Ontario Mathematics Gazette

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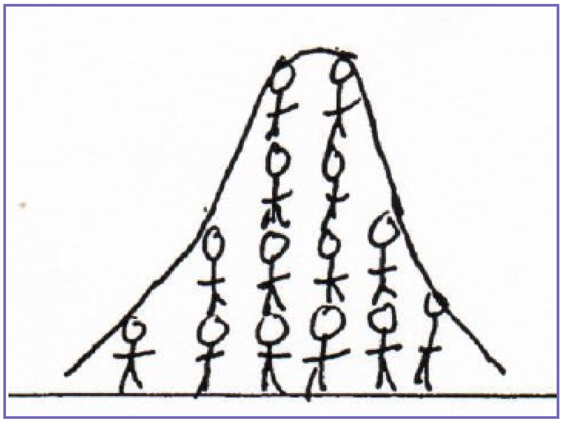
OAME – ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION

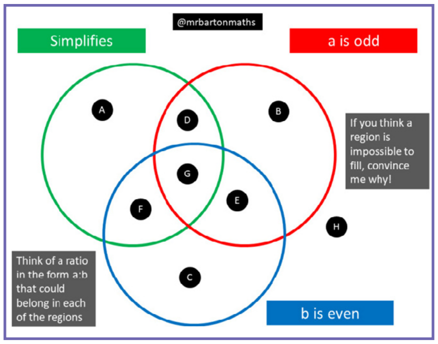
AOEM – ASSOCIATION ONTARIENNE POUR L'ENSEIGNEMENT DES MATHÉMATIQUES

Vol. 57 #4
June 2019

IN THIS ISSUE

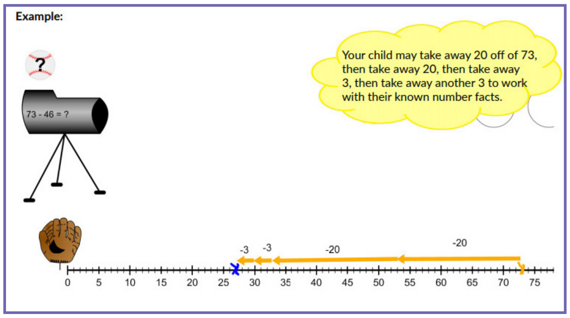


See Strategies for Enhancing Mathematics Learning for Students Who Are Kinesthetic Learners



See Technology Corner: Venn Diagram Activities

See Editor's Report



See Hey, It's Elementary: Building Parent Engagement in Math: A Project to Support the Implementation of the Renewed Math Strategy

See Provincial Digital Learning Resources – What's New? WINS – Putting the Fun in Fundamental

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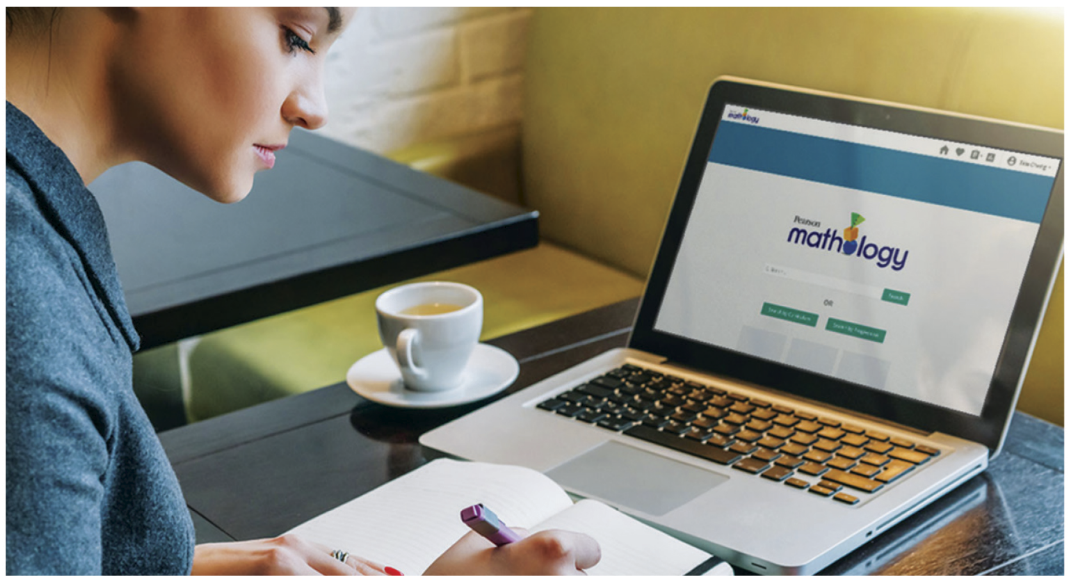

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Submission of Articles

The *Ontario Mathematics Gazette* (OMG) is looking for news items, articles, and good ideas that are useful to mathematics teachers and mathematics teacher education. We are seeking submissions, preferably from mathematics teachers K–12 and other mathematics education professionals, that describe innovative and creative approaches to mathematics teaching.

Please keep in mind the following criteria when making submissions to the *Gazette*:

- The ideas/activities must be of interest to the readership.
- The ideas/activities must be fresh and innovative.
- The mathematics content must be appropriate for the readership.
- The mathematics content must be accurate.
- The article must be well written and easily understood.
- The article and its ideas must be free of sexual, ethnic, racial, or other bias.
- The article must not have been previously published, nor should it be out for review by other publications.
- The article must be original.

Articles are to be word-processed, MS Word is preferred, and prepared according to the *Publication Manual of the American Psychological Association*, Sixth Edition (2009). However, please use single-line spacing (not double) and only one space after each period. Articles should not exceed five numbered pages of text, and figures, images, and photographs should be placed in the text close to where they belong, with captions. The photographer's permission is required, and for photos of students under the age of 18, the written permission of a parent or guardian is required.

Please submit your article in one blind file (i.e., identity of author is not evident), and include author names, contact information including email and mailing addresses, photos, biographies, and all content removed for blinding in a second file. Please email these two files to Tim Sibbald at **gazette@oame.on.ca**.

Upon review, you will be notified whether your article has been accepted for publication (as is, or pending minor or major revisions) or rejected. The Editor reserves the right to edit manuscripts prior to publication. Once an article is published, it becomes the property of OAME.

▲ Table of Contents

▲ About the *Ontario Mathematics Gazette*

The *Ontario Mathematics Gazette*—ISSN 0030-3011—is indexed in the Canadian Education Index and is published four times per year. Its Canadian Publication Mail Product Sales Agreement Number is 40051074.

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Advertisements for publication in the *Ontario Mathematics Gazette* should be sent to **Robert Sherk** by email or at the above address. Deadlines for advertisements are January 23 for the March issue, April 1 for the June issue, July 1 for the September issue, and October 1 for the December issue.

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▲ Editor's Report

TIMOTHY SIBBALD, OCT, PhD
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Tim Sibbald is the current Gazette *editor and a Past President of OAME. He is an associate professor in the Schulich School of Education, Nipissing University, with a focus on mathematics education in both pre-service and graduate programs.*

One of the things I love about teaching is the opportunity to learn. This happens daily in the classroom, where students bring a wealth of creative energy and there is always something new capturing their attention. Professionally, time for learning is not necessarily available on a daily basis, but it is available at times, such as during the approaching summer. While you could sit around contemplating the Thue-Morse sequence, 01–10–1001–10010110–…, there are many more opportunities to consider.

Having moved to post-secondary education, I have had my first experience of a sabbatical–essentially, a year with no teaching or meetings so that I can focus on a research agenda that is specified in the approval process. During the year, I have found time to travel. This column takes inspiration from my voyages because while I was looking at tessellated pottery in Nicaragua, it occurred to me that we rarely talk about the benefits of travel which may be related to mathematics.

One opportunity that is invigorating is the National (U.S.) Museum of Mathematics ("MoMath") in New York City. I happened to visit on a Saturday morning in February, and it was uplifting to see children and their parents engaged in a wide variety of math-focused activities. The museum is primarily oriented to K–12 students in a hands-on manner, but it has computer screens with explanations that begin in an introductory fashion before offering a little more detail to engage parents, then more detail for teachers, and typically ending with some sense of the place of the mathematics in current mathematical thought. The impressive aspect of the museum is the type of content that engages young minds. It is not curricular mathematics with a few exceptions, and where curricular mathematics is involved, it is rendered differently—for example, a square box had a four-piece tangram on the top of it, and while the box was suited for sitting, the pieces of tangram were hinged prisms that could be stretched out as four seats (Figure 1).

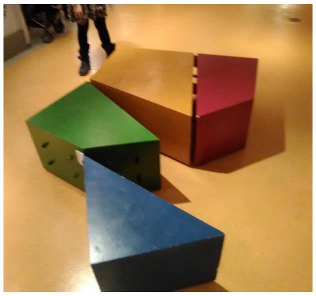

Figure 1: *MoMath tangram*

Figure 2: *MoMath fastest track activity*

There is a basketball launcher, where you specify the angle and speed for experimentation. That experimental flavour is pervasive throughout the museum. Consider an adjustable track, where a race car is released vertically and has to end up horizontal in the fastest time (Figure 2). There is an interactive video floor (Figure 3) that alternates between different activities and, whether a maze ("see if you can solve only turning left"), contours, or a shape generator that is responsive to movement, it was always busy.

Figure 3: *Interactive video floor*

The math is accessible and varies from one activity to another. A classic probability experiment, where balls roll down a triangular course being moved left or right at each level, was rendered in a slightly different way, where a slight bias could be introduced, and the outcome of many balls dropping showed the impact on the resulting distribution.

Figure 4: *Console showing perspective from car on a twisted track*

A centrepiece to the museum is a pair of tracks with twists in them (Figure 4). One is a Mobius strip. The cars are driven at a video console, and a camera mounted on the car shows the perspective from the car as it goes around the twisted track. If it is too exciting and you run for the washroom (Figure 5), you are confronted by a geometric sink!

There is an interactive set-up, where you are displayed as a fractal! A wall that shifts as a sine wave! A little gem that is easily overlooked, but clearly fascinated many middle school-aged children, has a rectangular box, where a board remains flat while rolling over oddly shaped solids.

I could go on, but suffice it to say that if you want an excuse to visit New York this summer, or your school plans a trip to New York, then the Math Museum is a way to infuse that trip with exciting and significant mathematics learning.

Figure 5: *Even the washroom has math!*

Figure 6: *Math from Kamloops*

There is, however, a world full of opportunities to see mathematics. In Kamloops, British Columbia, at Thompson Rivers University, I found a pair of doors with an indigenous motif (Figure 6). It has three circular arcs, with the outer arcs showing eagles and two inner arcs with fish travelling in opposite directions. In the middle of the circles is a bear paw. Or, is it actually as described? Notice that the arcs for the fish on the left and right do not quite align. This seems to be due to the eagles being formed from two circles with different centres (i.e., they have a comet shape). Perhaps the fish are on two different circles. It is a math problem of artistic proportions.

Figure 7: *Financial literacy Santa Monica*

Santa Monica has a series of acknowledgements of teachers with inverted bell statues that carry education quotations. An example is a quotation by Marian Edelman: *"Education is for improving the lives of others and for leaving your community and world better than you found it."* Los Angeles has some interesting gas pricing, as shown in Figure 7. The price on the left is for cash and on the right for credit cards, which can lead to discussion of credit cards having significant fees for vendors. (This is not a new issue, and was the original reason for Canadian Tire money—it was only given as a discount to those who paid cash.) One gas sign is all you need to also draw out the use of fractions with decimals, conversion of gallons to litres, as well as conversion of American dollars to Canadian. It is rich with financial literacy.

Figure 8: *The Murano icosahedron*

Figure 9: *Slope in Panama*

It really does not matter too much where one goes—there is an opportunity to find math that can infuse your future classes. In Murano, Italy, there is a glass icosahedron (Figure 8). France, Croatia, and Panama, to name just a few places, have modern suspension bridges well suited to teaching about slope (Figure 9). In Naples, you can view the famous math painting of Luca Pacioll (Figure 10). Morocco is a wonderful place to find examples of patterning (Figure 11), and Tunisia has astonishing plaster designs with very elaborate geometric features (Figure 12).

Figure 10: *Luca Pacioll in Naples*

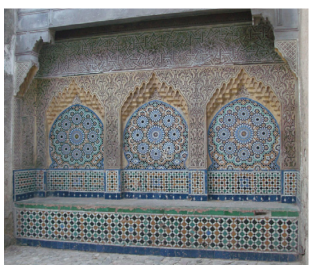

Figure 11: *Patterning from Morocco*

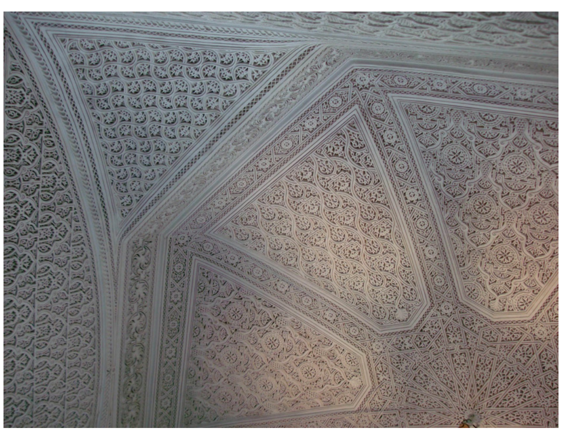

Figure 12: *Geometric designs in Tunisia*

Closer to home, in Sudbury, there is an interesting statue in the shape of a circle, that contains miners (Figure 13). While it may not hold a lot of mathematics, it affords an opportunity to include a meaningful photo in the classroom, and supports conversation about an important economic activity in Ontario.

Figure 13: *Sudbury mining statue*

Figure 14: *Tessellated hexagonal socks!*

All this is to wish you a restful and adventurous summer. Whether you hang out at a cottage and knit some hexagon-tessellated socks (Figure 14), or you travel further afield, consider it an opportunity to collect some artifacts that will allow for connections and add some zest to your future classes. All the best!

Errata: While we do not intentionally make mistakes in the *Gazette*, it does happen from time to time. If you received a hard copy of the March issue, you may have noticed it erroneously said 2018 on the cover when it should have said 2019. This was corrected on the electronic version, so perhaps the hard copies will become a collector item for their scarcity.

Special Request

I am interested in receiving accounts of experiences (of both teachers and students) of writing “Departmental exams” that were used in Ontario between 1940 and 1967 by the “Department of Education.” If you know of anyone who wrote any of the math or physics exams, I would appreciate an email (**gazette@oame.on.ca**) or a phone call (705-474-3461, ext. 4653). These will inform a future editorial on an important piece of our math education history.

In This Issue

David Petro’s *President’s Message* is a reflection of his year as President. There is an obituary for Jack LeSage. In the *Fields Institute MathEd Forum Report*, Angelica Mendaglio recounts the Margaret Sinclair Memorial Award talk given by Peter Liljedahl. Jacqueline Hill draws attention to what is going on in the NCTM. We have an invited reflection from a long-standing OAME member, Bob McRoberts.

The *MB4T* column, by Jennifer Holm, speaks about instruction around probability across many grades. *Assessment Abby* draws attention to a developmental continuum and its implications for instruction. *In the Middle* sees Carly Ziniuk finding rich approaches to STEM. David Costello continues his *Linking Literacy and Mathematics* column with further examples of comprehension strategies that can be leveraged from language into mathematics. Parent engagement is the focus of the *Hey, It's Elementary* column by Lynda Colgan.

The Provincial Digital team, Agnes Grafton, Ross Isenegger, and Markus Wolski, focus on "Working In Numbers Sense" (WINS) and resources to support this. In *Technology Corner*, Mary Bourassa provides various resources for using Venn diagrams to provoke thought. Shawn Godin, in *What's the Problem?,* explains a pyramid scheme and gives homework regarding descriptive statistics of dates.

By way of articles, Jeff Irvine draws attention to Processes and Learning Skills. Alex Barking writes about the importance of conceptual learning. A further article by Jeff Irvine looks at methods to address kinesthetic learning in mathematics. You will also find a book review.

Three items are not found in this issue. *Mathematical Snapshots* by Ron Lancaster and Ann Arden's interview are expected to return in the Fall. The other item that is not in this issue, and is concluding, is the *Executive Directors' Corner*. I would like to thank our Executive Directors, Fred and Lynda, for making time to provide an interesting perspective. The strength that Lynda and Fred bring to the organization is attention to what is coming, and understanding it in terms of the larger picture—be that political, historical, or practical. In addition, their remarks carry an implication of policy and organizational positions. Those two stances lay at the two ends of the Executive Directors' table and are the reason the executive is such an interesting place to be. Thank you, Fred and Lynda, for your contributions to the *Gazette*.

Acknowledgements

As usual, I want to thank a terrific team: Associate Editors Anne Yeager and Jacquie Foster, who due to injuries and the onset of gardening season, also had assistance from Liz Mulholland and Lisa Corbett. Gitta Berg has, as always, made sure we are consistent in style and grammatically correct (if, you, think, it, is, easy, think, again!). Lastly, Penny Clemens, who composes the wonderful covers, brings all the pieces together, and assembles each issue. Thank you all for the teamwork! ▲

▲ My OAME Recollections

BOB MCROBERTS
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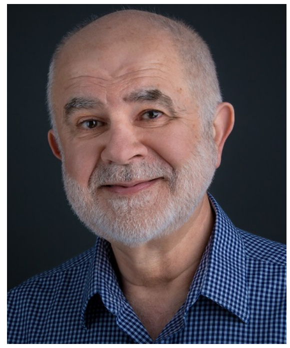
An invited opinion from a long-standing member of OAME.



My membership in OAME began in 1975 while I was still a university student. Back in 1975, we were made aware of the Ontario Association for Mathematics Education and advised that it would be a smart thing to belong to it. When you became a member back then, you got a certificate and an annual sticker—just like medical doctors have on their walls. I began teaching high school mathematics in 1976, and in June 2019, I will have been retired for ten years. I am still a member of OAME.

Personally, I don't understand why a mathematics teacher in Ontario would not become a member, i.e., every math teacher in Ontario should be a member. How can one go into a profession for three decades or so and not want to join the professional organization that provides ongoing support for you in that profession? It only makes sense to join!

Through the years, I have attended the OAME annual conferences all over Ontario. What an excellent opportunity to get rejuvenated and re-enthused about what you do every day by attending workshops and listening to keynote speakers from across the continent! Bring home the handouts and try some new ideas. Some things may fit your style and some may not. Talk with publisher representatives about their textbooks and resources. Pick up some free stuff! Meet some new people and some "old" folks you may not have seen for a few years. Share what works for you and what doesn't. Networking has benefits!

In addition, local chapters of OAME hold after-school conferences on a smaller, yet still effective, scale. I belonged to both GVMA and Y4MA (I'm a founding member) for many years. The OAME website and *Abacus* and *Gazette* publications are also a wealth of curriculum supports.

A few years after I retired, I decided I had many books, worksheets, old tests/exams, etc., that I was ready to part with. I packaged things up into 15 boxes and set up in a hallway at the OAME Conference that Spring. I was able to help about 30 math teachers that day by passing on some of my stuff.

Among other things these days, I tutor students in math. I keep my OCT and OAME memberships active to maintain my professional affiliations. Help yourself and some others by joining OAME! Tell your colleagues what they're missing! ▲

▲ President's Message

DAVID PETRO
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David Petro, the current President of OAME, is the math, science, and IT consultant at the Windsor Essex Catholic District School Board. He is a large proponent of exploiting technology for the educational benefit of students.

This being my last column as President means that I guess I should reflect upon the year. Technically it isn't finished yet (not as I write this or as you read it), but I can say that it has gone by fast. It seems, however, that I'm just getting used to what my role is, and soon I will be done. It's clear that being the President for just one year doesn't seem to be enough to effect change. To that end, we have started a new model going forward so that future presidents will serve a term of two years. This means our current President-elect, Paul Alves, will be the last President to serve a one-year term. We just finished this year's elections, and Judy Mendaglio will be the new President-elect, once Paul starts his presidential term in September. You might recognize her name, as she was president two years ago. She will be the first OAME President to serve a two-year term.

