8 PAGE INSERT ELEMENTARY MATH

Ontario Mathematics Gazette

OAME – ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION

AOEM – ASSOCIATION ONTARIENNE POUR L'ENSEIGNEMENT DES MATHÉMATIQUES

Vol. 54 #1
September 2015

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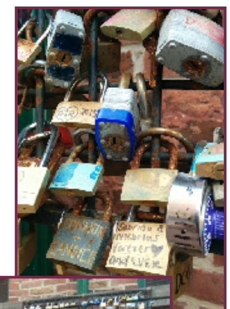

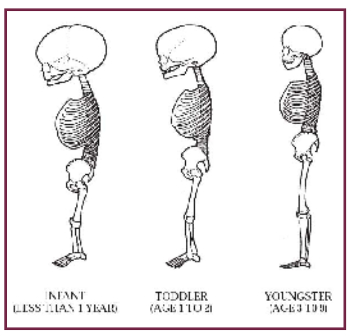

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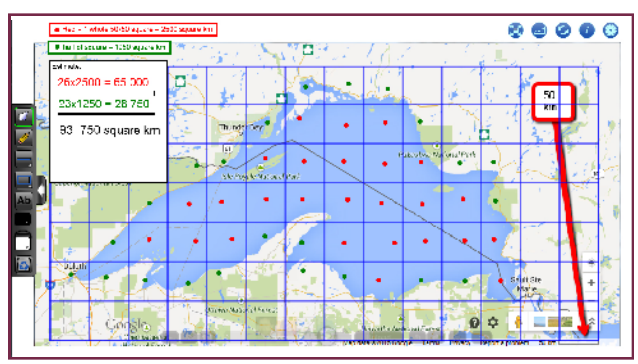

OAME/AOEM AWARDS FOR EXCELLENCE IN MATHEMATICS EDUCATION

The purpose of the Ontario Association for Mathematics Education's Awards is to formally recognize individuals and educators who demonstrate an outstanding contribution to mathematics education in Ontario. The successful candidate of each award enhances the learning environment, providing opportunity for students to do, see, hear, and touch mathematics in a profound and meaningful way. If you know someone who qualifies for this honour, please nominate her/him/them for one of the following awards.

OAME AWARDS

OAME/AOEM Life Membership Award recognizes an educator who has contributed in a significant way to OAME/AOEM; demonstrates outstanding leadership in mathematics education; and has accumulated ten or more years of membership in OAME/AOEM.

Award for Outstanding Contribution to OAME/AOEM and Mathematics in Ontario recognizes an individual who has made a significant contribution to OAME/AOEM, but who is not necessarily an educator or a member of OAME/AOEM.

Award for Exceptional and Creative Teaching in Elementary Mathematics recognizes an exceptional and creative elementary teacher who demonstrates excellence in mathematics education and contributes to the overall development of students.

Award for Exceptional and Creative Teaching in Secondary Mathematics recognizes an exceptional and creative secondary teacher who demonstrates excellence in mathematics education and contributes to the overall development of students.

Award for Leadership in Mathematics Education recognizes an educator who has demonstrated leadership by contributing in a significant way to the development of mathematics teachers and enhancing mathematics education in Ontario.

Secondary School Department Award for Exceptional and Collaborative Mathematics Teaching recognizes a secondary mathematics department which fosters collegiality, teamwork, and excellent classroom teaching; contributes to the overall development of students; and demonstrates leadership in the mathematics education community.

Elementary School Staff Award for Exceptional and Collaborative Mathematics Teaching recognizes an elementary school (a school accommodating any or all of Grades JK–8), whose staff fosters collaborative and excellent mathematics teaching; contributes to the overall development of student learning of mathematics; and demonstrates leadership in the mathematics community.

