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Ontario Mathematics Gazette

OAME – ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION

AOEM – ASSOCIATION ONTARIENNE POUR L'ENSEIGNEMENT DES MATHÉMATIQUES

Vol. 56 #1
September 2017

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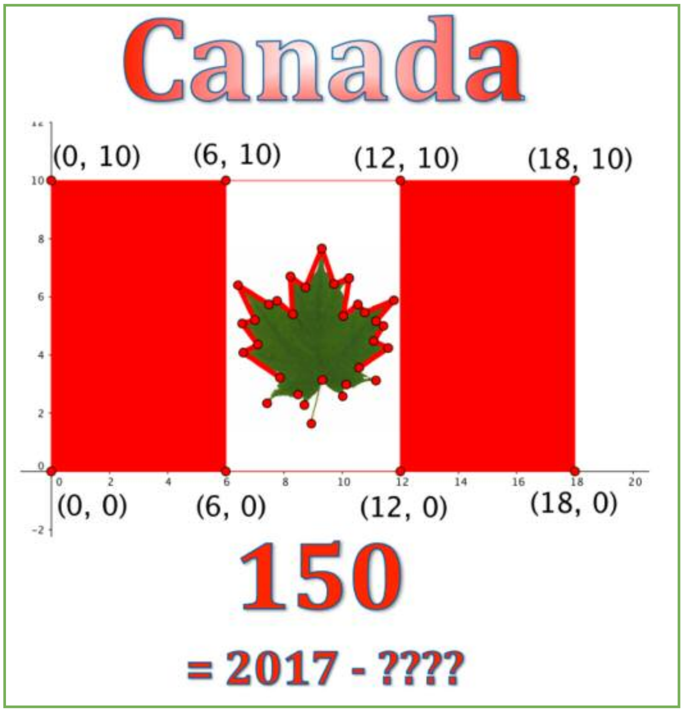

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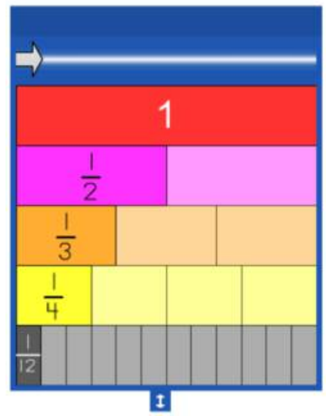

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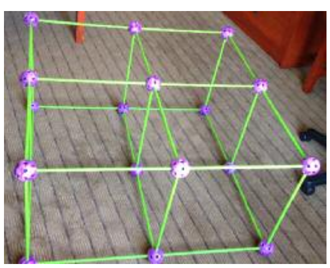

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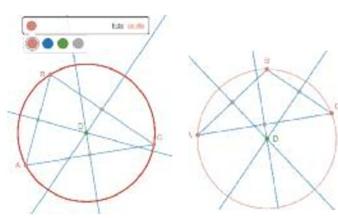

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Submission of Articles

The *Ontario Mathematics Gazette* (OMG) is looking for news items, articles, and good ideas that are useful to mathematics teachers and mathematics teacher education. We are seeking submissions, preferably from mathematics teachers K–12 and other mathematics education professionals, that describe innovative and creative approaches to mathematics teaching.

Please keep in mind the following criteria when making submissions to the *Gazette*:

- The ideas/activities must be of interest to the readership.
- The ideas/activities must be fresh and innovative.
- The mathematics content must be appropriate for the readership.
- The mathematics content must be accurate.
- The article must be well written and easily understood.
- The article and its ideas must be free of sexual, ethnic, racial, or other bias.
- The article must not have been previously published, nor should it be out for review by other publications.
- The article must be original.

Articles are to be word-processed, MS Word is preferred, and prepared according to the Publication Manual of the American Psychological Association, Sixth Edition. However, please use single-line spacing (not double) and only one space after each period. Articles should not exceed five numbered pages of text, and figures, images, and photographs should be placed in the text close to where they belong, with captions. The photographer's permission is required, and for photos of students under the age of 18, the written permission of a parent or guardian is required.

Please submit your article in one blind file (i.e., identity of author is not evident), and include author names, contact information including email and mailing addresses, photos, biographies, and all content removed for blinding in a second file. Please email these two files to Tim Sibbald at **gazette@oame.on.ca**.

Upon review, you will be notified whether your article has been accepted for publication (as is, or pending minor or major revisions) or rejected. The Editor reserves the right to edit manuscripts prior to publication. Once an article is published, it becomes the property of OAME.

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▲ About the *Ontario Mathematics Gazette*

The *Ontario Mathematics Gazette*—ISSN 0030-3011—is indexed in the Canadian Education Index and is published four times per year. Its Canadian Publication Mail Product Sales Agreement Number is 40051074.

Submission of Advertisements

Advertisements for publication in the *Ontario Mathematics Gazette* should be sent to **Robert Sherk** at the above address. Courier is recommended to avoid possible delays. Deadlines for advertisements are January 23 for the March issue, April 1 for the June issue, July 1 for the September issue, and October 1 for the December issue.

Full-page advertisements are to be on 8.5" by 11" paper with a minimum of 0.5" margins and single sided. Each advertisement should be print ready, and colour advertisements should have no bleeds.

Advertising Rates

Advertising rates are available by telephoning, emailing, or writing to the Advertising Manager.

▲ Editor's Report

TIMOTHY SIBBALD, OCT, PhD
EMAIL: Gazette@oame.on.ca

Tim Sibbald is the current Gazette *editor, co-chair of finance for the 2018 annual conference, and former president of OAME. He is an associate professor in the Schulich School of Education, Nipissing University, with a focus on mathematics education.*

Many years ago, I recall chatting to a person more knowledgeable than I was about what the role of *Gazette* editor entailed. It is slowly becoming clear, but in many ways, I have everyone who has gone before me to thank for the advice they have provided in the craftsmanship of the *Gazette*. Amy Lin, the most recent editor, has demonstrated that talent, and I offer my sincere acknowledgement of her tenure as editor. Other editors, Dan Jarvis and Jack Weiner, whom I chatted with during the summer, have also given glimpses into what the role entails.

This issue of the *Gazette* is quite striking because of the way the executive rallied around the newly appointed editor. I was humbled by the support and, in the first 48 hours, it was beneficial. Understanding the role was, naturally, the first endeavour—not as a reader, or consumer, of the *Gazette*, but understanding the tools that are used in developing the issues. That is why I turned to an item that has been in the works for some time, called the critical paths. This provided clarification, but rather than dwell on why it helped and the value of the "paths," I asked Bill Otto to share his expertise by clarifying some of the organizational details. When a transition takes place, the preparation embodied in the critical paths helps maintain the process expectations.

I am delighted to see this issue reach publication. As a reader, I always saw the *Gazette* as good professional reading that I could relate to—particularly with its Ontario focus. As an

WANTED

The *Gazette* editorial team has an opening for an associate editor. This requires a good eye for details, such as wordings that can be improved, grammar improvements, and proper use of punctuation. Knowledge of APA6 is useful, but not essential.

Interested? Email the editor and, using formal English, please articulate your interest.

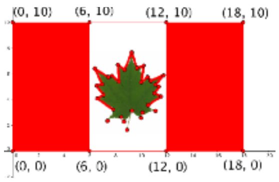
editor, it looks a little different; it is a team success that includes all the contributors you will find among the pages, but also the hidden associate editor work of Anne Yeager, proofreading and quality assurance of Gitta Berg, and the expert guidance and layout work of Penny Clemens. They are a strong team! Yet, it is a team that lost a member, Marilyn Hurrell, who was an associate editor at the time of her passing. An obituary is included in this issue. While Marilyn will be missed, we hope that there is an OAME member who will step up and join the team—and help address an overly heavy workload that Anne has had in preparing this issue. (Thank you, Anne.)

With more collaborative opportunities taking place in our classrooms, whether a Professional Learning Community or perhaps something with the Math Knowledge Network, or you simply want to share something that works in your classroom, consider writing about your experience! This might be a letter to mention something briefly or as an article. As the editor, I bring the view that part of my role is to support new authors with constructive assistance. The experience of becoming published is excellent professional learning. What always concerns me is that people may shy away because of the quality they see in the *Gazette*, but that is due to the editorial process that aims for consistency regardless of the author's skill. It also hides the fact that every published item has been examined by a minimum of four sets of eyes, and sometimes seven (it depends on the type of item). Seeing the polishing taking place is part of the learning.

"Writing for the Gazette is very similar to asking your own students to engage in a communication task."

Writing for the *Gazette* is very similar to asking your own students to engage in a communication task. It is not uncommon to find teachers who will tell you about an interesting experience, but ask them to write it down, and you might start to wonder if that is how their students learn to hesitate about writing about their thinking. When I started writing in the *Gazette*, it became natural to show my students that I modelled what I was saying about writing down the thinking process. They would point out I was a teacher and had experience, and I would point out that the level of expertise may change the audience, but it doesn't change the importance of it! I can promise teachers who embark on their first article for the *Gazette* a challenge of organizing their own thinking, a challenge of how to say what they mean, and a stronger understanding of what their own students go through when they tackle a communication task. (You can also read *In the Middle* for some ideas about communication tasks.)

This year, a theme of Canada 150 has been highly visible. In keeping with that notion, the cover picture comes from a graphing activity I made years ago. I tend to lose things over time and so I had to reinvent the idea. Using a rectangular box split into thirds, the rectangles at either end were filled with red. To make a maple leaf, I originally had a list of coordinates that I have misplaced, so, while out hiking one day, it occurred to me that a scan of a real leaf could be used to generate a set of coordinates. This is a good activity about reading coordinates! Note that if you wish to include negative numbers, simply move the coordinate system and/or the boundaries for the rectangle.



Pedagogically, I also like the use of the scan or photograph because it models a real maple leaf. This is distinct from generating an idealized graphic arts maple leaf—that is symmetric and has congruent shapes around the three primary leaf vertices. The use of a real leaf is authentic particularly if students choose their own leaf with which to work. Furthermore, there is likely a good data-management lesson waiting to be discovered by averaging coordinates for a collection of maple leafs. It might look like this: Lock the aspect ratio of the picture and then fix the lowest and highest points on the axis of symmetry to points such as (0, 0) and (0, 10), and then, for each vertex, average each coordinate for the collection of leafs. I look forward to hearing how you play with the idea in your classes, whether a letter to the editor or as an article.

CANADA 150

Related to the Canada 150 cover is the logo that was adopted for related events. The design, by Ariana Cuvin (CBC, nd), consists of five prominent kites. A kite is a quadrilateral, where there are two pairs of adjacent sides that have equal lengths. In the design, each pair of overlapping kites creates a smaller, but mathematically similar, kite. A good geometry problem is to figure out the four angles in each kite. (I get 45°, 90°, 112.5°, and 112.5°. Do you agree?)

Within this issue, we have the inaugural message of Jill Lazarus' presidency that establishes her vision of promoting collaboration and connectivity within the Ontario mathematics education community. This is sure to be evident at the Annual Leadership Conference, which she has planned, and which will take place November 10, 2017 in Ottawa.

The announcement of the award winners is found in this issue of the *Gazette*. Hopefully, this prompts all members to consider nominating inspirational educators this year.

The Executive Directors, Past-President, and a Vice–President have all contributed columns, which speak to different aspects of the organization, and they fit well with a letter to the editor. The Executive Directors' article is quite informative about the Board of Directors and Chapter Representatives. This fits well with the *Past-President's Retrospective*, which speaks to the importance of the membership and some struggles the organization has had. Rounding this off is an article by Bill Otto that provides some light professional learning about the OAME Constitution and related matters. Related to these is a letter to the editor regarding OAME elections.

An obituary is included for Marilyn Hurrell. It is never easy to write something acknowledging such a level of dedication, and I hope it conveys the spirit she encouraged.

There are several reports of activities within the OAME. These include the reports about the Ontario Mathematics Olympiad, National Council of Teachers of Mathematics Annual Conference, the Math Knowledge Network, and the 2017 Canadian Mathematics Education Study Group Conference. You will also find information about the 2018 Leadership Conference, and Annual Conference.

Regular columns also appear. *Assessment Abby* addresses a question about diagnostic assessment. In *Technology Corner*, Mary Bourassa highlights new features of Desmos. Shawn Godin explores the pirate problem he was introduced to by Peter Liljedahl (who will be speaking at the Leadership Conference). *Mb4T* continues with a guest column by Tim Sibbald that provides a number of ideas. The Provincial Digital team addresses addition and subtraction of fractions, using the Fraction Strips Tool in **mathies.ca**. (Editor note: They also wanted to tell everyone about how powerful it is for multiplication and division of fractions, but that will have to wait until the next edition.) *In the Middle* demonstrates the way questions from an older resource can be refreshed to suit the current curriculum. In *Hey, It's Elementary*, Lynda Colgan discusses issues in Kindergarten math instruction. In addition, you will find an interesting report about the Grade 9 Applied project, and an article about Frobenius numbers (also known as McNugget numbers).

As a final word, I want to reiterate my thanks to everyone involved in this issue of the *Gazette*, particularly Anne Yeager, who has filled both Associate Editor roles, handled columns that came in during a period of my absence, and has provided strong collaborative, and constructively critical, support that I sincerely appreciate. Anne, along with the rest of the team, has begun the process of training a new editor! And I promise I won't go on holiday during the final weeks of preparation for the next issue.

Reference

CBC. (nd). Canada 150 logo contest finalists. Retrieved from **www.cbc.ca/news/politics/canada-150-logo-contest-finalists-1.3155808** ▲

▲ Letter to the Editor: OAME Elections

In the last OAME Election, I noticed with some interest that Tim Sibbald was running for a second term as President of OAME. At a personal level, that interest stems from the fact that, at the moment at least, I'm OAME's only "two-term President," and with a limited number of claims to fame, I'm always keenly aware when I might have to share one. At an organizational level, the interest stems from the many discussions that this event has likely triggered and will quite possibly continue to trigger.

For example: *Experience versus New Blood*—this discussion has come up countless times in the past, and I have no doubt it will continue to do so. It usually goes something like this: OAME is an "aging" organization, and those who have served OAME for a long time should step aside and make room for new, young members to run the organization and move it forward. While this makes perfect sense in theory, the catch is that we've never had the luxury of a large cadre of OAME members chomping at the bit to serve on the Board of Directors or Executive. In fact, I would say in looking at the current Board and Executive, there appears to be a reasonably good mix of experienced and new.

In my own case, over a decade had passed between my first term as President and when I took on the role again. Tim didn't wait that long to try again, and that fact, I'm sure, led to other discussions—such as, should there be some required "waiting time" before seeking a second Presidential term? (There's nothing in the Constitution/By-Laws about this.) Should there be a limit as to how many times you can hold office? My personal opinion is that I don't think it makes sense to formalize many of these things, but rather, have a sensible discussion on the rationale and merit of individual cases as they come up (as I assume was done this time).

Another discussion that has come up frequently in the past, and likely will again, is whether OAME should move to the model that the National Council of Teachers of Mathematics (NCTM) has, where their President is elected for a two-year term. Many OAME Presidents in the past have observed that in their one-year term, they are really just "getting a feel" for the office when their time is up. I like the idea of a two-year term for the President, but for those running for the office, it does mean a four-year commitment—a year as President-Elect, two years as President, and a year as Past-President. The idea once got as far as having a group of us design a plan for how this would be implemented; perhaps it is time to dust it off.

Ralph Connelly

▲ Incoming President's Message

JILL LAZARUS
EMAIL: Jill.Lazarus@oame.on.ca

Jill Lazarus, the current President of OAME, is a high school teacher in the Renfrew County District School Board. She is also completing a PhD in Education at the University of Ottawa. Jill has a range of interests in mathematics education, including curriculum development, student communication, educational technologies, assessment, and professional learning.

When I was first approached with the idea of becoming President of the OAME, I felt honoured. Participating in OAME activities has been an important part of my own professional growth. Opportunities to connect with dedicated members of the mathematics education community, and with the wonderful ideas that are shared each year at the OAME Annual Conference, have inspired my teaching. Since attending the annual conference for my first time a decade ago, this event and the many other opportunities that arose from my involvement in the OAME have become an integral part of my professional learning.

In addition to being part of an amazing annual conference, there are many ways to get involved in the OAME. Regional events are organized by local chapters; the Ontario Mathematics Olympiad (OMO) offers a fun and challenging collaborative mathematics competition for teams of Grades 7 and 8 students; an annual Leadership Conference is held in the Fall; the OAME website and Twitter feed (@OAMECounts) include up-to-date information for the mathematics education community; the OAME awards offer opportunities to formally recognize educators who demonstrate outstanding contributions to mathematics education in Ontario; and all kinds of great ideas are published regularly in the OAME *Gazette* and *Abacus*. Special projects also continue to arise. This past year, for example, the OAME has been part of two projects that are reported on in this issue of the *Gazette*: The Grade 9 Applied Project (**www.math4thenines.ca**) and the Math Knowledge Network (**www.mkn-rcm.ca**).

It is the hard work of dedicated OAME members who volunteer their time to make these opportunities possible. Members of regional chapter councils organize local events. Conference organizers and presenters contribute to an exciting conference each year. Members of ad hoc committees collaborate to address special situations. The OAME Board of Directors and Executive come together throughout the school year to contribute to various activities and committees. The editors, reviewers, people from across the province who submit articles to the OAME *Gazette* and *Abacus*, website team, and Mathematics Conference Information System (MCIS) coordinators are working hard to keep us connected. Members from across the province take the time to come together for our regional and provincial conferences. With a recent transition in *Gazette* editors, I would like to specifically thank Amy Lin for her contributions to the OAME *Gazette* over the last year, and I would like to welcome Tim Sibbald, who will be filling in as interim editor this year.

Over the past year as President-Elect, I have had the opportunity to work closely with members of the OAME Executive. In my transition into this new role, I have learned from my successors and mentors: Past-Presidents Judy Mendaglio (2016–2017) and Tim Sibbald (2015–2016), Vice-Presidents Bill Otto and Sandra Jean Price, and Executive Directors Lynda Ferneyhough and Fred Ferneyhough. Everyone on this committee invests their time to make significant contributions to this organization. One name that is likely familiar to many OAME members is Lynda's. She is often the person we go to when issues arise. Lynda goes above and beyond in making significant contributions to all aspects of this organization, supporting members in many ways, and handling all kinds of situations as they arise. I look forward to continuing to work with the dedicated members of the OAME Executive, and I would like to welcome David Petro, who is our new President-Elect.

In addition to feeling honoured to become President of the OAME, I was humbled. I began to consider this role more carefully and to develop my vision during my year as President-Elect. Planning for the 2017 OAME Leadership Conference involved developing a vision that would be reflected at this conference. As I thought more carefully about what "Leadership" means to me, Bruce, Jarvis, Flynn, and Brock's (2011) findings concerning "Lead Teachers" in collaborative action research came to mind. More specifically, these Ontario researchers conjectured that contradictory statements made by teachers may reflect conflicting views of leadership. The more "distributed" style that is articulated in their conjecture reflects the vision that I hope to promote this year:

> We conjecture that one possible reason for these apparent contradictory statements lies within the perceived definition of what 'leadership' involves. Whereas traits-based leadership theory has typically emphasized the charismatic, authoritative (not necessarily authoritarian) leader image, more recent

leadership models such as ‘distributed’ (Spillane, 2005), allow for a more shared leadership style wherein truly capable leaders are those seen as possessing the ability to acknowledge and to emphasize the best and most creative contributions of those around them, rather than commanding obedience or even leading by example (Bruce et al., 2011, p. 44).

This type of leadership style is evident in various small- and large-scale mathematics education initiatives across the province. Consider, for example, the view of leadership that is reflected in the Grade 9 Applied Project and the Math Knowledge Network activities that are reported on in this issue of the *Gazette*. As current President of the OAME, my vision for this year is to support this style of leadership by supporting collaboration, and by making efforts to connect and highlight some of the amazing things that are already happening within the Ontario mathematics education community.

One responsibility of the OAME President-Elect is to design a program for the next OAME Leadership Conference that reflects his or her vision. The theme of the 2017 OAME Leadership Conference is Connecting Leading Ideas. My goal for this year is to reflect my vision by emphasizing “creative contributions” of OAME members. In particular, with a focus on *Thinking Classroom* and *Leadership*, the event will feature a keynote session by Peter Liljedahl, to offer background on the idea of a *Thinking Classroom*, a concept that has been gaining momentum in Ontario (e.g., the Grade 9 Applied Project report). *Thinking Classroom* breakout sessions will be facilitated by Ontario teachers, in both English and French. The afternoon will include a panel on Leadership followed by elementary and secondary breakout sessions. For the first time, this exciting event will be coming to Ottawa for Canada 150!

I look forward to another exciting year with the OAME, and I would encourage you to explore different ways to get involved. Consider attending and/or submitting a proposal to present at a conference, join your regional chapter council and/or participate in chapter activities, interact with us on Twitter, nominate someone for a position on the OAME Board or for an OAME award, and/or submit an article to the OAME *Gazette* or *Abacus*. Feel free to contact a member of your local chapter, or myself, if you have questions about getting involved.