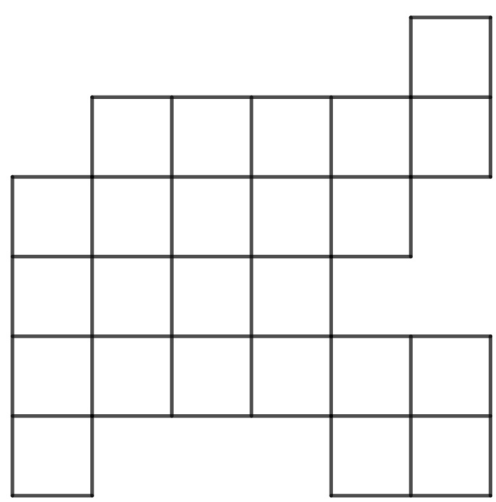
Being President has also given me opportunities that I wouldn't have had in the past. As President-elect, last year, I was sent to the NCTM annual conference to represent OAME. The NCTM annual conference is a massive gathering of thousands of math teachers, and it was interesting to be part of the Delegate Assembly, where various areas of North America had their chance to talk about possible changes within the NCTM. OAME is part of the Canadian Delegation, and I was able to take part in that process both last year and this year.

After attending the NCTM annual conference last year, I applied and was accepted to speak at this year's annual conference last April in San Diego. What was marvelous to see was that I wasn't the only Ontario teacher who was speaking. There was a long list (Jessica Bodnar, Cathy Bruce, Tara Flynn, Aleda Klassen, Rhonda Hewer, Ron Lancaster, Petra Le Duc, Matthew Oldridge, Jon Orr, Kyle Pearce, and Sunil Singh – I hope I didn't miss anyone). We only made up a small portion of the overall sessions, but we definitely were able to treat the conference goers with a healthy dose of Ontario Mathematics. Not only that, but it was clear, you don't have to go all the way across North America to participate in a great math conference. What we put on here in Ontario is consistently of high quality, and anyone who has been to an OAME annual conference would have no trouble agreeing.

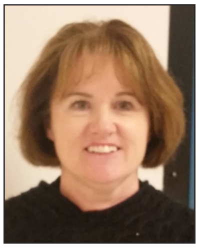
The other thing that has been good about this journey is being part of the OAME Board of Directors again. For years I was the SWOAME chapter representative and took part in the OAME board meetings. However, once I became my local chapter president, someone else was the chapter representative. So it has been nice returning to the board meetings to help the inner workings. Whether it has been the organizational strength of our executive directors Fred and Lynda Ferneyhough, or the way the board members take their roles seriously and are willing to take a stand for what they believe is right—this is exactly the kind of board that we need right now. These are very uncertain times in Ontario education, and you should know that the OAME will always represent the best mathematical interests of students and the teachers who serve them.

The 2019 OAME Leadership Conference will be held on November 1, 2019 at the Pearson Convention Centre in Brampton. The theme of the conference is "Paying Attention to Mathematics Leadership." The day will support teachers and teacher leaders (including administrators) in the different domains that they may be focusing on in the next instructional year. We hope to foster and support a network for leaders as they progress through the instructional year. Confirmed to speak at the time of this submission are Nat Banting, Marian Small, and Doug Duff. More speakers will have been confirmed by the time you read this! We hope to see you there for a great day of learning! ▲

Icebreaker 1 Puzzle
Your task is to break this iceberg (i.e. collection of squares) into four identical shapes!



▲ OAME/NCTM Report: Dessert Options!



JACQUELINE HILL
EMAIL: jacqueline.hill@bell.net

Jacqueline Hill is a Grades 9–12 teacher of mathematics, as well as an online instructor for York University. She is a Past President of OAME and OMCA, as well as the recipient of the award for Exceptional Teaching in Secondary Mathematics. She also wrote the "Director's Dialogue" for the Gazette *for a number of years.*

"If you don't eat your dinner, you don't get Dessert!" This phrase, along with the expression "You need fuel to fill your tank," are probably ones that you heard growing up. They both really speak to the fact that you have to keep yourself healthy in order to be good to others. Last summer, you were given a top ten list to consider. I would like to continue in that same vein, but suggest that this summer is an opportunity to work on you!

The NCTM is dedicated and devoted to helping mathematicians delve deeper into their craft. Whether you are a classroom teacher, retired teacher, university professor, or person in a position of responsibility in a school board or with the Ontario Ministry of Education, NCTM is ready to help you make the most of your summer.

Take time to organize your own professional reading list for the summer (as well as a few good beach books). Under the publications tab on the NCTM website, you will find a variety of resources:

- Books
- Journal for Research in Mathematics Education
- Mathematics Teacher
- Mathematics Teacher Educator
- Mathematics Teaching in the Middle School
- Rights and Permissions
- Teaching Children Mathematics
- Write, Review, Referee

As a member, you have access to all of the articles for free. If you are not a member of NCTM, you can still search the archives for articles and purchase them for a nominal fee. I recently differentiated my classroom activities with a group in each class responsible for reading the September 2017 NCTM article entitled *Financial Education: Increase Your Purchasing Power.* The only direction I gave each group was to find the math. My Grade 9 Academic group came up with the consideration as to "When to use proper discounts, 10% off or a $10 discount coupon," while my Grade 9 Applied group came up with "Coupons are good." My Grade 12's summarized that, "You need to understand the value of each product and service to see if you are getting ripped off." All of the groups spent some time looking at the extensive reference list included in the article with regard to financial literacy.

Perhaps the fuel you need to fill your tank or that of your colleagues is to be found on the NCTM calendar of 2019 dates:

- Georgia Council of Teachers of Mathematics: Summer Academies - June 25–26 in Augusta, Georgia
- Georgia Council of Teachers of Mathematics: Summer Academies(2) - July 9–10 in Tyrone, Georgia
- 2019 Conference for the Advancement of Mathematics Teaching (CAMT) - July 10–12 in San Antonio, Texas
- Georgia Council of Teachers of Mathematics: Summer Academies(3) - July 16–17 in Brunswick, Georgia
- Colorado Council of Teachers of Mathematics Annual Conference - July 29–30 in Denver, Colorado
- Michigan Council of Teachers of Mathematics Institute and Conference - July 30–August 1 in Grand Rapids, Michigan
- Pennsylvania Council of Teachers of Mathematics Annual Conference - August 7–8 in Harrisburg, Pennsylvania
- Nebraska Association of Teachers of Mathematics Joint Conference - September 20–21 in Kearney, Nebraska

Yours in mathematical fun,

Jacqueline ▲

Picture Equations

▲ Fields Institute MathEd Forum Report

ANGELICA MENDAGLIO
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Angelica Mendaglio is an instructional designer at Vretta Inc. in Toronto, Ontario, where she helps to create interactive digital mathematics lessons and activities for middle school students.

The February meeting of the Fields MathEd Forum centred around the winner of the 2018 Margaret Sinclair Memorial Award, Peter Liljedahl. Peter is a Professor of Mathematics Education at Simon Fraser University in Vancouver. His research into "Thinking Classrooms" and perhaps most famously, the use of vertical non-permanent surfaces (such as blackboards and whiteboards) has garnered attention from educators around the world.

Following the award presentation, Peter gave a talk to the attendees at the Fields Institute MathEd Forum called "Thinking about Thinking Classrooms." It began with a brief overview of where Peter's current research in math education began. About 15 years ago, while he was working on his doctorate, he visited 40 classrooms that were recommended to him by colleagues as being "good classrooms." He found a few patterns across all of them, the most striking of which was student disengagement. The students were very passive in the classroom, primarily mimicking what their teacher had just demonstrated. Peter also noticed that the layout was very similar across the different classrooms, with desks arranged in rows or perhaps in small groups, and one blackboard or whiteboard that the teacher could use to present to the class.

Peter found that these patterns could be described as "institutional norms." They were different from "classroom norms" because the patterns extended beyond an individual classroom or even a school. He also sensed that teachers did not feel that they had control over these institutional norms, and so they were in a sense "non-negotiated institutional norms." Peter then thought that if student disengagement was related to these non-negotiated institutional norms, then the norms would need to be broken. The result would be what Peter calls a "Thinking Classroom," where students are actively engaged in their learning.

Working with a large team of teachers, Peter tried different ways of breaking these norms and seeing what effect they had on student thinking and engagement. Through these experiments, the team was able to identify 14 practices, shown in Figure 1, which seemed to have a positive effect on student engagement.

The team also tested different ways of sequencing these practices that would make the transition from a passive classroom to a Thinking Classroom smoother for both students and teachers. Some practices are grouped together because the teachers reported from their trials that certain practices were co-dependent and did not work as well if they were implemented in isolation.

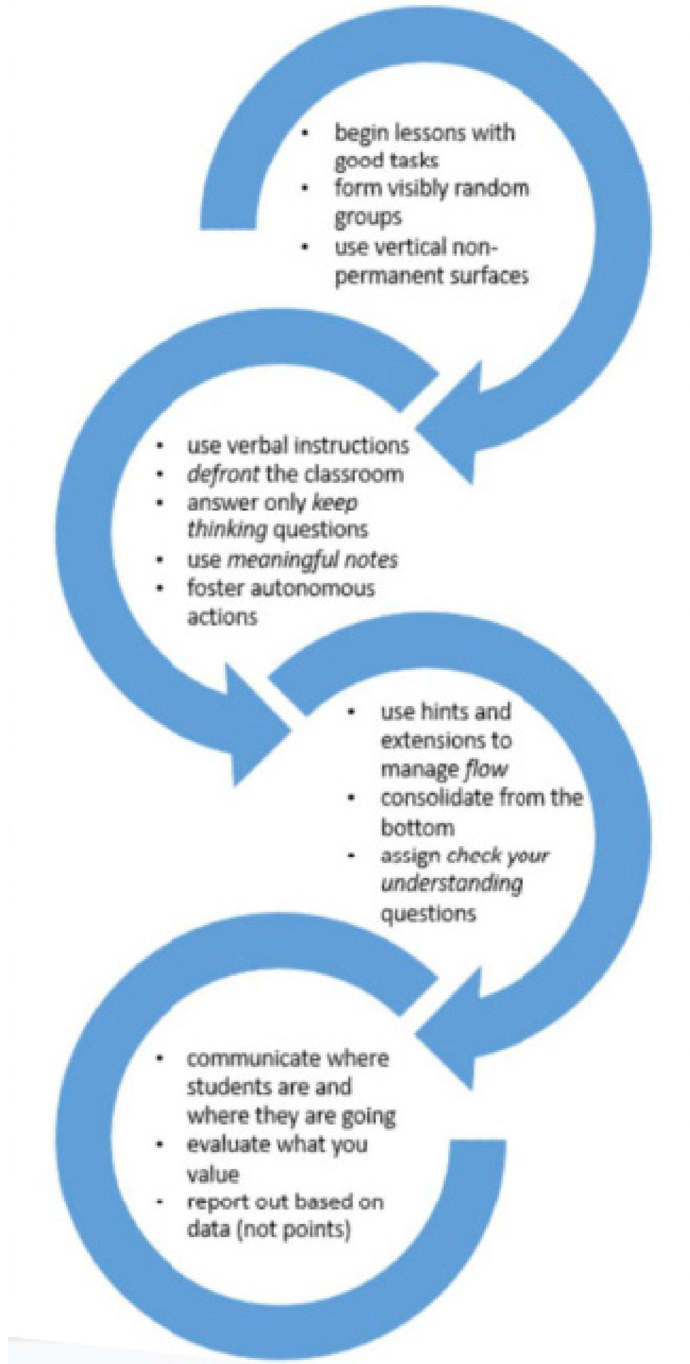

Figure 1: *Peter Liljedahl's 14 practices for thinking classrooms*

During the talk, Peter explained the importance of some of the practices and their sequencing. For instance, he finds that the first three practices (begin lessons with good tasks, form visibly random groups, and use vertical non-permanent surfaces) together seem to create a strong enough disruption to the institutional normative structure of the classroom that it does not elicit as strong of a resistance to it from students.

Visibly random groups (that is, putting students into groups randomly in such a way that students know the groups are random) are effective at least in part because they show students that the teacher has confidence in their abilities, increasing their own sense of confidence.

When a task is given through verbal instruction (as opposed to textual instruction), students begin engaging with the task almost immediately, and it is on a mathematical level. When students are given a task in text, their first engagement with the task is usually questioning what the task is trying to ask, which is engaging with the language of the task, rather than the mathematics.

Vertical non-permanent surfaces are effective for a number of reasons, one of which is that the student feels less anonymous and more autonomous when standing, rather than sitting. Peter noted that this works best when there is only one “pen” per group, so if blackboards are being used, it needs to be established that each group can only have one piece of chalk, lest the students break their chalk to gain more “pens.”

The rest of the February MathEd Forum was devoted to teachers from across Ontario who had been implementing these practices for a Thinking Classroom to share their experiences. A number of the presenters focused on how they handle assessment in their classes. Dave Lanovaz from Trenton High School spoke about how he went about giving group tests to his students, since they were doing all of their classwork in groups as well. Erin Marsella from York Region District School Board shared her experiences with creating markless, outcomes-based assessments in her Thinking Classroom. Suzanne Papuga from Halton Catholic District School Board shared her experiences implementing the Thinking Classroom practices in both Academic and Applied courses. Erin also shared that she initially tried to create a Thinking Classroom on her own, but has since found that having a colleague who is also working on implementing the practices is immensely helpful.

The Margaret Sinclair Memorial Award is presented each year to an educator in Canada who has demonstrated innovation and excellence in mathematics education at any level. The winner of the award delivers a talk at the Fields Institute during the following academic year, and it is always a very engaging talk to attend. The award winner for 2019 was announced at the March meeting of the MathEd Forum to be Nat Banting, a mathematics teacher and consultant from Saskatoon, Saskatchewan. Nat’s social media presence on Twitter and his blog has received attention internationally and has helped to make mathematics more accessible for students at many levels.

Nominations for the Margaret Sinclair Award are accepted until the end of February each year. If you would like to nominate someone for this award, visit **www.fields.utoronto.ca/honours-and-fellowships/margaret-sinclair-memorial-award** for more information.

The Fields Institute MathEd Forum convenes on the last Saturday of January–April and September–November at the Fields Institute in Toronto. If you are not able to travel to Toronto to join in person, you can watch the presentations remotely through the Fields website, at Fields*Live* (**www.fields.utoronto.ca/live**). You can also find a recording of previous Forums in the archive, including Peter Liljedahl’s Margaret Sinclair Award talk.▲

▲ Provincial Digital Learning Resources – What's New? WINS – Putting the Fun in Fundamentals

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Working in Number Sense (WINS), activities are designed so parents, guardians, and caregivers can engage with their child to build knowledge and skills in Number Sense, and deepen understanding of fundamental mathematical concepts. Each activity uses mathies learning tools or games to engage students in a fun, interactive, and meaningful way, while also supporting the development of fundamental skills. Embedded in this column you will also find information about recent updates to mathies tools and games.

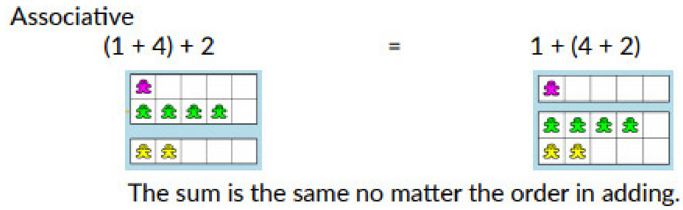
WINS activities focus on four key whole-number topics:

- Counting
- Composing and decomposing
- Representing, comparing, and ordering
- Operations

These topics are developed for various grade levels, starting first with skills related to whole numbers up to 5, and extending those to skills needed to deal with numbers up to 100.

WINS activities can be found by going to **mathies.ca**, selecting *Home Supports*, and clicking the WINS logo next to "What mathematics activities can we do together?" Each WINS activity has a link that opens a PDF file on the Home Supports page that includes the following:

- A *Mathematical Ideas* page, which explains the key mathematical concepts in parent-friendly language. Here is an example from the addition activities, highlighting the associative property:

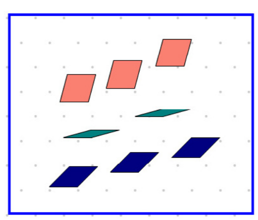


- A *Helpful Information* page, which includes tips, definitions, and materials for parents, guardians, and caregivers when working with their child
- A collection of activities with detailed instructions, including visual examples and suggested questions for parents/guardians/caregivers to discuss with their child

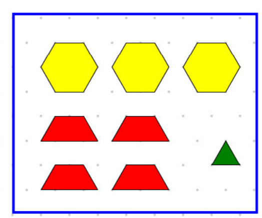
WINS activities focus on developing a broad range of Number Sense skills, using a variety of strategies to help students understand concepts. These activities can be used to support students as they develop the skills outlined in the Ontario Ministry of Education's *Focusing on the Fundamentals of Math* document (available at **www.edu.gov.on.ca/eng/teachers/teachers-math-guide.html**). Let's look at a few examples.

Working with Numbers

Composing and Decomposing Whole Numbers to 10 – Activity 7 has students using pattern blocks to create various representations of a randomly selected number, using different types of blocks. In this example, using the new mathies *Pattern Blocks+* tool, the number 8 has been represented using three different kinds of blocks in two different ways.



8 is made up of
3 salmon blocks,
2 teal blocks and
3 navy blocks



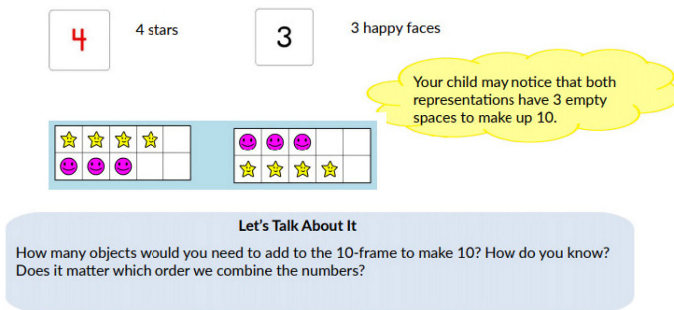
8 is made up of
3 hexagons,
4 trapezoids and
1 triangle

As students build and describe each representation, they are practising their composing and decomposing skills. How are these sets of blocks the same? How are they different?

Update Note: Three rhombus shapes have been added to the *Pattern Blocks+* tool, as seen in the example on the left.

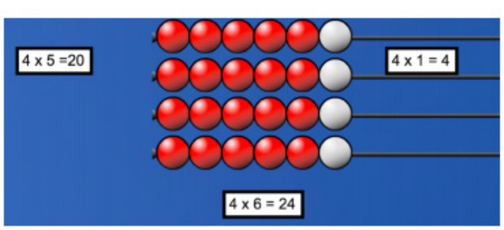
Recognizing and Applying Understanding of Number Properties

The mathies learning tools can be used to support students in understanding and applying various number properties. *Addition to 10 – Activity 3* focuses on helping students understand that the sum will be the same, regardless of the order in which we add the two numbers (the commutative property). Students are asked to use the mathies *Set* tool to represent two values in a ten frame in two different orders.



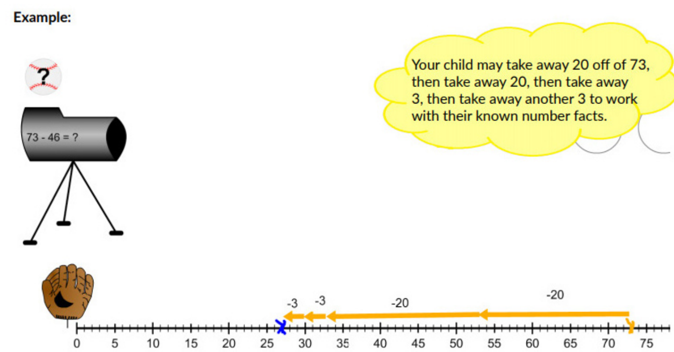
Students could use a variety of counting strategies to determine that the sum in both cases is 7. For example, they might pair a star with a happy face and count 2, 4, 6, and 1 more star makes 7. The discussion questions lead students to reason that the result is the same in both cases, since there are the same number of empty spots in the ten frame. Making ten is an important strategy in our base-ten number system. Repeating the activity for several values leads them to understand that the order in which numbers are added does not matter.

In *Multiplication Facts to 7 x 7 – Activity 2*, students use the mathies Rekenrek to represent a fact of five, in this case 4 x 5. They are then asked to add one extra bead to each row and determine the new multiplication expression and product. Students then return to the original fact of five, and this time, add on two beads to each row. This activity helps students understand that one way to determine facts for 6 and 7 is to start with the corresponding fact for 5 and then add on one more or two more sets, respectively. This strategy is an illustration of the distributive property at work, e.g., 4 x 6 = 4(5 + 1). The two colours of beads on the Rekenrek provide a memorable illustration of this property.