OAME/AOEM Awards Nomination Process

1. Complete the Award Nomination Cover Page appropriate to the specific award, found at **www.oame.on.ca**, or contact the Executive Directors at **EDs@oame.on.ca** for an electronic version. You may include a one-page letter of recommendation.
2. Provide up to three letters of support, of no more than two pages each. These supporting documents may be gathered from a variety of sources to reveal a full picture of the nominee. The support documents will provide precise information about the nominee's contributions as they relate primarily to the specific Award.
3. Include the nominee's Curriculum Vitae of up to five pages. (C.V. not required for Secondary School Department Award and Elementary School Staff award)
4. Send the completed cover page and supporting documents by January 15, 2016 to: OAME Executive Directors

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DEADLINE: Nominations are due by January 15, 2016 to the Executive Directors (contact info above).

The awards will be presented at the Annual OAME Conference in May 2016.

Submission of Articles

The *Ontario Mathematics Gazette* (*OMG*) is looking for news items, articles, and good ideas that are useful to mathematics teachers and mathematics teacher education. We are seeking submissions, preferably from mathematics teachers K–12 and other mathematics education professionals, that describe innovative and creative approaches to mathematics teaching.

Please keep in mind the following criteria when making submissions to the *OMG*:

- The ideas/activities must be of interest to the readership.
- The ideas/activities must be fresh and innovative.
- The mathematics content must be appropriate for the readership.
- The mathematics content must be accurate.
- The article must be well written and easily understood.
- The article and its ideas must be free of sexual, ethnic, racial, or other bias.
- The article must not have been previously published, nor should it be out for review by other publications.
- The article must be original.

Articles must be word-processed in MS Word, double-spaced with wide margins, not exceeding 10 numbered pages of text, and prepared according to the *Publication Manual of the American Psychological Association, Sixth Edition*. Figures and diagrams should be drawn by computer, if possible, or drawn in black ink in camera-ready form. Embedded images must also be submitted separately in jpeg or tif format. Proof of the photographer's permission is required, and for **photos of students** under the age of 18, the written permission of a **parent or guardian is required**.

You must submit **one complete copy** of your article, embedded with any tables, figures, and captioned photographs or graphics, to the Editor, Dan Jarvis, along with **separate files for each of the text, graphics, and/or photographs**. Please e-mail all files to Dan Jarvis at **dan.jarvis@oame.on.ca**.

Your name should not appear anywhere in your article, including websites, so that your article can be sent out for blind review. Your name, full mailing address, and e-mail address must be included on a separate sheet. Upon review, you will be notified as to whether your article has been accepted for publication (as is, or pending minor or major revisions) or rejected.

The Editor reserves the right to edit manuscripts prior to publication. Once an article is published, it becomes the property of OAME.

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▲ About the *Ontario Mathematics Gazette*

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Editor

- ▲ **Dan Jarvis**
  E-mail **dan.jarvis@oame.on.ca**

Associate Editors

- ▲ **Anne Yeager**
  E-mail **anne.yeager@oame.on.ca**
- ▲ **Marilyn Hurrell**

***Abacus* Co-Editors**

- ▲ **Mary Lou Kestell**
  E-mail **marylou.kestell@oame.on.ca**
- ▲ **Kathy Kubota-Zarivnij**
  E-mail **kkz@oame.on.ca**

Design and Production

- ▲ **Penny Clemens,** Graphic Designer

Printing & Binding

- ▲ **Pole Printing,** Box 69, 1-89 King Street East
  Forest, ON N0N 1J0 (519) 786-5112

Advertising Manager

- ▲ **Robert Sherk**
  E-mail **robert.sherk@oame.on.ca**

Submission of Advertisements

Advertisements for publication in the *Ontario Mathematics Gazette* should be sent to **Robert Sherk** at the above address. Courier is recommended to avoid possible delays. Deadlines for advertisements are January 23 for the March issue, April 1 for the June issue, July 1 for the September issue, and October 1 for the December issue.

Full-page advertisements are to be on 8.5" by 11" paper with a minimum of 0.5" margins and single sided. Each advertisement should be print ready, and colour advertisements should have no bleeds.