References

Bruce, C., Jarvis, D., Flynn, T., & Brock, E. (2011). Lead teachers in collaborative action research: Perceptions of role and responsibility. *Canadian Journal of Action Research, 12*(3), 29–46.

Spillane, J.P. (2005). Distributed leadership. *The Educational Forum, 69*(2), 143–150. ▲

▲ Marilyn Hurrell Tribute (1944–2017)

Marilyn Hurrell passed away May 17, 2017. She was committed to the OAME in a wide variety of ways. Marilyn was one of the longest-serving editors of the OAME *Gazette* (2003–2008). She also served as an associate editor until her death and had been a member of the OAME Board of Directors. In her chapter, Marilyn chaired the Ontario Math Olympics in 2001, when they were held in Thunder Bay, and was very active within the chapter.

Marilyn was born April 2, 1944 to Vern and Paulyne Sims. She pursued academics and cheerleading during her school years at Pine Street School and Port Arthur Collegiate Institute. This was followed with a mathematics degree from Lakehead University and further studies at York and Queen’s Universities years later. That facilitated a career teaching mathematics and English, as well as occasional lecturing at Lakehead and being a ski instructor. The latter led to her involvement in the Ontario Federation of School Athletic Associations’ Provincial Skiing Championships and Surrey Secondary Schools Athletic Association Alpine Ski League.

Marilyn made many volunteer contributions that include the Wesway Board of Directors, the Alzheimer Society, Diabetes Association, and the Thunder Bay Regional Health Sciences Centre. She also found time to enjoy the Thunderwolves hockey games, the golf course, gardening, bridge, and book clubs. Since retiring, she enjoyed trips to Vancouver, Alaska, the Maritimes, Ireland, and New Zealand.

Marilyn’s family was particularly important to her. She enjoyed visiting family and friends on Lake Superior and special time spent with her grandkids. She had two children and three grandchildren.

Marilyn will be missed by the mathematics community.

▲ Past-President's Retrospective

JUDY MENDAGLIO
EMAIL: judy.mendaglio@oame.on.ca

Judy Mendaglio is the Past-President of OAME. She joined the Board as a Director, becoming Vice-President after a few years on the Board. She is also active in her chapter organization, CHAMP, where she was President for three years. She began her association with CHAMP through her work organizing the OMO regional event.

As I prepared for the June Board meeting, my last as President, I reflected on the past year. I shared some of my thoughts with those who were at the meeting. Here are my thoughts.

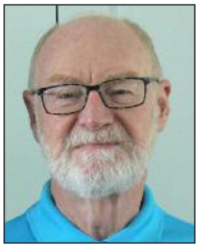
It was truly my privilege and honour to represent OAME this past year. This organization is populated by the most wonderful, smart, funny, and committed mathematics educators one could ever hope to meet and work with. Without a single exception, wherever I went as President of OAME, this organization was held in the highest regard. You may be unaware that OAME is frequently represented at meetings of the Subject Associations, the Faculties of Education Forums, the Ontario Ministry of Education, and the Communities of Practice of the Math Knowledge Network. The excellence of the work we do—including our Annual Conference, our publications (the *Gazette*, the *Abacus*, and the website), our Leadership Conference, the Ontario Mathematics Olympiad, and our regional conferences—are all well known in the education community. It is only as President that I came to know the extent of the admiration garnered by OAME.

The high regard in which OAME is held is, of course, because of its members. Everything the organization is known for is done by the energetic and committed educators who support the work of OAME in their "spare time." I put quotation marks here because I know full well that the people we depend on at OAME have no spare time. Every minute is accounted for, and yet they squeeze a moment (or twelve) to do the work required to make OAME the great organization it is. It is you, the hard-working and dedicated members of this amazing organization, who are the reason the organization has garnered such a profile. You are the face of OAME. The work you do with your students represents mathematics education at its best. Please know that the work you do is noticed and valued. As educators, we are told all too often what we are not doing "right," and all too rarely lauded for the great work we do.

There is a consequence to the high esteem in which OAME is held—our organization has undergone explosive growth in a very short time. Our Annual Conferences now bring in more than twice as many participants as compared to a few years ago. In each of the past five years, we have had to close registration when we reached the venue's capacity. Our Leadership Conferences have had wait lists and our Math4theNines sessions have filled up, requiring more sessions to be opened. These are wonderful problems to have, but they are still problems. Our mandate is to support teachers, and our popularity has the potential to jeopardize our ability to reach all of the teachers who seek our support. A plea for a space at the Annual Conference is a tough email to answer when a space is no longer available. We are a sleeker organization than in our past, with fewer members on our Board and Executive, yet, because of the high regard in which we are held in the education community, we are called upon to do more. This growth, in our membership as well as in the demands placed on our Board members, chapter members, and Executive, was not a carefully planned direction that was mapped out by the Board. Rather, it is a positive phenomenon that happened to the organization. Since uncontrolled growth has the potential to create gaps in any organization's ability to retain its quality and standards, it is my opinion that we now need to pause, take stock of how we got here, and to plan: Where do we want to be going in the future?, How do we propose to get there?, and What needs to be put in place to ensure that the excellence of the organization is maintained? By the time you read this issue of the *Gazette*, some of that work will have already begun.

A personal note: When I became involved in OAME, as "just a classroom teacher," I never, ever imagined that I would be elected to the Board of OAME, let alone take the position of President. At every stage along my personal journey, I said, "Me? I have nothing to offer. You want….," and I would name any of the wonderful math people I knew. In the interim years, I have come to realize that, with few exceptions, we have all responded to the call to lead with the same thoughts—"Me? What do I have to offer?" Please think about this. We are all "just classroom teachers." We all see the strengths in those around us, but minimize our own strengths. But the folks in the OAME organization are wise, and generous, and kind, and patient, and they will help you bring out the best in yourself. They will assist you as you build your leadership capacity. The organization needs you, both at the provincial and regional levels. Our students need you. Their futures need you. Please, continue to volunteer in this amazing organization as we build our future together, and encourage any of your colleagues who may think of themselves as "just a classroom teacher" to model what we expect of our students—that they stretch their thinking, that they push themselves outside their comfort zone, and that they take on new challenges with the joy that comes from learning. ▲

▲ Executive Directors' Report



LYNDA AND FRED FERNEYHOUGH
EMAIL: eds@oame.on.ca

Lynda and Fred Ferneyhough have been the Executive Directors of OAME since 2010. They taught in the Peel District School Board for over 30 years and had a three-year stint in the United Arab Emirates. Both of them served OAME as Chapter Representatives for CHAMP, Directors, and Vice-Presidents. During their career, they were Department Heads, and authors for McGraw-Hill Ryerson. They are both certified as instructors for Texas Instruments (TI), and Fred continues to coach for TI in the United States.

Lynda and I have now been in the role of Executive Directors for seven years. One of the aspects of the organization that we enjoy is that this is a volunteer organization, and there are many people who generously give their time for the benefit of math teachers in Ontario.

In this column, we want to recognize some people who have provided outstanding service to OAME. Let's start with our board members and chapter representatives.

OAME has fifteen chapters across the province. Fourteen of the chapters are defined by geographical boundaries and generally include a few school districts. The only exception to this is ISOMA, which represents all of the independent schools in the province. Each year, five of our chapters will conduct an internal election to appoint a person to represent them over the next three years. In this manner, OAME has continuity from year to year.

There are also six elected directors, three from K–6 and three from Grade 7 and above. Each year, one of each is elected for a three-year term, again ensuring continuity from year to year.

At the Executive level, we have a Past-President, President, and President-Elect. These people serve a three-year term, moving from President-Elect to President to Past-President.

Finally, the Executive has two Vice-Presidents, each elected for a two-year term.

These twenty-six elected people meet three times a year—October, March, and June—to conduct OAME business. Meetings begin on Friday night and continue through late Saturday afternoon. The board is broken up into four committees. Each committee has an executive liaison and several chapter representatives and/or directors. A brief description of some of the duties performed by each committee follows:

1. The Communications Committee handles the *Gazette*, *Abacus*, OAME website, and promotional materials.
2. SPaRC is an acronym for Strategic Planning and Renewal Committee. This committee takes care of revisions to the Constitution, the By-Laws, and the Terms of Reference.
3. The Futures Committee handles the annual OAME awards and tries to find chapters who will host the annual conference in the Spring, and the Ontario Mathematics Olympiad.
4. The Curriculum Committee formulates OAME's thoughts on the Ontario Mathematics curriculum, and has often formed writing teams to respond to a need for resources for a particular topic.

The board also has several non-voting members: the editor of the *Gazette*, the editor(s) of the *Abacus*, the Webmaster, and an NCTM (National Council of Teachers of Mathematics) representative.

These people freely give up their time for these committees and participate in the business portion of the three OAME meetings. The Executive members also hold meetings in September, January, April, and May, thus giving up even more of their time.

If no chapter steps forward to volunteer to host the annual conference three years in advance, our policy is that the board becomes responsible for organizing that event. This happened in 2015, and the board is also planning for the 2018 conference next Spring. In the years when this happens, board members use half of the meeting time for the various annual conference planning committees, as well as handling the regular board and committee business. Often, board members take more personal time to complete the business either from the board committee and/or the annual conference planning committee.

Members of OAME who are interested in serving on the Board of Directors can allow their name to stand for nomination for the elections that take place in March each year. Information on nominations, which close in January, can be found in the *Gazette*. ▲

(Editor - Note that some expense coverage and minimal honoraria come with some roles.)

▲ OAME/NCTM Report – The 2017 NCTM Annual Meeting and Exposition

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Jill is the President of the OAME and attended the NCTM 2017 Annual Meeting and Exposition as the OAME President-Elect.

One objective of the OAME is “to maintain affiliation with the National Council of Teachers of Mathematics [NCTM], and to promote projects of mutual concern.” Since September 2011, Todd Romiens—an OAME past president, OAME life membership award winner, and past editor of the *Gazette*—has taken on the role of NCTM Representative for the OAME. He has played an important role in facilitating and reporting on the affiliation in this column of the *Gazette*. On behalf of the OAME, I would like to thank Todd for his dedication to maintaining this connection over the years. I would also like to welcome Jacqueline Hill, who has agreed to step into the role this year. In this time of transition, and with my recent attendance as an OAME representative at the NCTM annual conference, it seemed fitting to report here on my conference experience.

As the OAME President-Elect, I attended the 95th NCTM Annual Meeting and Exposition, which was held at the Henry B. Gonzalez Convention Center in the beautiful city of San Antonio, Texas from April 5–8, 2017. The theme was “Creating Communities and Cultivating Change.”

Figure 1: NCTM Central

As a representative of an NCTM affiliate group, one of the first events that I participated in was the Delegate Assembly and Regional Caucus. In the Canadian affiliate group, which was also attended by NCTM President, Matt Larson, I met with other representatives from across Canada. The discussion concentrated on on three questions that the NCTM is focusing on this year: access, equity, and empowerment. The discussion concerned: (i) how we address equity, access, and empowerment in our region, and how the NCTM might support us in addressing issues; (ii) what we feel have been the benefits of our relationship with the NCTM and what NCTM and affiliates can do together to make this relationship more productive; and (iii) recommendations concerning mathematics education issues or NCTM operations issues.

Many ideas arose during this discussion, including the suggestion that there should be a more interactive flow of resources that are available in editable formats, and that there is a need for more NCTM presence in Canada, perhaps even an NCTM annual conference being held here. Looking at the conference archive, the last time this happened was in 1982, with the 60th annual conference being held in Toronto.

The NCTM offers many resources and events for members of the mathematics education community, and it was clear from this event that they are continuing to grow and explore ways to support membership. For anyone who is not yet an NCTM member, I would recommend becoming one (select OAME as your chapter and the OAME gets a rebate), and if you ever have the opportunity to attend the NCTM annual conference, definitely take it. It was such an amazing experience.

Thousands of delegates attend the NCTM conference, and many high-profile speakers are featured. I attended sessions, for example, by Andrew Stadel (**www.estimation180.com**), Dan Meyer and Robert Kaplinsky (**www.robertkaplinsky.com/need-know-applying-speak-nctm/**), Jo Boaler (**www.youcubed.org**), Peg Smith, Chris Shore (**www.clotheslinemath.com**), Dylan Wiliam, and Simon Singh (**www.simonsingh.net/**). Even some of our very own OAME members were featured, including Marian Small and Mary Bourassa. Mary represented Canada during an Ignite session titled “Newton, Curie, & Dr. Seuss.” Ignite, shown in Figure 2, featured ten classroom teachers, who each had five minutes to quickly “enlighten” the audience, with 20 slides that automatically advance every 15 seconds. For a recording of the Ignite sessions, check out **www.nctm.org/ignite_am17/** (Mary is on at 21:15).

Despite the time pressure of an Ignite session, which I am sure makes this a very uncomfortable experience for some, particularly Mary, who admitted during her talk to being both shy and introverted, Mary agreed to do this Ignite presentation a second time for OAME members at the OAME 2017 Conference in Kingston. She acknowledged during her NCTM Ignite session that her intention, with stepping out of her own comfort zone, was to give voice to those quiet students in our class. I would encourage you to check out the Ignite sessions, including Mary's five lessons for creating community. Also check her OAME 2017 Ignite session, along with many others at **www.youtube.com/playlist?list=PLYmB4P-NnvCPOCZLcP8gkvuNQtOEp4YDt**.

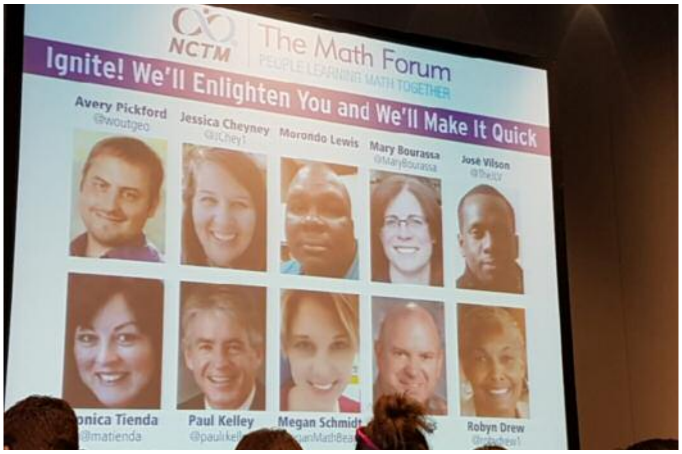

Figure 2: The Ignite sessions

A unique teacher-led session at NCTM 2017, hosted by Dan Meyer, Zachary Champagne, and Michael Flynn, was *ShadowCon* . ShadowCon is a teacher-led mini-conference designed to "highlight some of NCTM's best speakers." Each speaker had ten minutes to preview a free ten-week online course, which starts in the Fall of 2017. Video recordings of these sessions are available at **www.facebook.com/TeachersofMathematics/videos/10156139266873747**. Registration for these *free* courses is at **www.Tinyurl.com/shadowcon17**.

Figure 3: Reflection coves

Another unique feature of the NCTM annual conference was the opportunity for more intimate gatherings with delegates, presenters, and authors. *Reflection Coves* allowed for small-group conversation with invited speakers like Jo Boaler and Dylan Wiliam. These speakers announced the small-group meeting locations during their sessions. As you can see in Figure 3, these coves were set up with a small number of couches and chairs, in places like hallways or the NCTM Central location (see Figure 1), where delegates could drop in for more informal discussions.

The *Reflection Cove/Networking Lounge/Learning Lounge* sessions allowed delegates to connect with peers, speakers, journal editors, and authors. I was excited to join Peg Smith, for example, an author of *The Five Practices for Orchestrating Productive Mathematics Discourse* (2011). In a small group of only a few teachers, we learned more about the five practices. As a high school teacher, I was part of a school team at Valour JK–12 School in Petawawa, that participated in the Math4theNines Adobe Connect Professional Learning Series, *Five Practices for Orchestrating Productive Mathematics Discussions*. So for me, it was an exciting experience. The opportunity to join a Reflection Cove session with Dylan Wiliam was also fascinating. As a leading contributor to the area of formative assessment, his work has significantly influenced my thinking about classroom assessment and my own research in mathematics education.

Using real-life images to spark mathematical curiosity is an idea that arose on several occasions during the conference. During a session titled "Find your inner author—write for the mathematics teacher!" (Margaret Coffey, Roger Day, & Rob Lancaster), for example, Ron Lancaster shared some images that could be used to inspire articles in this publication. (*Editor note:* Ron is a Past President of OAME and had a Photo Math column in the OAME *Gazette* from 2000 to 2008. You can find it in the *Gazette* archives, which is in the members-only area of our website.) Andrew Stadel offered ideas for using visuals to tap into students' mathematical thinking (check out **www.estimation180.com**). During my time in San Antonio, a couple of mathematically exciting visuals that I encountered are shown Figures 5 and 6. I will put the question to you: How might we use such images to support learning goals in different grades?

For even more about the 2017 Conference, check out featured webcasts, including Matt Larson's President's Address, *Empowerment WITH Access and Equity*, at **www.nctm.org/2017SanAntonio**. More information about the NCTM and upcoming events is also available at **www.nctm.org**. Given their current emphasis on promoting access, equity, and empowerment, the NCTM will be featuring an Innov8 Conference, "Breaking Barriers: Actionable Approaches to Reach Each and Every Learner in Mathematics" this November 15–17 in Las Vegas, Nevada.

Finally, the NCTM Annual Meeting and Exposition will be held April 25–28, 2018 in Washington, DC. More information is available at **www.nctm.org/annual/**.

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▲ Frobenius Numbers in Grade 3: Learning from Watching Children at Work

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Matthew Oldridge is a father, mathematics educator of 15 years, thinker about things, TEDx speaker, facilitator of professional learning, and Tweeter. Current interests include making mathematics full of surprise and wonder for all children, and helping children and parents to see that "math is play."

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Melissa Pennarun is an educator and educatee, truly at home in the presence of our little humans. As a mother to young daughters, teacher of 13 years, and Math Lead at Brisdale Public School, her first passion is to connect with children and then to make mathematics accessible and wondrous for all learners. It is her goal that students want to do math all day long on the last day of school.

This is a story of giving students an interesting problem, and letting them work. Let a class loose on interesting problems, and they will always show us the power of their mathematical thinking. In this co-taught lesson, the empty McNugget boxes disguised something far more interesting about the structure of numbers: as co-teachers, we learned about Frobenius numbers alongside our Grade 3 students.

We learn the most about the teaching and learning of mathematics by watching students work, listening and talking to them while they work. While many professional learning structures favour PowerPoints, theories, and expert-driven top-down approaches, we gain insight into how students learn mathematics by watching them work on mathematics. A quotation about music of unknown origin goes something like: "Writing about music is like dancing about architecture." Talking about teaching mathematics without watching students do actual mathematics is a lot like "dancing about architecture" as well.

Collaborative inquiry is the framework commonly used by school boards to learn from students at work. At their best, school boards are able to give powerful learning experiences, where teachers collaborate together on interesting lessons, co-teach lessons, document the students' work, and talk about what was learned. Professional development can, and should, bring about greater teacher efficacy, which is the belief that they have the power to cause student learning to happen. As Bruce and Ross (2010) note, linking teacher efficacy to collaborative inquiry, vicarious experience is a key factor: "Teacher efficacy increases when teachers observe their peers bringing about student learning."

As stated in the Ontario Ministry of Education monograph (2014) on Collaborative Inquiry, "there is no prescribed protocol for collaborative inquiry, nor is there a single path that educators should follow" (p. 3). Collaborative inquiry structures can be more, or less, formal; this is a story of an informal collaborative inquiry. Melissa and I decided I would visit and try a new lesson with her students. They knew me as "the math guy," as they had provided their thoughts on mathematics for a keynote I did for parents at a weekend conference. We intended to document the learning thoroughly, and to push the students a little further in their mathematical thinking than they had gone before.

As a guest of Ms. Pennarun's Grade 3 class, I knew I had a chance to try out something interesting and new. We discussed over email and Twitter what the students knew about multiplication; our intent was to build on and go a little further with the skills, strategies, and thinking models they had learned this year.

Precision in planning comes with starting from what children already know. In this case, I was interested in the learning landscape of multiplication, which is very important in Grade 3. Once we had decided on setting a learning goal around multiplication, Melissa sent me these thoughts (Figure 1), and two anchor charts (Figures 2 and 3), full of ideas and strategies, which the students had used in her classroom.