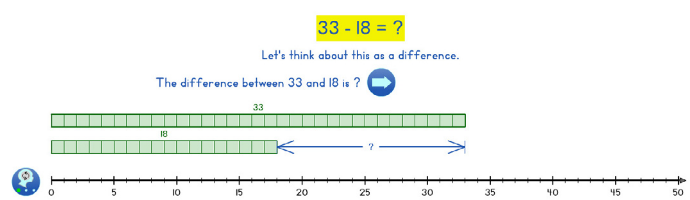


Mastering Math Facts

The list of WINS activities related to operations has recently been expanded to include addition and subtraction activities, as well as the already existing multiplication and division activities. Many of the Operations activities include a *Catch a Bouncing Ball – Operations* activity. This game is designed to allow students to use the built-in annotation features to share their thinking. On the number line, students can illustrate the various strategies they applied to determine unfamiliar facts, as shown in this example taken from *Subtraction Involving Number to 50 – Activity 6*.



Students can also use the game as a way to practise various math facts. As they drag the glove to the correct location on the number line, they receive instant feedback and are then able to revise their answer, if necessary. Once all ten questions are completed, students have the opportunity to review their answers. Each operation provides visual representations of different ways one might think about the operation. This image is one step of the feedback that illustrates subtraction as a difference.

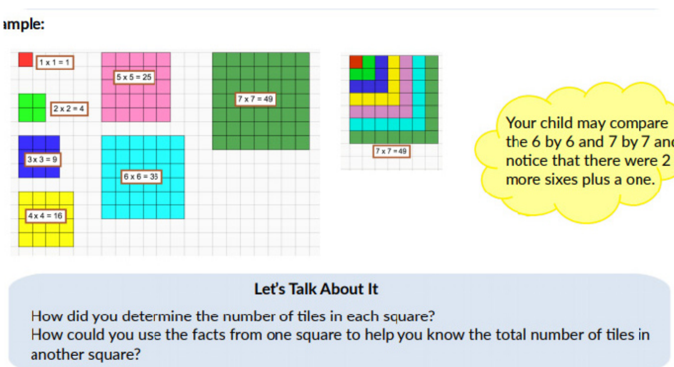


Update Note: *The Catch a Ball Operations* game has been updated this Spring to include two noticeable improvements to the addition and subtraction levels. First, there is a *Settings* button that allows the user to determine whether to work with questions that have two values, three values, or a mix of two and three values (the default). Second, visual representations on the number line are now included for the addition and subtraction games. As indicated above, these representations are part of the review process after the player has completed the game. For addition, the representations support the commutative property by illustrating the sum in different orders. Players are encouraged to think about subtraction as either a difference, a take-away, or an overall change. For all operations, players press the thinking-head button to see the representation illustrated in a different way.

For all types of games, there is also a new setting that allows the player to determine whether or not the ball is pitched automatically once the glove has been positioned. This setting defaults to “off,” which requires the player to first place the glove and then press the ball to activate the pitching animation. This allows players to revise their thinking or change the position of the glove, if it has accidentally been released in the wrong location.

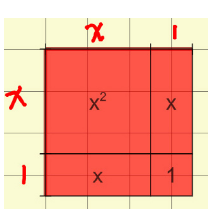
Developing Mental Math Skills

Using visual tools when learning to perform mathematical operations allows students to draw on these mental models and visualizations to help perform mental calculations. *Multiplication Facts to 7 x 7 – Activity 3* explores perfect squares, using mathies *Colour Tiles*. Students are asked to build a series of different-coloured squares, and then consider how knowing the value of one square can help to determine the value of the subsequent square.



In later grades, students can be encouraged to generalize this numeric result to understand why $(n + 1)^2 = n^2 + 2n + 1$. Being able to visualize subsequent squares will help students to avoid the common misconception that $(n + 1)^2 = n^2 + 1$.

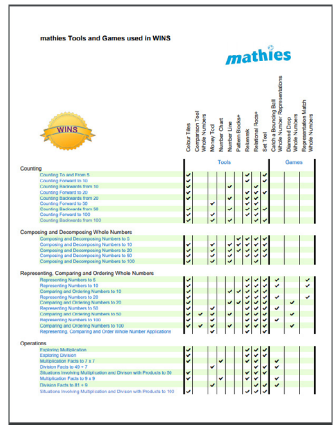
The mathies *Algebra Tiles* tool will provide other ways to visualize number patterns and operations. This tool is expected to be released by the time of publication.



The illustrations in this article show how students can make connections among visual representations of number properties and computational strategies. These connections support students in recalling various strategies to use when performing mental calculations.

Developing Proficiency with Operations

A fun way to improve proficiency is to use mathies games (**mathies.ca/games.php**). In addition to *Catch a Bouncing Ball*, WINS specifically mentions *Diamond Drop* and the *Representation Match* games for whole numbers. In later grades, student can play games to develop proficiency with integers (e.g., *Battle of the Integers*) and fractions (e.g., *Drop Ball*).



Visit **support.mathies.ca** for more examples and detailed descriptions of the functionality of each tool. In the *Links* section for relevant tools (e.g., Colour Tiles), look for the link to “mathies Tools and Games used in WINS,” which provides a summary of which mathies tools and games are used in each WINS activity.

EQAO

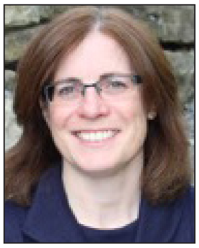
In addition to the WINS activities, which make extensive use of mathies tools and games, it is exciting to note that EQAO has revised and clarified their policy regarding the use of virtual manipulatives for the Grades 3, 6, and 9 math assessments. The *Grade 9 Administration and Accommodation Guide, 2019* specifically mentions mathies tools. It states: “Access to virtual manipulatives (e.g., some on **mathies.ca**) or math applications (e.g., calculators) that require Internet connectivity to function is now permitted during the assessment, as long as these tools are not instructional.” Be sure to check the relevant *Administration and Accommodation Guide* for more details.

Feedback and Future Requests

The mathies.ca site has games, learning tools, activities, and supports, like WINS, for students, educators, and parents/guardians/caregivers to explore. Feel free to use the Contact Us button to provide feedback or suggestions.

You can send your comments about mathies tools and games to **WhatsNew@oame.on.ca**. You can share your experiences on Twitter, using the hashtag #ONmathies, and follow or message us at @ONmathies. There is an increasing set of interesting posts of student and teacher work on Twitter. To be among the first to find out about the latest digital tool developments, sign up for our email list at **www.mathclips.ca/WhatsNewEmailList.html**. ▲

▲ Technology Corner: Venn Diagram Activities



MARY BOURASSA
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Mary Bourassa teaches mathematics at West Carleton Secondary School in Ottawa. She is a strong advocate for the appropriate use of technology in the classroom. She has presented workshops internationally, authored mathematics resources, is a Past VP of OAME, and a Past President of COMA. An award-winning teacher, Mary continually strives to learn new and better ways of helping students learn and love mathematics.

I recently discovered a website (**www.mathsvenns.com**) dedicated to using Venn diagrams to provide unique opportunities to showcase what students know about a topic. Each activity has a low floor, allowing all students an entry point, and has a challenge level that increases as you progress. Craig Barton (**@mrbartonmaths**) is the site's curator and describes these as "rich, Venn diagram maths activities." He was inspired by the task shown in Figure 1, from Don Steward's blog (**www.donsteward.blogspot.com**). Here students needed to place each of the given equations in the correct region of the Venn diagram.

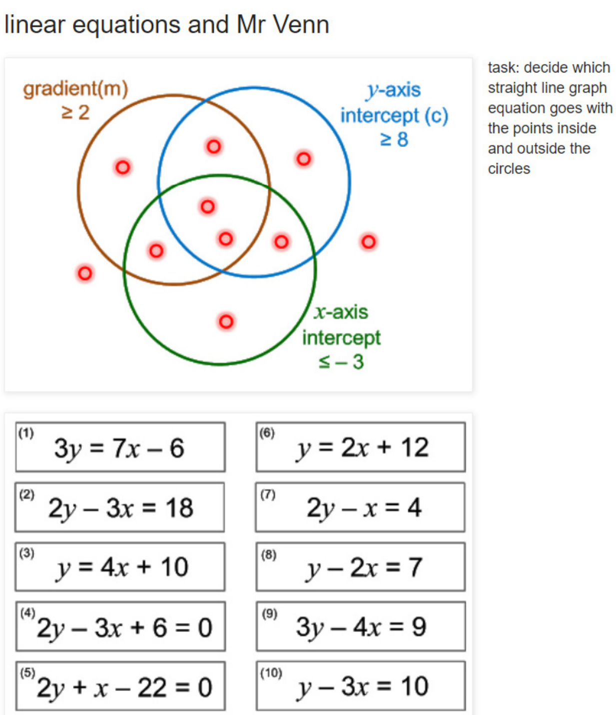

Figure 1

On Mr. Barton's site, the activities are more open, allowing students to create their own equations. These activities allow students to think deeply about the mathematics and make important connections. The first example is shown in Figure 2.

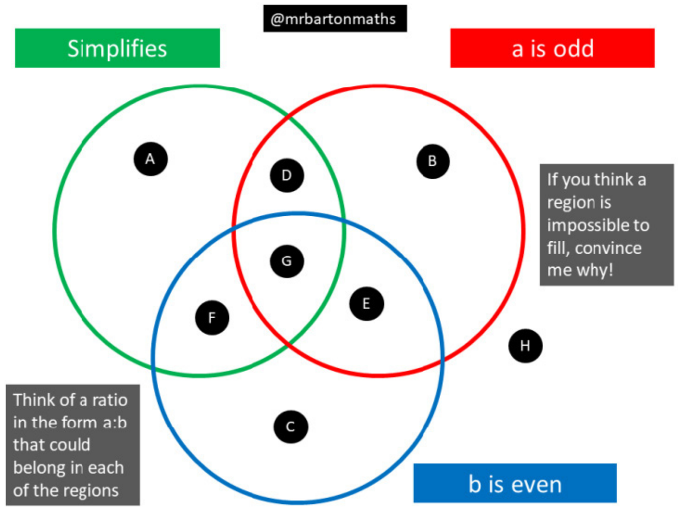

Figure 2

The goal is to have students come up with an example for each part of the Venn diagram. Students can start with any ratio they like and determine where in the Venn diagram it belongs. If you chose the ratio 1:2, then "a" is odd, "b" is even, and the ratio does not simplify, so this ratio belongs in region E. The ratio 3:12 would belong in region G because "a" is odd, "b" is even, and the ratio does simplify. Students must demonstrate a solid understanding of the concepts to complete the Venn diagram and to find a case that does not fit in any of the regions (H).

Mr. Barton considers these examples a source of purposeful practice, but they could also be used as tasks after a topic has been completed. Regardless, students must be able to explain their strategies and the order in which they completed the regions. In addition, there are many good questions that follow. Which region was the most difficult to complete? Perhaps there was one region for which students had many examples; why was that the case? Is there a region that is impossible to complete? Convince me why.

Mr. Barton suggests working through one together with your students as an introduction to this type of activity. This provides a fantastic opportunity for students to discuss their answers and how they found them. It would then become a class challenge to complete all eight regions of the diagram.

The previous example was taken from the "Number" category, which has 21 entries. The other topics are Algebra, Geometry & Measures, Statistics & Probability & A Level (British school leaving exam), which includes Advanced Sequences and Functions.

An example from the Algebra section is shown in Figure 3.

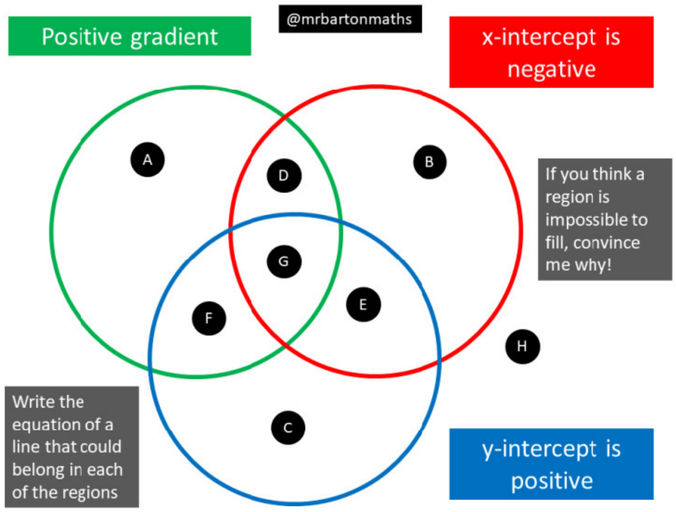

Figure 3

Students must find linear equations that fit each region based on their slope (gradient), and the sign of their x-intercept and y-intercept. An extra constraint could be added that the equation must be written in a particular form (slope-intercept form or standard form). An example for region A could be $y = 2x - 6$, as the slope is 2, the x-intercept is 3, and the y-intercept is -6. Region B would require a negative slope, negative x-intercept, and negative y-intercept, so $y = -2x - 6$ would work. However, it would be impossible to have a line with a negative x-intercept and negative y-intercept with a positive slope, so there are no examples for region D. Students could include a sketch, as in Figure 4, to show their thinking to support this statement.

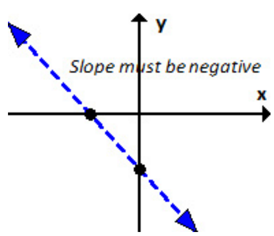

Figure 4

It is interesting to think about where horizontal and vertical lines fit in this Venn diagram, and the discussions that would ensue would help solidify students' understanding of these special lines.

The website also includes some lovely examples of student work, such as the image in Figure 5.

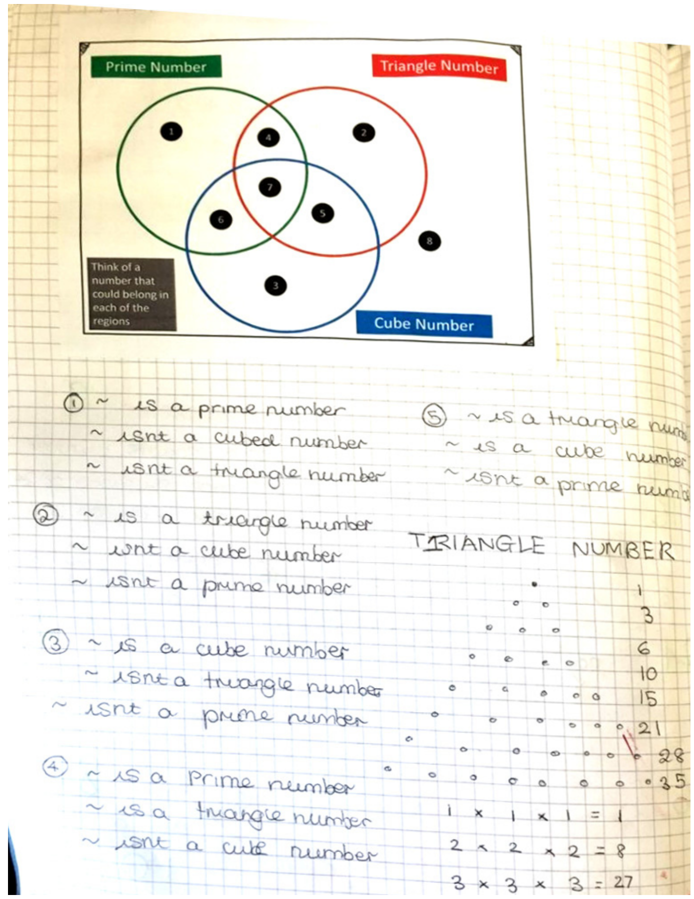

Figure 5

There is a PowerPoint template available on the site so that you (or your students) can create some of your own Venn diagram activities. Mr. Barton would be happy to add them to the collection, so be sure to send them his way.

I thought I would try creating one to demonstrate how quickly and easily these can be made. This one, shown in Figure 6, is intended for my calculus class. I think it would be really interesting for students to add a fourth circle, with the criterion of their choice, to increase the level of difficulty of this challenge.

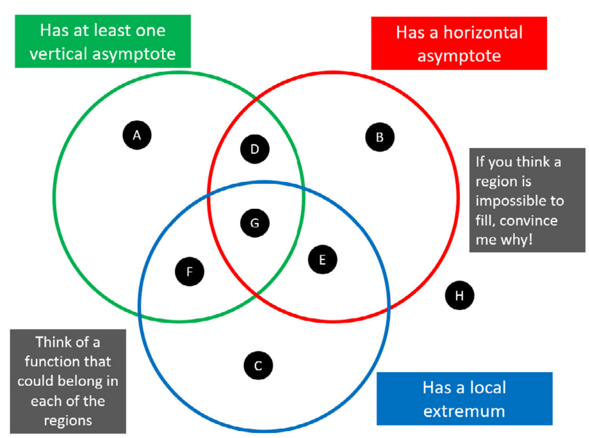

Figure 6

I have found functions that fit all the regions, except for B, as shown in Figure 7. I checked my equations on Desmos as I worked through the regions to ensure that they fit the criteria. If you find a function that would work for region B, please let me know!

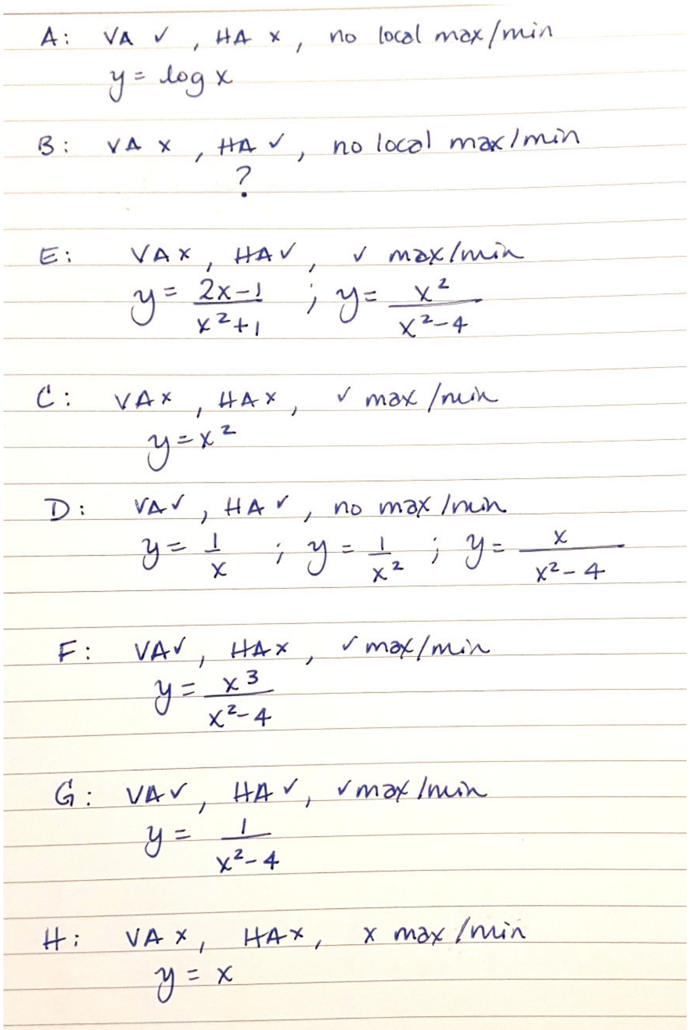

Figure 7

These Venn diagram activities can easily be adapted to almost any topic and, unlike many other tasks, they are quick to make. They can be adjusted quite easily if they are proving to be too much of a challenge or not enough of a challenge. Students can also create their own, using the template or simply by drawing their own Venn diagram. These will become part of my set of easily adaptable tools, like *Open Middle* and *Which One Doesn't Belong*, to draw out the mathematical thinking from my students. I encourage you to check them out too. ▲

Grade 13 Geometry Exam 1949

Eight persons are to travel in two 4-passenger cars. If only four of them can drive, in how many ways can a party of four be chosen for the first car?

▲ Hey, It's Elementary: Building Parent Engagement in Math: A Project to Support the Implementation of the Renewed Math Strategy

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Dr. Lynda Colgan is a Professor at the Faculty of Education, Queen's University. In addition to her teaching responsibilities in the BEd and Graduate programs, she is involved in research and knowledge-mobilization projects with the Council of Ontario Directors of Education, TVO/TFO, the Ontario Ministry of Education, and the Mathematics Knowledge Network.

Our team at Queen's Faculty of Education, Drs. Lorraine Godden, Michelle Searle, Sandy Youmans, and myself, as well as Research Assistants (and M.Ed. candidates) Tyler Ashford and Barbara Mendes, are in the last busy days of preparing the final "deliverable" to the Ontario Ministry of Education for our two-year project, *Building Parent Engagement in Math*, a website rich in resources for researchers, educators, and school council members. We look forward to its official launch in the near future. The goal of the website is to *mobilize knowledge*, in other words, disseminate what we learned as widely as possible so that others may build upon our learning and embark on their own localized initiatives with demonstrated supports from research *and* practice.

