondary level of an axiomatic treatment radically different from that now being followed at Nova High School. Probably this treatment should be based in part on the concept of transformation. Nova should not postpone too long an attack upon this problem."

Excerpt of a talk given by Julius H. Hlavaty at the Laurentian Leadership Conference NCTM held at Mont Gabriel, Quebec, October 7, 8, 1966 "Mathematics Education 1966".

"Geometry has been one of the main stumbling blocks. Why? Is a break-through coming? The Euclidean tradition was imported, and we have clung to it longer than the English and the French. In U.S. schools most geometry work is concentrated in one year. There has been a real obsession in deductive geometry, refining axioms, reducing the number of axioms, etc. This has not been such a problem in the U.K. They are more concerned with the applications of mathematics. There is not so much rigour, and they are more relaxed in their attitude. They deal more with symmetry, reflections, vector methods for deeper understanding. We are coming around to this way of thinking, and many experiments under way are pointing toward a broader and richer understanding."

* Made at a conference at Nova High School, Florida, on implementing a non-graded curriculum in mathematics based on the Cambridge Report.

A MATHEMATICS LEAGUE

by

R.G. Scoins

Waterloo Collegiate Institute

At one of the Junior Math Contest meetings, we discussed the possibility of expanding our activities to other Mathematical endeavors. R. Dunkley suggested the possibility of starting a Math League. After some discussion, Dunkley agreed to set down some basic guide lines for such a league.

I found there was enough interest in Kitchener-Waterloo to go ahead with plans to form a league. Four schools joined the League. A meeting was held at which a representative from each competing school attended. Using the suggested guide lines, we formulated a set of rules and a schedule for our league.

Why did we form a Math League?

The primary motive for forming a league was to further the students' interest in Mathematics and to encourage students to do some research in areas of math not covered on the curriculum.

Who could be a member of a school Math team?

Membership was restricted to students of grades 9, 10, and 11. Five members constituted a team and of those 5 members, no more than 3 could be from grade 11. The team did not have to use the same members for every game.

How was the League run?

1. Schedule
   Each school palyed each of the other schools once. (3 games) On a given day, two of our schools hosted the other two schools and both games used the same set of questions.

2. Format of Game

Each game consisted of 7 essay type questions. The questions fitted into two categories: (i) those testing general math knowledge and reasoning ability for which no specialized preparation was needed and (ii) questions selected from topics beyond the course of study. (The students were told in advance which extra topics would be tested on a given day. This was designed to motivate the students to do extra reading and study beyond the course.) The extra topics used were (i) number bases other than ten (ii) construction-deduction (iii) dophantine equations and (iv) modular arithmetic.

3. Scoring

Each question was worth 1 mark.

For a candidate to get the mark, the answer had to be entirely correct. This meant that for a question with several parts, all parts had to be correct. Hence on each question, a team could get 5 points and a perfect game score for a team was 35 points.

NOTE: Team scores for the league were in the range 6-15.

4. Running a Game

Each question was on a separate 8 1/2 x 11 page. Question 1 was handed out face down. On a given signal the contestant turned the page and attempted the question. There was a time limit on each question. At another signal the students stopped writing, the questions were collected, evaluated, and the score was posted. Then question No. 2 was handed out etc.

A Sample Set of Questions

Following is a set of questions which constituted one of the games.

1. A ladder whose foot is 9 feet from the side of a heuse reaches a window 40 feet above the ground. How long is the ladder?
Time: 2 minutes.

2. A 4" cube has all its faces painted. The cube is then cut into 1" cubes. What is the number of these 1" cubes that will have paint on at least one face?
Time: 2 minutes

3. Harry and Joe started working for different firms at the <u>same</u> salary. Last year Harry had a raise of 10% and Joe had a drop in pay of 10%. This year, Harry had a 10% drop and Joe the 10% raise. Who (if any) is making more money now?
Time: 3 minutes.