Advertising Rates

Advertising rates are available by telephoning, e-mailing, or writing to the Advertising Manager.

Promoting Excellence
in Mathematics Education

Promouvoir l'excellence dans
l'enseignement des mathématiques

▲ Editor's Message

DANIEL H. JARVIS, O.C.T., Ph.D.
E-MAIL: dan.jarvis@oame.on.ca

Dr. Daniel Jarvis is Professor of Graduate and Mathematics Education in the Schulich School of Education at Nipissing University, North Bay, Ontario. His research interests include instructional technology, integrated curricula, and mathematics of the workplace.

Welcome back to school/college/university and to another academic year of mathematics teaching and learning that lies before us—one full of potential in terms of efforts to improve our reflective practice.

When one packs up and heads out on an adventure during one's scheduled holiday time, it is known as a *vacation*. Alternately, if one forgoes the busyness of planning and travelling altogether, one might instead decide to remain at home, this situation commonly being referred to as a *staycation*. However, if one decides to tackle one or more major home improvement projects during one's break, we could perhaps refer to this experience as a *renova(ca)tion?!* Maybe you've taken part in one of these recently?! For us, it involved long overdue hardwood floor refinishing this past summer. Thanks to multiple YouTube "how to…" videos (isn't this a remarkable teaching/learning tool?!), a number of helpful local haberdasher employees, and some very long and dusty/toxic work periods, we were ultimately able to successfully accomplish this exciting and terrifying task with satisfactory (for us at least) results.

In 2006, I had the opportunity, along with a number of fellow Canadian math education researchers, to attend and participate in the 17th International Commission on Mathematical Instruction (ICMI) conference in Hanoi, Vietnam entitled *Mathematics Education and Technology––Rethinking the Terrain*. Dr. Seymour Papert, renowned mathematician, computer scientist, and former Lego Professor of Learning at Massachusetts Institute of Technology (MIT), delivered a fascinating keynote address, and later that day was involved in a near tragic accident, wherein he was hit by a motorcycle while walking back to the hotel from the university. Papert has accomplished many things during his long career, including being one of the pioneers of artificial intelligence, and a co-inventor, with Wally Feurzeig, of the Logo programming language. For me, one of Papert's most intriguing concepts is that of "hard fun," which he discusses in the following excerpt:

> The first [big lesson I have learned from computer games], which I have already noted, is echoed by kids who talk about 'hard fun' and they don't mean it's fun in spite of being hard. They mean it's fun because it's hard. Listening to this and watching kids work at mastering games confirms what I know from my own experience: learning is essentially hard; it happens best when one is deeply engaged in hard and challenging activities. The game-designer community has understood (to its great profit) that this is not a cause for worry. The fact is that kids prefer things that are hard, as long as they are also interesting. The preoccupation in America with 'Making It Easy' is self-defeating and cause for serious worry about the deterioration of the learning environment. (Papert, 1998, p. 89)

In both the *hardwood* and *hard fun* scenarios described above, there appears to be this sort of balance of (or tension between) *mystery* and *mastery* which often serves to generate a very positive context for new learning. Soviet psychologist and educational theorist Lev Vygotsky's definition of his Zone of Proximal Development (ZPD) presents us with a powerful and related idea: "[The ZPD is] the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance, or in collaboration with more capable peers" (Vygotsky, 1938/1978, p. 86). Hard fun is not accidental or happenstance; rather, it is carefully orchestrated, monitored, and revised by teachers of mathematics as they daily define the ZPD for individual students, and for their class as a whole.

The September 2015 issue of the *Gazette* includes three articles, nine regular columns, the Abacus insert, and several special features.

In his article *Thinking about Teaching, Learning, and Mathematical Mindsets Leads Me to Learning Skills*, author Jamie Pyper discusses growth mindsets and habits of mind, and also highlights a learning skills rubric that he has developed for student assessment.

Co-chairs Melissa Black and Linda LoFaro contribute a full report of the Ontario Mathematics Olympiad (OMO) 2015 competition, including photos of the organizing team, candid student shots, and the winning student team.