Note that throughout this article, the term "parent" is used as defined on the website, which is "anyone who shares responsibility for the well-being of a child and is actively engaged in his/her education."

Figure 1: *Photo during a Ministry-funded initiative, called* Building Parent Engagement: A Project to Support Ontario's Renewed Mathematics Strategy

The *Building Parent Engagement in Math* project (BPEiM), which began in Fall 2017, took place in six schools from three school districts across the province—ranging from Windsor in the west to Selby in the east. As the team completes its monographs, infographics, case studies, and resource packages, we have identified many important factors that contributed to what we believe has been the success of the initiative.

First and foremost, our project demonstrated that positive outcomes are possible through collaborative inquiry (CI) designed to engage parents in their children's mathematics learning. The program we developed promoted collaboration among educators, students, school/district leaders, university researchers, and, most significantly, parents who play a key role in their children's mathematics learning, beliefs, and dispositions. Each school community participated in a continuous and effective manner by producing plans, resources, organizing events, and activities that enabled participation of families and students, while ensuring reporting, evaluation, and sharing of supports and challenges to initiatives that endeavour to realize and promote parent engagement in math.

Through extensive collaboration with the purposefully selected partners of this project, the research team collaborated with the schools to create customized plans to be carried out by each school action team in order to implement, self-assess, and evaluate parent engagement. Ultimately, the research team liaised with all participants to conduct multiple case studies that investigated and recorded meaningful moments and outcomes of this study.

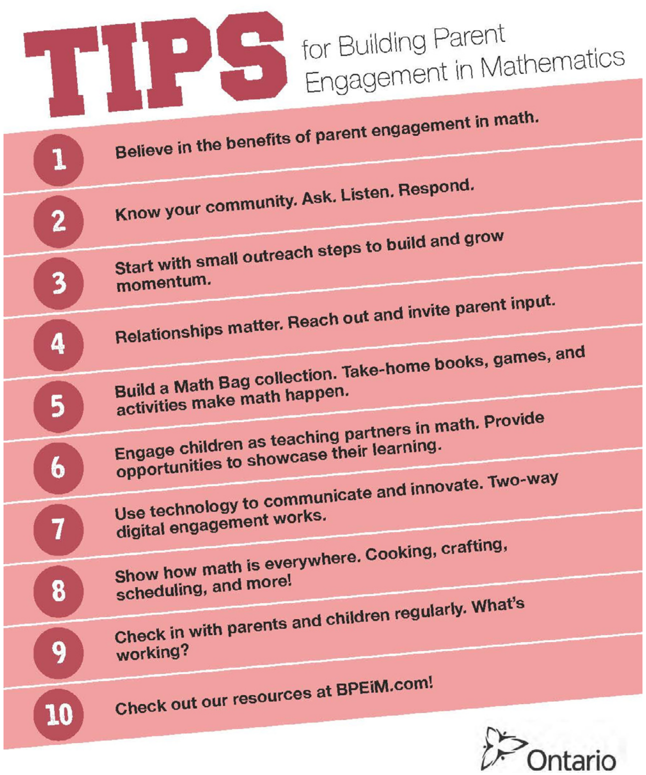

Figure 2: *An example tip sheet from the "Getting Started" collection from the BPEiM project*

It is these case studies and the materials generated by all collaborators that will populate the website so that others might have unrestricted access to the wide array of resources, ranging from surveys and templates to tip sheets and social media-ready interactive mathematics posts.

We also learned much about the conditions and supports that sustain practices geared toward eliminating barriers and creating positive school climates around mathematics teaching and learning, which are two of the strategies that are a focus of the Ontario Ministry of Education's policy document, *Parents in Partnership* (2010). Specifically, schools are required to:

- foster and sustain a positive, welcoming school climate in which parent perspectives are encouraged, valued, and heard
- identify and remove barriers to parent engagement that may prevent some parents from fully participating in their children's learning, and to reflect the diversity of our students and communities

As the six schools began this project, the knowledge they held of their parent communities was instrumental in shaping what strategies they might adopt to engage with their parent cohorts.

Many of the initial considerations came from their pre-existing knowledge of their parent community, especially in regard to what strategies might be welcomed or rejected by parents. To be successful in designing and implementing mathematics outreach activities and programs that met the individual needs of staff, students, and families, the school teams asked the parent community for input and suggestions, and reflected critically about their communities in order to design unique plans for parent engagement in mathematics that included implicit goals of inclusion, respect, building parental confidence, mutual support, and trust.

For one rural school, a critical component was to understand the diversity of its students' family demographics, and how this shaped communication and planning of activities suited to its particular climate. Knowing that many of the parents worked in a nearby city, and many grandparents assumed active, daily childcare roles within family units, one project school invited these extended family members to its math games event, held in the afternoon to accommodate seniors' schedules and driving preferences. Grandparents and other family members appreciated the event, sharing their gratitude for being made so welcome in the school, and for the opportunity to ascertain how math-based games supported the children's learning.

Figure 3: Grandparents and Games *was an afternoon event held at Prince Charles School, Verona. Every family took home a set of* Uno *cards to continue the experience at home.*

In the second strand of the Parents in Partnership policy, schools are required to identify and remove barriers that might prevent some parents from fully participating in their children's learning. In addition, schools are called on to reflect the diversity of their student and parent communities. In the *Building Parent Engagement in Math* project, two schools responded to this requirement by adopting a longer-term structured approach that allowed their English as a Second Language (ESL) parents time to develop confidence in the school's parent-engagement initiatives and by producing materials in both English and Arabic. An example of a social media post in both languages is shown in Figures 3a and 3b below.

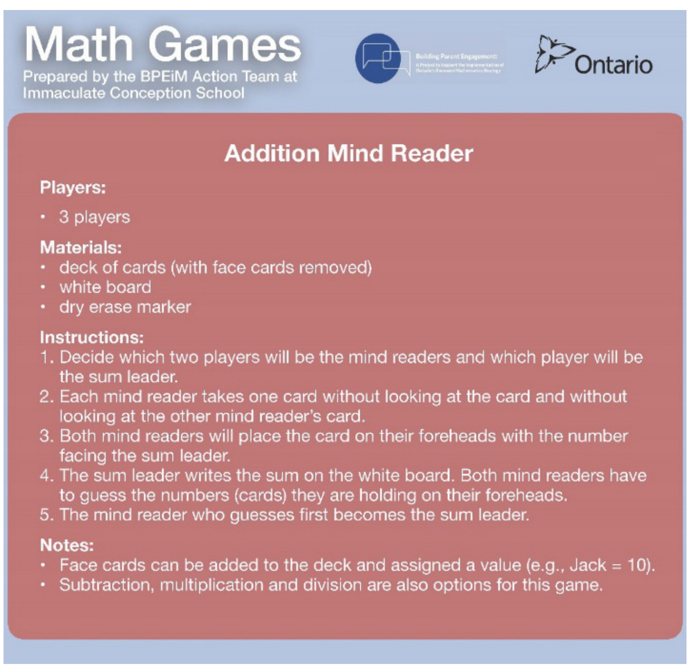

Figure 3a: *An example of a social media post developed by a project school for a weekly math challenge for families in its school community*

Figure 3b: *An example of a social media post developed by a project school for a weekly math challenge for families in its school community, which has been translated into Arabic in order to extend the "reach" of the communication*

In the case of this school, the impact of the mathematics outreach program was profound:

> We have a big ESL demographic here. In our Grades 1 to 3, 90 percent are newcomers between the last one to five years of being here, 90 percent of those who came and participated in our math events. It made them comfortable to come into the school and talk about it. They are seeing the teachers more often. We had two sessions a month and they seemed to really value this. It has built great relationships among the students, parents, and teachers. This project has been so valuable for our school, especially with getting parents involved here. Some of them still feel intimidated coming into school, especially because there is that language barrier; they feel they can't communicate to us what they want to know, or what they want to ask. But now, since they have come to those sessions, many of them we see them weekly, sometimes three times per week, and they are comfortable seeing us now. The knock-on effect in our relationships is really noticeable (school principal).

While it is impossible to capture all of the findings of this two-year project, the website will be a virtual repository, introducing our team and our schools; sharing our findings from the project through technical reports, infographics, monographs, scholarly papers, and slides from conference presentations; offering research-based suggestions for getting started by describing best practices from the six project schools; presenting detailed case studies of each school's particular successes in different areas of parental engagement in math; and sharing field-tested resources for school teams, parent councils, educators, and parents.

The Building Parent Engagement in Mathematics project hopes that by sharing its materials with concrete practices that foster productive parent engagement and parent–school partnerships in mathematics, the claim can be challenged.

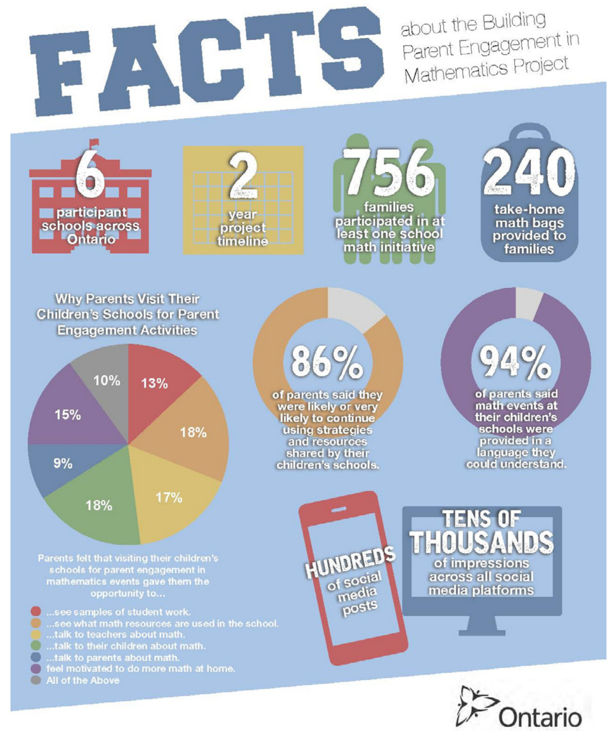

Figure 4. *An infographic summarizing the impact of the* Building Parent Engagement in Math *project*

Reference

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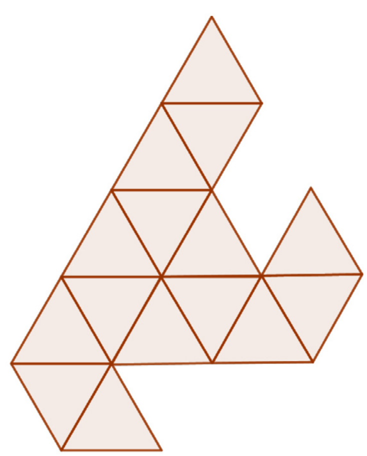
Icebreaker 2 Puzzle

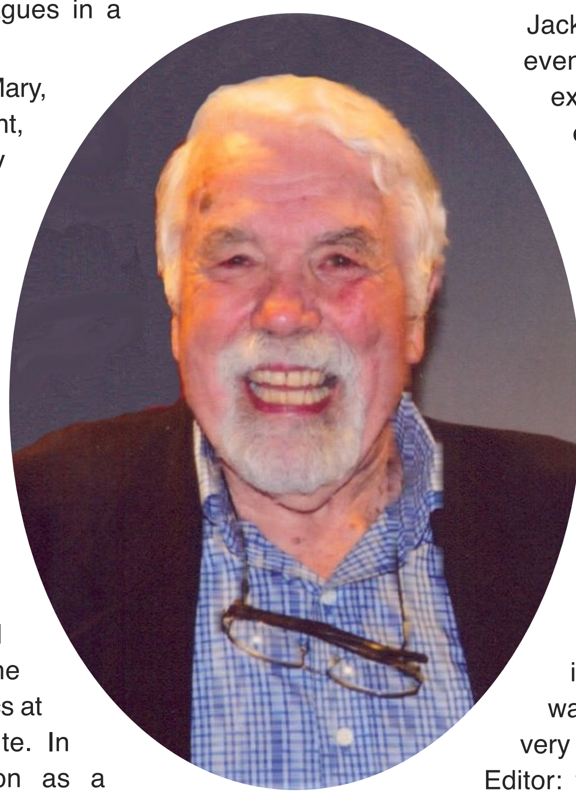
Your task is to break this iceberg (i.e., collection of triangles) into four identical shapes!



Jack Wilfred LeSage

BY BILL MORRISON AND GREG CLARKE

Jack was a role model and mentor for many teachers and widely recognized as an outstanding educator in mathematics. He loved teaching and shared his vast knowledge of mathematics with students, teachers, and colleagues in a quiet, unassuming manner.



He was devoted to his wife, Mary, and his family. After his retirement, Jack and Mary enjoyed many trips to various destinations. Jack loved driving and took trips to both the East and West coasts to visit family.

After graduating from the University of Toronto in 1953, he started his teaching career at St. Michael's College School. In 1955, Jack moved to Kapuskasing and taught there until 1957. He moved to Barrie and taught at Camp Borden High School until 1962, when he became the Department Head of Mathematics at Barrie North Collegiate Institute. In 1969, he accepted a position as a Mathematics Consultant with the Ontario Ministry of Education. Jack returned to what he loved to do, teaching students, at Eastview Secondary School in 1975, where he was also the Assistant Department Head of Mathematics. He continued in that role until his first retirement in 1988. Jack then taught and tutored many mathematics courses at Georgian College/Laurentian University until 2014.

Jack was a founding member of the MAC2 chapter. He was involved in the initial discussions regarding the formation of the OAME/AOEM chapter in 1994 and became part of the steering committee. When MAC2 became a chapter in 1995, Jack was the Secretary and held that position until 2006, but his contributions went far beyond that. He was the heart of our chapter. His experience, insight, and wisdom were invaluable in the development of the chapter over the first 20 years. Jack continued as a councillor for the chapter until 2014. He chaired the Registration Committee for the 2002 OAME/AOEM conference, co-chaired the Facilities Committee for the 2007 conference, and was a valuable member of that committee for the 2016 conference. Jack shared his knowledge of mathematics and unique ideas with colleagues by presenting sessions at the annual conference over most of the last 45 years.

Jack was an integral part of OAME even before it officially came into existence. He served as an executive member of the OATM (Ontario Association of Teachers of Mathematics) in 1970 as Publicity Representative, and also served as Councillor of OATM in the early 1970's. The OATM merged with the OMC (Ontario Mathematics Commission) to form OAME in 1973. Jack joined the Editorial Team for the *Gazette* in 1975, helped organize the OAME Leadership seminar for that year, and continued to be involved with OAME thereafter. He was instrumental in shepherding two very special editions of the *Gazette* as Editor: the Centennial Edition in 1991, which celebrated 100 years of mathematics education in Ontario; and the 40th Anniversary Edition in 2012, celebrating the 40th year of OAME.

He was an author of mathematics textbooks and a developer of Algebra tiles, which have been widely used in mathematics classrooms since the 1980's. He was even part of the committee that recommended the metric system.

Jack was a talented mathematician and had the extraordinary ability to make a person feel that he or she had solved a problem or come up with an idea when, in fact, he was guiding the person and process throughout. He was a gifted teacher of mathematics.

He challenged us, and made us better people for knowing him. Jack will be truly missed.

▲ Mb4T (Mathematics by and for Teachers): Is It Going to Snow Today? A Look at Probability

JENNIFER HOLM
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Jennifer Holm is an Assistant Professor at Wilfrid Laurier University and works with primary/junior and junior/intermediate pre-service teachers, as well as in the field supporting current mathematics teachers. She is interested in developing mathematics knowledge for teaching and analyzing beliefs about mathematics with both pre-service and in-service teachers. Her current focus is on the beliefs and opinions that pre-service teachers hold about mathematics and teaching along with the connection they have to past experiences. She uses this research to support future teachers in developing beliefs and knowledge that will encourage and support effective teaching practices.

When I first started teaching, I thought probability would be one of the easiest topics to run through right at the end of the year because there was "nothing to it." I had been taught that you just multiply fractions together and that would tell you how likely an event is to happen. It did not take me long to discover that there was substantially more to probability, and to teaching probability, than I had initially thought.

Although probability tells "how likely something is to happen," the nuances of how it can be taught and explored in schools can lead to some powerful understandings of mathematics. In Ontario, probability runs along a continuum throughout the years so that each grade builds on the understandings of the previous grades. In Grades 1 and 2, the goal is to describe probability, using words like "impossible," "unlikely," and "certain." This sets the base for Grade 3, when students try to predict the probability of an event occurring and look at how "fair" games are by seeing if outcomes are equally likely to occur. It is important to realize that students are not yet stating probabilities in Grade 3, only exploring them and what they mean. These early conversations have the potential to later support students' understanding of what probability means when the expectations are deepened in future grades.

In Grade 4, the expectations change slightly in that students are more focused on conducting experiments to test their predictions. An activity you could start using at this grade level would be "Balloons and Dice" (Kajander, 2007). I like to use this activity in multiple grades (as you will see throughout this column) because it can support developing understandings of probability, but has the added bonus of supporting better subitizing skills (by having to recognize the numbers from the dot patterns on the dice), as well as reinforcing addition facts. For instructions on the game, see Figure 1. In Grade 4, the idea is to ask students what they think the most common sum is, and then have them explain why they are making that prediction. Having students play the game a few times allows them to test their predictions while recording the data. Group discussions following the game allow for their predictions to be refined. The second expectation related to probability that is important in Grade 4 has to do with seeing how playing the game a greater number of times reveals changes in the occurrence of specific numbers. This is setting students up with a base understanding of the difference between a theoretical probability and an experimental probability, without explicitly introducing the concept. Having students play several iterations of "Balloons and Dice" allows them to see how the number of trials impacts the results.

"Balloons and Dice" (Kajander, 2007, pp. 90–94)

Materials: Game board like the image below

1	2	3	4	5	6	7	8	9	10	11	12

12 pennies (or tokens) for each student
2 dice per pair

Directions:

1. Place the pennies (balloons) under any numbers you choose in your stalls.

2. Each player takes turns rolling the two dice. You may "pop" a balloon if you have one in the stall that is the sum of your roll (e.g., if you roll a 3 and a 4, you can pop one balloon in the 7 stall).

3. The first player to pop 10 balloons (or all balloons) is the winner.

Figure 1: *Rules for Balloons and Dice (Kajander, 2007)*

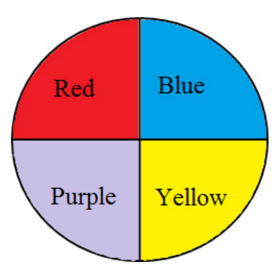

Figure 2: *Spinner showing red has 1/4 probability*

Red	Blue	Purple	Yellow

Figure 3: *Area model for spinner in Figure 2*

Grade 5 is the first time in the Ontario curriculum where students are using fractions to state what the probability is for a simple event. At this grade level, students start looking at how a probability is stated as the number of times an event can occur over the total number of outcomes. For example, the probability of red being chosen in the spinner in Figure 2 is (number of red sections divided by the total number of sections). In Grade 5, students can also begin creating simple probability trees by writing out the possible outcomes (e.g., for the spinner, the list would be red, yellow, green, blue). Creating area models, as shown in Figure 3, helps students build understandings that will be expanded in the intermediate grades. Another activity for Grade 5 students is to have them create their own probability experiments and try them out, recording the data in a way that makes sense. Spending time on these simple experiments sets students up for success in later grades, where the concepts get more complicated and more in-depth considerations of probability, such as bias, can be introduced. Another activity that works well in helping to lay the foundation for concepts is discussion based on the "Find the Right Bag" activity (Ontario Ministry of Education, 2007, p. 110). To set up the experiment, use four paper bags, labelled A through D, and put 20 cubes in each bag (see Figure 4 for how to set up the activity). The task for students is to ask, without looking in the bag, which one has exactly 10 blue cubes and 10 white cubes. Students are only able to remove one cube at a time, and the cube has to be replaced, by putting it back in the bag it came out of, before pulling another cube from any bag. Students should consider, as they are solving the problem, how many times they can pull a single cube from a bag and replace it before they can say with certainty which bag it is. Since one bag also contains pink cubes, it can be eliminated as soon as a pink cube is pulled. Identifying the other bags is more difficult, since they only include blue and white cubes, and hopefully students will realize that they cannot say for certainty which one has only 10 blue and 10 white cubes. A question to ask during this realization can be, "What would help you make the decision, other than looking into the bags and counting the cubes?" Students often suggest being able to keep some of the cubes out instead of having to put them back into the bag. The teacher can respond by asking, "How many cubes would you like to keep out so that you are sure you have the correct bag?" This activity leads to some in-depth conversations about the concept of probability and what it means within a practical activity, without being worried about stating probability values for the events.