4. The sum of two numbers is 12 and their product is 24. What is the sum of their reciprocals?
Time: 4 minutes.

5. The units digit of a 3 digit number is 0. The difference between the number and the number formed by reversing the order of the digits is 693. Find the largest 3 digit number which satisfies these conditions.
Time: 5 minutes.

6. A farmer requires at least two horses. He spends $7,520 buying cows at $230 and horses at $370 each. How many of each does he buy?
Time: 7 minutes.

7. (a) Write 423 in base 5.
   (b) Change 324_8 to base 10.
   (c) Perform the following computations in the bases stated.

$$\begin{array}{r} 246_8 \\ +\ 375_8 \\ \hline \end{array} \qquad \begin{array}{r} 524_6 \\ -235_6 \\ \hline \end{array} \qquad \begin{array}{r} 413_5 \\ \times\ 4_5 \\ \hline \end{array}$$

Time: 7 minutes.

Evaluation

The students' reaction to the League was favourable. The interest of the team members was high and I feel their understanding of math has been broadened. It was excellent preparation for the upcoming Junior Math Contest.

The League will be organized again next year. Some minor changes will be made but the format will be very similar to this year's.

NOTE: J. LeSage also organized a Math league in Barrie and used the same format and questions as we did in K-W.

REFLECTIONS ON THE O.C.E. SUMMER COURSES

by

Mrs. Dianne Goffin, Sir Wilfred Laurier High School, Ottawa*

"Abolish O.C.E. summer courses," (i.e., those leading to an Interim Type B) is a proposal that one hears with increasing frequency these days. It is made on the grounds that one eight-week course cannot adequately prepare a teacher for the experience of facing a classroom full of students, and that two such courses are hardly any better.

As a recent graduate of the O.C.E. summer courses, I have some fairly strong opinions regarding the value of these courses. The standard complaint from those who have not suffered through them is that a total of fourteen weeks just isn't sufficient time for adequate professional training. This, however, is far different from that of the average O.C.E. student - that not nearly enough is offered in the available time; that, in blunter terms, the allotted 14 weeks are a gigantic bore. To cite a specific - and fairly typical - example, I was able to pass easily all four required subjects of the second summer sessions with no more than <u>one hour</u> of extra-class work per week. Surely a course which demands so little of its students is sadly in need of overhaul.

A brief review of the courses offered indicates many areas in which they are lacking.

In the first 8-week summer session, the student alternates between a week of lectures and a week of practice teaching. On the surface, this seems reasonable. However, it is far from adequate. The "week" of practice teaching usually means 2 or 3 lessons of practice teaching; I have heard of instances where a student taught 1/2 a lesson during the entire week.

On alternate weeks the student is "challenged" by lectures

---

* Reprinted from <u>Math Memo</u>, published by the Mathematics Subject Council, District 26, Ontario Secondary School Teachers' Federation.

in specific subject fields and in School Law and Management (surely the dullest collection of facts ever compiled by a high school teacher.) With the exception of students taking Commercial or Physical Education options, most students attend lectures in the mornings only; their afternoons were completely free.

In the mathematics lectures, the student is drilled in two things: a) The lesson plan (Preparation, Presentation, Application) and b) Grade 10 mathematics. Although the former was of considerable value, this particular student at any rate had got the formula (Preparation, ...) down pat in somewhat less than the allotted four weeks. And as for teaching to University graduates the content of a Grade 10 course, that is little short of insulting.

At the end of eight such stimulating weeks, the student is told, "Go and teach." Of course he isn't adequately prepared! But, I suggest, this isn't due mainly to time limitations. Rather, it is largely due to the failure of O.C.E. to make adequate use of the time available.

The second summer was even more disappointing than the first. With a year of teaching under my belt, I felt I had the right to expect something more than the old Preparation-Presentation-Application formula. To my dismay, the only difference between the math course offered in the first and second summers was that the latter concentrated on Grade 12 course content rather than Grade 10.