- Skip counting and counting patterns, using number line and hundreds chart, fingers and toes, groups of 10, coins of equal value
- Repeated addition
- Arrays with shapes and grid paper
- Area
- Familiar facts
- Using knowns to find unknowns
- Cows and chicken legs, motorcycle, car, and bus wheels

Figure 1: Thoughts on multiplication strategies

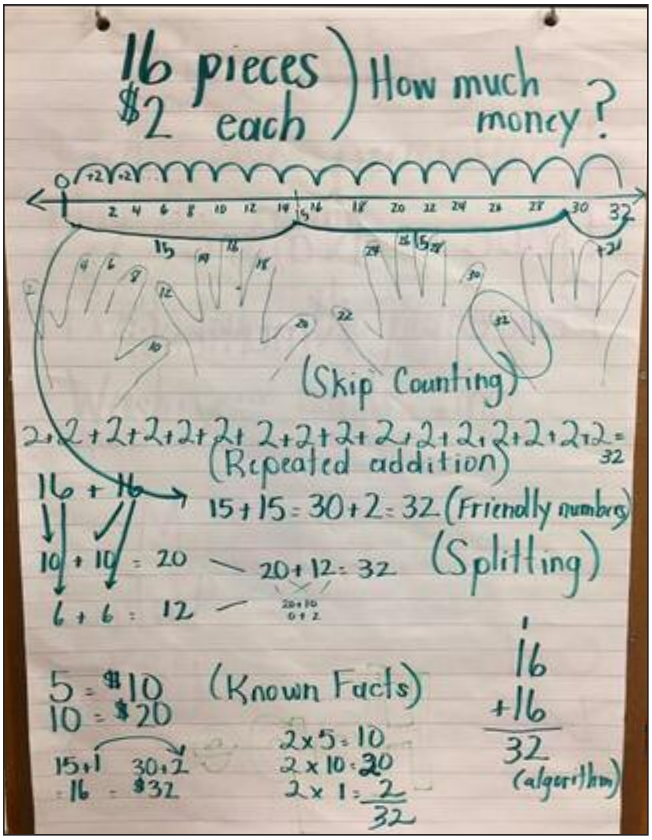

Figure 2: Anchor chart of multiplication ideas

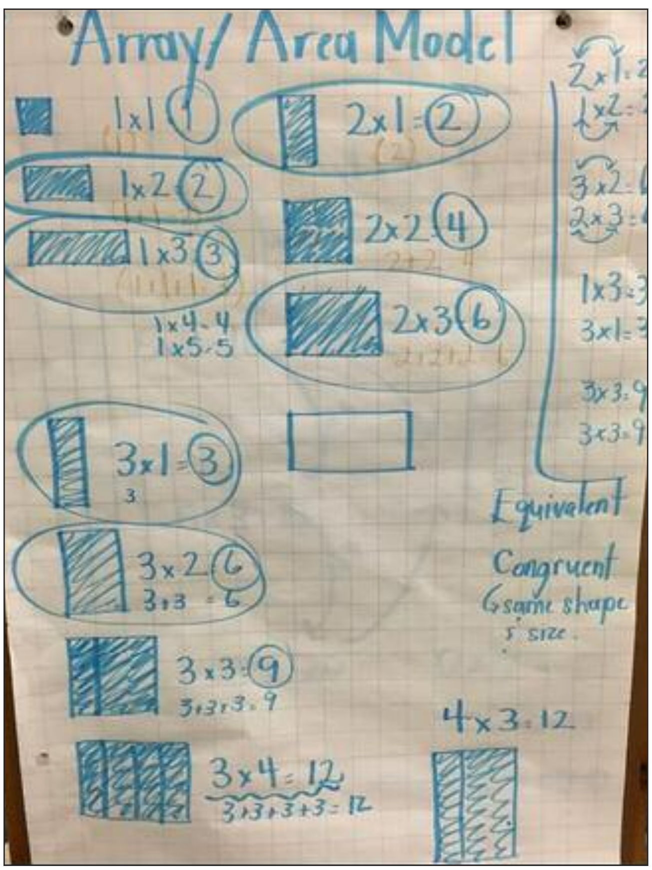

Figure 3: Anchor chart of a multiplication strategy

The class had access to a variety of tools and strategies related to multiplication. Prior knowledge must be harvested in order to create new knowledge. In the case of Melissa's class, the children had, over the course of the year, worked with skip counting, number lines, arrays, the area model, and number talks as part of building their understanding of multiplication.

We asked the class, “What is multiplication?” I like to ask this question to both the children and teachers, after seeing Dr. Brent Davis ask it in a session at OAME 2015: teachers in the room came up with 24 distinct meanings for multiplication. As Davis and Sumara (2009) note that teachers will often answer “repeated addition,” or “groups of,” but were surprised when asked, “And what else?” The students came up with a number of meanings, like “equal grouping,” “repeated adding,” and “skip counting.”

We chose to start with a number talk, to activate student thinking about multiplication. Number talks, as formulated by Sherry Parrish, are short 5–15 mental math talks, where students are given a number fact, and asked to explain their strategies, which the teacher records on a whiteboard or chart paper. As Parrish suggests, “these exchanges lead to the development of more accurate, efficient, and flexible strategies” (p. 199).

The number talk we did with these students was on 8 x 7. This is a good fact to address, as it’s a knotty fact for many children and adults. For many, it doesn’t tend to “stick.” It also doesn’t lend itself to “doubling and halving” (or rather, doubling and halving doesn’t help much when one factor is 7), which might be considered a drawback when looking at strategies. I realized later, when Melissa pointed it out, that the Grade 3 curriculum goes up to 7 x 7, although 8 x 7 still seemed to be in the range of comfort for most of the class.

Many had the answer, 56, quickly. Some referred to it as a “known fact,” which means they have worked enough with this fact to know it (whatever that means). Knowing, memorizing, and understanding are tricky topics for mathematics educators. Jo Boaler uses an anecdote about a British politician, who publicly stumbled on trying to evaluate 8 x 7. Critics seized on this as proof that all students should memorize times tables, but as Boaler notes, “this was ironic as his mistake revealed the limitations of memorization without ‘number sense’” (p. 1). Others went to 8 x 8, and then subtracted 8. The key move I wanted to make was to have children use the area model; i.e., 56 can be thought of as an 8 x 7 rectangle. We did not get around to using a number line.

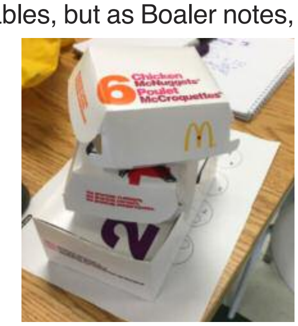

Figure 4: McNugget boxes of 6, 9, and 20

The task:

McNuggets only come in packages of 6, 9, or 20. Can you buy exactly 43 McNuggets?

Can you buy exactly 44 McNuggets?

Pick a number bigger than 44. Can you buy that number of McNuggets?

The task we used was inspired by the interesting Numberphile video called “How to Order 43 McNuggets” (**www.youtube.com/watch?v=vNTSugyS038**), which concerns the fact that 43 is the largest integer that cannot be made from combinations of boxes of 6, 9, and 20 McNuggets (see Figure 4). In other words, the so-called “McNugget numbers” are really just a contextual disguise for something more properly called Frobenius numbers. A Frobenius number is the largest number that can’t be made from a set of combinations of positive integers. The Frobenius number of 6, 9, and 20 happens to be 43. Yes, this means that all integers larger than 43 can be made with packages of 6, 9, and 20 McNuggets.

The tricky part was that I didn’t know they had changed the 9 McNugget package to 10. This ruined the math because with groups of 6, 10, and 20, no odd number combinations are possible. (The mathematics of McNuggets was further weakened by the creation of a 4-McNugget box for children). So I went and got 3 empty packages. I crossed out 10, and put 9 to restore the original mathematics challenge.

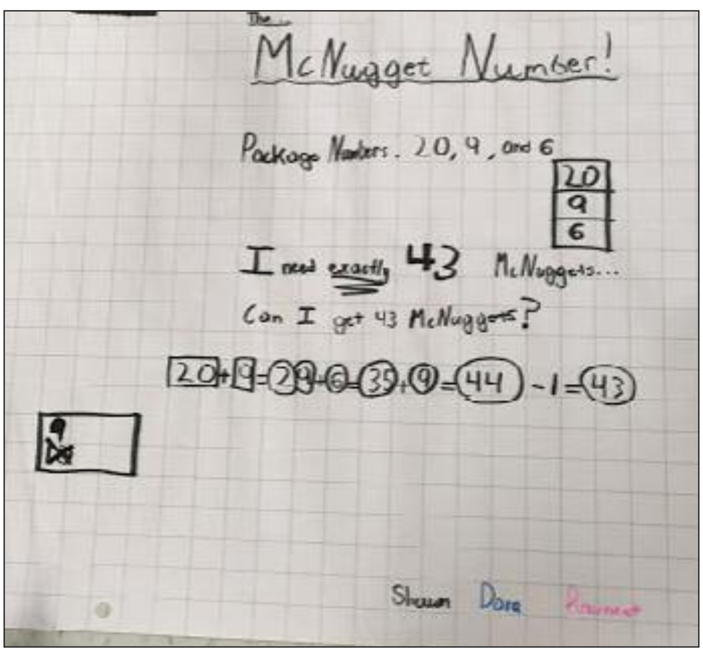

Figure 5: Calculating the McNugget number

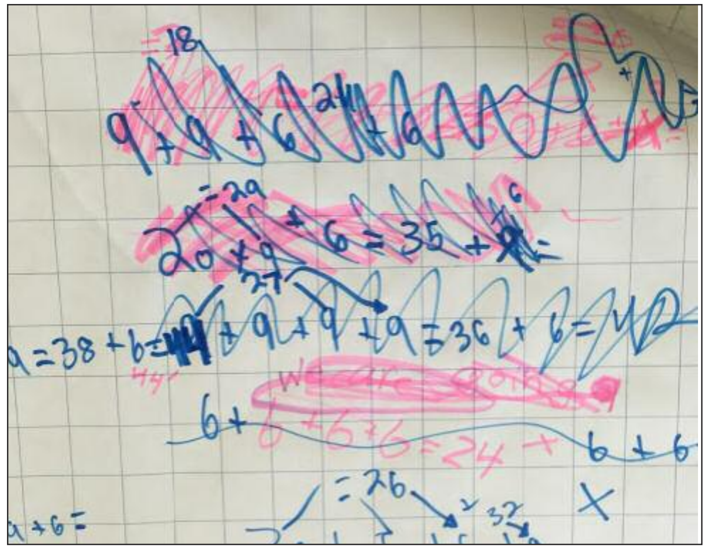

Figure 6: The beautiful messiness of mathematics

Here is where we needed our “mathematical world.” Of course, in the real world, you just buy 44 because 44 is possible with the possible boxes, and eat one. We had to tell students that that was not allowed in our mathematical world. They seemed to accept this constraint.

Many children spent a long time writing out combinations of numbers, trying to make 43. Trial and error is exactly the initial strategy you would expect to spring up. There was lots of adding of various combinations.

Figure 6 illustrates the beauty of watching children at work. It’s a beautiful mess of mathematical experimentation and investigation. Combinations were strewn all over the page, scribbled out, as they continued the search for the right one.

At this point, we realized that children are accustomed to proving things true. In other words, I suspected they really thought they would find a combination to make 43. That was the first surprise, and there are always surprises when we unleash students’ thinking and watch them at work.

At one point in the middle of working on the problem, Ms. Pennarun took one child aside and worked with a number line to start mapping out combinations. We realized the hundreds square would help visualize what was happening with the combinations. We pulled out a large-sized hundreds square and gathered the class together.

This was what I considered the big pedagogical move. The students already knew the problem was about the structure of numbers (as a matter of fact, none of them were really interested in the idea of McNuggets at all; they were too busily immersed in their own mathematical thinking). Connecting one representation, the McNugget boxes, to another, the hundreds square, was the big move—we wanted to move their thinking toward the general structure of the problem.

We started by taking children's numbers that they knew would or wouldn't work, and crossing them off, or circling them. For example, 35 works because it is 20 + 9 + 6, and 27 works because it is 9 + 6 + 6. The students in the class easily saw that many smaller numbers, like 10 and 11, were impossible.

Most students spent a large amount of time on 43. They knew it was the key to solving the problem. We agreed that 43 was impossible. We ruled in some larger numbers, like 56 and 78. We ruled in 44. We started showing them how to add forward by 6, 9, or 20, from numbers we had already ruled in.

In our consolidation as a class (see Figure 7), we crossed off more and more numbers above 43 that the students had found to be possible. Revelation hit at this point. I realized that we would be able to "get" all numbers out to infinity.

Why? Consider this:

> If I am at 100, I can add 6, 9, or 20. But then I can go back to 99, and add 6, 9, or 20. I can keep going backwards and forwards (skip counting!) and never miss any numbers. This is the part I am not sure that children really "saw" yet. If I followed up, I would try and get them to think about the magic of infinity. Look at this picture and see if you can see it. As soon as you have 6 consecutive integer combinations that work, you can skip count forward forever, hitting all the numbers.

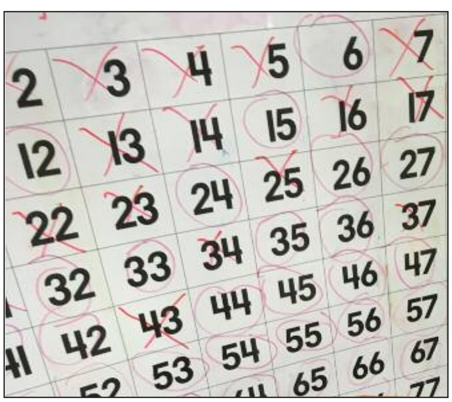

Figure 7: Recording results on a 100-chart

Educators, what would you do next? The pedagogical ball is in your court. Learning from children at work means knowing where they are, and a good place to go next. Many possible pedagogical paths are open to you. You could look at prime numbers, for example, or prime factorization, depending on the grade with which you are working. You could give students a different combination of numbers, and ask them to find the largest number that can't be made (for example, 5, 8, and 12).

Responsive mathematics pedagogy is created by purposefully choosing an interesting task based on prior knowledge, watching students work, documenting their thinking, and engaging them in purposeful consolidation. Learning alongside students is powerful and humbling work. In our opinion, professional development should start and end with examining student thinking about interesting mathematics. We learn the most about the teaching and learning of mathematics by watching children work, listening to them while they work, and talking to them while they work.

The classroom teacher's perspective on the lesson:

> Students played with the given numbers, initially to make 43 from combinations of 6, 9, and 20, and they stayed engaged and explored for a surprisingly long time—they multiplied, added, and then tried to subtract, but it was made clear that we weren't allowed to take any away. I noticed students drawing arrays on grid paper, writing number sentences and drawing number lines. They used calculators and grouped snap cubes to investigate. There was a common theme in student thinking—the guess and check... and then the scribble. It was important to remind students, at this point, that mess is okay, but it is helpful to see what is scribbled out in order to track errors and ideas that did not work. Because we found that my students have not been exposed to providing proof for something that does not work, we decided to regroup. Not one student was able to prove that 43 worked; therefore, we concluded as a class that 43 cannot be made with combinations of 6, 9, and 20.
>
> The next step was for students to test numbers greater than 43. My mission as co-facilitator, in addition to solving the challenge myself, was to find ways to move students toward organizing how they chose to record their thinking, if we were going to get anywhere. They were testing and proving numbers; however, the numbers were getting lost on the page. I prompted some students to access other tools available in our classroom, including the dry-erase number line to 100 and hundreds charts to 1000. Another student chose to document his thinking using a T-chart. We were beginning to get somewhere. The tools helped to keep track of proven numbers. Furthermore, students were beginning to develop more efficient strategies for generating numbers that worked, like skip counting on a number line by 6, 9, and 20, and finding visual patterns (multiples of 20) on the hundreds charts. When we met again as a class, we were able to consolidate numbers students had tested systematically by circling numbers that worked and crossing out numbers that didn't. Visually, we were able to find the McNugget number 43, with the exclamation of my favourite sound, "OOOOH!"

Conclusion

Typically one would anticipate student responses in order to plan how the lesson would move them forward. However, there is power in engaging in math challenges alongside students. The co-facilitators learned about Frobenius numbers with them. The challenge easily lends itself to parallel instruction, and the initial numbers that students are asked to prove could be smaller or larger in order to accommodate for the different needs of students. An extension for higher grades would be to explore the value of different combinations by first finding the McNugget unit cost.

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▲ What's the Problem? Playing with Parting Pirates

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Shawn teaches and is a department head at Cairine Wilson Secondary School in Orleans. He strongly believes in the central role of problem solving in the mathematics classroom. He continues to be involved in mathematical activities: presenting workshops, writing articles, working on local projects, and helping create mathematics contests.

Welcome back, problem solvers. As you read this, a new school year has begun. I wish you all the best in this new school year. It seems ironic to me, since I am writing this during the last few days of the school year in June. In the last issue, I left you with the following problem:

> *A band of 8 pirates is going to disband. They have divided up all of their gold, but there remains one GIANT diamond that cannot be divided. To decide who gets it, the captain puts all of the pirates (including himself) in a circle. Then he points at one person to begin. This person steps out of the circle, takes his gold, and leaves. The person on his left stays in the circle, but the next person steps out. This continues with every second pirate leaving, until there is only one left. Who should the captain point at if he wants to make sure he gets to keep the diamond for himself?*

I encountered this problem at a workshop presented by Dr. Peter Liljedahl from the Faculty of Education at Simon Fraser University in Burnaby, British Columbia. Professor Liljedahl was brought to Ottawa to be the keynote speaker and session presenter at the Ottawa District School Board's secondary mathematics PD day last February. While he was in Ottawa, he also did an afternoon session with a number of teachers who had been using "his methods" the day before the PD day. It was at this afternoon session that I encountered this month's problem.

I first came to hear of Professor Liljedahl's work through a colleague, Bruce McLaurin. Bruce had attended Professor Liljedahl's session at the 2014 Canadian Mathematics Education Forum in Ottawa. At the next district meeting of mathematics department heads, Bruce told us about vertical non-permanent surfaces and visible random groupings. In his work, Professor Liljedahl had studied the effect of the recording instrument—permanent (pencil/paper, or marker on chart paper) versus non-permanent (whiteboards or chalkboards); as well as the medium placement: horizontal (on a desk) versus vertical (on a wall)—on how students were engaged and persisted in mathematical problem solving. He found that students were quicker to engage and more likely to stay engaged, if the surface was non-permanent, as "errors" were easily erased. Similarly, if the surface was vertical, so that students needed to stand, students stayed engaged for longer periods of time.

When Bruce reported these findings to the math heads group, it made sense to many of us. I brought it back to my department, and one of my colleagues, Lynn Miller, and I experimented with it for the rest of the semester, using laminated chart paper as our non-permanent surface. We both loved how it changed our classroom dynamic. As a result, I managed to convince my principal to buy us a number of small whiteboards, which we glued to bulletin boards and attached to chalkboards with magnets. Since then, I have followed the lead of Bruce and his colleague, Al Overwijk, from Glebe Collegiate Institute, and have covered all the walls in Lynn's and my classrooms with whiteboards. Bruce and Al are instrumental in our board, and beyond, in exposing people to Dr. Liljedahl's work and giving practical examples of how to implement a thinking classroom, especially at the Applied level. For their work, Bruce and Al were very deservedly awarded the Secondary Staff Award at OAME 2017.

You can read about Dr. Peter Liljedahl's work and see a number of great resources (including the problem from this issue that I will eventually address) on his website, **www.peterliljedahl.com**. You can find some other great resources at Al Overwijk's blog, **slamdunkmath.blogspot.ca**, as well as numerous entries on vertical non-permanent surfaces, visible random groupings, activity-based teaching, and many more topics. I strongly suggest you visit both sites.

Back to the pirate problem. Unlike many PD sessions that I have attended, Professor Liljedahl spent very little time talking at us. He started the session by having each of the approximately 30 teachers pick a card. He then gathered us around him and presented the problem as an anecdote. We were asked to find our group and get to work. He circulated around the room, observing, answering questions, and not answering questions. He explained, in his keynote address the following day, that he has classified student questions into three categories: proximity questions, stop-thinking questions, and keep-thinking questions. The students ask proximity questions just because you are close. They are things that they probably wouldn't go out of their way to call you over to ask, but since you are there…. Stop-thinking questions are questions designed to take the workload from the student and put it on the shoulder of the teacher, such as "We're stuck, what should we do now?" Keep-thinking questions are questions that need to be answered for the students to continue to work. These include questions of clarification and extension. Professor Liljedahl suggests not answering the first two types of questions, but to answer the third. For the first two types of questions, he would usually smile politely, turn around, and walk away. Although annoying, from a student perspective, they soon learn to ask questions of value. With keep-thinking questions, where possible, he tries to pull the answer out of the group by responding to their questions with questions of his own.

As our group arrived at "the answer" to the problem, we called Professor Liljedahl over to ask him if it was correct. He never did answer us; he just asked us questions about our solution until we convinced ourselves that we were correct. He asked us, "What would happen if there were 11 pirates?," then turned around and walked away. This led us on the path to generalize and find the solution for any number of pirates.

Our group attacked the problem by playing with it. We drew a circle of pirates, eventually numbering them 0 for the captain, and 1 through 7 for the other pirates, numbered clockwise from the captain (i.e., to the captain's left), as shown in Figure 1.