Using 4 paper bags that cannot be seen through. Put the following amounts of cubes in the different bags:

1. 15 blue cubes and 5 white cubes
2. 15 white cubes and 5 blue cubes
3. 5 white cubes, 5 blue cubes, 10 pink cubes
4. 10 white cubes and 10 blue cubes

Label the Bags A-D to allow for discussion.

Figure 4: *Directions for "Find the Right Bag" (Ontario Ministry of Education, 2007)*

In Grade 6, students start to learn that fractions are not the only way to express a probability. With decimals and ratios being introduced in this grade, they can be used to describe the likelihood that an event will occur. For example, looking at "Balloons and Dice" again, the students can state the probability of getting a 2 (or 12) in the game is $\frac{1}{36}$ or 1:36 or $0.02\overline{7}$ or $2.\overline{7}\%$. They are still predicting events as in previous grades, and the previous activities would be appropriate to continue the discussions around probability concepts.

Grades 7 and 8 put everything together, and students formalize their learning about theoretical versus experimental probabilities. If you return to the "Balloons and Dice" game once again, it can now be expanded for use in intermediate classrooms beyond just playing the game. In these grades, you can add the extra layer of having students try and finish the game as quickly as they can by suitable placement of the 12 pennies. At this point, the theoretical probability tells students to place the most balloons in the centre numbers, since the probability of 6, 7, 8 tells us these are more likely to occur. Using a probability tree or area model (see Figures 5 and 6), students can calculate the probabilities of these numbers. Students can now compare their theoretical probabilities with experimental probabilities by playing the game at least twice and collecting the data. What they should understand from Grade 4 is that the more times the game is played, the closer it should resemble the theoretical probabilities. Consider the two graphs in Figure 7. The one on the left was created from a single pair playing the game, whereas the one on the right shows the entire class compiling the data. Replicating this exercise with intermediate students makes it very clear that the larger data set helps to see that the probabilities do begin to resemble the theoretical probability. This is an important conversation; the results are not exact, so students who seem to roll a 10 more often when they play, may often do better at the game by playing their own "odds."

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Figure 5: *Area model for "Balloons and Dice," with options for die 1 along the top and die 2 down the side. The inside shows all the possible sums based on the two dice.*

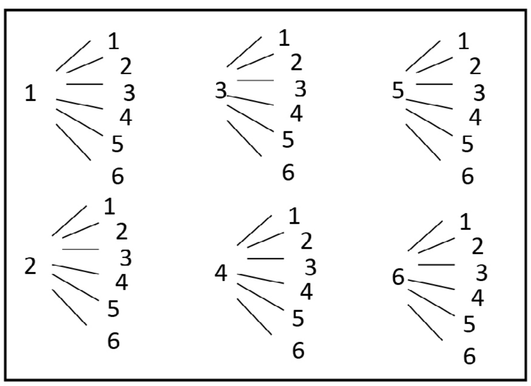

Figure 6: *Probability tree for "Balloons and Dice," listing all the possible outcomes on one die, with the possible outcomes on the second. There would need to be an added step of determining the combinations for each sum to use for finding the probability of a sum.*

It is also important to create larger probability experiments where ideas are combined together. Making it more complicated helps students explore how to find probabilities where the results are much more detailed. For example, a complication can be introduced by using a coin, one die, and a five-coloured spinner, which produces an interesting probability tree (see Figure 8) and a lot of in-depth questions. In looking at the game, one could ask what the probability is of getting an even number $\left(\frac{30}{60}=\frac{1}{2}\right)$. What is the probability of getting a green section $\left(\frac{12}{60}=\frac{1}{5}\right)$? The added complexity of the game gives a natural starting point for conversations about how weather fits into probability, or discussions around populations or diseases, and the real-life implications of knowing probability. Another activity for intermediate students to apply their probability skills is to plan a games day for the lower grade levels. They can create games and justify why the games are fair, and determine probabilities as part of the justification. A more interesting application is to have students create games where the probability is rigged against the player such that it is not obvious. For example, making it hard for the player to win makes it less likely he or she will win in the time available. By having to apply the idea of probability in a way that allows the "house" to win more often, without making it obvious, would add another layer of dimension to the discussions.

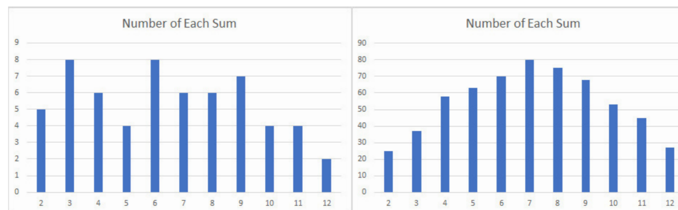

Figure 7: *Single game compared to results for the whole class*

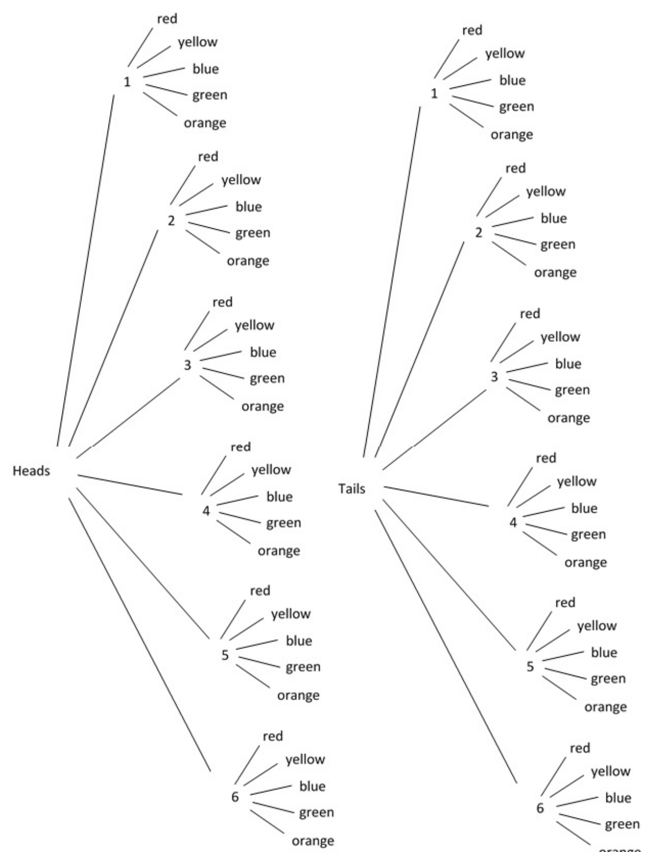

Figure 8: *Probability tree for a game using a coin, one die, and five-section spinner. This shows there are 60 possible outcomes in total.*

Probability serves as a rich area for conversation about the likelihood of an event to happen, and there are many other ideas that can be brought in to discuss ideas that influence probability. For example, Banting, Vashchyshyn, and Chernoff (2018) discuss adding elements of bias to discussions about probability in order to deepen understandings of probability within the secondary school classroom. Some of these conversations would also be interesting starting points for intermediate students. Following the curriculum trajectory from Grades 1 to 8 allows for students to continue building on initial understandings to deepen their knowledge of probability. Throughout elementary school, students need to engage in rich problems and discussions related to probability, as it increases their understanding of the concept beyond simply stating the likelihood of an event to occur. These conversations and activities move students beyond knowing that they just "multiply the fractions together" to truly understand the real-life applications of probability and what that end fraction (or representation) means in a practical sense.

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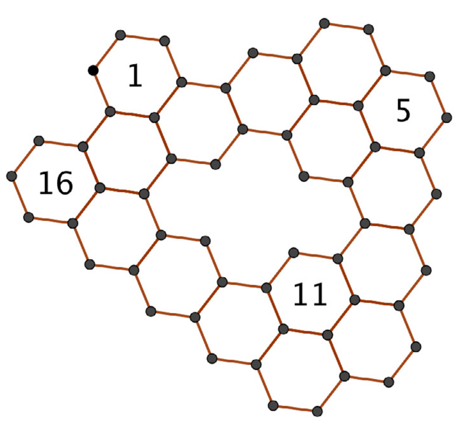
Hidato: Fill cells so that neighboring cells have sequential values.



▲ In the Middle: STEM as an Aspirational Medium

CARLY ZINIUK
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Carly Ziniuk teaches Grade 9 Mathematics, Grade 12 Data Management, and Advanced Placement Statistics at the Bishop Strachan School in Toronto, Ontario, Canada. She is very active in adopting real-life data to engage her students in solving problems.

I have noticed an increase in the prevalence of math resources that show diversity in STEM and between the STEM disciplines. An example is retired Airborne U.S. Army Colonel Patty Dooley, who served in Afghanistan and is now a Chemistry professor at Bard's College, New York. On her school's video profile, titled "I'm proud of being a geek" (Bard College, 2012), Dooley contends that STEM is the ideal place for curious people to meet each other, and her article, "How I convince teenagers to love STEM" (Dooley, 2015), has inspired me to explore more "wow" moments (as Dooley calls them) in my math classes. What follows is a collection of opportunities for "wow" moments through the use of STEM resources.

In her article, Dooley writes:

> I explain to my students that STEM is a source of hope. It reassures us that in the future, maybe in our lifetimes, we will progress. We will, one day, learn how to prevent or cure cancer, eliminate polychlorinated biphenyls (PCB's) from the environment, control greenhouse gas emissions, end hunger, and go faster-than-light through a wormhole to another galaxy. Maybe in our lifetimes the nature of dark matter and dark energy will be discovered (Dooley, 2015).

Considering STEM as a driver of wonder and a provocateur of hope is an aspirational stance, and I agree with Dooley that these approaches to learning math motivate teenagers.

I started looking for specific small hooks to show how math explains captivating phenomena. One example is Victor Powell's *Explained Visually* project, which he describes as an "experiment in making hard ideas intuitive." The project has produced visualizations of pi, exponents, and the Pythagorean theorem. Another example is Shodor Education Foundation's *Interactivate*, which includes online activities, visualizations, and games illustrating mathematical relationships. One favourite in my Grade 9 class is a version of the classic board game *Connect Four*, requiring algebraic simplification with common factors to place each piece. Shodor's *Interactivate* is also a source for a variety of applets like tessellation creation or directing a robot through a minefield grid, using coordinates. Both *Explained Visually* and *Interactivate* have engaged my students' curiosity as warm-ups and as opportunities to consolidate or to generalize larger overall concepts.

The Public Broadcasting Service (PBS) Infinite Series (2018) contains presentations of perplexing problems like "Why computers are bad at algebra" and "How to break cryptography." The questions at the beginning of each episode are invitations to further learning on topics such as modular arithmetic and rearranging multiple variables. "Kill the mathematical Hydra" is a favourite of my classes. For a slightly easier version of it, I adapted a problem from Chris Smith's March 22, 2019 Maths Newsletter as follows:

> You meet a magical monster with three heads and three tails. In your effort to defeat this beastie, each swing of your sword can chop off EITHER one or two heads OR one or two tails. If you cut off one head, however, another grows back. If you cut off two heads, nothing happens (except the heads are gone!). If you cut off one tail, another two tails grow back. If you cut off two tails, one new head will appear! To slay the monster, you need to remove all of its heads and all of its tails. Can you be the victor in this battle?

The PBS Infinite Series episodes "The cops and robbers theorem" and "How many cops to catch a robber?" surprise students with the ideas found in planar and Hamiltonian graphs. The link between practical and theoretical game-theory applications is cleverly shown in the episode "Splitting rent with triangles," a problem that Albert Sun (2014) also explained in a more traditional way in the *New York Times*.

In her article, Dooley (2015) states, "(STEM) reassures us that there is a valid explanation for what is seemingly inexplicable." This occurred when I showed my students an animation, from NASA's Dr. James O'Donoghue (n.d.), of the distances between Sun–Mars–Earth–Moon at the speed of light. O'Donoghue's goal is, "(to) convey context instantly, as real as possible." When my students viewed the speed of light as it travels around Earth in 13 seconds, the scope of those numbers and the value of using operations in scientific notation were clearly relevant. I also used astronomical values and the work from NASA's Jet Propulsion Laboratory (JPL), particularly their "Pi in the sky challenge" (NASA/JPL Edu, 2019) to explain the importance of being careful when combining negative exponents according to exponent laws. They have beautiful printable posters with short inquiry problems, as well as separate teacher resources and answers.

I recommend this "Deadly Dust" poster question that can be accompanied with July 2018 footage from NASA and National Geographic (NASA/JPL-Caltech/MSSS, 2018; National Geographic Society, 2018):

> In the summer of 2018, a large dust storm enshrouded Mars, blocking visibility over a large portion of the planet. The thick dust covered almost all of the Mars surface, blocking the vital sunlight that NASA's solar powered Opportunity rover needed to survive. In fact, the storm was so intense and lasted for so long that Opportunity, which had spent 14.5 years traveling around the Red Planet (with a 21 344 km circumference), never managed to regain consciousness and the mission had come to an end. During the height of the storm, only the upper caldera of one of the solar system's largest volcanos, Olympus Mons, peeked out above the dust cloud. The diameter of Olympus Mons' caldera is approximately 70 km. What percent of the Mars surface was covered in dust at this time?

Teachers can make additional NASA connections by downloading the complete *Hidden Figures – Curriculum and Discussion Guides* for free from Journeys In Film (2017). This curriculum guide presents mathematical connections to the film, such as lessons on scientific notation and exponents, as well as problems involving conic sections along with similar and congruent figures. These problems use NASA research and sections from the film as the context for middle school math content learning.

These include (p. 83):

- "How big would your scale model of the solar system have to be if you did use a basketball to model the sun?
- What happens to the area of a triangle if you double the base and double the height?
- What if you double the base of a triangle, but reduce the height to half its original value?
- What effect does doubling the radius have on the area of a circle?"

Planet Money is an economics radio show/podcast on National Public Radio (NPR) that has determined "Why a pack of peanut butter M&M's weighs a tiny bit less than a regular pack" (Kestenbaum, 2014). This economics and math puzzle has been a source of fascination to my data management and statistics students this year, and I recently shared it with my Grade 9 students, who were eager to gobble up the experiment! The students argued strongly with the proposed answers in the problem and their contradictions.

Many math teachers have done a lot of work to put STEM applications into the many topics we teach. Dooley's article reminded me of how important it is to show where the content is used, and to revel in the "wow" moments that STEM integration provides as we examine the world. It provoked me to look more frequently for these moments to inspire my students and to show mathematics as an agent of change as well as a tool of hope. I received feedback from both students and parents that this perspective was motivating in students' daily learning and showed up regularly in family conversations. Using these resources to describe phenomena and present connections helped students to see beyond algorithms and processes that I thought I was presenting to support the Applications category of achievement. Seeing math as a valuable tool to explain phenomena gives my students many more moments of "wow" and, I hope, convinces them to love the math in STEM all the more.

Answers

Adapted Monster problem: 9 cuts is the minimum. The solution is well explained visually on The Math Forum (National Council of Teachers of Mathematics, 2007).

Deadly Dust problem: 99.97% (The steps to the answers are provided with the JPL/NASA poster series.)

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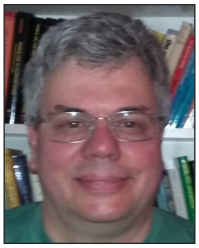
Grade 13 Trigonometry and Statics Exam 1949
Name two respects in which the concept of angle, as used in synthetic geometry, is enlarged in trigonometry.

▲ What's the Problem? Predicting Pyramids



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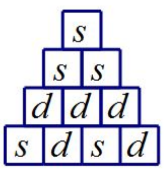
Welcome back, problem solvers. Last time, I left you with the following problem:

In a self-describing pyramid, a cell gets an *s* if the two cells below it are the same, and it gets a *d* if the two cells below it are different. The diagram below illustrates a self-describing pyramid with four levels.



How many possible ways are there to fill the four cells in the bottom row to produce an *s* at the top of the pyramid?

This problem was inspired by problem 19 from the 2018 AMC 8 competition. The American Mathematics Competitions (AMC) runs a series of mathematics competitions in the United States, available to Canadian and International students. You can access information about the competitions on the Mathematical Association of America (MAA) website, **www.maa.org/math-competitions**. The Art of Problem Solving has a section of their website dedicated to AMC competitions. You can find past contests problems and solutions at **www.artofproblemsolving.com/wiki/index.php/AMC_Problems_and_Solutions**.

I chose this particular neat little problem because it just begs to be played with. We could draw our pyramid, put an *s* on top, and fill the other spaces with *s*'s and *d*'s in all possible ways to see which ones "work." The problem with this approach is that since there are two ways to fill each empty space, there are $2^9 = 512$ ways to fill the remaining nine spaces. Hopefully there will be a better way than doing an exhaustive search of all those pyramids.

A powerful problem-solving strategy is to look at an easier problem. Thus, instead of looking at a pyramid with four layers and an *S* on top, we will look at the shortest pyramid that has two layers. By the very definition of how the pyramid is made, we can see that there are two possibilities, as shown in Figure 1. Similarly, with a *D* on top, there are two possible ways to create a pyramid with two levels. Since there are $2^2 = 4$ ways to create a row of two letters, using only *S* and *D*, we have accounted for all possibilities.

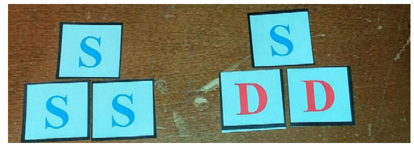


Figure 1: *All possible pyramids with two levels and an S on top*

Next, we can move up to three levels. If we start with the pyramid with two levels and every square containing an *S* from the left in Figure 1, we can create two different third levels, as is shown in Figure 2. Similarly, there will be two different bottom layers (*DSD* and *SDS*) that will have the top part the same as the right picture from Figure 1. Thus, there are a total of four ways to create a pyramid with three levels that have an *S* on top.

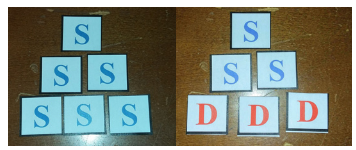


Figure 2: *Possible third levels for pyramid with S on top two levels*

If we went a bit further, we would see that there are also four ways to make a pyramid with three levels with a *D* on top. It seems at this point that we could construct any row, using *S* and *D*, and they would uniquely define a pyramid. When thinking about how a pyramid is created, we can see why this is true. Apart from the bottom row, each cell sits on two cells in a lower row, so its value is completely determined by those two lower cells. You may want to try creating a pyramid with six layers that has bottom layer *SDDSDS*. Its solution can be found in Figure 4 at the end of the column.

You should also have noticed at this point that there seems to be an equal number of pyramids with an *S* on top as with a *D* on top. We might assume that this pattern continues, and then the answer to our problem would be half the number of ways that we can create a row of four, using *S* and *D*, which is 8 (i.e., half of 2×2×2×2). A difficulty with this approach is that the pattern we see from a couple of examples may not be the genuine pattern. As such, if we can understand how and why the pattern works, we can address this difficulty and be confident in making our conclusions.

The original problem from which I drew inspiration used "+" and "–" rather than *S* and *D*. If we reformulate the problem, where each *S* is replaced by +1, and each *D* is replaced by –1, our rule for creating the pyramid can be turned into a mathematical operation. For example, *SD* leads to a *D*, just as (+1) × (–1) = –1. Each entry, other than the bottom row, is the product of the two entries underneath it. Thus, if we let the bottom four entries be *w*, *x*, *y*, and *z*, the pyramid entries will be as shown in Figure 3.

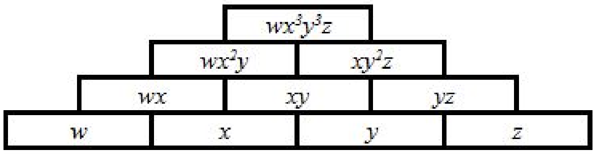


Figure 3: *Four-layer pyramid with variables*

As all of the variables are equal to +1 or –1, and $(+1)^2 = (-1)^2 = +1$, the top entry can be simplified to $wx^3y^3z = wx^2xy^2yz = w(1)x(1)yz = wxyz.$

From the simplified expression, we can deduce that if an even number of entries in the bottom row are –1, the top entry will be +1. Similarly, if an odd number of entries are –1, the top entry will be –1. A quick check shows that 8 of the 16 ways that we can create the bottom row yield a+1 on the top row; the other 8 give a–1.