There was no room in the course for discussion of the numerous topics of vital interest to teachers, topics such as the following:

1. How do we motivate 2 and 4 year students. (Believe it or not, these students are <u>never</u> mentioned at O.C.E. Apparently they simply don't exist. Yet almost every teacher is required to teach them.)
2. What about the changes being made in the curriculum? Who is responsible for them, and where will they ultimately lead?

3. What is the role of Programmed Instruction in High School Teaching?
4. What use can the teacher make of visual aids - T.V., the overhead projector, film loops, etc.?
5. Possible methods of teaching the subject matter other than the straightforward "Prep-Pres-Apply" technique.

But of course there is no time to discuss all these things because the student is far too busy learning Grade 12 math.

No wonder O.C.E. is a bore! To those who wish to abolish summer courses, I propose a far more difficult - and worthwhile - task: improve the summer course. Utilize efficiently the available 14 weeks; then your argument that the time is too short will be far more convincing.

*****

O.I.S.E. POSITIONS

The Ontario Institute for Studies in Education has professorial positions to fill for the coming academic year in its various departments. Most positions involve graduate teaching, research, and development in particular specialized fields of education. Applications stating qualifications and experience are invited by the Chairmen of the departments listed below:

Adult Education	Dr. J. R. Kidd
Applied Psychology	Dr. F. G. Robinson
Curriculum Research & Instructions Techniques	Dr. Marion Jenkinson
Educational Administration	Dr. T. B. Greenfield
Educational Foundations	Dr. Willard Brehaut
Educational Planning	Dr. Cicely Watson
Information & Data Systems	Dr. L. D. McLean
Measurement & Evaluation	Dr. V. R. D'Oyley

The Ontario Institute for Studies in Education
102 Bloor Street West
Toronto 5, Ontario

PROBLEMS AND SOLUTIONS

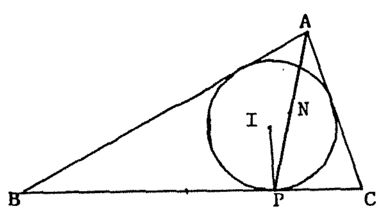
Send problems and solutions to:

The Editor, Problems and Solutions,
Ontario Mathematics Gazette,
c/o Department of Mathematics,
Queen's University,
Kingston, Ontario.

PROBLEM NO. 3 (R.D. Alexander, Riverdale Collegiate Institute, Toronto)

In the diagram, N is the midpoint of AP and M is the midpoint of BC . Prove that N,I,M, are collinear.

(Mr. Alexander has an algebraic proof. He challenges the reader to find a Euclidean proof.)



PROBLEM NO. 4 (A.W. Harris, Althouse College of Education)

Five men check their hats at a restaurant.

While returning the hats after the meal, the check girl becomes confused and hands them out at random. What is the probability that no man receives his own hat?

Solution: If there are no restrictions the hats may be arranged in $5!$ ways = 120 ways. These cases can be broken down into groups according to the number of exchanges involved.

a) Two pairs of exchanges: This is obviously impossible as it would leave the fifth man receiving his own hat.

b) One pair of exchanges: The pair may be selected in C(5,2) ways. After this has been done, the remaining three, say, A, B and C may be dealt with as follows:

A can pick a hat in 2 ways

B then can pick a hat in one way

C then can pick a hat in one way

∴ the number of ways in which there can be one pair of exchanges is $C(5,2) \times 2 \times 1 = 20$

c) No exchanges: The successive men may pick a hat in 4,3,2,1 and 1 ways.

∴ the number of cases involving no exhcanges $= 4 \times 3 \times 2 = 25$

∴ the total number of cases = 44

∴ Probability that no man receives his own hat (assuming random distribution) $= \frac{44}{120} = \frac{11}{30}$

Comment: A year ago this problem was going the rounds of people who like puzzles, It is similar to a question in Algebra 13 and one which was on last year's Problems paper. The above solution was suggested to me by a young man who was visiting us from Denmark last summer. He thought of it within a couple of minutes of being presented with the problem. I would be interested to hear of other solutions.