And finally, in Meth Devendra's *Exponential Function and Tangents*, he demonstrates how a single common tangent line to two given functions can be achieved, using several distinct examples.

Regular columns include the following highlights: OAME President, Tim Sibbald (President's Message), shares his inaugural President's Message; Assessment Abby (eponymous) answers a submitted question regarding teachers' communication with parents/guardians; Mary Bourassa (Technology Corner) highlights the online resource known as the Mathematics Assessment Project (MAP); Carly Ziniuk (temporarily replacing Mirela Ciobanu as a guest columnist for the *In the Middle* column—thank you, Mirela, and welcome aboard Carly!) presents a sample math investigation focusing on the Pont des Arts pedestrian bridge in Paris, which has garnered international attention regarding the symbolic and controversial love padlocks placed on its side rails; and Greg Clarke, Agnes Grafton, Ross Isenegger, and Markus Wolski (Provincial Digital Learning Resources) discuss e-mail list notifications, images in Notepad, partitioning sets tool, and wiki supports.

Lynda Colgan (Hey, It's Elementary) delves into forensic anthropology and how mathematics is used by experts to identify bone remains; Shawn Godin (What's the Problem?) elaborates on the previous tiling problem and poses a new puzzle regarding buried pirate treasure; Ann Kajander (Mb4T) looks at how to introduce students to integers; and Todd Romiens (OAME/NCTM Report) shares some ideas around stimulating a child's curiosity and also how parents/guardians can be involved in their child's math learning. As usual, this issue also features the rich contribution of ideas for elementary math teaching as found in the Abacus insert, co-edited by Mary Lou Kestell and Kathy Kubota-Zarivnij, and focusing here on equivalence and relational thinking.

Volume 54 Issue 1 also includes several special features: the 2015 OAME Awards recipients; a review by Tim Sibbald of Richard Hoshino's book, *The Math Olympian* (2015); Ontario mathematics education researcher profile highlights; and the several more Canadian mathematics education researcher excerpts from within Lerman's *Encyclopedia of Mathematics Education* (2014).

Hard times ahead, perhaps, for some of our elementary, secondary, and tertiary teaching colleagues in terms of collective agreement negotiations and related actions in Ontario schools and universities. In the midst of that potential turmoil may we each be mindful of *hard fun* and *hard-wood* type new learning as we continue to challenge our students and ourselves, respectively.

References

Lerman, S. (Ed.). (2014). *Encyclopedia of mathematics education.* London, UK: Springer.

Papert, S. (1998, June). Does easy do it? Children, games, and learning. *Game Developer*, 88–89.

Vygotsky, L. S. (1938/1978). *Mind in society: The development of higher psychological processes.* Cambridge, MA: Harvard University Press. ▲

▲ OAME AWARDS SUMMARY 2015

Secondary School Department Award for Exceptional and Collaborative Mathematics Teaching – **Upper Canada College**

The committee was unanimous in its selection of Upper Canada College (UCC). Many of UCC's initiatives to engage students are truly innovative (student generated: e.g., created an invitational math contest; reached out to inner-city schools involving test design and lectures; also, math dragons podcasts). It was particularly noteworthy that the UCC group's undertakings were truly a team effort, with considerable leadership by example.

OAME/AOEM Life Membership Award – **Bill Otto**

Bill Otto was the runaway selection for this year's award in this category. His many, varied contributions over his long-time involvement with OAME are self-evident and collectively speak for themselves in the high-quality operation of OAME.

Award for Exceptional and Creative Teaching in Elementary Mathematics – **Najwa Chalabi**

The committee recognized the significance of Najwa Chalabi's nomination from a student, and the succeeding nominations resulting from this beginning. She was particularly recognized for her innovation and creativity in the classroom, and development of students through her involvement in a diverse number of activities, not just in the mathematics classroom.