Something interesting happens when we look at the exponents in the algebraic expression for our top entry: 1, 3, 3, 1. Hopefully many of you will recognize that as corresponding to the third row of Pascal's triangle. Upon closer inspection, the exponents of each square along a layer are a row of Pascal's triangle (e.g., "121" occurs in each square of the second row). This should not be too surprising, as the way the pyramid is constructed is similar to Pascal's triangle, with each entry depending on two other entries. As we move up the pyramid, the exponents are adding in the same way that elements of Pascal's triangle are added as we move down.

If we are looking to generalize the behaviour of the top entries in our problem, some interesting things happen. If we added another row to our pyramid, and looked at the exponents of the top entry, they would be 1, 4, 6, 4, 1—the fourth row of Pascal's triangle. When we think about how we are representing the problem, those three even numbers (exponents)—4, 6, and 4—mean that the middle three entries of the bottom row have no influence on the top entry! That means if you had a pyramid with five rows that had an *s* on top, and you changed one of the middle three entries on the bottom from an *s* to a *d* or vice versa, some of the entries in the pyramid would change, but not to one on top! For certain numbers of rows, there will be some even entries in Pascal's triangle, which means that the corresponding entries on the bottom row have no effect on the top entry.

One important thing to note from Pascal's triangle is that every row begins (and ends) with a 1. This means that the first (and last) entry of the bottom layer will always influence the top layer. That is, if we have a pyramid with an *S* on top, and we change the first entry of the bottom layer, the top will now switch to a *D*. For any arrangement of *S*'s and *D*'s, we will define its complementary arrangement to be the arrangement obtained by switching the first element from *S* to *D* or vice versa. This gives us a way to justify our conjecture. For a pyramid of any height, if we wrote out every possible arrangement of the bottom row, we could pair them up with their complements. That is, in a pyramid with five layers, the bottom row of *dsssd* would be matched up with *ssssd*. Since the pairs differ in the first entry only, they will each have a different entry on top. Thus, exactly half of all possible pyramids of any height will have an s on top, and half will have a *D* on top. Since there are 2^n ways of creating a row of *n* elements using *S* and *D*, then there will be $2^n \div 2 = 2^{n-1}$ ways to create a pyramid with *n* layers with an *s* on top. Thus, there are $2^{4-1} = 2^3 = 8$ ways to create our original pyramid.

An interesting connection to computer logic is mentioned in the solution on the Art of Problem Solving web page, where it is noted that if you let the *D* be 1 and the *S* be 0, then the logical operation "exclusive-or," abbreviated XOR, returns a 1, if exactly one of the numbers is a 1; otherwise, it returns 0. I have created a spreadsheet with the Boolean operators XOR, OR, and AND. There is a check box for each of the two inputs *A* and *B.* When the check box is checked, it makes the value *true* (1); otherwise, it is *false* (0). You can see how the three Boolean operators work by experimenting with different inputs.

On the same spreadsheet, I have created pyramids with three, four, five, and six layers. You can change the entries in the bottom row by checking or unchecking the check boxes. You can explore the problem yourself by accessing the spreadsheet in my "What's the Problem?" Google drive folder at **www.drive.google.com/drive/folders/0ByDlaUaj8StpanhnUWo2bEV6ZE0?usp=sharing**.

Figure 4: *Solution to the six-layer pyramid*

Now it is time for your homework:

Melanie computes the mean μ, the median *M*, and the modes of the 365 values that are the dates in the months of 2019. Thus, her data consist of 12 1's, 12 2's,…, 12 28's, 11 29's, 11 30's, and 7 31's. Let *d* be the median of the modes. Arrange the numbers μ, *M*, and *d* from smallest to largest.

Until next time, happy problem solving! ▲

▲ Linking Literacy and Math: Comprehension Strategies to Support Mathematical Understanding (Part Two)

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Dr. David Costello is a vice-principal who has held many roles before this. He taught in the primary division before assuming roles of numeracy interventionist, numeracy coach, numeracy leader, and curriculum consultant for Prince Edward Island. David has also instructed university courses focused on curriculum, differentiation, mathematics, and language arts. He facilitates professional learning sessions in the areas of mathematics and school development. His research situates mathematics within a literacy perspective and examines leveraging literacy-based instruction in mathematics.

This column is an extension of my column published in the last issue. In this column, I continue to emphasize that a highly literate person is someone who can make meaning when engaged with a task (Fountas & Pinnell, 2006; Keene & Zimmermann, 2007). When one is making meaning, using in-the-head problem solving, it is called comprehension (Clay, 1991). What is referred to as comprehension for reading is often referred to as conceptual understanding in mathematics (Small, 2008; Van de Walle, Lovin, Karp, & Bay-Williams, 2014). Leveraging comprehension strategies that students are already using in their reading experiences for use in mathematics can be beneficial.

In my previous column, I outlined the strengths of identifying comprehension strategies as thinking strategies versus limiting them to reading strategies. That broadening of scope facilitates applying the comprehension strategies to mathematics. Teachers do not need to teach conceptual understanding as a new direction or entirely new processes. Instead, teachers can leverage successful instructional practices from reading into mathematics.

Three examples of processes that students learn in reading are *making connections, synthesizing,* and *questioning.* I will use these as examples of comprehension strategies that can be transferred from language arts classes to mathematics classes.

Making Connections

Each student brings a collection of prior experiences when engaging with a text. It is through the purposeful recall of relevant prior knowledge that students make meaning, whether it is before, during, or after they read the text (Pinnell & Fountas, 2007; Keene & Zimmermann, 2007). Relevant prior knowledge, also referred to as schema, is gained from the student's personal experiences, learning about the world, and/or reading other texts. Through this activation of prior knowledge, students can make connections with the text and deepen their understanding beyond the literal level.

The connections that students use to deepen their understanding in reading can also be applied to mathematics. Making connections to a past problem, a concept, and/or event or experience can provide students with strategies to approach the task at hand. These connections impact the inferences made while working through a problem, as well as how students comprehend the material. Connections permit students to internalize the material being worked with, and reinforce their background knowledge used to support their understanding. It is the connection of new experience to previous experiences that supports the learner as he or she moves forward.

Connections, through schema, act as a support system for students. When students make a connection, they internalize the concept and/or strategy and are more likely to use it in the future. The more experiences a student can bring to a situation, the more his or her schema builds into a comprehensive network. It is through this network that students can engage with new information, without starting from scratch. The more connected this network is, the easier it may be to remember and retain concepts and/or strategies. The student can rely on previously learned concepts and/or strategies to assist in learning new concepts and/or strategies.

The following is an example of a primary-aged child making connections between multiplication and repeated addition.

The teacher begins by reading the problem aloud:

Today we are going to read about a squirrel that is collecting nuts for the winter. A squirrel walked through the forest, looking for nuts to store in a tree for winter. The squirrel could carry 4 nuts back to the tree at a time. If the squirrel made 7 trips with full loads to the tree, how many nuts does it have stored for winter?

Teacher: *So, boys and girls, what can we do to solve this problem?*

Alice: *We could draw a picture showing the nuts* *collected. That's what we did to solve the problem of the day this morning.*

Teacher: What would the picture look like?

Alice: It would be 7 circles with 4 dots in them.

Teacher draws the diagram.

Teacher: Like this?

Alice: Yeah.

Teacher: Is there another way to solve this problem?

Alex: Could we do an array?

Teacher: Yes, Alex, go ahead.

Alex: It would be 7 rows of 4.

Will: Both ways give an answer of 28.

Henry: What about adding the numbers?

Teacher: What would we add?

Henry: We would do 4 + 4 + 4 + 4 + 4 + 4 + 4. That is like addition—what we did in the Fall.

Teacher: Yes, Henry, we call that repeated addition.

Crystal: Couldn't we just multiply 7 x 4?

Teacher: Yes! That is another way to solve this problem.

In the above example, the students make many connections between multiplication and other ways to solve the problem. In particular, Alice commented on an approach used that morning, and Henry referenced using addition to solve the problem—an approach the class worked with earlier in the school year. The students relied on connections to support their understanding of the problem at hand. The teacher was able to determine how students used previously learned concepts to support the current concept being explored.

Synthesizing

When reading a text or working through a problem, new information tends to arise that needs to be addressed by the student. Whether this new information supports or contradicts previously understood concepts and/or strategies, the new information must be recognized and interpreted. To be able to synthesize as one reads or works through a problem, means that background knowledge and new information are put together to create new understandings (Keene & Zimmermann, 2007).

Students who can synthesize new information with background knowledge are aware of changes in their ideas and understanding as they work toward the conclusion of the task. This comprehension strategy necessitates that the students can articulate when their understanding has evolved, either through confirming or contradicting background knowledge.

The ability to synthesize is an important comprehension strategy. As a student reads or problem-solves, she or he is able to actively revise her or his cognitive synthesis through a constant state of monitoring new information against background knowledge (Pinnell & Fountas, 2007). What this means is that some earlier decisions are confirmed, while others may be deemed incorrect. A student strengthens his or her mathematical understanding by combining new and old knowledge about concepts and/or strategies through synthesizing these two points of information.

The following is an example of a primary-aged child working with increasing patterns for the first time.

Teacher: So, Tariq, what can you tell me about patterns?

Tariq: Patterns repeat, like ABABAB or CDDCCDDCCDDC.

Teacher: Can we have other types of patterns?

Tariq: I don't know.

Teacher: How about this?

Teacher displays a chart with every fourth number to 20 covered.

Tariq: That's not a pattern.

1	2	3	●	5	6	7	●	9	10
11	●	13	14	15	●	17	18	19	●
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Teacher: What if I tell you that yes it is?

Tariq: How?

Teacher: Well, in this pattern, what numbers are covered?

Tariq: 4, 8, 12, 16, 20.

Teacher: What could be the next covered number?

Tariq: 24?

Teacher: How did you know that?

Tariq: I just counted four because they're all counting four.

Teacher: So, what would come after 24?

Tariq: 28.

Teacher: Yes, you have uncovered the pattern. You add four. But, where does the pattern start?

Tariq: At 4.

Teacher: Yes, so what is the pattern rule for this?

Tariq: I know! Start at 4 and count 4 more.

Teacher: So, what can you tell me about patterns now?

Tariq: There are two kinds. One is like the first one I told you about (referencing repeating patterns). It just keeps the same thing going over and over, and one is with numbers that keep getting bigger.

In the above example, Tariq is met with a pattern that he is not familiar with and that contradicts his current understanding of patterns. It is in discussion with the teacher and working through the problem that Tariq adjusts his understanding of patterns to incorporate increasing patterns. The teacher was able to recognize this growth in Tariq's understanding of patterns.

Questioning

Within the reading process, the reader generates numerous questions. These questions are purposefully generated before, during, and after reading (Keene & Zimmermann, 2007) to clarify meaning, bring attention to certain events, ideas, characters, and to consider what is yet to be read. The act of raising questioning is something that evolves over time as the reader becomes familiar with text features and understands how generating and asking questions can be used to deepen comprehension of a text.

Using the raising of questions as a comprehension strategy is also beneficial in mathematics. For example, when working through a problem, the student will encounter instances when he or she hits a stumbling block and needs to re-examine his or her approach or the application of the approach he or she is using. In such instances, questioning students' thinking and approach can lead to clarity and strengthen their decision-making process throughout problem solving.

While generating questions can be beneficial to deepening comprehension, listening to the questions of others can prompt students to reflect on their thinking and act as a generative measure for creating questions. Questions asked by others can trigger a thought, connection, and/or uncertainty that leads the student toward generating additional questions needed in solving the problem.

Those who ask the question are those who are thinking. When the teacher is the one generating and asking questions, he or she is leading the students toward an approach to solve the problem. It is only when students generate questions that are pertinent to their experience with the problem, that they are actively thinking about the problem.

Students need to be aware of the purpose of asking questions—to clarify their thinking when engaged with a problem. If the student is not able to generate the question, how is he or she able to identify their next step in the problem-solving process. When students say, "I need help," it is imperative that they can articulate what they need help with to continue their engagement with the problem. If they wait for the teacher to ask the question, they are waiting for the teacher to lead them in their thinking, which is passive learning.

The following two examples approach questioning using teacher-led and student-led approaches. Both examples focus on the same concept.

Teacher-Led Example

In this example, the teacher models the use of questioning as an effective strategy to represent the following word problem as an equation.

Susan is collecting money for her soccer team. The team is heading to the provincial championships and needs to sell cookbooks as a fundraiser. Every student on the team receives $50 as a donation from the school to fund the trip to the championships. The price of each cookbook is $18. If Susan sells 15 cookbooks, how much money does she now have for the provincial championships?

Teacher: So, let me read aloud the word problem.

The teacher reads the problem aloud.

Teacher: There is a lot of information in this problem. Let me read it again.

Teacher rereads the problem aloud.

Teacher: So, what does the $50 donation mean? What does that mean for an equation?

I guess Susan will have at least $50, no matter how many cookbooks she sells. So, she has $50 PLUS however much money she makes selling cookbooks.

Now, she will sell cookbooks for $18 each. In an equation, would I just put + 18 like the donation?

No, because $18 is what the student gets for each cookbook sold. So, it will be $18 times however many cookbooks she sells. Do I know what that would look like in an equation? I could put 18x, *with x being the number of cookbooks sold.*

*Do I know how many cookbooks Susan sold? Yes, she sold 15. So the part of the equation about money raised from cookbook sales is 18*x*, with x being 15.*

The equation is 50 + (18 × 15) = 320.

Does this make sense? If each book cost $20, 15 books sold would make $300. Then I would add $50, giving me a total of $350. Since the cookbooks are less than $20 each, my answer of $320 makes sense.

Student-led example:

In this example, the student uses questioning as an effective strategy to understand the following word problem as an equation.

Jaxon shovels driveways in the winter. He makes $100 a month plus $12 each time he shovels a driveway. If Jaxon shovelled driveways 22 times in January, how much money did he make that month?

Gary: Okay. So, what do I know already? I know that Jaxon will get the $100 in January, no matter what. So he has $100 plus whatever he makes from doing the driveways. So how can I make that into an equation?

I will put $100 + driveways = how much he makes.

Okay so how can I show how much he makes for driveways? If he gets $12 a driveway, I can put 12 × # driveways. So I can put 12d and say d = # driveways.

So what does that all look like?

I think I have $100 + 12d = how much he makes.

So since he does 22 driveways, I can put $100 + (12 × 22) = how much money he makes.

So what does that give me?

So 100 plus…(Gary works out 12 × 22 to equal 264) 264 equals $364. WOW! I'm gonna start shovelling driveways this winter!

Maybe that's too much. Maybe I made a mistake. How can I make sure that would be right?

Well 22 times 10 would have been 220 plus the 100 would get me 320. So that seems right.

In the teacher-led example, the teacher models how to use questioning as a strategy to support understanding. Specifically, the teacher uses questions to understand a word problem and represent it as an equation. By doing this, the teacher is modelling to students how questioning can be used at multiple points throughout the problem-solving process, to frame understanding and work toward a solution.

In the student-led example, the student uses questioning to represent the given word problem as an equation. Through this questioning, the teacher is able to observe student thinking throughout the problem-solving process and assess how the student worked through any stumbling blocks. This form of comprehension strategy provides the teacher opportunities to go beyond checking the answer by assessing the thinking that framed the answer.

Final Thoughts

The three comprehension strategies provided in this column—making connections, synthesizing, and questioning—can support student mathematical understanding as she or he works through a problem. Through the exploration of the comprehension strategies, students are able to make meaning as they engage with the task. It is through these strategies that students can rely on cognitive actions to assist them in making meaning in the three stages of problem solving: before, during, and after.

What I am advocating is that these comprehension strategies are not to be considered as reading strategies, but as thinking strategies. When viewed in this manner, educators and students can identify and see the importance of cognitive actions in not only reading, but in mathematics. I look forward to continuing the discussion of linking literacy and math via Twitter (@dr_costello) or email.

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Grade 13 exam questions, 1940

10. Find the term independent of x in the expansion of

$$\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9.$$

11. Using the binomial expansion for fractional exponents, find the value of $\sqrt[3]{123}$ correct to six significant figures.

▲ Assessment Abby Help with Struggling Students

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Ask Assessment Abby A³ is a regular column in the *OAME* Gazette, where teachers can share concerns and best practices about assessment, evaluation, and reporting of mathematics. Please send your questions to Ask Abby at **assessmentabby@oame.on.ca**.

Dear Assessment Abby,

I read your letter in a previous edition of the *Gazette* about the importance of figuring out where my students are in their learning. I have done some baseline assessments to see where my students are with Number Sense. I now realize that many of my students are struggling with subtraction. They tend to tally out the total number, then cross out the amount they are subtracting, and finally, recount what is left. The expectation for my students by the end of Grade 3 is that they need to be able to subtract three-digit numbers—that is going to be a lot of tally marks! Help!

Thanks for this question. There is actually a researcher in Ontario, Alex Lawson, who has done considerable work on understanding the development of students' mathematical development in numeracy (see Figure 1). Specifically, she has developed a continuum of mathematical strategies and models that students move through. The strategy that your students are using is referred to as "counting three times," which is in the *Direct Modelling* and *Counting* stages of the continuum. Rather than expecting your students to jump ahead and learn a standard algorithm, there are some logical next steps, such as counting back from the larger number and progressing into taking larger leaps back. This understanding can become deeper by constructing the number line, which promotes flexible thinking with both addition and subtraction. By moving through these strategies, students develop a deeper understanding of subtraction and improve their readiness to make sense and use a standard algorithm.

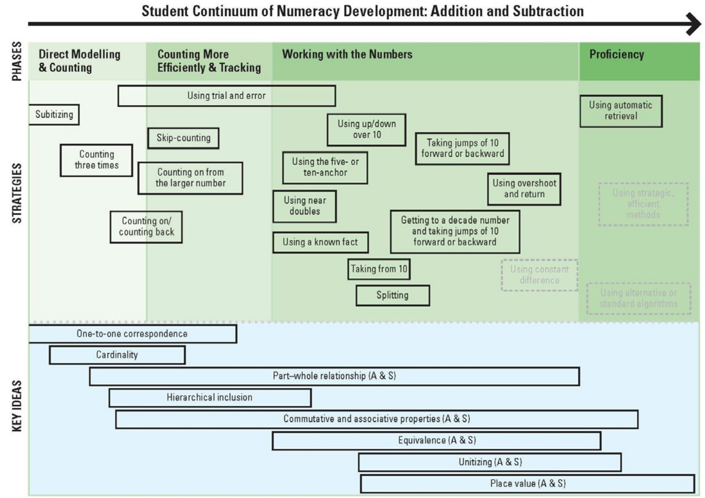
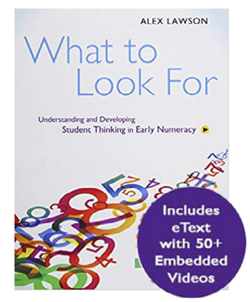


Figure 1: *Sample Developmental Continuum from "What to Look For"*

I am so glad you asked this question! This is a common struggle with so many educators.

The following are some resources that may help:

- Alex Lawson's continuums: Follow this link, **Developmental Continuum**, or if using print, go to **www.pearsoncanadaschool.com**, do a product search for *What to Look For*, and then look for the Developmental Continuum link on the Explore menu.



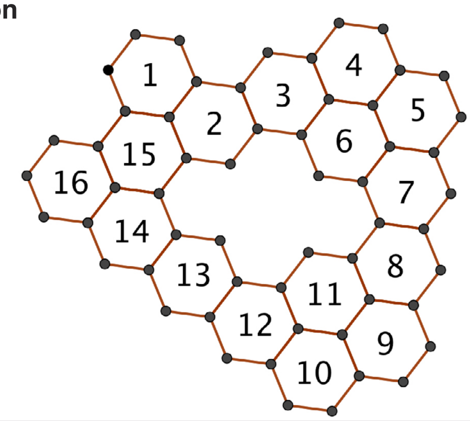
Alex Lawson talking about models and strategies:

- **www.thelearningexchange.ca/?s=alex+lawson**

Ontario Ministry of Education resources:

- **www.edu.gov.on.ca/eng/literacynumeracy/inspire/research/ww_modelling_proficiency.pdf**
- **www.edu.gov.on.ca/eng/literacynumeracy/inspire/research/Lawson.pdf** ▲

Hidato Solution



▲ Activities for Assessing the Math Processes and Learning Skills

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The mathematical processes, which are critical to mathematics education, are found in the front matter of each curriculum policy document. Assessing how well students demonstrate their understanding and utilization of the math processes is an important dimension of our assessment practices. Teachers can find supports for teaching and assessing the math processes in several locations on the EduGAINS website (**www.edugains.ca**). Specifically, the math process cards that can be found in the Differentiated Instruction portion of EduGAINS identify what students are expected to do for each math process, to demonstrate understanding, sample teacher questions, and sample teacher feedback. The Mathematics section of the EduGAINS website provides more in-depth supports, including sample lessons that focus on the math processes and full workshops to better develop teacher understanding and assessment of the math processes. In addition to the content expectations, teachers are expected to assess students' understanding of the seven mathematical processes, as well as assess the six generic learning skills identified in *Growing Success*. It is possible to assess all these criteria without sacrificing attention to the content expectations. Teachers can use checklists to identify students who demonstrate knowledge of the math processes and/or learning skills during regular class activities. There are several activities that I have proven useful in this regard. This article describes three classroom-tested activities.