The approach he used illustrates an important aid to problem solving: "When the problem is too complicated, think of a simple one with a similar structure." In this case if one immediately says to oneself, "It would be easy if there were 2 ; there would just be an exchange of hats!" "I wonder what it would be like for 3 ?", then he has the basis for an elegant solution.

THE ONTARIO INSTITUTE FOR STUDIES IN EDUCATION

NEW PUBLICATION

GEOMETRY: KINDERGARTEN TO GRADE THIRTEEN

Curriculum Series No. 2 $2.00

A report of the Geometry (K-13) Mathematics Committee, formerly of the Ontario Curriculum Institute (now Office of Development at OISE). Proposes far-reaching changes in the content and approach to geometry at the primary, intermediate, and senior levels in the schools, as well as in the relation of geometry to the rest of the curriculum. Suggests a suitable structure for the school geometry program; provides material, both topics and methods of presentation, on which course outlines may be based; and gives suggestions on how to effect these changes in the schools. Illustrated. An important publication in view of the current concern for long-term curricular planning and revision.

Pp. viii + 118. Paperbound

TWO HELPFUL BOOKLETS

Many teachers may not be aware of two booklets which can be of real help in the continual struggle to improve the quality of teaching in Mathematics.

SUGGESTIONS FOR TEACHERS OF MATHEMATICS IN SECONDARY SCHOOLS, prepared by the Inspectors of Mathematics, is available to heads of Departments from the area Inspector.

It contains material of interest to all teachers, from the head of the department to the greenest novice. Adequate course outlines; the timing and planning of lessons; the art of questionning; the use of models, overhead projectors and other visual aids; physical details of the classroom; methods of achieving a learning atmosphere; and discipline problems are discussed. Special sections are devoted to the problems of teachers of occupational and 4-year classes. The articles on review lessons and testing are thoughtful, practical and stimulating. Most teachers will be interested in the opinions pupils form of teachers and even more in the "likely terms of reference" for ratings by principals, inspectors, and other supervisory personnel.

MATHEMATICS - A RESOURCE BOOKLET, published by the Ontario Secondary Education Commission of the Ontario Secondary School Teachers' Federation is available from O.S.S.T.F., 1260 Bay St., Toronto 5, Ont.

The material in this booklet was prepared by top teachers still in the classroom. Section 11, "The Mechanics of Teaching a Mathematics Class", covers the standard topics - course outlines, timing, texts, types of lesson, homework, testing discipline, etc. What makes the booklet so valuable is Section 111 in which the general ideas of the previous section are illustrated by specific topics from the Departmental course of studies. For example, a course outline is given for the Trigonometry of Grade 11 in the 5-year programme. This is followed by three consecutive plans and then six possible tests in Trigonometry. Sample Grade 9, 11, and 12 June examinations, with a complete marking scheme for the Grade 11, are provided. Section IV, "Miscellaneous", advances helpful hints to beginning teachers, comments on Mathematics clubs and projects, and discusses 'bad form'.

H.C.H.

****

IMPRESSIONS OF THE OATM 76TH CONVENTION

by

Helen C. Hay

Loyalist Collegiate and Vocational Institute, Kingston

What highlights have survived the week since the close of the OATM-76th Convention? First, Miss Biggs; second, the seminars; third, the pleasant arrangements. Other features follow in indefinite order.

The committee is to be congratulated on the setting and on the social side of the gathering. The Inn-on-the-Park is certainly a far cry from the institutional smells, sounds, seats and parking of O.C.E. The informal reception on Monday evening and the dinner on Tuesday were very pleasant opportunities to meet old friends and make new ones among the mathematics teachers of the province.