We then proceeded by picking a person at random, going through the process, and seeing what happened. We started with person #2, two to the left of the captain, and found that when we were done, person #1 got the diamond (see Figure 2).

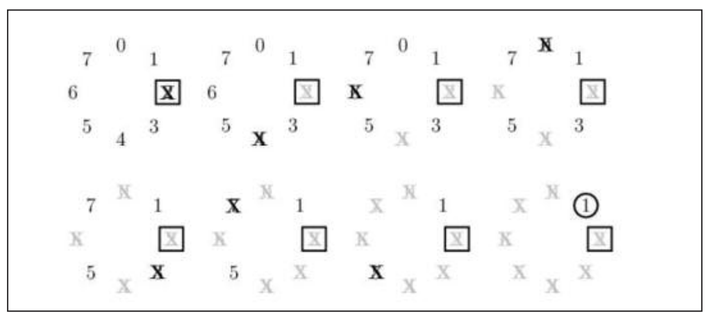

Figure 2: Starting with pirate #2

Similarly, Figure 3 shows that when we started with person #3, we ended with person #2.

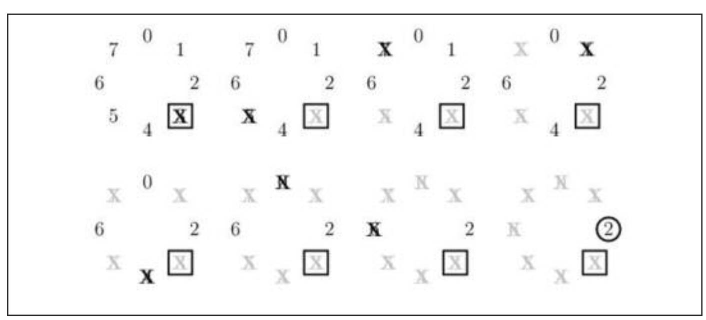

Figure 3: Starting with pirate #3

At this point, we noticed that we always end one person "behind" our starting point. As such, if the captain points to the person to his immediate left, he will get the diamond (try it for yourself).

As indicated earlier, Professor Liljedahl subsequently had us proceed with 11 pirates. We conjectured that position #1 would be the best again, so we tried it out, and unfortunately, #6 was left (left diagram in Figure 4). It should have made sense that it would not work, since the captain is sitting in position #0, which, if we count to his left, will be 9 positions to his left. If we start on #1, we will eliminate all the "odd" numbers on the first round, which means that the captain will be eliminated after #9. If we just concentrate on our starting and ending positions, we see that we ended up 5 spaces to the left of our starting point, or the start position is 5 spaces to the right of where we end. Using this logic, we should start at #6, which is 5 positions to the right of the captain (11 – 5 = 6), which you should check for yourself to see that it works.

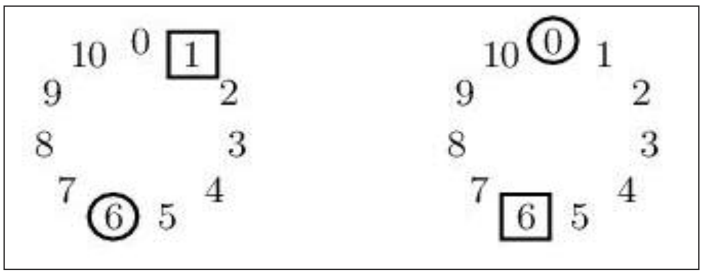

Figure 4: The 11-pirate scenario

We realized that we didn't need to call Professor Liljedahl over, since we knew this was heading in the direction of generalization: given the number of pirates, can we quickly tell where the captain should point? We proceeded to gather more data systematically. Using our same numbering system, we created a table similar to the one in Figure 5.

Number of pirates	Starting position
2	1
3	2
4	1
5	4
6	3
7	2
8	1
9	8
10	7
11	6
12	5

Figure 5: Where the captain should point

At first, the numbers seemed to be at random, but as our table filled up, a striking pattern appeared. The numbers decreased until they reached 1, and then "reset" to the end of the list, since it would make no sense for the captain to point to himself first.

As I looked around the classroom, I got an idea of other groups' strategies. There were many similar strategies, but different groups used different representations. Although most, if not all, groups settled for a numbering system to explore the problem, even these varied. Some groups used #0 as the captain's position, while some used #1. Some groups numbered the positions and moved the captain to different positions for different trials. It was reassuring to see similar methods from other groups. At the same time, it was thought-provoking to observe the differences between our work and the work of other groups. One of the benefits of the vertical surfaces is that when students are stumped, they can be inspired by other groups. In most cases, what appeared on the boards as "thinking in progress" was not necessarily a model solution. As such, when looking at other groups' work, a lot of thinking has to take place to decipher the calculations and representations used by others to help *their* thinking. Of course, a student could also just walk over and talk with members of another group.

Although we have reached "the answer," there is a lot more that we can explore with this problem. If we look at the "reset" positions, they occur with 9, 5, and 3 pirates. This is not a lot of data with which to work. Notice that 3 and 5 are 2 apart, while 5 and 9 are 4 apart. We may suspect 15 is the next place where we get a reset, as 9 and 15 are 6 apart, but this is not the case. If we kept working things out, we would see that 17 is the next reset, 8 more than 9. Could the pattern be 2, 4, 8, 16? If you try 33, you would see that it does correspond to a "reset," that is, if you start with the last person to the left of the captain, or the person to his immediate right, we do end up with the captain leaving with the booty.

We could explore the reset cases more and try to understand why they work, but a more fruitful exploration may come from looking at the last case before a reset. When there are 2, 4, or 8 pirates, you should start with position #1, to the immediate left of the captain. This seems like a more recognizable pattern with the powers of 2 staring us in the face. If we look at the next power of 2, that is 16, we can see what happens if we start in position #1.

I will switch my representation at this point to help illustrate the pattern. Instead of putting the pirates in a circle, I will put them in a line, but remember that the person at the end of the line is beside the person at the beginning. If we look at this as we go through a complete circle, a pattern begins to form. Starting with 16 pirates, and beginning to the left of the captain, we see that by the time we get to the end of the line (i.e., one pass around the circle), all the odd positions have been eliminated, as in Figure 6.

0	X	2	X	4	X	6	X	8	X	10	X	12	X	14	X

Figure 6: The process modelled as a line

This leaves us with multiples of 2. Also, since the last person in line was eliminated, the next person to go will be the person to the immediate left of the captain. On the next round, the ones who are eliminated are those who are not multiples of 4. This process continues, leaving the captain, in position #0 (or, if you prefer, 16), the only position that is a multiple of 16. See Figure 7 for how this plays out.

Round 2:	0 2 4 6 8 10 12 14 ⟶ 0 **X** 4 **X** 8 **X** 12 **X**
Round 3:	0 4 8 12 ⟶ 0 **X** 8 **X**
Round 4:	0 8 ⟶ 0 **X**

Figure 7: What happens with 16 pirates

Logically, we can see why this works. If we start with a power of 2 pirates, and start by eliminating the person to the immediate left of the captain, by the time we have gone around the circle once, all the "odd" positions have been eliminated, the last of which is the person to the immediate right of the captain. As such, with the next "round," we have half as many pirates, which is still a power of 2, and are starting with the person to the immediate left of the captain. Therefore, in *all* cases where we start with a total number of pirates that is a power of 2 pirates, and start to the immediate left of the captain, at the next round, the scenario will have a total number of pirates that is the next smaller power of 2. Thus, in a sense, all of these situations are equivalent. Hence, if we look at the solution to one of these cases, it will be the same for all. So if we start with 2 pirates and eliminate the one to the left of the captain, we can easily see that the captain takes home the booty, so this strategy works for all powers of 2. We could be a bit more rigorous and prove this with induction, but a student would probably be more comfortable with a story similar to the one presented.

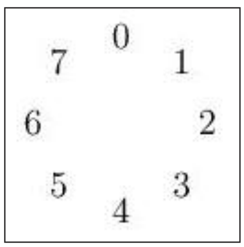

Figure 9: Arrangement of the captain (#0) and pirates

We can even convince ourselves that the strategy will work for non-powers of 2. If we start with the situation where there are $2^n + 1$ pirates, then we can arrange our choice so that the second person picked is the one to the immediate left of the captain. If we do this, we will have 2^n pirates remaining and be starting in the correct place, so it will work. The way to accomplish this is to have the pirate to the immediate right of the captain, in position 2^n, be the first elimination. When that happens, there will be 2^n pirates remaining. We will skip the captain and next eliminate the pirate to his immediate left, which makes us equivalent to the powers of 2 situations, so we know it is a solution. We can carry this argument further to see that the first person to be eliminated should then move to the right, to the next lower number, until we are back to position #1 for the next power of 2. Figure 8 shows how the cases with 9, 10, and 11 pirates can reduce to the same starting position as the case with 8 pirates by having the starting pirate move down one each time.

8 pirates:			0 **X** 2 3 4 5 6 7
9 pirates:	0 1 2 3 4 5 6 7 **X**	⟶	0 **X** 2 3 4 5 6 7
10 pirates:	0 1 2 3 4 5 6 **X** 8 **X**	⟶	0 **X** 2 3 4 5 6 8
11 pirates:	0 1 2 3 4 5 **X** 7 **X** 9 **X**	⟶	0 **X** 2 3 4 5 7 9

Figure 8: Cases with 9, 10, and 11 pirates

This is a beautiful little problem that opens itself to many opportunities for exploration. We could change the elimination criterion to see what happens. You could try to come up with a mathematical description for the problem. I came up with

$$f(n) = 2^{[\log_2(n-1)]} + 1 - n,$$

where n is the number of pirates and $f(n)$ represents the first place that should be eliminated, as the number of positions to the left of the captain. Note that the [ ] is called the floor function (or a truncate function, as it truncates the decimal); it returns the largest integer less than or equal to the expression inside. I have graphed this function on Desmos, and you can check it out at **www.desmos.com/calculator/qg0hgnw7rp**. When I revisited my graph, I noticed that it didn't match my solution, and I saw that our group must have used a different numbering system than I have presented (we used the captain at position #1, the person to his left would be #2, and so on). I am not sure if the solution presented lines up with my original way of seeing the problem, or if I was influenced by other solutions that I saw from other groups. It just goes to show, even though I have seen the problem before, there is still something new to be discovered. Now for your homework:

> A line intersects the parabola with equation $y = -x^2 - 3x$ at points with $x = 2$ and $x = 4$. Determine the y-intercept of the line.

Until next time, happy problem solving! ▲

▲ Provincial Digital Learning Resources – What's New? Fraction Strips Tool




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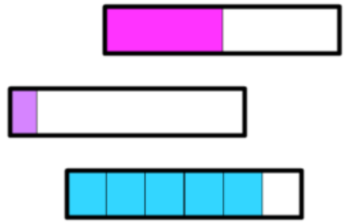
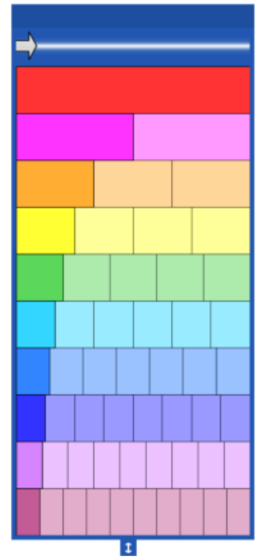
Agnes (Brant Haldimand Norfolk Catholic District School Board), Ross (Near North District School Board), Markus (Bluewater District School Board) and Greg (Simcoe Muskoka Catholic District School Board) are all currently on assignment with the Ontario Ministry of Education as Provincial Math Leads, working on the digital resources found at ***mathies.ca****.*

The most recent addition to the mathies collection (**www.mathies.ca**) of digital learning tools is an updated version of Fraction Strips Tool, which is now available for mobile devices as well as desktop computers. The revised version includes the often-requested annotation tools, image import, and undo/redo functionality. In addition, this version of the tool includes an infinite workspace that is pannable/scrollable and the ability to Zoom In, Zoom Out, and Zoom to Fit.

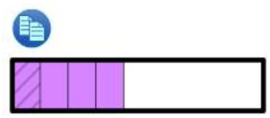
Let's explore the functionality of the Fraction Strips Tool by doing some math!

Representing Fractions

The tower contains fraction strips that are equi-partitioned into pieces. As students reason about the part-whole relationship represented by each strip in the tower, they count the number of pieces that make up a whole and name them as fractions; e.g., there are five equal-sized green pieces that make up a whole, and so one green piece represents $\frac{1}{5}$.

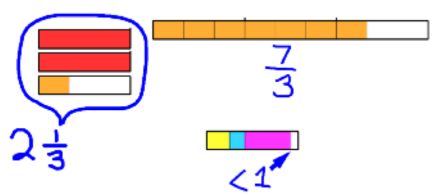



Drag fraction pieces from the tower into the workspace to create representations of various fractions. Drag the first piece on the left of a strip, to create a unit fraction. Starting with a piece further to the right in the tower creates other proper fractions. For example, drag the fifth piece in the row that is partitioned into sixths to create five-sixths. Pieces are always placed within a whole so that the part-whole relationship is reinforced.



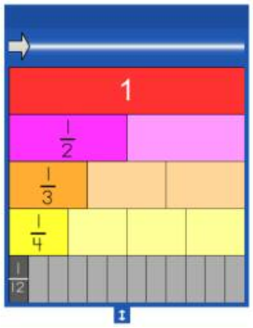
When fraction pieces are selected, a copy button appears. Students could practise their unit fraction counting skills by selecting a unit fraction, like one-ninth, and pressing its copy button repeatedly, counting 2 one-ninths, 3 one-ninths, etc.

The tool can also be used to create mixed fractions, improper fractions and fractions made up of unlike pieces.



As students represent various fractions, they may wish to adjust some of the tool options to assist them in their work.

Use the Customize Tower button to choose a colour palette (Rainbow, Original, or One Colour) or to change the colour of all the fractions pieces of a specified size. It is also possible to include or exclude strips from the tower. For example, you might want to set up a tower that shows only a specific set of strips, as illustrated here.



Additionally, students can control whether or not labels are showing. Press the label button to toggle between showing: no labels, the labels in the tower only, or labels in both the tower and workspace.

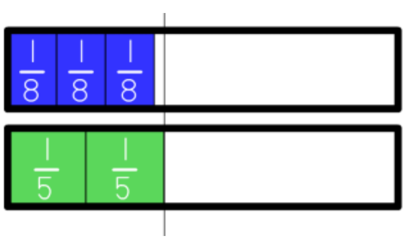
Suggested Representation Activities:

1) Represent various fractions in more than one way.
2) Represent a fraction that is really close to 1. How do you know?

Comparing and Ordering Fractions

The Fraction Strips Tool has been designed to facilitate the comparison of fractions. While fraction strips are technically an area model, students are able to compare fractions by attending only to the length, since the height is constant. A key feature of the tool is that the size of the whole is consistent for all representations. Consider that one-half of a large pizza and one-half of a small pizza are quite different quantities, since the wholes are different. Also, fraction strips can easily be aligned using the built-in snapping feature, which ensures that two fraction strips have the same starting position.

Students can use the Zoom In button to examine representations with greater precision. They might also activate the Vertical Comparison Bar to help visually track from one fraction strip to the other.



After zooming in, press the Zoom to Fit button to re-orient the workspace so that all fraction pieces are once again visible on the screen.

Once students have compared fractions, they can arrange them in the workspace to order them, using whatever rules they would like.

Suggested Comparison Activities:

1) Name two fractions. Which do you think is larger? Why? Use the tool to verify.
2) Solving problems like the *Bike-a-thon Task*:

   In a bike-a-thon, cyclists will find:

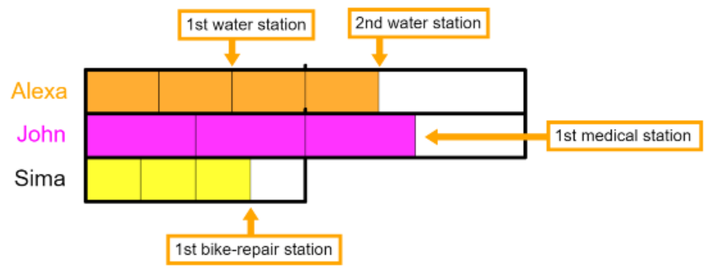
   - water stations every two-thirds of a kilometre
   - medical stations every three-halves of a kilometre
   - bike-repair stations every three-fourths of a kilometre

   John has reached the first medical station,

   Alexa is at the second water station, and

   Sima is at the first bike-repair station.

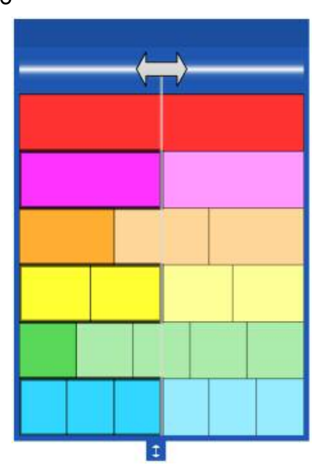
   Who is furthest along the course? (Adapted from Nelson Mathematics 6, 2006)



A detailed analysis of one solution of the bike-a-thon problem, including mathematical and learning needs considerations, is available on the support wiki page for the Fraction Strips Tool (**www.mathclips.wikispaces.com/FractionStripsTool#PDF**).

Equivalent Fractions

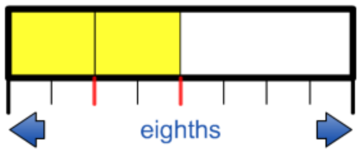
Understanding the concept of equivalent fractions is one of the fundamental building blocks needed for deep fraction understanding. Drag the Equivalency Bar, located at the top of the Fraction Strip Tower, to explore various equivalencies. Equivalent fractions are highlighted as the bar moves across the tower. This image shows that $\frac{1}{2}$, $\frac{2}{4}$, and $\frac{3}{6}$ are equivalent.



In the workspace, use the Rulers button to explore various equivalencies. The Fraction Strips Tool helps students understand two important equivalent fraction concepts.

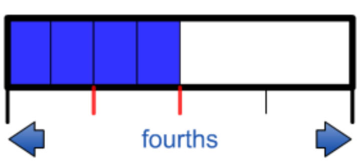
1) Splitting:

This image shows that $\frac{2}{4}$ is equivalent to $\frac{4}{8}$. Here, 2 one-fourth pieces have been dragged to the workspace, the rulers have been set to show the number stepper, and the stepper has been changed from fourths to eighths. Two ticks have turned red to indicate that the end of the piece lines up exactly, allowing a student to conclude that $\frac{2}{4}$ is equivalent to $\frac{4}{8}$. Students can continue to change the stepper. Notice which fractional units allow 2 one-fourths to be named nicely and which do not.



In fact, the representation supports students in understanding why multiplying both the numerator and denominator of a fraction by the same value determines an equivalent fraction. In this case, the eighth ruler shows the partition, or split, of each one-fourth piece into two equal pieces. This effectively doubles the number of pieces that are shaded yellow (the numerator), as well as the number of parts in the entire whole (the denominator).

2) **Merging:**
Another way to see that $\frac{2}{4}$ is equivalent to $\frac{4}{8}$ is to start by representing 4 one-eighth pieces. Next, adjust the steppers while watching for the red tick marks to indicate an exact match. The representation illustrated shows that every 2 one-eighth pieces can be merged to form a one-fourth piece. This is what happens when we form equivalent fractions by dividing both the numerator and denominator by the same value. Here we divide by two, which halves both the number of shaded pieces (the numerator) and the number of parts in the entire whole (the denominator). Students can continue to adjust the stepper to find other equivalent fractions that are created by merging smaller parts to make larger parts.



Fraction Operations

The Fraction Strips Tool can also be used to model the various operations and help students to develop conceptual understanding of these operations.

We will consider each of the four operations, using the fractions $\frac{4}{5}$ and $\frac{1}{4}$.

Addition of Fractions

Let's start with the sum of $\frac{4}{5}$ and $\frac{1}{4}$.

Before asking students to actually determine this sum, ask them to decide whether the result will be less than, greater than, or equal to 1. If we ask students to use their reasoning and proving skills to estimate the value of an answer prior to calculating it, they are more likely to reflect on the reasonableness of their answer after employing a standard algorithm.

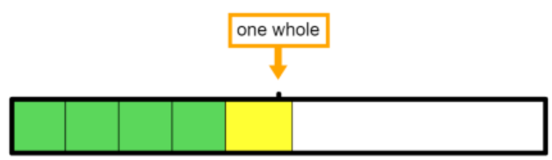
In this case, students can use the tool to reason that the sum of $\frac{4}{5}$ and $\frac{1}{4}$ will be greater than 1. Their reasoning might go something like this:

$\frac{4}{5}$ is one-fifth away from 1.