Yarn It

This activity is well suited to reviewing definitions while assessing both learning skills and math processes. It can be used in Grades 6 through 9 and can be extended to Grades 10 and 12.

- Students stand in groups of five to seven.
- From the ball of yarn, unroll about 5 m or 6 m of yarn for each student in the group, and knot the two ends to form a large circle. An efficient way to do this is to have the students stand in a line with arms spread, give the first student one end of the yarn, extend the yarn to the other end of the students, loop back to the first student, and tie a knot to make the circle.
- One student (not in a group) is designated as the judge. He or she has one word of dialogue: "Justify." Once a group has constructed a given shape, they must justify their claim to the judge before moving to the next shape.
- In junior grades, you may also want to assign the role of information officer. This student has a computer tablet and can be consulted by a group if they need the definition or properties of a geometric figure.
- All students in a group must keep two hands on the yarn at all times.
- For Grades 6 to 9, each group must make the following shapes, in order, and justify their configuration to the judge before moving to the next shape.
    - A triangle
    - An isosceles triangle
    - A right triangle
    - A rectangle
    - A square
    - A parallelogram that is not a rhombus
    - A hexagon
    - Three equilateral triangles
    - Four congruent triangles
    - Five non-congruent triangles
    - Three parallelograms that are not rectangles
    - Three hexagons with the greatest possible number of shared sides
    - A cube

This activity can be modified to focus on parallel lines (Grade 9); triangle properties (medians, etc.) for Grade 10; and circle properties (MCT4C, MAP4C). For example, circle properties would include sector angles subtended by the same arc, central angles and sector angles subtended by the same arc, right triangle inscribed in a semicircle, and others including possibly tangents to a circle, depending on the prior knowledge of the class. It is interesting that even Grade 12 students find this activity engaging and fun, as well as conducive to learning.

While most of the math processes and learning skills can be assessed during this activity, the learning skill that is particularly obvious during this activity is *Collaboration*. Student interactions allow the teacher to assess all the bullet points listed under "Collaboration." In the Appendix, there is a checklist for learning skills that I have used for a long time, noting the names of students who demonstrate the particular learning skill during the activity. Having the checklist on a tablet makes this an efficient method of assessing.

COLLABORATION

- Accepts various roles and an equitable share of work in a group
- Responds positively to the ideas, opinions, and values of others
- Builds healthy peer-to-peer relationships
- Works with others to resolve conflicts and build consensus
- Shares information, resources, and expertise, and promotes critical thinking to solve problems and make decisions

This is also an excellent activity for assessing the mathematical process of *Reasoning and Proving*, since students discuss how to verify that a particular configuration satisfies the requirements, such as for an isosceles triangle.

REASONING AND PROVING

- Hypothesizes/makes connections
- Makes inferences, conclusions, and justifications: uses models to infer/conclude
- Reasons inductively by considering specific cases
- Analyzes and evaluates the mathematical thinking of others
- Presents arguments in a logical and organized manner
- Tries multiple examples
- Looks for counter-examples

How Much Hot Chocolate?

This group activity fits Grades 8 and 9. Most math processes and most learning skills can be assessed during this activity. However, this activity is particularly useful for assessing the mathematical process of *Reflecting*, as well as *Problem Solving.*

REFLECTING

- Checks that data being gathered are appropriate
- Reflects on new skills, concepts, and questions to see how they connect to prior knowledge
- Applies and extends knowledge to new situations
- Examines questions and demonstrate flexibility in choice of strategy
- Self-monitors progress while problem-solving, and revises, if necessary

PROBLEM SOLVING

- Plans
- Collects data related to the problem
- Selects and applies a problem-solving strategy

As groups work through and critique the various problem-solving strategies, students have the opportunity to demonstrate all of the bullet points listed under "Reflecting" in the math processes. The Appendix has a math processes checklist that can be used for this, noting the names of students who are clearly demonstrating reflecting as they work with their groups. Again, the teacher having this checklist on a tablet simplifies this procedure.

Here is the activity:

Part A: Can you and your group devise a plan to figure out how much hot chocolate would be in the cup shown? You have available the following materials: graphing calculator; graphing software; ruler; graph paper; pencil; string; protractor; scissors; and, of course, a coffee cup (empty). Outline the steps in your plan. Do not actually carry out your plan.

Part B: Critique the plan that has the following steps.

1. Use the ruler to measure the height of the coffee cup.
2. Compute the midpoint of the height.
3. Use the string and the ruler to measure the circumference of the cup at the midpoint of the height.
4. Compute the radius of the cup from the circumference you measured.
5. Compute the volume of a cylinder with the radius you computed in step #4, and the height of the cup you measured in step #1.

Outline the logic used in designing this plan. Now carry out the plan, and compute the volume of this cylinder. How is this volume related to the amount of hot chocolate that could go into the cup? Did you encounter any problems in carrying out this plan?

Part C: Critique the plan outlined below.

1. Use the scissors to cut the cup into 1 cm wide rings.
2. Use the string to find the circumference of each ring.
3. Compute the radius of each ring.
4. Compute the volume of each ring, considered as a disk.
5. Add up the volumes of all the disks. It is claimed that this is the volume of hot chocolate that could go into the cup.

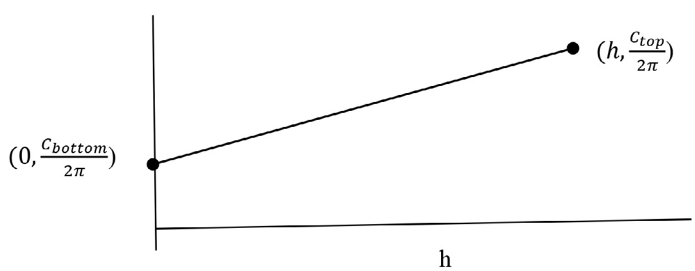
Carry out the plan, and compute the total volume.

Compare your group's answers from the plans in Parts B and C.

Write a brief summary of your results. [Hint: You may want to use percent differences to do your comparison.]

Which plan do you think gives a more accurate estimate of the hot chocolate that could go into the cup. Justify your decision.

Part D: Suppose a student tried to relate the hot chocolate cup problem to the work that has been done in class. He or she draws a graph (shown below), but doesn't know what to do next. What would your group suggest? Give reasons for your suggestions. [I find this part really interesting, since in general, you would need integral calculus to compute volume of a solid of revolution. However, many ingenious suggestions come from the students—a great opportunity to assess student reflecting.]



Were any of the plans similar to the plan your group designed? Which plans, and how similar were they to yours?

Are Monarch Butterflies Endangered? Problem Posing, Using Newspaper, Other Print, or Internet Articles

Here is an activity that lends itself to assessment of most of the mathematical processes as well as most of the learning skills. An exception is the learning skill *Self-Regulation*. Self-regulation is a learning skill with an internal lens. It is difficult to assess externally, although having students write focused reflections on their perceptions of their own self-regulation is one way of assessing this learning skill. This can be added to the activity as a journal entry or ticket to leave.

Newspaper articles provide fertile ground for problem posing. In addition to frequently being more interesting to students than routine textbook problems, newspaper articles allow math to meet the real world, with its attendant data issues, such as missing data, contradictory information, sometimes incorrect information, and generally "fuzzy" information at best.

One topic that interests many students is related to endangered species. For example, the monarch butterfly population may have become endangered due to man-made interventions such as pesticides, destruction of plant food stocks, and reduction in suitable mating areas due to housing construction. Here are some articles on this topic, which could serve as resources for student problem posing:

- **www.stuff.co.nz/science/9666559/Stunning-monarch-migration-in-peril**
- **www.thespec.com/news-story/6116004-mexico-cites-indications-of-3-4-times-as-many-monarch-butterflies-thisseason/**

When you first use this problem-posing strategy in class, I recommend that the teacher seek out articles that provide sufficient richness to serve as a base for problem solving. However, very quickly, students will find articles that address their own interests and bring them to class, either in print or electronically, for their own problem posing. This actually created an issue with our school librarian, since some of my students were cutting articles out of the library copy of the daily newspaper to bring to our math class. I resolved the problem by having one student responsible for taking a pristine copy of the daily paper to the library to replace any copies that my students had defaced in their enthusiasm to do math.

For the monarch butterfly article, groups of students were asked to:

- pose a problem of interest to the group
- identify what information was required to formulate the problem mathematically
- identify an appropriate mathematical model
- fit the data to the model
- solve their problem
- write a report or make a presentation

This allows the teacher to assess most of the math processes as well as most of the learning skills, except self-regulation, during the students' working time. While it is possible to assess most of the math processes and almost all the learning skills with this one activity, my recommendation is to narrow what you assess at any one time. For example, with this activity, one can assess math processes of *Problem Solving, Connecting,* and *Reflecting.* Learning skills that can also be assessed include *Organization*, *Collaboration*, and *Initiative*. By narrowing what is assessed, this becomes a more reasonable task. Next time we do an assignment similar to this one, I would assess a different subset of math processes and learning skills.

In my experience, students selected a model based on information given in the article, such as a linear model or an exponential model. Some students used Internet searches to obtain data beyond what was given in the article. This allowed them to refine or even reject their initial model to better represent the reality of the situation.

While the monarch butterfly article is especially rich for problem posing, almost any copy of the daily paper provides articles that can be used as a basis for problem posing. Allowing students to select the article with which they will work is an excellent way to motivate students and give them some level of control over their own learning, which research has shown to be an important dimension of motivation and engagement.

The math processes, which appear in the front matter of each curriculum policy document, sometimes receive less attention than is appropriate due to the pressure to give priority to content expectations. This is unfortunate, since the math processes are closer to the big ideas of mathematics than most of the content expectations. Similarly, the learning skills, which are arguably the real goals of education, can be treated in a superficial manner. By integrating assessment of the math processes and learning skills into regular classroom activities, we can give the appropriate attention to these critically important areas without sacrificing time deemed needed for math content. Using activities in which students have some level of choice also encourages engagement and has positive implications for student attitudes toward mathematics.

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Appendix

MATHEMATICAL PROCESSES CHECKLIST

Math Process	**Student Names**
PROBLEM SOLVING • planning • collect data related to the problem • select and apply a problem-solving strategy	
REPRESENTING • select an appropriate representation and defend their choice • understand that various representations can be used to represent the same situation • understand that there may be different variations of one representation	
CONNECTING • apply a strategy or reference system that draws on previous learning in another context • make connections between new and prior knowledge to make sense of what they are learning • apply mathematics to contexts outside of mathematics • use different models to best convey mathematical information • make connections between different representations	
SELECTING TOOLS AND COMPUTATIONAL STRATEGIES • understand when a computational strategy is appropriate • use technology to explore, gather, display, manipulate, and present data in a variety of ways • use manipulatives and/or technology to develop understanding	
REFLECTING • check that data being gathered are appropriate • reflect on new skills, concepts, and questions to see how they connect to prior knowledge • apply and extend knowledge to new situations • examine questions and demonstrate flexibility in choice of strategy • self-monitor progress while problem-solving, and revise, if necessary	
REASONING AND PROVING • hypothesize/make connections • make inferences, conclusions, and justifications: use models to infer/conclude • reason inductively by considering specific cases • analyze and evaluate the mathematical thinking of others • present arguments in a logical and organized manner • try multiple examples • look for counter-examples	
COMMUNICATING • respond to instructions: explain, discuss, describe, justify, compare, suggest, write, tell, read, share, demonstrate, present • use correct mathematical language and vocabulary • respond clearly and with sufficient detail so that thinking can be understood • interpret and summarize information from charts and graphs • use mathematical symbolic language correctly • communicate mathematical learning by combining various representations	

Based on the Math Process Cards found on the EduGAINS website under "Differentiated Instruction." **www.edugains.ca/newsite/di/enhancements2008/dienhancementsdimathcards8x11.html**

LEARNING SKILLS CHECKLIST	
Learning Skill	**Student Names**
RESPONSIBILITY • fulfills responsibilities and commitments • completes and submits work according to agreed-upon timelines • takes responsibility for and manages own behaviour	
ORGANIZATION • establishes priorities and manages time • devises and follows a plan and process for completing tasks • establishes priorities and manages time • identifies, gathers, evaluates, and uses information, technology, and resources to complete tasks	
INDEPENDENT WORK • independently monitors, assesses, and revises plans to complete tasks and meet goals- • uses class time appropriately to complete tasks • follows instructions with minimal supervision	
COLLABORATION • accepts various roles and an equitable share of work in a group- responds positively to the ideas, opinions and values of others • builds healthy peer-to-peer relationships • works with others to resolve conflicts and build consensus • shares information, resources, and expertise, and promotes critical thinking to solve problems and make decisions	
INITIATIVE • looks for and acts on new ideas and opportunities for learning • demonstrates the capacity for innovation and a willingness to take risks • demonstrates curiosity and interest in learning • approaches new tasks with a positive attitude • recognizes and advocates appropriately for the rights of self and others	
SELF-REGULATION • sets own individual goals and monitors progress toward achieving them • seeks clarification or assistance when needed • assesses and reflects critically on own strengths, needs, and interests • identifies learning opportunities, choices, and strategies to meet personal needs and achieve goals • perseveres and makes an effort when responding to challenges	

Based on Ontario Ministry of Education (2010). *Growing success: Assessment, evaluation, and reporting in Ontario schools.* Toronto, ON: Queen's Printer for Ontario. ▲

▲ Fractions – The Importance of Conceptual Understanding

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In my role as a math coach at a private elementary school in Toronto, I have had the privilege of working with exceptional teachers and their high-achieving students. In the pursuit of excellence, both groups struggled with the concept of fractions, particularly in the junior grades. This was not my first time observing the challenge. In teachers' college, many of my peers lacked confidence in this important area of mathematics. As we studied the issue by looking at data, we came to realize that a key missing component and barrier to success was a sound conceptual understanding. Students were overly focused on following step-by-step procedures at the expense of understanding "why" the procedures work.

"No area of elementary school mathematics is as mathematically rich, cognitively complicated, and difficult to teach as fractions, ratios, and proportionality" (Litwiller & Bright, 2002, p. 3). "It is clear that a weak foundation in fractions can eventually cut students off from higher mathematics and we must make strides through mathematics educational research and classroom practice to ameliorate this situation" (Bruce, Chang, Flynn, & Yearley, 2013, p. 7). So what must we do to help our students? Before recommending strategies to effectively support student understanding, we must understand the underlying factors that often makes the study of fractions so intimidating and difficult for students.

Why are fractions so confusing?

Students often think about fractions as two whole numbers.

Children learn about whole numbers and their quantitative relationships from a very young age. Some children can count by the time they are two or three. They then learn to add, subtract, multiply, and divide whole numbers, before being introduced to fractions at the most basic level (Ontario Ministry of Education [OME], 2005). Research has indicated that when a well-established understanding of whole-number quantities has been formed, it is difficult for children to integrate fractions. Emphasizing counting by whole numbers at an early age reinforces a strong concept of numbers as whole numbers, and because it is much later when most students are introduced to fractions, they often find it difficult to think about a continuous system that is not based on whole numbers (Bruce, Bennett, & Flynn, 2014).

A common spoken reference that teachers use when teaching fractions is "out of." For example, instead of saying "four-ninths" (4/9), a teacher might say "4 out of 9." This further reinforces students thinking in whole-number quantities. They think about 4 things out of 9 things. This description will tend to cause confusion when students are exposed to improper fractions such as eight-fifths (8/5). It may similarly lead to perplexity when students are required to compare fractions. They may think that two-eighths (2/8) is greater than one-third (1/3), simply because two things are greater than one thing (Siebert & Gaskin, 2006).

There is too much focus on the part–whole relationship.

Initial fraction learning in North America is usually focused on the proper fraction that typically represents a part–whole relationship. This one representation can be an obstacle for students in deeply understanding fractions that are greater than one whole (Bruce, Chang, Flynn, & Yearley, 2013). The progression of the Ontario curriculum introduces different "types" of fractions in different grades. Grades 2 and 3 present the concept of dividing a whole into equal parts, without using fractional notation. The Grade 4 curriculum, which outlines several more expectations than in previous years, includes proper fractions only. All examples in the Grade 4 curriculum documents have fractions where the denominator is greater than the numerator. Students study proper fractions for a full school year, without being introduced to fractions that are greater than a whole, mixed numbers, or the relationship between fractions and decimals, which are concepts only introduced in Grade 5 (OME, 2005).

In Korea and Japan, two top performing countries in international mathematics assessments, students are taught decimals and improper fractions alongside proper fractions. This early exposure prevents students from developing misconceptions, such as thinking that all fractions must be less than a whole (Bruce, Chang, Flynn, & Yearley, 2013; Watanabe, 2007). In a widely used Ontario textbook, *Math Makes Sense 5,* decimals are included in the fourth of 11 units, and fractions are not introduced until the eighth unit (Morrow & Connell, 2005). Although teachers do not always teach in the order of the units in a textbook, for many educators, textbooks are their initial resource for both content and planning template for long-range plans. This means that if the textbook is followed, students may not work with fractions until close to the end of the school year, and that the decimal unit may initially be taught in isolation.

Language and notation are not intuitive.

Another explanation for student confusion with fractions is that names and notation are counter-intuitive to what students already know to be true. A seventh-grade student is bigger than a sixth-grade student, but one-seventh is smaller than one-sixth. Students require repeated exposure in various contexts, where the wholes are equal, to understand the concept that the greater the number of pieces, the smaller those pieces and therefore the smaller the fraction. Hence, when numerators are equal, a greater denominator means a smaller fraction, and when denominators are equal, a greater numerator means a greater fraction. This concept is difficult to comprehend without a sound foundational understanding of fractions.

Procedures are taught before mastery of conceptual understanding.

Many students struggle to understand the key concepts of fractions and are taught to follow procedures without knowing why. If students do not have a solid understanding and foundation of fraction concepts, then their progress will be hindered when solving problems involving fractions or fraction concepts. This can lead to later problems in other areas of mathematics, such as algebra and probability (Clarke & Roche, 2009; Hackenberg & Lee, 2012). Trying to memorize the numerous and counterintuitive algorithms for fraction operations without a foundational understanding will be more difficult for most students.

Effective approaches to teach the conceptual understanding of fractions

Place more emphasis on unit fractions.

Teachers and textbooks must emphasize the concept of a unit fraction. For example, students should understand that 4/5 is four times 1/5. When students see the number 4/10 or 0.4, they should initially name these numbers as four one-tenth units (Strother, Brendefur, Thiede, & Appleton, 2016). Understanding unit fractions is the most important link between different fraction constructs as well as the idea that fractions are relative quantities (Bruce, Chang, Flynn, & Yearley, 2013).

Use bar models and number lines to represent fractions.

"In Ontario, it is particularly perplexing where, although students in primary grades use circle representations when studying fractions, the concept of area of a circle is not formally addressed until intermediate grades" (Bruce, Chang, Flynn, & Yearley, 2013, p. 21). Teachers should be using bar models and number lines as initial representations of fractions. When they are used to compare fractions or to model fraction concepts, the unit lengths are more easily compared. This is the most common approach used in many high-achieving countries. Area models, especially circles, are less effective (Strother, Brendefur, Thiede, & Appleton, 2016), as it can be very difficult to partition circle models equally. This often leads students to focus more on the number of parts and less on their congruency. It can result in students becoming confused about whether or not the parts must be congruent. The earliest representations that students are taught or exposed to tend to be the representations that students hold on to as they ground them for conceptual understanding (Bruce, Chang, Flynn, & Yearley, 2013). Therefore, representations should be well thought out, and teachers should ensure that they will establish a foundation for success in future years as problems increase in complexity.

Allow students to realize how and why common conceptions are misconceptions.

Students should be given questions or tasks where they are required to consider the reasonableness and correctness of another person's thinking. Present a question with an answer that is flawed based on a deep conceptual misunderstanding. "By asking students to first determine why a fictional peer might reasonably determine an incorrect answer, teachers create an opportunity to alleviate potential misconceptions and misunderstandings before they become solidified in students' thought processes" (Strother, Brendefur, Theide, & Appleton, 2016, p. 136).