Award for Exceptional and Creative Teaching in Secondary Mathematics – **Heather Theijsmeijer**

Heather Theijsmeijer was recognized for her creativity and innovative strategies in the classroom. Not content with success, it was clear she is a reflective practitioner, who is constantly looking for even more effective strategies, including technology (e.g., BYOD). Heather has a portfolio with a huge spectrum of subjects taught, interests, and approaches to learning. It is highly evident she is trying as many things as she can to keep students from falling through the cracks. ▲

▲ President's Message

TIM SIBBALD

The OAME is a vibrant community because of the volunteers who share so many ideas and experiences. As the new president, I find value in a charitable organization that fosters support among educators in a way that allows us to collectively explore our practice without being impeded by the boundaries of our respective teaching roles.

With a goal to explore our collective practice, chapter events are diverse and are an opportunity to discuss our teaching practices. To new members of the OAME, and those who have been enjoying the *Gazette*, but have been otherwise passive members, I would like to encourage you to find out what your chapter has to offer. It is inspiring to the chapter leadership to see interest expressed by simply being an available voice at a chapter meeting. The sharing of new and fresh ideas often leads to interesting conversation for everyone who participates. It is an opportunity to meet a broader range of math colleagues and see beyond the walls that may confine one's daily math thinking.

If you have attended past events, I encourage you to look for opportunities to present what you have found useful in your classroom or teaching environment. Often presenters find that new inspiration emerges from the process of explaining what they have done. It is in our professional best interests to share our inspiration. If you have presented at OAME events, I encourage you to continue that practice because of the professional learning you are sure to have realized. It may also be worthwhile to consider turning your presentation into an article for the *Gazette*—a challenge that highlights a different form of professional growth. If you know of someone in your chapter who has a good presentation, perhaps you can encourage him or her to consider proposing it for the annual conference (see MCIS for the timeline for proposals). I would also encourage you to consider helping within your chapter so that others have opportunities for the presenter-mentorship that you have had.

The value of our chapters should not be underestimated. They are, in many ways, scaled versions of the board of directors and executive. It is through a process of "proportional reasoning" that experiences in our chapters prepare members who are up to the challenge of running for a position on the democratically elected board of directors and executive. It is not for everyone, but there is no doubt that it is an extraordinary math-education opportunity for those who take it on. In some cases, chapters look for initiatives of their own making, e.g., has your chapter considered taking up the Provincial OMO or the annual conference in the coming years? There is also some money available for special projects if your chapter is looking to do something special.

Within our organization, changes to the structure of the board of directors and executive have taken place over the past few years, and it is my vision that, over the course of the coming year, growing the organization with respect to the modified structure will be a central theme. Already this vision is being enacted in several ways. The OAME Leadership Conference is taking place November 5–7, 2015. It is no accident that registration for this event is on MCIS, a tool that can facilitate members visiting different chapters. I would encourage all chapters to consider the value of sharing information about their events by means of MCIS. It may not always be the easiest tool to use, but the support provided for it is second to none. (Your chapter representative will receive details at the upcoming board of directors' meeting in October.)

In terms of my presidency, I felt I should start by expressing some elements of my vision. It is very much about moving beyond understanding the changes in the board of directors and executive and looking at how we can most effectively engage ourselves in the vision and mission of the organization. Last year, the board of directors planned the annual conference. We now have the opportunity to focus that energy on assisting chapters and dealing with developments from the larger mathematics-education community.

While I wanted my initial message to start solidly with a focus on the forward motion of the OAME, it is important to acknowledge events that have moved us to this point. Paul Alves, whose presidency I follow, demonstrated a dedication and work ethic that was remarkable. He faced a variety of challenges with grace and poise, and was an excellent role model. Fred and Lynda Ferneyhough, our remarkable executive directors, have been enablers at all levels of our organization, and truly embody the mission and vision. Bill Otto has been a strong contributor to keeping conversations and discussions on an even keel, and brings a wealth of experience. Judy Mendaglio has been instrumental in supporting the initial stages of the vision for the year and in developing her vision for next year.