$\frac{1}{4}$ is larger than $\frac{1}{5}$ (since a whole partitioned into fourths has fewer pieces than the same whole partitioned into fifths, so the fourth pieces must be larger than the fifth pieces). *Notice in the representation that the yellow one-fourth piece is definitely bigger than the green one-fifth piece.*

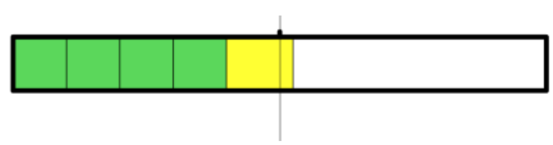
So adding a piece that is larger than $\frac{1}{5}$ to $\frac{4}{5}$ will result in a fraction that is larger than 1.

The operation of addition can be modelled by dragging the $\frac{1}{5}$ piece into the same whole as the $\frac{4}{5}$ piece to combine the two fractions.

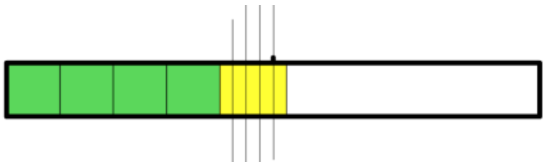


The visual representation quickly confirms the prediction that $\frac{4}{5} + \frac{1}{4}$ is larger than 1. To find the precise sum, we need to determine how much larger it is.

Using a Vertical Comparison Bar to indicate the whole can help with reasoning about the size of the piece that is to the right of one whole.

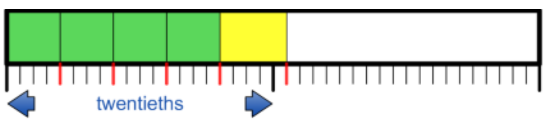


Use additional vertical comparison bars to equally partition the yellow one-fourth piece.



Visually, it would seem that the piece we are trying to name is $\frac{1}{5}$ of $\frac{1}{4}$. *Here, we intentionally use non-specific language, since we are working with pretty small pieces and it is difficult to be certain if our partitioning is exact.* If each of the 4 one-fourth pieces in a whole is partitioned into 5 equal pieces, there would be 20 pieces altogether. So, the sum of $\frac{4}{5}$ and $\frac{1}{4}$ looks like it could be $1\frac{1}{20}$.

Use the Ruler feature of the Fraction Strips Tool to confirm this reasoning. Clicking the button once shows the ruler; clicking again shows the fractional unit stepper.



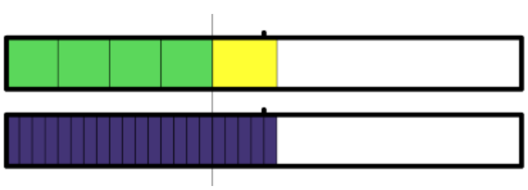
Adjust the stepper to twentieths. As the stepper is changed, notice that some ticks are black and some are red. Recall that the red ones indicate the piece matches that tick mark exactly. So, the red tick mark at the end of the yellow piece indicates that this strip is exactly $1\frac{1}{20}$.

Students can also use this representation to see that:

- $\frac{1}{5} = \frac{4}{20}$ (each green piece is split into 4 equal parts),
- $\frac{4}{5} = \frac{16}{20}$ (4 green pieces, each split into 4 equal parts gives 16 pieces), and
- $\frac{1}{4} = \frac{5}{20}$ (the yellow piece is split into 5 equal parts).

Hence, this representation might lead students to discover the standard algorithm for adding fractions, finding a common denominator. $\frac{4}{5} + \frac{1}{4} = \frac{16}{20} + \frac{5}{20} = \frac{21}{20}$

The model can also be used to show converting to twentieths.

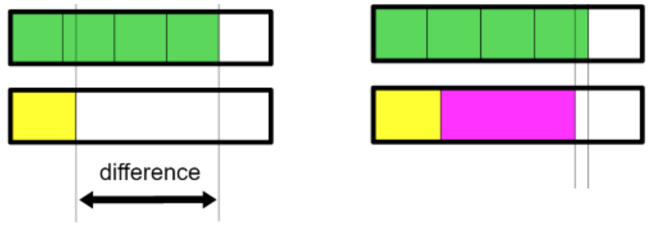


Subtraction of Fractions

First let's consider subtraction as the difference between $\frac{4}{5}$ and $\frac{1}{4}$.

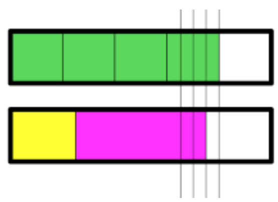
Once again, before computing, encourage students to estimate an answer.

The Fraction Strips Tool visual representation shows us that the difference is a bit more than $2\frac{1}{2}$ fifths, so the difference is just a bit larger than $\frac{1}{2}$ of the whole.



Confirm this reasoning by dragging a one-half piece into the strip beside the yellow one-fourth piece, to see that there is an additional white piece needed to make up the difference.

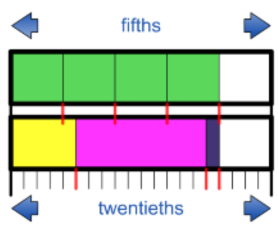
Students can reason about the size of this piece by creating additional vertical bars and using them to equally partition the one-fifth piece, much as we did for addition.



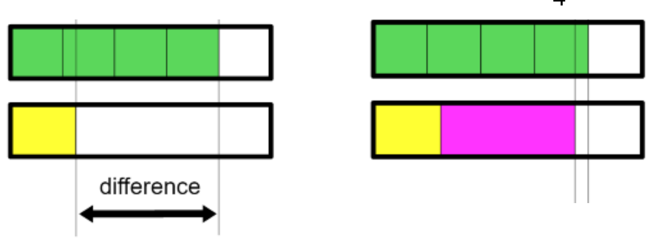
It seems that the missing piece we are looking for is one-fourth of one-fifth, which is one-twentieth (5 pieces per whole, each cut into 4 equal pieces).

So the difference appears to be $\frac{1}{2}$ plus $\frac{1}{20}$. Using their previously developed understanding of equivalent fractions, students can rename $\frac{1}{2}$ as $\frac{10}{20}$. An additional $\frac{1}{20}$ gives a total difference of $\frac{11}{20}$.

As with addition, turn on the rulers to confirm this reasoning and think about the common-denominator algorithm.



Alternatively, to model subtraction as the act of removing or taking something away, start by creating an equivalent representation of $\frac{4}{5}$ that includes a $\frac{1}{4}$ piece. Reasoning similar to the difference thinking above can be used to model this representation. Then, simply remove the yellow $\frac{1}{4}$ piece.



File Operations

One of the exciting new features of mathies tools is the ability to open and save files. Once you have used the

Fraction Strips Tool to solve a problem, open up the Settings and press Save.

Pressing the Save button will bring up the system dialog with a suggested file name and file type (e.g., .FractStrips for Fraction Strips). It is recommended that you do not change the file type, since the *Open* operation only finds files with that extension (or with .txt and .xml). The suggested file name is based on the date and time of saving and should be unique.

To retrieve this saved file at a later time, use the Open File button in Settings. Pressing this button will bring up the system dialog that allows you to find the folder in which you have saved the file.

Open and Save functionality is also available on mobile devices. Files saved on one device can be opened on another! For detailed support, you can visit **www.mathclips.wikispaces.com/File+Operations**.

Currently, Fraction Strips saves and opens only the actual state of the tool at the time of saving. When you open a file, the representations are entirely interactive and you can continue to add to your work. In the near future, you will also be able to undo back through the history of steps that led up to the point of saving. The development team believes that this feature will support a variety of assessment opportunities for students to explain their thinking.

Feedback and Future Requests

Please feel free to send us your feedback about any mathies tool, using the Feedback Form button inside the Information Dialog, accessed from the button. Visit the support wiki page for more examples and detailed descriptions of the functionality of the tool.

You can also send your comments to **WhatsNew@oame.on.ca**. You can share your experiences on Twitter, using the hashtag **#ONmathies**, and follow or message us at **@ONmathies**. There is an increasing set of interesting posts of student and teacher work on Twitter. To be among the first to find out about the latest digital-tool developments, sign up for our email list at **www.mathclips.ca/WhatsNewEmailList.html**. ▲

▲ Hey, It's Elementary: Why We Need Specialized Preparation for Teaching Kindergarten

LYNDA COLGAN
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Dr. Lynda Colgan is a Professor at the Faculty of Education, Queen's University. In addition to her teaching responsibilities in the BEd and Graduate programs, she is involved in research and knowledge-mobilization projects with the Council of Ontario Directors of Education, TVO/TFO, the Ontario Ministry of Education, and the Mathematics Knowledge Network.

You may have heard about the Math Knowledge Network (MKN), hosted by the Fields Institute for Research in Mathematical Sciences (Fields) through its Centre for Mathematics Education (CME). As one of the Knowledge Networks for Applied Education Research (KNAER), the MKN will bring together diverse mathematics education stakeholders from across Ontario, in both official languages. One of its goals is to mobilize evidence from research and professional practice in mathematics education and facilitate the use of evidence-based practices for mathematics instruction to support improved educational achievement. This will contribute to the Ontario Ministry of Education's Renewed Mathematics Strategy (RMS) in partnership with educators, researchers, and organizations across Ontario.

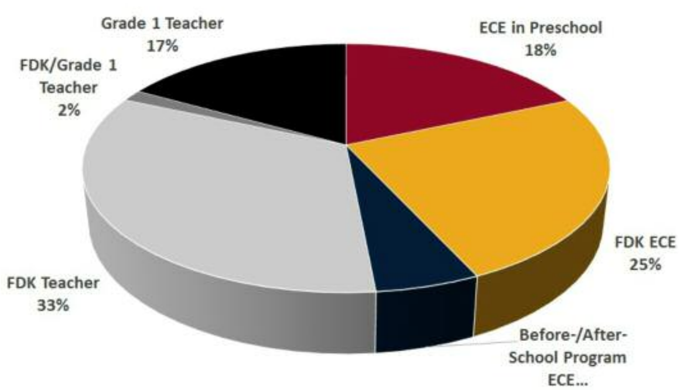
I am honoured and privileged to be the Lead for the Community of Practice (CoP) that is focusing on *Critical Transitions in Student Mathematical Development*. Currently, this CoP has four initiatives related to critical transitions: Early Childhood (which I coordinate); Grade 8 to Secondary School (Dr. Daniel Jarvis, Nipissing University); Grade 9 Locally Developed to Grade 10 (Dr. Ann Kajander, Lakehead University); and Secondary School to Post-Secondary (Dr. Peter Taylor, Queen's University). Over time, each of the individual projects within this CoP will report on its activities, resource development, and knowledge-sharing enterprises through multiple venues, and I will distribute summaries through this column in the *Gazette*. All materials generated by each unique project within the *Critical*

Transitions CoP will be available on the MKN website (**www.mkn-rcm.ca**) and through TeachOntario (**www.teachontario.ca**).

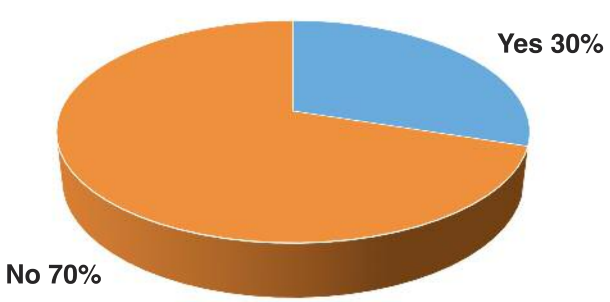
I am excited to report here on one of the early activities undertaken by the Early Childhood CoP: the results of an online Survey for ECEs, Kindergarten teachers, and Grade 1 teachers about their Early Math Education training. The survey also collected data about beliefs and practices. For this first phase of our work, participants were limited to personnel in the Greater Essex County District School Board and active members of the Association of Early Childhood Educators Ontario. The survey will be distributed provincially in Fall 2017 and the results published in Winter 2018.

There were 178 participants for the Phase I survey; however, 27 participants were removed from the data because of incomplete responses or consent not being given. For data analysis, 151 participants' responses were included. On average, the Early Childhood Educators (ECEs) had 13 years of experience, and the teachers, 15 years (11 of which were in early primary division, i.e., Kindergarten or Grade 1).

To get a sense of the demographics of the participant pool, we asked survey-takers to identify the role that they currently held. The graph below summarizes this information. (Note: FDK is Full-Day Kindergarten.)



The graph below shows participants' responses to the question, "Did you learn about early math development and education (preschool to Kindergarten) during your educator training?"



While we will report on the complete findings of the survey at a later time, it was the stunning response to this question that was the catalyst for an important event for the Early Years CoP.

On August 17, 2017, we hosted individuals from Faculties of Education, who teach Primary/Junior curriculum courses, and instructors from Community College ECE programs, at a full-day event to learn about the importance of early mathematics development (courtesy of Dr. Jean Clinton). We heard about the correlation between specialized professional preparation for pre-K and K educators and student achievement. Further, we will design an action plan for stronger mathematics preparation requirements for those who will be teaching in specialized early childhood environments.

Teaching in the Kindergarten program may be the most important assignment in a teacher's or ECE's professional career. However, in Ontario, we are doing little to prepare and support our Kindergarten educators to provide the robust mathematical knowledge children need in their earliest years of formal schooling. Here is our CoP's rationale for action:

1. An important predictor of children's achievement is the quality of the Early Years ECE and classroom teacher (e.g., Darling-Hammond, 2000; Darling-Hammond & Youngs, 2002; Hanhushek, Kain, & Rivkin, 1998).
2. There is compelling research to suggest that teacher preparation influences teacher effectiveness (Barnett, 2003).
3. Specific training in early childhood development, in particular, influences educator effectiveness in pre-K and K. Many studies have found significant relationships between specialized training in early childhood education and positive results for children. Teachers with specialized training have been found to provide more appropriate direction, build upon children's prior knowledge, "scaffold"—or layer—activities to develop emerging understanding and skills, and engage students in activities that are appropriately challenging, rather than merely repetitive. There is an abundance of research to demonstrate that teachers with training in early childhood development are better equipped to facilitate young children's language, cognitive, social-skills, and mathematics development (Pianta, Barnett, Burchinal, & Thornburg, 2009).

Why is teacher effectiveness in early mathematics so important? Because mathematics achievement is correlated to academic success in all subjects. Mathematical thinking is cognitively foundational, and children's early knowledge of math strongly predicts their later success in math. It is surprising to note that preschool mathematics knowledge predicts achievement into high school, and also predicts later reading achievement even better than early reading skills (Duncan & Magnuson, 2011).

In spite of the fact that much is known about how young children think about and learn math, that research is not finding its way into the pre-service or professional development of those who are in pre-K and K classrooms. In numerous countries, professionals in multiple educational roles vastly underestimate beginning students' abilities. One study showed that groups of teachers, teacher educators, and ECEs who worked with preschoolers underestimated the mathematical competencies of these very same students when they entered Kindergarten. For example, more than 80 percent of the students could count out nine marbles, but the adults' estimates were from 20 percent to 50 percent. More than 40 percent of the students could calculate the subtraction 10 – 8 without objects, but all adults estimated less than 10 percent. If teachers and those who work with teachers underestimate what students already know and can learn, they will not present appropriate, challenging mathematics activities that move the children along a positive learning trajectory (Van den Heuvel-Panhuizen, 1990).

While the previous examples emphasize the fact that many early childhood professionals are not aware of the mathematics developmental milestones that young children can meet, other research suggests that play alone is insufficient as a learning strategy. Mathematics concepts may be learned and conveyed through activities that children experience as play—but mathematics learning does not automatically happen through play. It is true that play or games can effectively reinforce and expand upon what children learn during more focused instructional times (Ginsburg, Lee, & Boyd, 2008; Klibanoff, Levine, Huttenlocher, Vasilyeva, & Hedges, 2006), but the success of the play-based environment is a function of teacher effectiveness and the implementation of *higher-level* free play. Well-prepared teachers (i.e., those who have specialized knowledge about how young children think about and learn math, and who have a clear sense of the mathematical goals for their students) are more sensitive and responsive to young children's mathematical thinking. According to researchers, it is only this specialized cadre that does not need to employ a structured curriculum in the early primary grades (Carpenter, Ansell, Franke, Fennema, & Weisbeck, 1993).

In conclusion, high-quality instruction has meaningful effects on children's mathematics knowledge, and high-quality instruction is designed and delivered by ECEs who have specialized knowledge. It is time to take a long, hard look at what we, as teacher educators, at the university (pre-service) or college level, or the in-service and AQ level, are doing to ensure that children in Kindergarten learn, and learn to love, mathematics. For that to happen, we need effective educators in every Kindergarten classroom in Ontario. For that to happen, their preparation must include mathematics education as a mandatory component. The importance of strong mathematics educators, especially in Kindergarten, cannot be overstated.

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Van den Heuvel-Panhuizen, M. (1990). Realistic arithmetic/ mathematics instruction and tests. In K.P.E. Gravemeijer, M. Van den Heuvel-Panhuizen, & L. Streefland (Eds.), *Contexts, free productions, tests and geometry in realistic mathematics education* (pp. 53–78). Utrecht, The Netherlands: OW&OC. ▲

▲ Ask Assessment Abby: Diagnostic Assessment

ASSESSMENT ABBY
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Ask Assessment Abby A³ is a regular column in the OAME *Gazette*, where teachers can share concerns and best practices about assessment, evaluation, and reporting of mathematics. Please send your questions to Ask Abby at **assessmentabby@oame.on.ca**.

Dear Assessment Abby,

My principal has asked me to do a diagnostic at the beginning of the year, but I am unsure of what it should look like. Please advise.

Well, let's start with the purpose of a diagnostic. *Growing Success* says that diagnostic assessment "occurs before instruction begins so teachers can determine students' readiness to learn new knowledge and skills, as well as obtain information about their interests and learning preferences" (p. 31). So it is more than assessing knowledge and skills; it should be well rounded and offer opportunities for students to demonstrate both their strengths and areas for growth. Effective diagnostics reveal student learning styles, interests, mindset, knowledge, and skills. All of this information (likely gathered from more than one diagnostic) contributes to the student learning profile.

Diagnostics reflect what we value as mathematics teachers. They may look different, for example, if we want to emphasize mathematical processes (e.g., problem solving, communicating, representing) or if the focus is on content knowledge. Even though paper-and-pencil tasks are an option (there are many commercial diagnostics available), there are other ways to elicit student thinking. If we want to get a sense of problem-solving processes and mindset, for instance, consider observations and conversations as students engage in problem-solving activities; ask questions, observe, and listen to students as they share. If you would like to document some of these activities, perhaps consider using apps, videos, and/or photos to capture student thinking. Always keep in mind the purpose of your diagnostic.

Diagnostics inform instruction and learning opportunities by identifying student strengths and areas for growth. For this, ensure diagnostics are done regularly and that they are well rounded, including more than just pencil-and-paper tasks. For more information, consider resources like *Growing Success*, *Learning for All*, or even *Sandra Herbst and Anne Davies'* blog.

Stay tuned for more on what to do with information that is gathered from diagnostics!

Keep math rich with students in mind,

Assessment Abby

Resources

Herbst, S., & Davies, A. (2013, March 20). Preparing diagnostic assessments [Weblog post]. Retrieved from **sandraherbst.blogspot.ca/2013/03/preparing-diagnostic-assessments.html**

Ontario Ministry of Education. (2010). Growing success: Assessment, evaluation, and reporting in Ontario schools. Retrieved from **www.edu.gov.on.ca/eng/policyfunding/growSuccess.pdf**

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▲ OAME Award

Secondary Teacher Award

Teresa Mosher, of Assumption College School in Brantford, has been recognized as the 2017 Outstanding Secondary Teacher. Teresa has had a sustained impact on students' learning and she shares her skills broadly, including with other schools and districts. "Her classes included a mixture of concept explanations, examples, and 'experiments'" (student words). Her colleagues also echoed these comments: "[She] is careful to include various forms of technology and practical applications into her lessons to make her students' learning experience a positive one." Teresa approaches people with respect, dignity, and inclusivity that inspire team building and a cohesive vision that all students can succeed. Words used to describe her throughout the testimonials included: dedicated, inspirational, genuine, engaging, creative, supportive, passionate, and superior. She changes students' lives and enhances the lives of the teachers and the community she supports. ▲

▲ Technology Corner: Desmos Geometry

MARY BOURASSA
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TWITTER: @MaryBourassa

Mary teaches mathematics at West Carleton Secondary School in Ottawa. She is a strong advocate for the appropriate use of technology in the classroom. She has presented workshops internationally, authored mathematics resources, is a past VP of OAME and a Past President of COMA. An award-winning teacher, Mary continually strives to learn new and better ways of helping students learn and love mathematics.

Last May, Desmos released a beta version of its geometry tool. It is, and will remain, free. You can find it at **desmos.com/geometry**. It is a clean, simple tool that will help students learn about geometry. Like all Desmos products, it is designed to be effortless and a pleasure to use, allowing all students to intuitively use it to immediately start creating. The Desmos team does not forsake that ease of use for power, so this geometry tool has begun with a small set of tools, which will expand in a thoughtful way over time. It will also integrate with the rest of their products, creating some amazing possibilities within Activity Builder.