Miss Biggs, said the program notes, is one of Her Majesty's Inspectors on special assignment to train teachers to use the methods which she herself had developed to teach mathematics in elementary and intermediate schools. What an inadequate preparation that was for the impact of her personality and vitality as she related story after story of the joy and triumph of pupils discovering by themselves mathematical facts and relations, of the skill with which patient and understanding teachers guided even the most hopeless and defeated pupils to success. String, paper, scissors, ribbons, coins and a dozen-and-one other homely articles were the tools for measuring and sharing and recording some of the activities to provide the experience necessary for the child's discovery of the generalization waiting to be made. Most assuredly, she said, such "discoveries" required more time than it might take to set them forth by traditional methods, but they never had to be retaught and, once made, were a firm foundation for other discoveries. Miss Biggs' illustrations, demonstrations and sketches rarely went beyond the elementary grades, but the rapt attention of her audiences told me that many besides me could see how much of what she put before us could be adapted to secondary classes.

Unfortunately, one can only attend two seminars. Partly because I have had very little opportunity to use calculus since University and partly because of the high regard in which I hold Father Egsgard as a teacher, I chose his seminar on "The Solution of Rate Problems Using the Chain Rule". I was not disappointed. Before we knew it, he had his overflow audiences working together to solve his illustrative rate problems just as each of us would want our pupils to do after we had copies his excellent technique of presentation. A very profitable hour!

My next choice was Mr. W.H. Baxter's discussion of the present Grade 11 and Grade 12 Trigonometry. He thinks (and many present agreed with him) that the sine and cosine laws should be developed and used much earlier and proving identities should be deferred. Methods of introducing the trigonometric functions were considered. Opinion was fairly evenly divided between introducing them by the general rotation angle and as circular functions of a real number. Teachers of Math A have found, I was interested to hear, that the mapping approach involved in the use of the winding function has eliminated problems that frequently arose when the rotation angle was used.

I do like to browse in the publisher's displays and this year I had plenty of scope, for they were both conveniently arranged and most interesting.

The trouble with mathematicians, say my non-mathematical friends, is that they always have to prove things. Dr. Fryer is no exception. He spent his hour proving that whenever he wants to stop being a mathematician he can always become an entertainer.

******

C.T.F. MATHEMATICS TEACHING CONFERENCE

by

D.C. Attridge

Thornhill Secondary School

At the 1966 Annual general meeting of the Canadian Teachers' Federation it was agreed to sponsor a mathematics teaching conference in Ottawa during March 16-18, 1967. The purpose was to discuss the feasibility of establishing a Canadian Council of Teachers of Mathematics, and, if approved, begin the implementation of such a body.

Approximately 50 people, representing most of the affiliated bodies of C.T.F. and visiting organizations, attended the conference. Rev. J.H. Conway, first vice-president of C.T.F., chaired the meetings at the Chateau Laurier. The routine was to first stimulate thought and second to act accordingly with regard to the establishment of such a council of Canadian teachers of mathematics.

What Should The Schools Be Doing?

A panel consisting of Dr. G.F.D. Duff from the University of Toronto, Dr. R.B. Gwilliam, Chief, Technical Services Div., Department of Manpower and Immigration, Ottawa, and Miss Nazla L. Dane of the Canadian Life Insurance Association, reacted to this question. Dr. Duff gave a resumé of the various changes in our schools' new mathematics programmes showing that teaching mathematics is now many-sided. He felt that the major aims were to develop comprehension, satisfaction, and reliability in teaching our programmes in mathematics.

Dr. Gwilliam mentioned that a sub-committee on mathematics visited 40 institutes training technicians and received several complaints about our high school students: (a) most have never really thought in an applied sense for they had studied mathematics only in the pure sense; (b) students have not always covered the topics on the curricula; (c) the background of the students reaching these technical institutes is extremely varied and more standards are desirable. Dr. Gwilliam noted that the Federal Government plans to retrain the entire labour force in the near future. Since there is barely a skill today that doesn't need mathematics, topics in mathematics will be very prominent in this retraining programme.

Miss Dane felt that "good old-fashioned" arithmetic should still be taught since she had found that most of our school's products are not adept in the manipulation of figures. This was evident from meeting cashiers, waitresses, clerks, etc., and from interviewing various employers. Dr. Duff replied to this stating that the old method of constant drill led to memorization of tables without thought and had no better results than the discovery approach used today. Our job today should be to foster initiative and independence in our students.