To the directors and chapter leaderships, thank you for your service. Your roles are pivotal to our organization and, in many ways, define who we are to many members. Lastly, but most certainly not least, are our Abacus editors, Mary-Lou and Kathy; *Gazette* editor, Dan; NCTM affiliate representative, Todd; and the webmastering triumvirate, Greg, Kathy, and Claudio. This is a remarkably talented group who support the membership in so many ways. ▲

▲ Hey, It's Elementary: The Tales Dead Men (and Women) Tell

LYNDA COLGAN

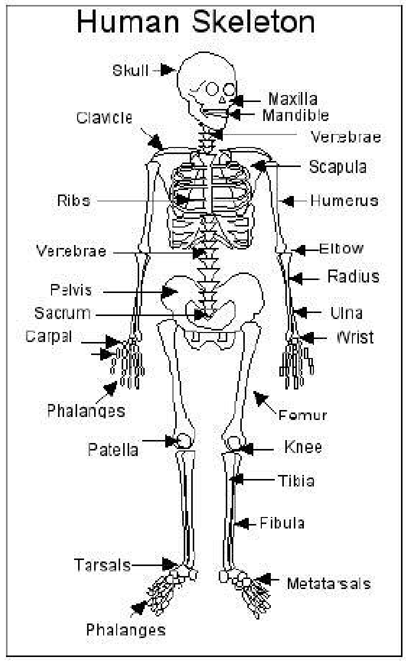
Lynda Colgan's career has included roles as a classroom teacher, a university professor, and newspaper columnist. Her contributions to mathematics and its teaching have been recognized through awards such as the Marshall McLuhan Foundation Distinguished Teacher Award. Lynda always exhibits a passion for mathematics and views her professional mission as dispelling the myth that math is the bad guy.

I sincerely hope that the summer months provided you with the opportunity to recharge your batteries, rejuvenate your body, and restore your soul. Each one of us needs to be ready to hit the ground running when September rolls around, and that is possible only when we are ready—physically, emotionally, and intellectually—for a new school year, complete with the myriad responsibilities and demands that come with "being a teacher."

I was fortunate to spend some time, once again, in Petite Rivière, Nova Scotia, in a cabin by the ocean: no phone, no Internet, and no television. That afforded much opportunity to indulge my secret passion—reading forensic murder mysteries.

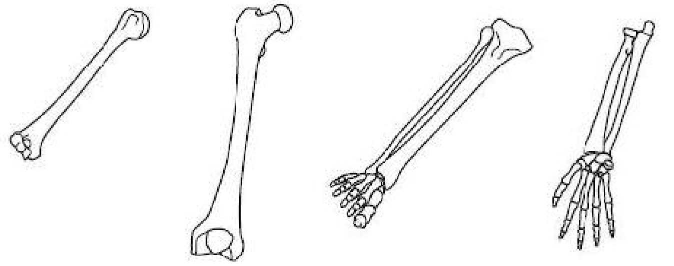
And those murder mysteries were the inspiration for this column, in which I hope that you will find some ideas upon which you can build some hands-on classroom lessons that incorporate curriculum expectations from measurement, early algebra, proportional thinking, and problem solving. In this example, mathematics is integrated with the science curriculum to give authenticity to the activities. The information presented here was obtained from a wealth of medical sites on the Web and cross-checked for accuracy, and the diagrams are used with permission from *Positively Aging*, a product of the University of Texas Health Science Center at San Antonio. You may choose to have the students do their own online research, since the exercise of extracting the information from various tables, graphs, and charts is an important exercise in numeracy and mathematical literacy.