The current menu features the following tools:

 The pointer tool allows you to select objects.

 The point tool allows you to place points on the canvas.

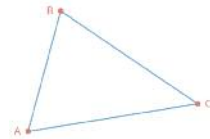
 The line tool allows you to add segments, lines, rays, and vectors to the canvas.

 The circle tool allows you to create circles based on the centre and radius.

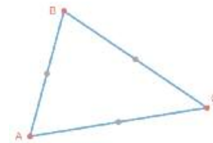
 The construction tool brings up a menu featuring midpoint, parallel line, perpendicular line, and compass.

Let me walk you through constructing the circumcentre of a triangle (intersection of the perpendicular bisectors) to show off some of the features.

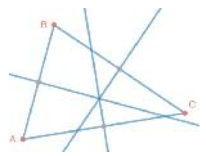
Start by creating three lines segments that join end to end. You can label the vertices by clicking and holding on each, then checking the label box, after which you have the option to edit the label (the labels automatically come up as A, B, C, etc., respecting the order in which they were created). This same pop-up menu allows you to change the colour of your object.



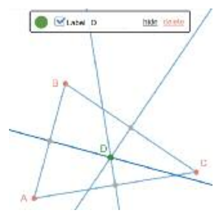
Next, use the midpoint tool (on the construction menu) and click on each segment to create its midpoint.



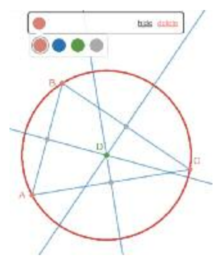
You can now use the perpendicular bisector tool to construct a line through each midpoint, perpendicular to each line segment.



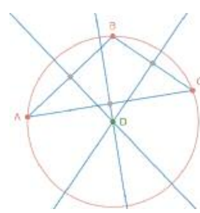
To make the intersection more obvious, add a point, which I labelled D. I also changed its colour. Notice that within this same pop-up menu, you can hide objects or delete them.



Finally, to show that point D is the same distance from each of the triangle's vertices, I created a circle centred at point D through A, B, and C. Again, it is simple to change the object's colour.

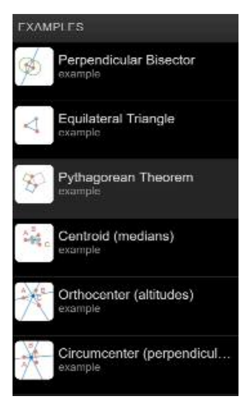


Because this was constructed, all of its properties hold true when you move the vertices around the canvas.



As with all of Desmos' tools, this is easy to use and intuitive. Students will need very little, if any, guidance from their teacher to get them started. They can create, construct, and explore.

There are also examples provided to help you and your students begin. These will let your students explore the mathematics without having to create the constructions themselves.



What's coming next? By the time you read this, you will be able to add measurements and angles to your files, as well as save them and share them. Enjoy playing with this new tool and all the new features as they roll out. To keep up with the latest, be sure to subscribe to the Des-Blog at **blog.desmos.com**. ▲

▲ Report from the 2017 CMESG Conference

JUDY MENDAGLIO AND JILL LAZARUS

Judy Menaglio is the Past-President of OAME. Jill Lazarus is the President of OAME.

The Canadian Mathematics Education Study Group, more commonly called the CMESG, had their 41st Annual Meeting from June 2–6, at McGill University in Montreal. Even though it was unintentional, we both happened to register this year so OAME was well represented.

The conference had more than 200 delegates from across Canada and the United States, and a contingent from Nigeria. Participants were mainly from the mathematics education research community, but there were many math teachers from all levels: pre-K, elementary, secondary, college, CEGEP (Collège d'enseignement général et professionnel), and universities. A unique feature of the CMESG Conference is that it is fully bilingual.

CMESG brings together such a diverse group of people because they all share a deep interest in how students learn mathematics and how educators can support their learning. The main environment for this sharing and collaboration is the Working Group, which is a wonderful place of sharing and collaboration. For three mornings, the same group of participants shares, brainstorms, discusses, and analyzes a single topic. The Working Groups are facilitated by two or three leaders, who produce a report after the conference has ended. The report summarizes the discussions, ideas, and recommendations of the group. This year's Working Groups were:

A: Teaching first-year mathematics courses in transition from secondary to tertiary,

B: Elementary pre-service teachers and mathematics anxiety: Searching for new responses to enduring issues,

C: Social media and mathematics education,

D: Quantitative reasoning in the early years,

E: Social, cultural, historical, and philosophical perspectives on tools for mathematics.

We attended different working sessions and both thoroughly enjoyed the discussions.

In addition to working groups, the CMESG Conference included plenary talks, small-group discussions, PhD thesis presentations, ad hoc sessions (delegates can sign up on-site to facilitate a session on a topic of interest), and a panel discussion. Reports on these sessions will be available in the conference proceedings when they are available. In the meantime, archived reports from past CMESG conferences at **www.cmesg.org/past-proceedings/**.

We had the pleasure of hearing from math education legends, Annie and John Selden. Annie was a plenary speaker, while John led a topic session. Small-group sessions gave participants the opportunity to talk about the plenary and then compile a list of questions for the plenary speaker. The whole group then reconvened, giving the plenary speaker an opportunity to answer these questions.

On the final day, we watched a debate that included our own *Gazette* columnist, Stewart Craven. He was sporting a new T-shirt that challenged us to "Think outside the Klein bottle."

Then we heard an Elder Talk from Joel Hillel, who started us off with an interesting puzzle; something to think about if we lost interest in his talk! The puzzle is the Four-Tumbler Problem: Four tumblers (glasses) are on a Lazy Susan. Some are upright and some are upside down. You are blindfolded. Your task is get all four tumblers oriented the same way (all four right side up or upside down) in a finite number of steps. You achieve this through the following process: You spin the Lazy Susan. After each spin, you select two tumblers. After inspecting them (you can't see them), you can change the direction of both, one only, or none. When you achieve the goal, a bell will ring, telling you that you are done. The purpose of the puzzle is to come up with an algorithm, or process, which anyone could follow, that would guarantee that a blindfolded person could achieve the goal that all tumblers have the same orientation in a finite number of turns. Thus, you cannot be counting on luck!

This is just a sample of the whole program, which ended with preliminary reports from the Working Groups. We left exhausted (they made us work all day!), but invigorated. How wonderful to be with so many people of such diverse backgrounds, who are all eager to get together to further the field of mathematics education research.

Next year, the CMESG will be hosted by Richard Hoshino at Quest University in Squamish, British Columbia. You may remember Richard if you came to the 2015 Leadership Conference, where he spoke about his book, *The Math Olympian*. (A review can be found in *Gazette* 54(1).) Further information about next year's conference can be found at **www.cmesg.org/about-cmesg/**. ▲

▲ In the Middle: Area in a Different Way

CARLY ZINIUK
EMAIL: carlyziniuk@gmail.com

Carly Ziniuk teaches Grade 9 Mathematics, Grade 12 Data Management, and Advanced Placement Statistics at the Bishop Strachan School in Toronto, Ontario, Canada. She is very active in adopting real-life data to engage her students in solving problems.

In intermediate grades, the transition that increases the formality of algebraic thinking is challenging and it is difficult for students to communicate their thinking during the evolution. Reflecting on this led to my review of the questions I commonly use for instruction, and I found they demonstrated the achievement categories of communication and knowledge of mathematics. This was an important consideration because the questions were from the *Gage Mathematics Assessment Series* for Grades 7, 8, and 9 (Flewelling & Lemenchick, 1997), but I adapted them to fit the curriculum revisions that have taken place since then. It is the taking of a good idea from an older resource and revitalizing it to the current curriculum that is the key idea in this column.

This year, my Grade 9 students and I used these adapted questions to review Grade 8 algebra concepts. Some students had previously used hands-on approaches with algebra tiles, while others had a strong mastery of the skills, yet with less overall understanding of how the algebra could be represented visually.

The original questions just showed a rectangle and asked for the perimeter and area to be expressed algebraically. I liked the connection of area and algebra, which has been particularly useful to direct my students to express their thinking using algebraic communication in a variety of ways, and connect to their visual representation as well. I looked at some of the problems along with work I had done with a colleague to encourage small-group conversations in pairs and fours. The group learning was then used to bring student thinking into larger class discussions.

The long-standing use of the questions, which had been adapted through curriculum changes, brought to mind the possibility of further modifications. I considered ways they might be customized or utilized to encourage students to use multiple approaches to solve problems. We discussed in our class how students could start the same problems in different ways and how they would end up with the same result.

I used versions of these closed-ended questions to get at the communication of area and algebraic representation. As the students looked at these simpler problems, some students were able to explain their thinking well orally, yet had to be prompted to write down their thinking. Others found it easier to visually communicate their thinking with a diagram. Either way, they found the algebraic representations were consistent.

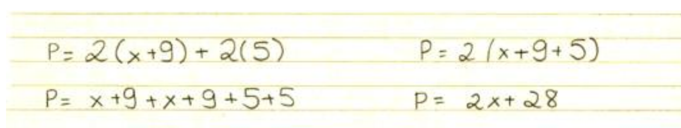
1. **C, K** Write an algebraic expression that represents the perimeter of this rectangle.

   5

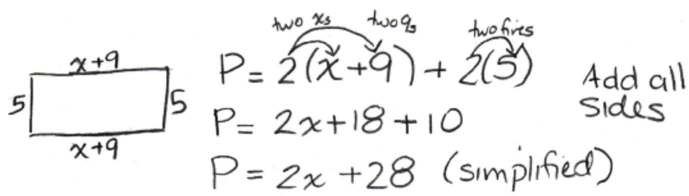
   $x + 9$

Seeing the students finding the perimeter in different ways was encouraging. Among the multiple approaches was 2 (x + 9 + 5) as well as 2 (x + 9) + 10 and x + 9 + x + 9 + 5 + 5. When groups compared solutions, they connected the approaches by discussing the collecting of like terms and the use of the distributive property. Using algebra to communicate their thinking showed clearly that, regardless of how they started, algebraic simplification resulted in the same answer.

One student started her work on the left, identified her partner's solution as quicker on the right, and then compared the two different versions. She took the time to show all the different ways she could get the same answer.



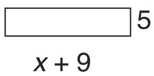
Another student demonstrated a comprehensive illustration of the distributive property.



She used words beside her work to clarify her overall strategy. In addition, she used arrows to remind her of the process for using the distributive property.

Since my students routinely present answers and questions in my Problem-Based Learning classroom, seeing different methods worked out the same was typical. However, this particular question led to comments such as, "I liked her way better than the way I did it." I hadn't asked the students to do this type of question in more than one way in the past, but this adaptation became the focus of my instructional use of this set of questions. It gave them many opportunities in the classroom to communicate their varied approaches and check that their answers, when simplified, all turned out the same.

2. **C, K** Write an expression for the area of the same rectangle. How is it different from the perimeter?

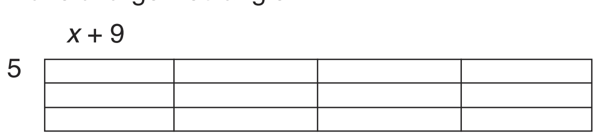


Following a review of the distributive property with our class, a discussion moved to area. Inviting classroom discussion about how perimeter and area were different brought conversations about the differences between the expressions $5x + 45$ versus $2x + 28$. Surprisingly, it also led to discussion that one expression was communicating squared units (of area), and the other, linear units (of perimeter).

The students recognized that using algebra tiles was a good way to determine if they got the same answer when solving in different orders or different ways. Some students tested out the distributive property by giving x different numerical values, and "proved" to themselves and to others in the class that it actually worked. Other students were able to reason, in class discussion, that it didn't matter what the numerical value of x was, and in fact that was an important characteristic of the algebraic communication. In the future, I see an opportunity to promote recognizing that algebra provides a stronger "proof" because it is true for all numerical values. Having students appreciate this generality over only a handful of tested values will make an interesting classroom conversation.

When I noticed how quickly students were using this multiple-approach strategy, I adjusted more questions to suit this instructional method. Moving away from closed-ended questions to ones requiring more thinking gave the students a chance to examine the many ways they could communicate their ideas. In some cases, I included an explicit expectation in the question, for example:

3. **C, T** Twelve of these rectangles are combined to make a larger rectangle.



How many different ways can you express the area of the largest rectangle?

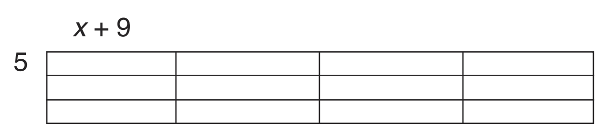
What happens when you simplify the different expressions?

With students divided into pairs, I gave them the task of figuring out two different ways of expressing the area and then asked them to explain in words. Students focused on finding many ways to obtain the area, including looking at the area of one small rectangle and then multiplying by 12 (i.e., $12\,(5x + 45)$). Another solution was to first find the larger rectangle's length (i.e., $4x + 36$) and the width (i.e., 15), which had the benefit of requiring them to practise adding like terms again! Other solutions arose when students visualized the rectangle being created from three long, thin rectangles or two "six-section" rectangles.

It is important to acknowledge that students don't always succeed. In my classroom, one student thought she had a solution, but her partner showed why it wasn't correct. By providing a safe, non-critical space in the classroom, dialogue about errors can be quite informative to students. It is similar to sharing multiple-solution strategies because students reap the benefit of seeing multiple ways things can go awry.

A student anticipated my lesson plan and predicted to the class that there would be a similar question that involved perimeter. She was quick to explain to her classmates that we could find the perimeter by adding the length and width of the overall rectangle and then doubling the sum. Her partner wondered if they could just multiply the perimeter of a smaller rectangle by 12, since they had found that worked for area.

4. **C, T** Twelve of these rectangles are combined to make a larger rectangle.



How many different ways can you express the perimeter of the largest rectangle?

What happens when you simplify the different answers?

As you can imagine the students felt ownership of what they were doing. There was a similar rich discussion that highlighted that, unlike area, perimeter of smaller rectangles does not sum to the perimeter of the larger rectangle. Rather than recount the details, I will simply encourage you to try it with your students!

To this point, I have presented questions that are based on *assessment for* learning, with a focus on the communication category of achievement. The questions were presented with a diagram and guided students in a direct fashion as to what to do. The key learning for students was to consider that multiple approaches could be shown to be the same by algebraic simplification. This led me to think about how to move to *assessment of* learning. How can question adaptations and lesson directions allow students to be more independent, engage in self-reflection, and express their thinking more effectively? One approach is to use questions that provide a statement, but no diagram. Students follow the earlier examples and naturally draw a diagram as well as introducing and working with variables of their own choosing.

In the following class, I provided students with a series of more open-ended questions to allow them greater independence in exploration, more time to work at their own pace, and a chance to address more than one question before checking in with the teacher. I have used this strategy for their homework, as well as classroom discussions of answers, rather than just working in class. As an example of an *assessment of* learning series of questions, consider the following:

5. A rectangle is twice as long as it is wide.

a. **C**, **A** Draw a diagram using the variable w to indicate the width.

b. **T**, **C** What does the equation $w + 2w + w + 2w = 39$ tell you about the rectangle?

c. **C**, **T** What other pieces of information can you figure out about the rectangle from the equation above?

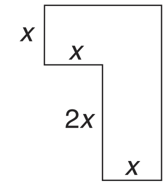
d. **T**, **A**, **C** How would the situation change if the original question had been written as, *"A rectangle is twice as wide as it is long."* Would your answers in Parts a, b, and c have changed? Why or why not?

e. **K**, **T**, **A**, **C** How would the situation change if the original question had been written as, *"A rectangle is half as long as it is wide."* What would have changed in your earlier answers?

This problem gave students an opportunity to prepare solutions based on their own learning and efforts to explain ideas to each other. It also allowed them to reflect on their own thinking. Notice that the series of questions connect to each other, providing students with the opportunity to record their ideas and develop them more fully before checking.

Another example that highlights the multi-step approach came from questions my Grade 8 students had previously developed when given just the diagram. Using the student-created questions with the Grade 9 students gave them the chance to consider an important goal of multiple representations.

6. For this diagram, the opposite sides are parallel with all angles at 90°, except for one interior angle of 270°.



a. **K**, **T**, **A** Determine an expression for the perimeter in a meaningful way; then simplify it, if possible.

b. **K**, **T**, **A** Determine an expression for the area in a meaningful way; then simplify it, if possible.

c. **K**, **T**, **A**, **C** Brodie discovered that this shape can be broken into two congruent parts. Find the area of one part and use that area to determine the area of the entire figure. Does that agree with your work above?

d. **K**, **T**, **A**, **C** Natalie, however, thought of it a different way. She thought that the shape could be broken into four congruent parts. Is that true and can you use this to determine the area another way? Does it agree with your work before and/or with Brodie's discovery?

e. **K**, **T**, **A**, **C** Taylor said it is easier to think of it as a big rectangle with dimensions of length ($3x$) and width ($2x$) and having a small rectangle cut-out of it. How big would the rectangle cut-out be? What would its dimensions be?

Among the learning goals in our Grade 9 Academic Mathematics class are: "I can find errors in algebraic work" and, "I can explain my algebraic thinking to others." These are clearly matched with our communication assessment goals. There is evidence of students also showing development of the learning skills of initiative and collaboration. The students worked to set goals independently and in small groups, endeavouring to move toward them, recognizing what needs to be done, and knowing what to do when difficulty arises.

At the beginning of Grade 9, the class looked at manipulating polynomial expressions by reviewing their Grade 8 understanding. It was very helpful to use some closed-ended questions in pairs and fours so that I could collect observations quickly. Those observations let me see that students' understanding was varied both in algebraic skills and experiences. Focusing on multiple representations is one of the big changes in the Ontario mathematical curriculum and expectations in classrooms. Not all questions can be done in this way, but taking those questions that can, and then opening them up for students to examine both independently and collectively, does refresh an older resource for our current classrooms.

EQAO often includes questions involving area and perimeter, using algebraic representation, and this adaptation does lend itself to some of those more closed-ended questions. Additionally, along with students, I was able to connect ideas, particularly the different units of area and length, to the exponent rules, and again when we revisited optimization of two- and three-dimensional shapes. Having the students start by checking in small groups, with smaller questions, empowered them so that they felt more confident when solving bigger problems. They were subsequently more likely to try to find a variety of ways to solve a problem, and looked at ways to extend problems by asking open-ended questions. It was also beneficial for me to recognize that a few small, older-style questions could be successfully reworked to make a multiple-representation introduction to students' algebra unit. So perhaps it's time to go back and have a look at a few of those older resources to find some of the gems that just need to be revised for today's students.

Reference

Flewelling, G., & Lemenchick, C. (1997). *Gage mathematics assessment series.* Vancouver, BC: Gage Learning Corp. ▲

▲ OAME Award

Secondary Staff Award

Alexander Overwijk and **Bruce McLaurin**, from Glebe Collegiate Institute, Ottawa, are the 2017 recipients of the Secondary Staff Award. Their nominators praised their professional efforts in the highest possible terms; a student gave particular praise. Alexander and Bruce were cited by all for their creative, student-centred approaches to learning; for their collaborative efforts that extended throughout their schools; and for their pioneering efforts in instructional practice that, as a student remarked, "[helped] students to become innovators outside of high school." Their work also extended beyond the physical boundaries of their own school, both through personal presence in professional activities as well as the production of instructional materials. ▲

▲ Ontario Mathematics Olympiad – 2017

OMO 2016 CO-CHAIRS
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Bart Vanslack is a Math Resource Teacher for the Toronto Catholic District School Board, where he supports teachers and schools in Numeracy. Bart is an active member of TEAMS and one of the Elementary Directors of OAME. He also is one of the Course Instructors for OECTA for the Math AQ courses.

EARL TOTTEN
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Earl Totten is a Mathematics Resource Teacher for the Toronto Catholic District School Board. Prior to education, he enjoyed a career as both a consultant and principal with an actuarial consulting company in Toronto. He has been known to shudder when someone says, "nine over ten" or "zero point nine" instead of nine-tenths!

The OAME Ontario Mathematics Olympiad (OMO) is a fun, challenging mathematics competition for Grades 7 and 8 students. The students are required to solve questions based on the Grades 7 and 8 Ontario mathematics curricula. The TEAMS (Toronto Educators Association for MathematicS) chapter hosted the 2017 OMO at Humber College, Lakeshore Campus, on May 26 and 27. Twenty-five teams of four students, two from Grade 7 and two from Grade 8, from OAME chapters across Ontario, competed in a series of math challenges.

The students participated in four challenges from three categories: individual, two paired activities, and a team activity. Mathletes demonstrated their understanding of the mathematical concepts, their ability to solve problems, communicate their knowledge, and work collaboratively with their teammates. The activities included:

Team Activity — A relay of short, but challenging, mathematics problems. A given problem needed to be solved before the team was given the next problem.