The New Courses

Dr. Norman France, Faculty of Education, McGill University, and Dr. A.L. Dulmage, University of Manitoba, introduced this topic. Dr. France felt that the elementary schools hope to involve the teacher and the student with the content to first develop a need for the skill in mathematics and <u>then</u> develop that skill. The development of the teaching of mathematics as an environmental study with an appeal to the intuition was a changing factor in the elementary schools. Dr. France hoped that the teachers of today would enjoy their freedom from the shackles of regimentation and learn to enjoy the "discovery method" of teaching.

Dr. Dulmage outlined the new topics instituted into the secondary school's curricula and what purposes they had in the student's background of mathematical knowledge. The use of functions as unifying concepts, set notation for clarity of definition and proof, and combinatorial mathematics to emphasize constructive or step-by-step proof, were a few singled out.

Future Directions

Professor A.J. Coleman, Queen's University and Dr. Z.P. Dienes, University of Sherbrooke, outlined their predictions as follows. Dr. Coleman presumed (a) that the emphasis of teaching mathematical "facts" would shift to the more difficult task of developing an understanding of mathematical ideas, (b) that it will be recognized that children love abstract ideas and can cope with them, (c) that we will be forced into more flexible patterns of streaming which will permit each student to develop at his own pace, (d) that the main pressure for mathematics reform comes, and will come, not from mathematicians, but from the needs of society. In deciding proper content for school programmes, serious consideration should be given to the opinion of YOUNG mathematicians, (e) that within a few years the attraction of that new toy, the computer, will fade, leaving the valid insight that the teaching of many parts of mathematics should be aimed toward the achievement of an explicit computable algorithm, (f) that the methods of training teachers and their roles will change, (g) that teaching should be a spiral approach and 50% of the mathematics taught in the classrooms throughout Canada should be directed from a central curriculum source.

Dr. Dienes gave an enthusiastic talk and demonstration about mathematics teaching using the experimental approach. He felt that the efficiency of teaching would be up by a factor of 5 if experimental ideas were employed and if the teacher was encouraged to get away from the front of the classroom. The National Film Board represented by Gordon Martin, Sid Goldsmith and Hans Moller, presented a film of a T.V. lesson and then invited criticisms and comments. A reception and dinner followed that evening.

The Role of the Canadian Teacher of Mathematics

Prof. D.H. Crawford, Queen's University, introduced this topic stating that mathematics teachers should be always teachers first and they do not live in a vacuum. The role of the mathematics teacher today is a Key one in shaping attitudes and understandings about the place of mathematics in this increasingly automated and technological society of ours. Quality is of the essence in education today meaning it must be up-to-date in content and method, it must teach a child how to think rather than what to think, and it must be concerned with attitudes and feelings, with teaching children to care. Three conditions are necessary for this quality education, (a) a good physical environment, (b) enough teachers, (c) good teachers.

Dr. Crawford outlined the teacher's traditional role of lectures and questioning and showed the factors tending to change it. There is a trend to greater individualization in learning as witness the interest in the ungraded school and, of course, T.V. programmed learning and computers are changing our ideas rapidly. The changes in content and approach all point to a definite shift of the teacher from a relatively isolated individual to becoming the Key member in a team of various specialists. The role of the teacher will tend to become that of a resource person, coordinator, and personal tutor, freed from much routine work, but still exerting a vital personal influence on students. Dr. Crawford felt it was a challenging time to be a teacher of mathematics and if we wish to preserve a distinctive Canadian brand of education, we will need each other as never before to share our common problems.