For example, a problem could state, "John believes that ½ + ½ = 2/4. Is he correct? Explain your reasoning. Why/how do you think John got this answer?" This is a clear example that would enable a student to reason that adding numerators and denominators would not work conceptually. Because 2/4 is equal to 1/2, students can readily prove that this process would not work. Here we are causing students to think and allowing them to articulate errors in understanding, while encouraging them to provide a justification.

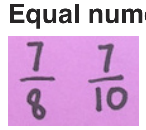
Teach conceptual understanding before the procedure.

This section will focus on teaching students how to compare fractions, using conceptual understanding. The most common way this has been taught in the past is to find a common denominator, then to find equivalent fractions, using that common denominator, and finally compare the numerators. This process can be time-consuming and requires a level of patience not always present in young students.

There are multiple resources available to support students' conceptual development of strategies to make sense of fraction comparisons. The *Gazette* has published several articles on the subject of fractions that include methods to teach conceptual understanding. The Ontario Ministry of Education also supports various effective resources such as Mathies and Critical Learning Instructional Paths Supports (**mathclips.ca**). In addition, award-winning and *New York Times* bestselling author Greg Tang has a free website with fraction games that focus on conceptual understanding (Tang, 2013) that I have found especially useful in my practice. Here we will explore his methods. Once the five methods he identifies have been learned and mastered, students can try an "expert" level that mixes different types of questions, where students have to choose an appropriate strategy. These are basic, but essential, strategies for supporting understanding and strategies in later grades.

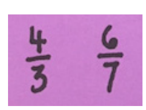
Although these methods may initially appear to be "rules," they are essential for a deep conceptual understanding, and unlike rules, these strategies require understanding to be applied. Instead of teaching these strategies, students can be offered opportunities to discover these concepts by being asked to compare fractions and examine various methods in groups.

Equal numerator



When two fractions have the same numerator (number of parts of a whole), the fraction with the larger pieces will be greater. Both fractions have seven pieces, but eighths are larger than tenths; therefore, $\frac{7}{8}$ is greater than $\frac{7}{10}$. It is worthwhile to note that this example is specific to part–whole, which is just one type of fraction representation, and should not be overemphasized.

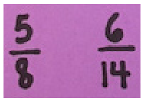
Benchmark of 1



If one fraction is more than a whole, and the other is less than a whole, the fraction that is greater than a whole will be the larger fraction. $\frac{3}{3}$ is equal to one whole, so $\frac{4}{3}$ is greater than a whole. $\frac{7}{7}$ is equal to one whole, so $\frac{6}{7}$ is less than a whole. Therefore

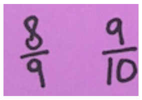
$\frac{4}{3}$ is greater than $\frac{6}{7}$.

Benchmark of 1/2



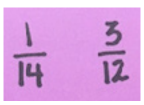
If one fraction is greater than one-half, and the other is less than one-half, then the fraction that is greater than half will be the larger fraction. Half of 14 is 7, so $\frac{6}{14}$ is less than half. Half of eight is four, so $\frac{5}{8}$ is greater than half. Therefore, $\frac{5}{8}$ is greater than $\frac{6}{14}$.

Missing piece/s



When both fractions are missing the same number of pieces, the fraction that is missing the smaller piece/s will always be the greater fraction. In order for each of the fractions above to be equal to one whole, one more unit fraction or "piece" must be added. Each fraction is missing the same number of pieces (in this case, one). Because tenths are smaller than ninths, a smaller piece is missing from the second fraction, therefore making it the greater fraction.

Unambiguous case



Sometimes a student will view two fractions, where one has a smaller denominator (larger pieces) and a greater numerator (more pieces). In this situation, that fraction will always be greater. In the example, twelfths are larger than fourteenths. Three pieces are more than one piece. In this case, the second fraction has larger pieces and more of them; therefore, it must be the greater fraction.

Conclusion

In the field of mathematics education, students have gaps in their understanding of fractions, leading to barriers in their progress. Students have difficulty learning fractions and integrating them into their already solidified understanding of whole numbers; there is too much focus on the part–whole relationship; language and notation is the opposite of intuitive; and students often forget confusing lengthy procedures for fraction operations that are often taught before mastery of conceptual understanding. In order to best support students, there are a number of worthwhile approaches that require adoption. Namely, teachers should put more emphasis on unit fractions and fractions as relative quantities by using bar models and number lines; teachers should allow time for students to realize why some common conceptions are in fact misconceptions; and teachers should ensure that students have mastered the conceptual understanding before teaching processes for fraction operations.

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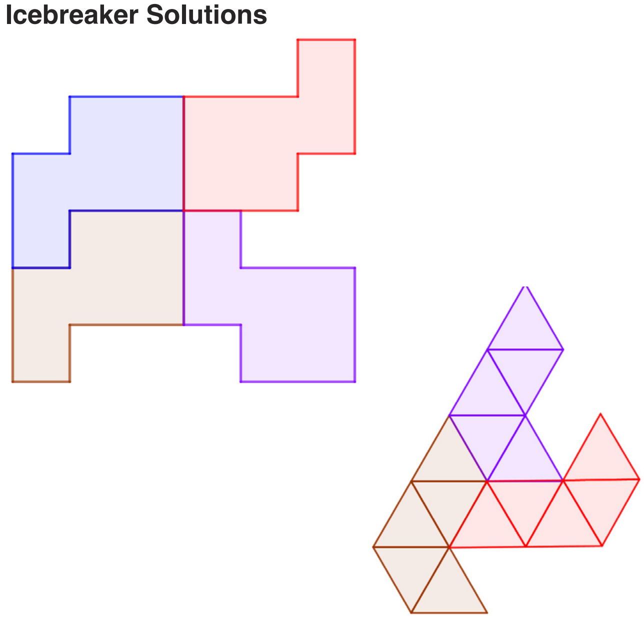
Icebreaker Solutions



▲ Strategies for Enhancing Mathematics Learning for Students Who Are Kinesthetic Learners

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"Tell me and I forget, teach me and I may remember, involve me and I learn." ~ Benjamin Franklin

According to Jennifer Weichel (Weichel, 2016) of Michigan State University, "we retain 10 percent of what we see, 30–40 percent of what we see and hear, and 90 percent of what we see, hear and do." The breakdown of primary learning modalities varies, but a typical K–12 classroom contains 30 percent visual learners, 25 percent auditory learners, and 15 percent kinesthetic learners, with the remaining 30 percent consisting of students with mixed learning styles (Earnhardt & Richter, 2017). In my experience, we often do a good job for the auditory and visual learners, but recent efforts have focused on improving the frequency with which teachers address kinesthetic learners. In addition, I have found that almost all learners tend to perform better when there is a less static classroom environment and students have the opportunity to move about rather than sitting for the entire period. This article presents a number of strategies that work particularly well for kinesthetic learners, while these also include other learning styles; the focus is the kinaesthetic benefits for students. Some of these strategies are mathematics specific, while others are modified from strategies used in other disciplines.

Quadratic Aerobics

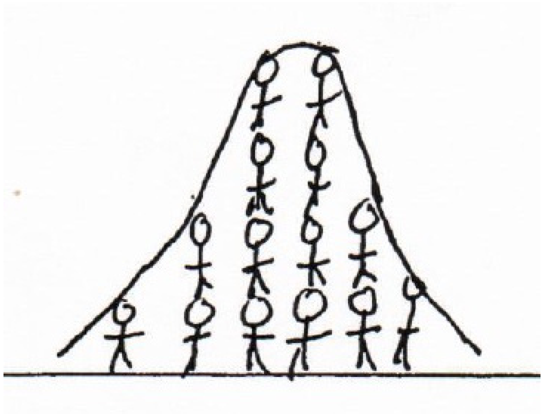
Students have an opportunity to "be" a quadratic function. This is an excellent activity for Grade 10 students to connect body movement to the curricular expectation of understanding parabolic shapes. The stick figure is demonstrating $y = x^2$. Students then progressively demonstrate other quadratics, such as $y = -x^2 - 2$, $y = 2(x - 3)^2$, $y = -\frac{1}{2}(x + 4)^2 + 1$ For example, a student "being" the last quadratic listed would have his or her arms down and wide, would be shifted four units to the left, and would be standing on tiptoe to simulate the "+1" vertical translation. This strategy can also be used with transformations of polynomial functions, but I have found that quadratics give the best results, since students find it difficult to model other functions such as cubics. Many Grade 10 students have indicated to me that they felt this activity really addressed their need for movement, and that they understood the concepts much better after doing this activity.

Live Normal (MDM4U)

This activity uses the normal distribution of student heights (even though one class is a small sample), and that one standard deviation from the mean encompasses about 68 percent of the data points, two standard deviations encompasses about 95 percent of the data points. Since a single class is unlikely to be exactly normally distributed, there is also an opportunity to discuss sample sizes, outliers, estimations of mean and standard deviation for the class, and more.

- Students line up along a wall in increasing order of height from left to right. I usually ask them to accomplish this without any student speaking.
- Take the middle $\frac{2}{3}$ of the students and have them form two rows advancing out from the wall.
- Take all but two of the remaining $\frac{1}{3}$, have them form two rows advancing out from the wall, on either side of the middle (larger) two rows.
- The remaining two students form two short rows on either side of the rows just created.
- Have the first student in each of the six rows hold onto a long piece of string/yarn, starting from the left and enclosing the entire array of students.
- You may want to have students take photos of the yarn for reference.
- Students return to the classroom and sketch the shape of the yarn.

- Discussion of sample mean, standard deviation, outliers, sample size, etc., follows.



Data Management students indicated to me that not only was this an enjoyable activity, they also were more engaged and motivated to study the normal distribution, since it had a clear connection to real life.

Jigsaw (any grade)

This is a popular co-operative learning strategy. Johnson and Johnson (1994) list five criteria for co-operative learning: mutual positive interdependence, individual accountability, face-to-face interactions, social skills, and group processing. The jigsaw steps are as follows:

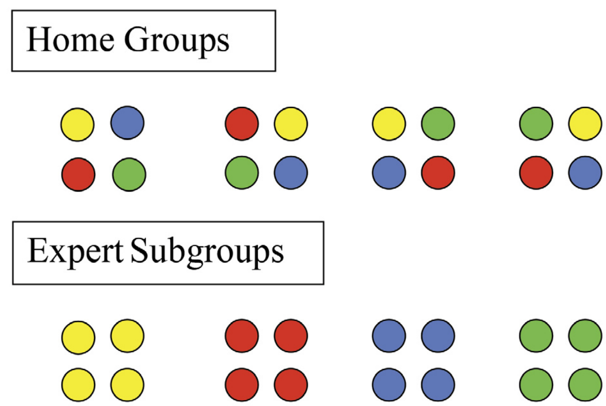
1. Students start in home groups.
2. Then students go to expert subgroups to learn or discover something.
3. Students return to home groups and teach/inform the other members of their home group.

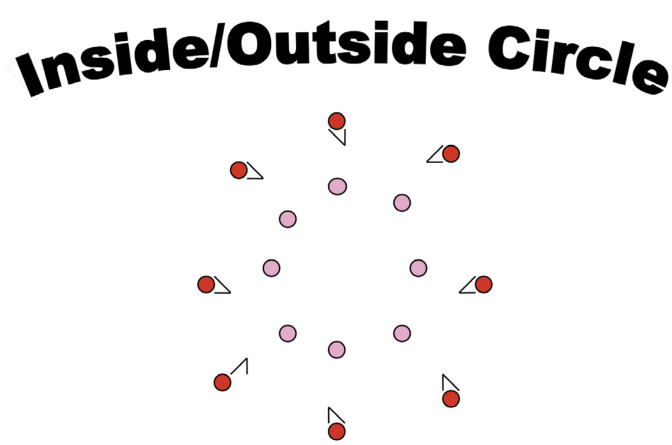
Here is one way that I have used jigsaw for the topic of word problems related to linear systems. Each group member gets a set of word problems, segregated by type (number, age, coins/money, cost analysis such as cellphone plans, mixture, mixtures involving percents, simple interest and investment allocations, and speed/distance/time problems). Each page has a worked example followed by six problems. Students start in home groups of four. They then choose which expert subgroup they will attend for the first four problem types. In expert subgroups, students learn a solution method for their problem type, and together complete the odd-numbered problems on their page. Students then return to their home groups and teach their problem type to the rest of the home group. Home groups then together complete the even-numbered problems for each type. A whole-class discussion checks for understanding. This is followed by formation of new home groups, and the process is repeated for the other four problem types.

Student comments on this activity are usually centred on the empowerment that they felt when they successfully taught their home groups and coached their peers during the problem solutions.

Considerations:

- Content needs to be minimally consecutive (e.g., transformations of functions: quadratic, exponential, logarithmic, trigonometric; applications of functions: quadratic, exponential, logarithmic, trigonometric; power laws in Grade 9; solution methods for systems of equations in Grade 10; intersection of planes in MCV).
- All members of expert subgroups need to contribute (mutual interdependence).
- All members of expert subgroups need to thoroughly understand their content, since they will be teaching their home group (individual accountability).
- An activity can be used inductively as well. [Each expert subgroup investigates a specific case. Then students return to home groups to pool their results and draw conclusions.]





Inside/Outside Circle (works for any grade or topic)

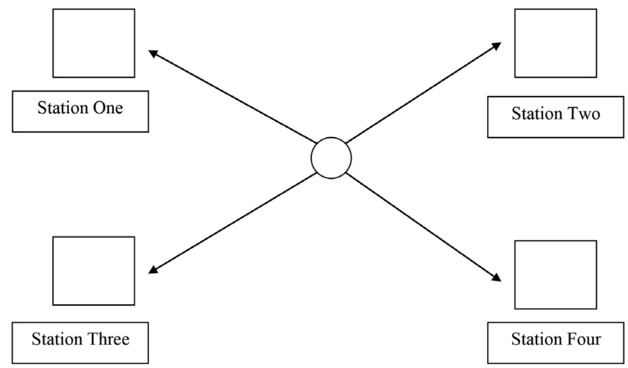
I have found that this is excellent for review. When I use this strategy in math, we start by having each student complete an index card. For example, if the topic is trigonometric graphs, the student writes a trigonometric equation of his or her choice on one side of the index card. On the back of the card, the student lists important information, such as amplitude, period, phase shift, and so on. Using think/pair/share, students trade cards to verify that the information is correct. Then we progress to inside/outside circle.

- Half of the students form a circle, facing outward.
- The other half of the students form a circle, facing inwards, so that every student is facing a partner.
- The student on the inside circle holds up his or her index card, showing only the equation side. The student on the outside circle then lists important information. (The inside student may prompt by asking, "What is the amplitude?," and so on.)
- Once all the information has been given by the outside student, he or she holds up the index card, and the inside student responds.
- After students from both circles have given their information, the inside circle rotates one student to the left. (A variation has them swap cards.)
- The new student pairs then repeat the index card activity.
- The inside circle can be rotated until everyone has shared with everyone, or until the teacher judges that sufficient time has passed.

For best results, a maximum of eight students should be in each circle. For larger classes, set up multiple circles. My students who participated in inside/outside circle activities enjoyed the movement and the chance to formulate their own problems and solutions. They also indicated that they felt more comfortable taking risks, since they were conversing only with a single peer each time, rather than speaking during a whole-class discussion.

Fraction Line (Grades 6 to 9)

Each student is given a card with a different fraction. Students must line up in order from smallest to largest fraction. An introductory activity might include fractions such as $\frac{1}{6}, \frac{1}{2}, \frac{5}{6}, \frac{1}{3}$, etc., and then proceed to more complex fractions and mixed numbers. It can also be done with negative rational numbers. The activity generates a lot of different strategies for comparing fractions, such as common denominators, conversion to decimals, common numerators. I have also done this activity with irrational numbers such as $\pi, \sqrt{2}$, and so on, to give students practice with estimation skills. Younger students especially enjoyed the movement, and frequently demonstrated good mathematical reasoning when deciding where "their" fraction should be placed in the line.



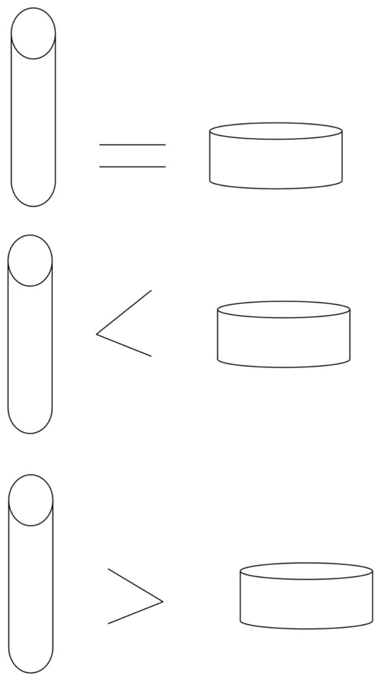
Four Corners (any grade or topic)

The traditional four corners strategy has students move to a corner in response to a declarative statement shown on the board or projector. The corners are labelled *Strongly Agree, Agree, Disagree,* and *Strongly Disagree*. Students then construct arguments in support of their position and debate other corners.

To modify this strategy for mathematics, present a problem, and label the corners: *Graph, Table of Values, Algebraic Solution,* and *Use Technology*. Students go to the corner of their choice. Corner groups then solve the problem, using their preferred method, and share solutions with the other groups. This process empowers students by allowing them to choose their preferred method of solution.

Another way I have used four corners is to present a system of equations, with the corners labelled *Comparison, Substitution, Elimination,* and *Graphing* (with or without technology, your choice).

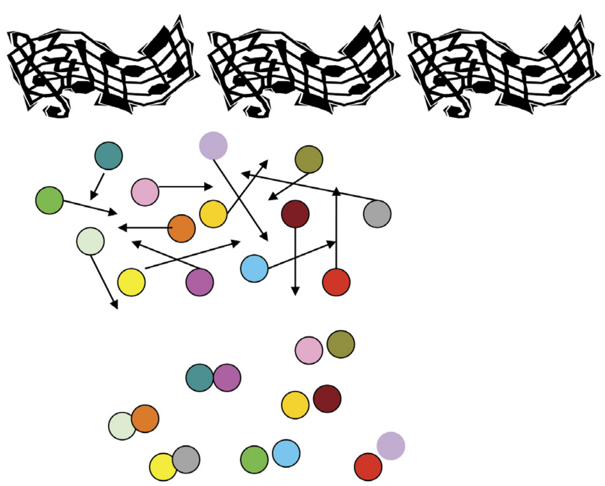
The following is another example, with only three corners. The objective is to have students consider cases involving volumes for a fixed surface area. Student groups are given blank pieces of paper and tape. They are asked to construct two different cylinders without overlapping the paper. The corners are labelled as shown in the diagram. Students choose a corner, and each corner group devises a method for proving its corner's position. The group then carries out the solution and compares it to the solutions of other corner groups. I have found this to be a very successful activity for Grade 9, since students in all corners come to the realization that shape affects volume, even with a fixed surface area.



Milling to Music (any grade or topic, but I have found it especially good for younger students such as Grades 9 and 10)

This is a social-interaction strategy that could be compared to “musical chairs” for learning. The objective is for students to formulate a position, opinion, or solution to a problem. While the music is playing, students wander around the classroom.

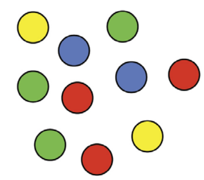
When the music stops, students interact with their closest peer(s). For math, students share problem solutions, or they can discuss information such as properties of a quadratic, sinusoidal, or polynomial function. They can also work together to pose problems based on print or Internet articles.



If you have a very social class or one with student cliques, you may need additional conditions; for example, students cannot hang around their friend and then share with the friend when the music stops.

Dotmocracy (any grade, especially useful with problem solving)

This is a strategy that reinforces higher-order thinking, particularly evaluation. After problem solutions are posted, students are given several coloured sticky dots. Prior to the dotmocracy, the whole class brainstorms criteria for what makes a good solution. Sometimes we generate a checklist for students to carry with them as they do the dotmocracy activity. Students make judgments about the solutions by placing dots on the ones that they think are the best, for example, most innovative solution, or a particularly useful line in a solution. I have students initial the dots and make notes in their notebooks, so that we can have a whole-class discussion after the dotmocracy. Student sharing during consolidation allows for the correction of misconceptions and proposes alternative solutions. This activity promotes communication skills and the proper use of mathematical language and form.



All of these strategies acknowledge that some learners need movement and activity to learn best, and that many students benefit from a less static learning environment. Luckily all these strategies work, no matter what student learning modality is involved. They also have the additional benefit of increasing student engagement and potentially modifying student attitudes to a more positive view of mathematics: necessary for some, good for all.

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Picture Equation Solution

😎=10 🔧=3 🕷=1

$$1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\cdots=\frac{\pi^2}{6}$$

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