Problems were taken from all strands of the Grades 7 and 8 curricula. Group discussions were encouraged in order to have the students communicate their understanding.

Embedded Cubes Paired Activity — This required students to build models to investigate patterns involving dimensions, area, and volume of cubes or parts of cubes. The students generalized the patterns with a formula, or with a written description, and extended to larger dimensions.

Probability Task Paired Activity — Students were required to determine the probabilities of different types of triangles (scalene, equilateral, isosceles, right, etc.) being formed using side lengths defined by the roll of three dice. The students provided sample diagrams and an explanation to support their calculations.

A special thank you to our sponsors, Spectrum Educational Supplies, Texas Instruments, and Future Design School.

Congratulations to ISOMA Team 2 (Daniel Yang, Emily Liu, Eric Shen, Halley Halim), who won the competition as best overall team.

Second place overall went to ISOMA Team 1 (Martin Tang, Orelia Pi, William Szeto, Jennifer Wang).

Third place overall went to SAME Team 1 (Jason Chen, Lucy Shen, Eric Du, Jaqueline Wang).

▲

▲ Update on the Math Knowledge Network

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Judy Mendaglio is the Past-President of OAME. Jill Lazarus is the President of OAME.

In June of 2016, the Ontario Ministry of Education, in conjunction with the Knowledge Network for Applied Education Research (KNAER), announced the creation of a new Mathematics Knowledge Network (MKN). In November 2016, the announcement was made that the MKN would be hosted by the Fields Institute for Research in Mathematical Sciences.

Information describing the MKN can be found in the June 2017 issue of the *Gazette*. Alternatively, there is a summary page at **www.knaer-recrae.ca/images/MKN%20Overview%202016_v.01_2016-11-08.pdf**.

The MKN has been very busy, despite being in its inaugural year. It has grown to include over 12 school districts, 13 universities, and 16 community organizations.

The first Math Knowledge Network Advisory Panel meeting took place at the Fields Institute on February 27 and 28. At this meeting Communities of Practice (CoP) met and started planning. OAME is a partner to all four CoPs:

- Computational Thinking in Mathematics Education
- Critical Transitions
- Indigenous Knowledge and Mathematics Education
- Mathematics Leadership

Since the advisory panel meeting, many CoP members have met, and their activities are described below for each CoP.

Those interested in Computational Thinking in Mathematics Education will be attending the Computational Thinking in Math Education Symposium at the University of

Ontario Institute of Technology from October 13–15. You can also see the CoP's Repeating Patterns Lesson Study Documentaries at **http://mkn-rcm.ca/repeating-patterns/**.

The Critical Transitions CoP held an event at the University of Guelph-Humber on August 17 called *Kindergarten Forum for Action: Building Mathematical Foundations for Young Children*.

Members of the Indigenous Knowledge and Mathematics Education CoP had a brief meeting at OAME 2017 to become acquainted with one another. Perhaps you participated in the surveys that were linked on the OAME home page? This group also had a planning meeting at Trent University from June 27–29. In October, there will be follow-up at the First Nations, Métis and Inuit Education Association of Ontario (FNMIEAO) Conference in London.

The Mathematics Leadership Network's (MLN) "Not a Book Study" was very successful and generated a lot of enthusiastic Twitter chatter. All the podcasts with Dr. Cathy Fosnot have been archived at **https://voiced.ca/not-a-book-study-with-cathy-fosnot/**.

TeachOntario is also a partner with the MKN. They will host sharing sessions for Ontario teachers through online discussion groups at **TeachOntario.ca**. Check out MKN resources at **www.teachontario.ca/community/explore** by looking for the picture below:

As you can see, there is a lot happening in mathematics education in Ontario, thanks in part to those at the Math Knowledge Network. For more information, visit **mathnetwork.ca**, watch the OAME home page and Twitter feed, or sign up for your own updates at **http://mkn-rcm.ca/**. ▲

▲ OAME 2018: Infinite Possibilities

WAYNE ERDMAN
CHAIR, OAME 2018: INFINITE POSSIBILITIES

Wayne is a long-time member of OAME and has been involved in the annual conference as a regular presenter and as an organizer. Wayne has now been part of five organizing teams, chairing or co-chairing three of them.

The OAME conference returns to the Toronto area in 2018, after two very successful conferences in Barrie (2016) and Kingston (2017). The theme of OAME 2018 is "Infinite Possibilities." Influenced by positive thinking, progressive teaching methods, and positive choices, Infinite Possibilities means that you can be both enlightened and achieve your dreams. How does this relate to a mathematics education conference? One of the defining features of the annual conference is the strength of the program that is offered to delegates (attending online and in person), and OAME 2018 promises to continue this tradition. OAME 2018 will be boasting a lineup of keynote and featured speakers, who will make this conference one that opens you up to infinite possibilities in furthering your professional career, and opens up a world of infinite possibilities for your students.


Peter Liljedahl

Thursday's program will kick off with your first breakout session, to get everyone into the action. We will then introduce keynote speaker Dr. Peter Liljedahl, Associate Professor of Mathematics Education at Simon Fraser University. Many classrooms across the province are building on the research Dr. Liljedahl has done on vertical non-permanent surfaces and visible random groupings. Rounding out the Thursday daytime program will be featured speakers Fawn Nguyen and Marian Small. This is Fawn's first time presenting at the conference, but she is well known to many educators in the province, primarily through her work on **visualpatterns.org**. Marian has presented many times and continues to challenge and help us integrate progressive methods of mathematics instruction and assessment. There will also be numerous breakout sessions from which to choose.


Jo Boaler

If you thought that there wasn't much more you could get out of a Thursday, you would be wrong. Once the sun goes down, the professional learning continues. Replacing the traditional Saturday program, Thursday's *OAME After Dark* will feature an abbreviated program, with Dr. Jo Boaler delivering a keynote that will bookend the day's events. Dr. Boaler, Professor of Mathematics Education at the Stanford Graduate School of Education, is the author of the acclaimed book *Mathematical Mindsets*. This will be a jam-packed day of great learning and networking.


James Tanton

Thursday will be tough to follow, but Friday will continue the rich professional learning. The keynote for Friday is James Tanton, Mathematician-in-Residence at the Mathematical Association of America. Have you heard of the Exploding Dots? James Tanton is the brainchild behind these, as well as Global Math Project, and Global Math Week (10/10/2017) See **www.youtube.com/watch?v= 1fXyW1vB4vI** to get a sense of Exploding Dots. Also appearing on the Friday program as featured speakers are regular *Gazette* contributor Mary Bourassa (also the curator of **wodb.ca**), and Dr. Cathy Bruce, whose work around spatial reasoning is at the forefront of much learning and research happening around the province. With this lineup of speakers, coupled with the strong lineup of workshops that is a staple of the conference, you could say that the learning is infinite.

For those of you who are outside a 100 km radius of Toronto, who cannot make it to the conference in person, we will again be offering an eConference. We will live-stream one day's major events, and individuals, schools, or departments can share in the professional learning environment with those attending the conference in person.

We invite those of you who are thinking about presenting a session at OAME 2018 to register by November 1 at the MCIS (Mathematics Conference Information System) website (there is a link on the OAME home page). We are always looking for interesting workshops on one of our strands: Technology, Assessment, Leadership, Instruction, and FNMI focused.

Visit **www.oame2018.ca** for more information about the conference. Place the OAME conference on your calendar—May 3 and 4, 2018, at Humber College North Campus in Toronto—and prepare to open up infinite possibilities.

Editor note: Pictures from:
Peter Liljedahl – **www.sfu.ca/education/faculty-profiles/pliljedahl.html**
Jo Boaler – **www.telegraph.co.uk/education/education opinion/12045884/Times-tables-are-not-how-you-teach-maths.html**
James Tanton – **www.dailyheraldtribune.com/2014/11/30/ gppsd-hosts-math-information-session-for-parents**

Some interesting further reading! ▲

▲ Teaching Grade 9 Applied Mathematics: A Collaborative Inquiry

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Dr. Chris Suurtamm is Vice Dean of Research and a Professor of Mathematics Education at the University of Ottawa. Her research focuses on the complexity of mathematics teaching, and the use of formative assessment to improve student learning. She has been involved in many large-scale research projects and is dedicated to connecting research and practice. She has been the Canadian Representative on the NCTM, Co-Chair of Assessment Topic Study Groups at International Congresses for Mathematics Education, and a member of the Ontario Ministry of Education Curriculum Council.

JILL LAZARUS
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Jill Lazarus is a PhD Candidate at the University of Ottawa, where she is working under the supervision of Dr. Christine Suurtamm. As part of the University of Ottawa research team, Jill was a research assistant for the Grade 9 Applied Project. She is also a high school teacher in the Renfrew County District School Board, and is the current President of the OAME.

KELLY MCKIE
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Kelly Mckie worked in the oil and gas industry before returning to school to pursue a career in mathematics education. She has worked with teachers while studying at the University of Calgary, where she partnered with the Galileo Educational Network Association (GENA). It was with GENA that she first worked with teachers, Professional Learning Communities, and was introduced to the lesson study model. She is pursuing a master's degree at the University of Ottawa under the direction of Dr. Christine Suurtamm, and was a research assistant for the Grade 9 Applied Collaborative Inquiry Project.

This article provides a brief report on the OAME Grade 9

Applied Project that took place during the 2014–2015 and 2015–2016 school years and involved Professional Learning Communities (PLCs) in ten schools across Ontario. The Ontario Ministry of Education (OME) provided funding during the 2014–2015 and 2015–2016 school years to the Ontario Association for Mathematics Education (OAME) to develop and research a professional learning initiative that would support ten school PLCs across Ontario. The Steering Committee that guided the project consisted of representatives from OAME, the Ontario Mathematics Coordinators Association (OMCA), the Ontario Ministry of Education, and a University of Ottawa research team, chosen to support and document the project. This article gives an overview of the project followed by descriptions of the types of activities that the PLCs engaged in that supported Grade 9 Applied Mathematics teaching and learning.

Each school PLC was comprised of a school administrator, a Mathematics Lead at either the school or board level, two to three teachers of Grade 9 Applied Mathematics, as well as a Special Education Resource Teacher and/or Student Success Teacher in the school. The main objectives of this project were for the school PLCs to focus on enhancing PLC participants' understanding of the Grade 9 Applied Mathematics curriculum, and to focus on implementing the curriculum in ways that would best meet the needs of their students. It is well recognized that the teacher is the key element in educational change and classroom improvements, and that support from administration and those supporting teaching and learning is crucial (Fullan, 2007; Hargreaves, 1998).

Each PLC met monthly, and during their initial meetings, they determined aspects of teaching and learning to focus their activities. They also engaged with research and resources that related to their area of focus and that provided evidence of effective practices. Members of the University of Ottawa research team attended meetings to collect data for research purposes and to offer resources and other support when needed; focus-group interviews and relevant artifacts and professional learning logs were also collected at the individual school meetings and at organized gatherings of all of the school PLCs. In order to better understand the collaborative inquiry process and ways to support Grade 9 Applied Mathematics teaching and learning, each school PLC was considered a case study, and the research team carried out a cross-case analysis of data that were collected during the 2014–2015 and 2015–2016 academic years.

As well as meeting monthly, all of the PLCs came together on several occasions. In October 2014, all of the school PLCs met at a project launch, where they had the opportunity to further define and clarify what they perceived to be challenges facing Grade 9 Applied Mathematics, to organize their plans for addressing these challenges, and for working collaboratively throughout the 2014–2015 academic year. There were also presentations on effective professional learning models and activities that helped to focus attention on the content and intent of the Grade 9 Applied Mathematics curriculum. Enhancing teachers' understanding of the Grade 9 Applied mathematics curriculum was an important focus of this project. This emphasis was reinforced by Macaulay's research (2015) that highlighted the characteristics of schools with successful Grade 9 Applied Mathematics programs. Her findings indicate that focused attention on unpacking and implementing the mathematics curriculum were part of these schools' success.

Near the end of the first year, the school PLCs met again for one day, at which time they shared their work, including both the challenges they experienced and what they observed to be effective practices. They met together again for a two-day Summer Institute in August 2015, before the start of Year 2, to further share their work, engage in workshops, and to plan for their second year of work. These cross-PLC meetings were significant to their continued work, as many PLCs were working on similar tasks and goals, and coming together allowed ideas to cross-pollinate.

Classroom Practices

Over the two years of the project, the school PLCs worked on several different ways to enhance their classroom practice. These included the use of rich tasks, assessment considerations, creating a positive environment for learning mathematics, going deeper with the mathematics curriculum, and exploring approaches to algebraic thinking.

Use of tasks

Many of the PLCs initially identified the need to design and use rich tasks. Throughout the first year, they grappled with the characteristics of a rich task, anticipating what implementation might look like, and understanding the effectiveness of these tasks with different students. Student work that resulted from engagement with rich tasks offered opportunities for school PLCs to examine students' thinking.

The Border Problem, from the *Connecting Mathematical Ideas* book by Boaler and Humphreys (2005), was a valuable resource for several PLCs who explored rich tasks. The description of how this problem evolved in Humphrey's classroom helped prompt a deep exploration of the mathematical ideas that could emerge from this problem.

Five Practices for Orchestrating Mathematical Discussions (Smith & Stein, 2011) was helpful for preparing to use rich tasks like the Border Problem, since it prompted members of the PLCs to try the task themselves and to anticipate student responses prior to implementation.

The use of rich tasks was an area that many PLCs wished to explore further during the second year of the project. In response to requests for more guidance and resources, time was dedicated to this topic when all school PLCs were together again for the August Institute in 2015. First, the components of rich tasks were reviewed:

- Rich mathematical tasks provide varied opportunities for learning and encourage high-quality student interaction (Bruce, 2007).
- Rich mathematical tasks provide multiple entry points so that all students can engage with the problem and experience success.
- Tasks that have high cognitive demand and multiple ways of solving problems provide opportunities for students' mathematical thinking and discussion, and should not scaffold too early (Boston, 2012; Smith & Stein, 2011).

Next, participants discussed these and other characteristics that make a task rich, and then everyone engaged in a carousel of tasks. In the carousels, which included tasks that were shared by some of the school PLCs, participants tried each, discussed how each might align with the Grade 9 Applied Mathematics curriculum, and considered how it might be used in the classroom. The influence of this work was strong. For instance, one Grade 9 Applied teacher stated that her thinking changed drastically based on the work of the Summer Institute, particularly through discussions of how another school PLC was using rich tasks to revisit concepts in many different contexts. A number of rich tasks that were shared by school PLCs are available on the resources page of the math4thenines website (**www.math 4thenines.ca/resources.html**).

Five Practices as a way to think about implementing rich tasks

The school PLCs expressed interest in learning more about implementing rich tasks. Hence, during the Summer Institute in 2015, the research team offered a workshop drawing on the work of Smith and Stein (2011), who define the *Five Practices for Orchestrating Mathematical Discussion* as anticipating, monitoring, selecting, sequencing, and connecting. This workshop, the discussions that followed, and a study of their choice of either an article (Stein, Engle, Smith, & Hughes, 2008; Smith, Hughes, Engle, & Stein, 2009) or book by these authors (Smith & Stein, 2011), greatly influenced the school PLCs' work in Year 2.

As PLCs engaged with these ideas, they commented on the significance of the first phase of anticipating student responses before implementing a task. Many school PLCs did the anticipation phase collectively during their meetings. They often used this work to develop an anticipation guide that informed their monitoring of student work during classroom observations. Several PLCs commented on the significance of the *Five Practices* in their use of tasks and in influencing the focus of their work. A member of one school PLC articulated, for instance, the importance of these practices in improving their consolidation phase during problem solving activities:

> When we do an activity we feel that our consolidation is not as strong as it needs to be and that is something that has been identified in the department as something we need to work on. It's great to have an activity, but how do we know what we've got out of it? [taps the book *Five Practices* in front of him to imply that they use the book to work on this] (Grade 9 Mathematics Teacher, Year 2, Case 7, Nov. 19, 2015).

Assessment

As teachers began to shift their classroom practices, they started to recognize a need to shift assessment practices. Some changes to instruction that prompted shifts in assessment included the use of rich tasks, emphasis on making student thinking visible through the use of Vertical Non-Permanent Surfaces (VNPS), and a greater focus on student thinking in general. Some examples include:

- Case 6 – This PLC focused on mapping rich tasks to curriculum expectations in a way that allowed students to revisit expectations and concepts throughout the course (spiralling). This PLC also explored alternative ways for students to express their thinking. The PLC's work on spiralling and activity-based learning influenced the emergence of a focus on how best to assess activity-based tasks. This focus arose early in Year 2. PLC members had all used various activities in their teaching prior to this project, but no one had ever taught exclusively through activities, a practice that led to questions related to assessment. This became one of the school PLC's struggles in Year 2. They discussed how they would assess learning through the activities (e.g., using checks, levels, marks), and decided that tests were less useful in aligning with activity-based learning and that a larger activity that encompassed many course concepts would be a more suitable culminating assessment.

- Case 7 – This school PLC was teaching primarily through activities, with students working on vertical surfaces. The PLC participants reported that this posed challenges for assessment as they felt that they had a lack of physical evidence of student learning. One Grade 9 Applied teacher suggested that he didn't have many marks to justify a grade that he might give a student, but through observations and discussions with students as they worked, he felt he had a fairly strong understanding of what they understood.

Many of the PLCs examined student thinking and problem-solving processes by considering hard copies and recordings of student work, and by having students use whiteboards in the classroom. Such activities prompted teachers to focus on student thinking. The use of whiteboards also seemed to become common across all cases, and many of the teachers installed whiteboards in all of their classrooms. Students worked in groups at these whiteboards to solve problems collaboratively. Some of this work was influenced by Peter Liljedahl's (2016) research on the use of Vertical Non-Permanent Surfaces (VNPS) as a way of creating a thinking classroom. Teachers noticed that having students working around the classroom on vertical whiteboards provided the opportunity to see what students were doing and thinking.

- Case 8 – In one conversation with a teacher from Case 8, the teacher had discussed the changes that she could see in student engagement with the use of the whiteboards. In the morning at the PLC meeting, this teacher described how she could walk around and have discussions with students and observe their thinking. It was interesting that later in the day, she was discussing a push by the school board to be sure that teachers are using conversations and observations as part of their assessment practices. She mentioned that she did not see how this could happen, and the researcher mentioned that it sounded as though the teacher was already having conversations and observations while students worked at the whiteboards. This realization was an "aha" moment for the teacher, and she became absorbed in thinking about how she could keep a record of what she was learning through conversations and observations.

Many PLCs also explored alternative forms of assessment. In addition to the opportunities for conversations and observations offered by having students work at whiteboards, apps like *Show Me* or *Explain Everything* made it possible to capture student talk and thinking, thus including a record of their thinking processes as well as the products they produced through a task. One Grade 9 Applied Mathematics teacher, for instance, explored the use of *Explain Everything* to offer students an alternative way to demonstrate their achievement of curriculum expectations:

- Case 5 – A Case 5 teacher incorporated a "re-try" process by which students could redo an assessment, or provide evidence in a different format, to demonstrate their achievement of curriculum expectations. Another teacher in Case 5 had students create their own screencast videos to incorporate their written response to a problem with a verbal description of their solution, which added a much richer dimension to understanding the student's thinking. He might use this format for a "re-try" assessment if the student did not perform well on a strictly written assessment. This allowed the teachers to see assessment as a process, rather than an event.

The research findings indicate that assessment practices were not only moving from merely assessment *of* learning to assessment *for* learning, but were also incorporating assessment *as* learning. One school PLC (Case 5), for instance, offered multiple opportunities for students to demonstrate their achievement of curriculum expectations. As part of a "re-try" process, students had opportunities to learn from the mistakes that were made on tests before taking another opportunity to demonstrate their achievement. This same school PLC also incorporated checkpoints and helped students to recognize these checkpoints as part of self-assessment. Case 8 also illustrated a practice that models what assessment as learning can look like:

- Case 8 – One way that Case 8 helped students develop self-assessment skills was through "Gap Days." A "Gap Day" is an activity in which students work on particular skills or concepts that have been identified as an area where they could improve. They work in small groups at a specific activity station that is supervised by an "expert," a fellow student who has mastered that concept or skill. Through this work, students start to self-identify areas of strength and areas that need improvement.

Some of the PLCs focused on developing learning goals and success criteria and working with students to make these goals and criteria explicit. Many participants emphasized the need for students to know what they are doing and what their target is. Case 5 presented a unique perspective on how they were approaching working with the curriculum in particular:

- Case 5 – At one point, one teacher in Case 5 suggested that the curriculum expectations are like a grocery list to prepare a meal (i.e., a lesson). The principal in the case suggested that the success criteria are the ingredients. Participants also considered that they must be reasonable about the amount of class time that is spent discussing success criteria, and that learning goals and success criteria should not be so narrow that they do not recognize that there is variety of ways to solve problems. They saw that it is important to use curriculum expectations, rather than merely learning goals and success criteria, to guide planning.