The Needs of the Canadian Teacher of Mathematics

Dr. H.A. Elliott, Royal Military College of Canada, felt the elementary teacher, while he cannot be a specialist, must teach the liking of mathematics and thus be a good actor. He should know where we are going in order to teach certain grades and relate mathematics with every other subject in the school. Like Dr. Crawford, Dr. Elliott felt that the secondary school teacher did not need high level mathematical courses to teach mathematics properly but they should have one full year of mathematical content and then take a full course every 6 or 7 years outside the classroom to become updated with the new changes. Teachers need time for doing research and time to discuss, as a department, everyday classroom problems. We should be allowed to make mistakes and then not be afraid to admit defeat and say, "I just cannot cope with this topic" and then learn from the ensuing discussions. Time should also be used for intervisitation. Dr. Elliott stated emphatically that every grade 9 teacher each year should visit and observe classes in mathematics at the grade 8 and grade 10 levels in order to understand the full role of mathematics that his students will be involved with. Only then will he appreciate the overall picture of his role as educator. When retraining classes are necessary, let them be in school hours and not after school in the evenings or during the weekends.

A vigorous discussion followed each of these "food-for-thought" topics. Then the various delegates were asked to explain their own provincial activities at the mathematics level and later their concerns and aspirations. The general feeling was that plenty was being done throughout the provinces and each affiliate felt that the common knowledge should be shared with all concerned. With this in mind the delegates got down to work to organize the Canadian Council of Teachers of Mathematics.

C.T.F. outlined that the association should be classroom oriented, should allow for diversity of cultures, and should not interfere with any existing organizations (e.g., N.C.T.M.). It was unanimously agreed upon that the association would be desirable but the financing for such a project provided quite a bit of concern. However, it was felt the funds could come from the various affiliated groups of C.T.F., N.C.T.M., Canadian Mathematics Congress or perhaps private companies.

Why the Canadian Council of Teachers of Mathematics?

It was considered to be a worthwhile project for the following reasons:

(a) it will be a central source from which to keep in contact with the other provinces

(b) it will provide a national rather than a provincial point of view

(c) it will provide a central structure for consultation for speakers, correspondence, and for the coordination of views on curriculum development and new mathematics programmes

(d) it will allow our central organization, C.T.F., to relate World Organizations to the proper channels concerning mathematics.

Having established solid reasons for such a council, the organization and structure took effect as follows:

(a) that the Dominion of Canada have 3 regions: West (B.C., Alberta, Saskatchewan, Manitoba), Central (Ontario, Quebec), and East (New Brunswick, Nova Scotia, Prince Edward Island, Newfoundland)

(b) that a planning committee of 4 be formed with the purpose of (1) establishing a constitution and (2) procedure for organization, (3) preparing a newsletter before June 30, 1968 (4) setting up a national conference in 1968 or 1969 and (5) assessing the finances of such a council

(c) that each provincial mathematics association of the C.T.F. affiliates be asked to contribute to the cost of this first organizational meeting to be held before December 31, 1967.

The meetings eventually closed harmoniously with the planning committee of Mr. Roy Craven from British Columbia, Father John Egsgard from Ontario. Mr. Doug Potvin from Quebec and Mr. Ed Murrin from Nova Scotia prepared to meet in the near future in the hopes of officially organizing this Council of Teachers of Mathematics.

The sessions were stimulating and rewarding and I cam away very much more aware of the various concerns and aspirations that some of my colleagues, outside my province, had, and with the hope that through this future association, we would be able to form a closer bond with each province and develop a truly national flavour in the role of mathematics. I personally feel that perhaps Ontario will give much more than it will receive for awhile, but why can't we be the role of the donor in order that the rest of our country may benefit through our successes and perhaps our failures? I join Dr. Crawford in requesting that we link ourselves vertically from Kindergarten through University, and physically from British Columbia to Newfoundland. Let us learn to share our common problems and work for a united profession and quality education for all provinces and not just that of our birth or choice.

Those delegates participating from Ontario were:

Donald Attridge, Thornhill S.S. (O.S.S.T.F.)
Ronald Edwards, North Bay (O.P.S.M.T.F.)
Father John Egsgard, St. Michael's College School, Toronto (O.E.C.T.A.)
Brother Pierre Gravel, Lasalle H.S., Ottawa (Assoc. Des Enseignments Franco-Ontariens)
Ken Jones, Anderson Street C.V.I., Whitby (O.S.S.T.F.)