Creating a positive environment for learning mathematics

Most of the PLCs began their journey with the goal of enhancing students' growth mindset and increasing student engagement. Most PLCs realized that in order for students to be successful, it would be important to create a positive environment, where students feel comfortable with taking risks and making mistakes.

The focus on enhancing student mindset was seen in different ways.

- Case 2 – This PLC worked to develop a better understanding of growth mindset and explored strategies to support students' growth mindset. Its members also explored ways to measure growth mindset through such things as student surveys, classroom observations, and individual student interviews. To better understand mindset, PLC members read two books during Year 1: *Mindsets in the Classroom* (Ricci, 2013) and *Mindsets, the New Psychology of Success: How We Can Learn to Fulfil our Potential* (Dweck, 2006). Their work on students' mindsets led them to consider teacher mindset, and they wondered if/how teacher mindset influenced student mindset.
- Case 7 – This PLC was also familiar with growth mindset and this became part of teachers' focus on developing metacognition. They also used mindset videos such as those from Jo Boaler and Carol Dweck. Teachers in this PLC believed that their combined Academic/Applied class, rather than a separate Grade 9 Applied class, helped to promote a more positive mindset.

In Case 4, even though "growth mindset" was identified as a problem of practice, "ideas about growth mindset were folded into the broader area of focus around meeting the multiple needs of students and thereby increasing their level of confidence and engagement." As Year 2 came to an end, the Case 4 PLC discussed the importance of students having a caring adult to support them and summed up their work on creating a positive environment as follows:

> We need to meet the emotional needs of the student through a consistent network of support. This feeling of safety extends to the staff, who also need support to build confidence and competence in their capacity as instructors of mathematics (Statement taken from one of Case 4's slides from a summary presentation at August 2016 conference).

Going deeper with the mathematics curriculum

One of the purposes of this OAME Project was to support teachers as they re-examined the Grade 9 Applied Mathematics curriculum and worked to implement the curriculum in ways that supported the needs of their students. We saw many ways in which the PLCs did this.

Recognizing the importance of the curriculum continuum – going across grades

As the project moved into Year 2, we saw that many PLCs requested the inclusion of teachers from Grades 7 and 8, and in some cases, Grade 6, in their PLCs. Focusing on the curriculum helped participants to see the similarities and the differences between the curriculum expectations of different grades and also between their different contexts. In some cases, the school PLCs planned lessons on a particular topic that crossed multiple grades, and teachers of the different grades tried out the lessons, bringing back student solutions for the PLC to discuss. In other cases, the context of the grade level opened up discussions about assessment or the use of tasks.

- Case 4 – This PLC highlighted the benefits of having the Grades 7 and 8 students and teachers within the same building as the high school students. The Department Head commented that this facilitated these students' transition to high school, and working with Grades 7/8 mathematics teachers was an important element of this team's planning. One of the areas that this PLC focused on was trying to bring greater consistency to teaching practices, which included focusing on the mathematical language used across the different grades in the curriculum. In addition to focusing on the Grades 7 and 8 students, and planning to ensure they were being well prepared for Grade 9, in Year 2, this PLC also began to focus on Grade 6 students. The Special Education Resource Teacher noticed that it was that transition from Grades 6 through 9 that was particularly challenging with respect to students' mathematics education. The Department

Head noted that in Grade 9, the emphasis should be on building on what students have previously learned, rather than feeling that reteaching was necessary. Their focus was on how the curriculum was connected and developing across the grades.

- Case 1 – In discussions about rich tasks, a Grades 7/8 teacher in Case 1 discussed having the assessment focus on the process expectations, and explained the difficulty that elementary teachers have, as they have to report on content strands individually, which is sometimes difficult when you are working with a rich task that crosses strands.

Spiralling, reorganizing the curriculum, scope, and sequence

Most of the PLCs examined and changed the way that they sequenced topics in the curriculum. They considered which topics to begin with, keeping in mind student mindset, connections to work in previous grades, and approaches that engaged students. For instance, the Case 8 PLC changed the sequence of units so students would experience more success earlier in the course, and they waited to introduce some of the more challenging topics later in the course.

As work moved into Year 2, many of the PLCs were organizing the Grade 9 Applied program around particular tasks, with a clear matching of tasks to curriculum expectations. Many of the PLCs were influenced by the work of Case 7, who used an approach to the curriculum that they call "spiralling." Case 6 developed a framework to match activities and curriculum expectations. Their thinking about spiralling was influenced by a Case 7 presentation at the Summer Institute, and they decided to visit Case 7 in Year 2 to see activity-based learning and spiralling in practice.

Approaches to algebraic thinking

During the end of Year 1, the school PLCs reported that they saw rich tasks connecting with other topics, but not necessarily with algebra. Hence, a focus of Year 2 for many of the PLCs was to see how they could make algebra more engaging.

- Case 1, in particular, saw that through the extensive exploration of the "Border Problem" and the book study of *Connecting Mathematical Ideas* (Boaler & Humphreys, 2005), the PLC participants shifted their entire focus from algebra skills to algebraic reasoning; that is, they saw algebra as more than the manipulation of symbols, but as a way of reasoning: recognizing patterns, generalizing, and comparing generalizations. Symbol manipulation was just a part of how they saw algebra.
- Cases 9 and 10 reported a similar shift in thinking. The PLC had been working with seeing algebra using geometric structures. For instance, Figure 1 represents a geometric structure for $4x^3$ and can be viewed as $(2x)(2x)(x)$ based on the length of its sides. A question to students might be to seek other ways to build $4x^3$ and to compare the geometric structures and algebraic structures to see how they are similar to the one presented.

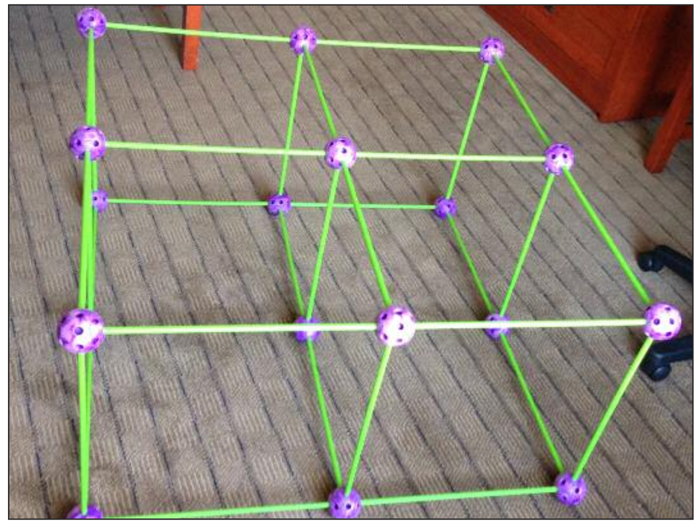

Figure 1: Geometric structure for 4*x* cubed

This work by participants in Cases 9 and 10 seemed to indicate that they started to see algebra as a set of relationships, rather than a set of rules to be memorized. One participant suggested that the structure work helped her realize that exponent "rules" are not rules to be memorized at all, but can all be justified and demonstrated and therefore do not need to be memorized. This shifted the school PLC focus away from teaching algebra as a set of rules and procedures, and they began teaching it with a focus on developing understanding.

How PLC Learning Was Facilitated

Participants reported different ways that their perspectives on mathematics, on mathematics teaching and learning, and on the curriculum shifted during the work in the PLCs. They suggested several ways their learning was facilitated. Some of these ways relate to the activities that the PLC engaged in, the makeup of the PLC, the ways the PLC worked, or the structure of this project itself.

The PLCs engaged in a variety of activities during their meetings, depending on the focus at that time, including classroom observations, lesson and book studies, co-designing new tasks, anticipating students' responses, examining students' thinking, participating in interviews with students, and discussions with teachers from other grades or schools. A great deal of time was spent co-planning, followed by classroom observations and/or sharing student work and debriefing after the planned lessons were implemented. Respectful dialogue was key, and with all PLC activities, to facilitate a strong connection between the work of the school PLC and the classroom.

Several participants suggested that the diversity of the school PLC team and their roles helped to contribute to the PLC members' learning. For instance, the Special Education Resource Teacher or Student Success Teacher might have had different perspectives toward mathematics or different views on ways to support students than the Grade 9 Applied Teacher. The role of the Administrator was critical to bring credibility, coherence, and a view and means for sustainability to the table. The structure of the PLC encouraged a distributed leadership model that seemed to help facilitate participant learning. Many commented that they felt they had a safe environment to share their thinking. The structure and responsiveness of the overall project also seemed to be a key component of the PLC participants' learning. Several participants suggested that there was great value in the sustained nature of the project. The responsive design of the project was essential for gauging participants' needs and for allowing for the emergence of new structures and ideas.

Recommendations for Supporting Grade 9 Applied Mathematics Education

Many things have been learned from this two-year OAME Grade 9 Applied Collaborative Project. The following list provides recommendations for enhancing mathematics teaching and learning in Grade 9 Applied Mathematics (and could be adapted to mathematics teaching and learning in other grades) that emerged from our research.

- **Focus on the curriculum** – Participants in PLCs used the curriculum as their basis. Their focus on the curriculum included recognition that the verbs in the curriculum (e.g., students will explain, solve, distinguish, determine through investigation) should drive the actions that students are doing in the classroom.
- **Provide an engaging and safe space to take risks** – The use of such things as erasable whiteboards, manipulatives, iPad apps, and collaborative work not only served as motivation for students, but also helped to allow students to make mistakes. Part of changing the classroom mindset is to provide space for students to take risks and to be successful.
- **Engage students in tasks and problem solving** – Some of the benefits of many of the student tasks included more student engagement and opportunities for teachers to pay attention to student thinking.
- **Use a variety of assessment strategies** – PLCs provided opportunities and a variety of assessment strategies that gave space to allow student thinking to emerge and be heard.
- **Have high expectations for students** – Focusing on growth mindset includes believing that all students are capable of engaging in mathematics activity. This study demonstrated that when teachers value students' ways of thinking, students are able to develop high-level skills and are capable of success.
- **Work with other teachers across grades** – As many of the PLCs moved into Year 2 of the project, they decided to include Grades 7 and 8 teachers. This was a great help to support all teachers' understanding of the continuum of concepts that are taught across Grades 7–9, as well as the ways they are taught and the vocabulary that is used. Teachers and students of all of these grades benefited from the collaboration.

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▲ Nothing Like a Good Constitution

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Bill Otto retired from teaching after 35 years. He was Coordinator of Secondary Programs with the Thunder Bay Catholic District School Board for six years, a Math Chair for almost two decades, and taught math, computer science, and special education at both the separate and public boards in Thunder Bay. He has been involved in the NWOAME Chapter for almost three decades and has served OAME in many different capacities for over two decades. He is currently serving as a Vice-President of the OAME, as Executive Liaison to the SPaRC committee (which manages the OAME Constitution), and now lives near Halifax, Nova Scotia.

Editor's note: The purpose of this article is to tap the brain of one of the longest-serving members of the OAME leadership, to help everyone understand why some of the organizational pieces are the way they are. The article is structured as a series of questions with responses from Bill Otto.

There have been mentions of the Board of Directors being different from the way it was a few years ago. Is that true? Why?

That's true. We have instigated some significant "restructuring" changes over the past few years in order to strengthen the Board of Directors and the OAME in general. Numerous amendments to the Constitution, By-Laws, and Terms of Reference were implemented, which served to streamline the size of the Board of Directors by reducing the number of elected positions. This modification not only ameliorated the OAME budget by reducing board meeting costs, but also augmented the role of the OAME Chapter Representatives and improved the connections between the organization and their province-wide chapters and events. It altered how the Board of Directors functions, how the various Standing Committees complete their tasks, and how board processes facilitate and affect change within the organization. For an organization to continue to remain vibrant, progressive, and support ongoing professional learning, it is worthwhile to regularly re-evaluate its Constitution, By-Laws, and Terms of Reference.

There has been mention of something called "critical paths." What are these, and how will they help the organization?

"Critical paths" are essentially the role descriptions, duties, and timelines for each position on the OAME Board of Directors. Historically, critical paths were utilized, but due to an ever-increasing workload, and the important projects undertaken by the OAME to aid mathematics educators, over time, they became "lost in the shuffle."

When do members consult the OAME Constitution, By-Laws, and Terms of Reference? Why are there three documents instead of having them together in a single document?

Members consult these documents any time they wish to know more about the organization, either with regards to the broad strokes or the specifics. If someone is contemplating becoming more involved in the organization, he or she might consult the Constitution. The Board of Directors typically consults the Constitution when making decisions about any aspect of the organization. It guides decision making in a consistent manner.

Each of the three sections serves a specific purpose. The Constitution is intended to be a clear set of statements about the mission, goals, and objectives (i.e., the *raison d'être*) of the organization. The By-Laws are the rules and regulations that govern what the organization actually does; think of it as you would a city's/town's/municipality's by-laws—the *laws*. The Terms of Reference contain how those laws are to be carried out (i.e., *the nuts* and *bolts*).

Do chapters need a Constitution, By-Laws, and Terms of Reference?

The short answer is *yes*! Chapters are a very important part of the OAME. The Chapter Representatives are an integral part of the link between the membership and the overall OAME organization, and these pieces help each chapter function. If the chapter executive and membership feel that developing all three is onerous, or will affect their capacity to offer exemplary professional learning to mathematics educators, then a chapter constitution is the priority. However, all three serve a definite purpose and can be helpful, if instituted.

Should chapters have an Annual General Meeting like the organization, as a whole, has in October?

Ideally, yes. In reality, many of the OAME Chapters serve very large areas of the province. Marilyn Hurrell (see obituary in this publication) would often say, "Well, NWOAME alone is the size of France!" Most OAME chapters serve members who are geographically spread out, which is challenging to local chapter executives who plan to hold an Annual General Meeting (AGM) somewhere that minimizes the travel and expense for members to attend. The OAME understands that this can be problematic, but trusts that every chapter is doing its best to aim for that goal. To this end, OAME is currently exploring the possibility of allowing members to join the provincial AGM virtually through their chapter.

If I am a member who knows very little about these things, what is your top five list of surprising details that are in the OAME Constitution, By-Laws, and Terms of Reference?

First, Article 4 of the OAME Constitution contains the Mission Statement.

1) The mission of OAME/AOEM is to promote excellence in mathematics education throughout the Province of Ontario;
2) Excellence for students by striving for a curriculum that will prepare them to meet the mathematical challenges in their lives, by fostering familiarity with the methods of mathematics and an admiration for its principles, and by giving recognition and respect to their own mathematical achievements;
3) Excellence for educators by promoting their role as facilitators and resource persons, by supporting a variety of teaching methods to meet the varied learning styles and needs of students, by encouraging their students to gain facility in learning both co-operation and independence, and by celebrating the mathematical excellence of both individuals and groups.

Second, Article 5 contains the Objectives.

OAME/AOEM is committed to the following objectives:

1) To aid the professional growth of mathematics educators.
2) To encourage and to engage in research, development and evaluation of curriculum design and curriculum teaching materials related to mathematics education.
3) To promote coordination in mathematics education at all levels, and effective communication within the mathematics community of Ontario.
4) To provide liaison with other educational organizations in Ontario and with mathematics education organizations in other provinces and countries.
5) To maintain affiliation with the National Council of Teachers of Mathematics [NCTM], and to promote projects of mutual concern.
6) To maintain an active and representative mathematics educator's voice with the ministry that is responsible for education in the Province of Ontario, the various other educational bodies in Ontario, and the public.
7) To encourage the development of local, regional, and special-interest mathematics organizations in Ontario, to grant such organizations chapter affiliate status as defined in Article 8, and to maintain a mutually supportive relationship with them.

Third, By-Law 7 mandates that a percentage of the annual OAME membership fee is given by the OAME to the local chapter designated by each member for use at the chapter level. Currently, that percentage is 30 percent.

Fourth, there are a maximum of 27 voting members on the OAME Board of Directors, including 15 Chapter Representatives, 6 Executive Committee members and 6 Directors, with an additional 5–8 non-voting members who hold specific roles within the organization (e.g., Website Coordinator, *Gazette* Editor(s), *Abacus* Editor(s)).

Fifth, in the Terms of Reference, the Outreach Committee of the OAME Board of Directors is charged with the responsibility of soliciting nominations for, and adjudicates, the selection process of worthy candidates for up to six different annual awards. Watch for more information in the *Gazette* regarding these awards, and consider making an appropriate nomination.

Constitutional matters can be tricky, as we know with Canada's challenges with its Constitution. How long does it take to alter the OAME Constitution?

Article 11 of the Constitution requires that amendments to the Constitution receive a two-thirds majority vote at the AGM. Currently, that means that a notice of motion for an amendment to the Constitution for the AGM in October 2017 would have to have been provided to the Executive Directors at least four weeks prior to the Board of Directors meeting in June 2017. Fortunately, changes to the Constitution occur infrequently.

Amendments to the By-Laws and the Terms of Reference are considerably simpler, as they can be handled at Board of Directors meetings, provided that the appropriate process is followed. If you are interested in knowing more about the Constitution and related amendments, go to the Members Only section at the OAME website, select "OAME Constitution" on the left side under "Board Links," and look up By-Law 23 and Terms of Reference 9. Enjoy, and have a great school year. ▲

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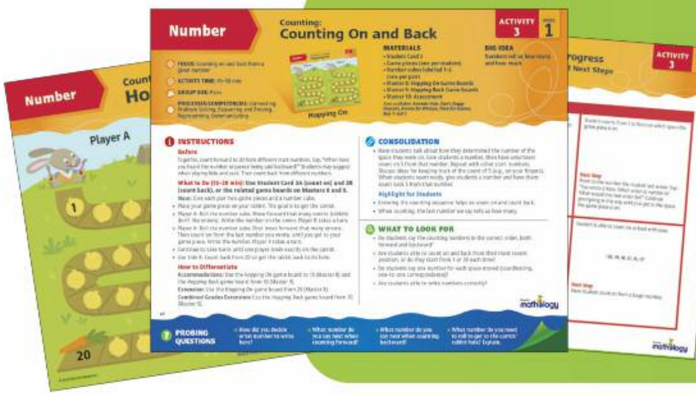

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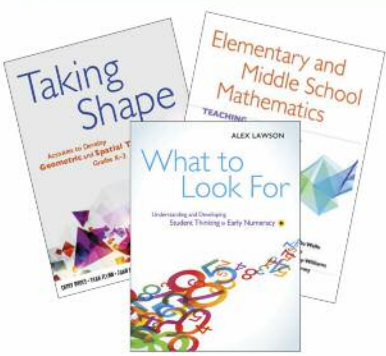

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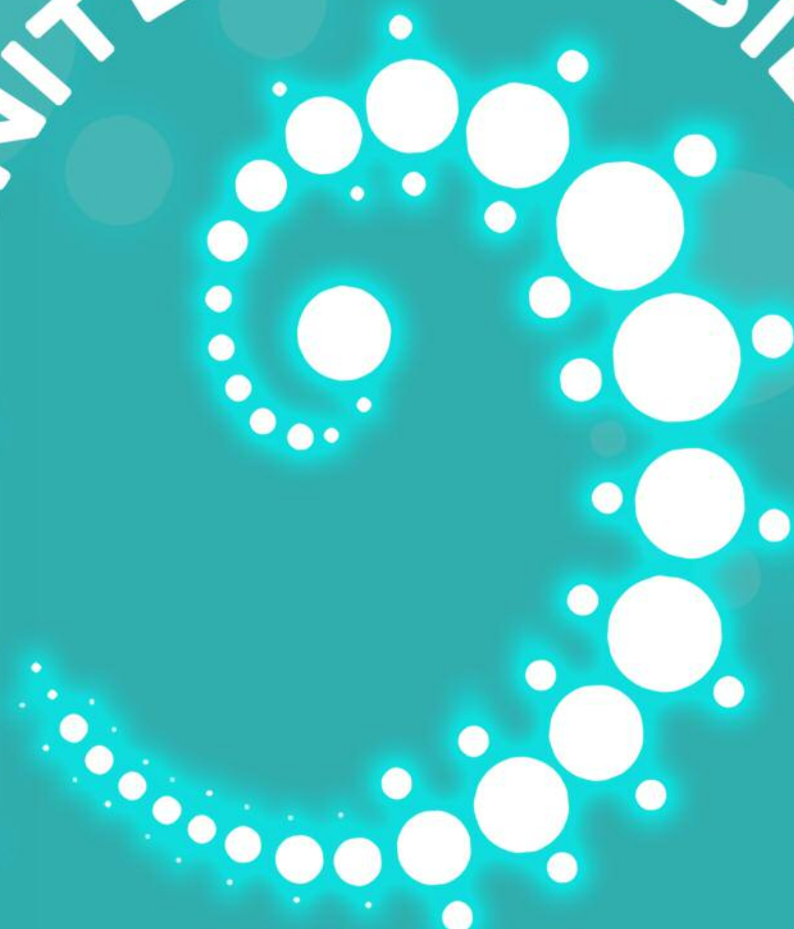

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