To set the stage for such a unit, one might make reference to television programs like *C.S.I.* and *Bones*, which often begin with the discovery of skeletal remains in a remote location. Typically, the first expert to be called by the police is a forensic anthropologist (a.k.a., a skeleton sleuth or Sherlock Bones), trained in osteology (the study of bones), who needs to determine whether the evidence is at all human, and if so, the gender, age, physique, and race of the person. Because our bones are living, growing tissues that respond to the factors affecting our life—nutrition, health, degree of physical activity, age, and genetic factors such as sex and race—they can be considered to be an unwritten diary of our life. In fact, it is precisely because there is such an abundance of information about each of us recorded in our bones that a large amount of forensic evidence can be obtained from skeletal remains found at the scene of a crime.



As an adult, your skeleton is made up of 206 bones, whether you are male or female; however, the size and weight of bones of female skeletons is generally smaller and lighter than those of males. In general, the weight of your skeleton accounts for 20 percent of your total body weight.

Determining the exact physical dimensions of a victim of a crime is one key to unlocking the identification of the victim; therefore, when a skeleton is found, one of the first things a forensic scientist does is to use the lengths of certain bones to calculate the height of the living person. The bones that may be used (pictured from left to right below) are the humerus (H), the femur (F), the tibia (T), and the radius (R).



For example, your upper leg contains a large, single bone called the femur. This long bone stretches from the hip (pelvis) socket to the kneecap (patella) and is generally considered to be the longest, strongest, and heaviest bone in the body. The length of this bone can be used to roughly estimate a person's height—the femur is approximately 1/4 of a person's adult height.

Forensic scientists require more accurate methods to calculate height; therefore, they use tables and formulas that have been developed using thousands of samples to arrive at a precise relationship between bone length and height, taking into account many factors, including gender and race. For example, if you measure the length of your femur in centimetres, multiply that measure by 2.6, then add 65 to this number, you should arrive at your approximate height in centimetres. How close was the calculation of your approximate height to your exact height? to your first approximation using the 1/4 rule?

To get a better approximation, it helps to add your gender to the equation. If you are female, you multiply the length of your femur by 2.47, then add 54.1 to the answer to get your height in centimetres. If you are male, you multiply the length of the femur by 2.32, then add 65.53 to get your height in centimetres.

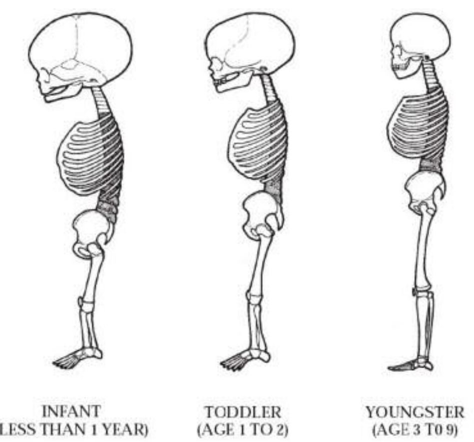
It is also possible to infer a person's height from a measure of the humerus, the single large bone that extends from the elbow to the shoulder socket. If you are female, multiply the measured length of your humerus (in centimetres) by 3.06, then add 64.26 to this number. This final number is your approximate height based upon the length of your humerus. If you are male, multiply the measured length in centimetres by 3.279, then add 59.41 to this number to calculate your approximate height based upon humerus length.

Scientists have discovered that the length of the tibia (the larger central bone of the lower leg that extends from just below the kneecap to the ankle) may also be used to calculate a person's height. For males, multiply the tibia length by 2.41, then add 81.93 to get an approximate height in centimetres. For females, multiply tibia length by 2.9 and add 61.53.

And since two bones are better than one, you can combine the measures for two of the long bones into one equation in order to infer a person's height: *Height* = 1.31 x (length of femur in centimetres + length of fibula in centimetres) + 63.05.

There are also growth and development changes in the skeletal system between infancy and adulthood. It is known that bones grow in length and thickness during part of our life, and that we lose bone in other phases. And while both heredity (genetics) and environmental factors (health and nutrition) contribute to the overall development of bone, there are some general facts about skeletons that appear to be true to everyone.

For example, as we age, our head becomes proportionately smaller. As an adult, the length of our head is about 1/8 of our body height; whereas, in infants, the head is about 1/4 of the body length.



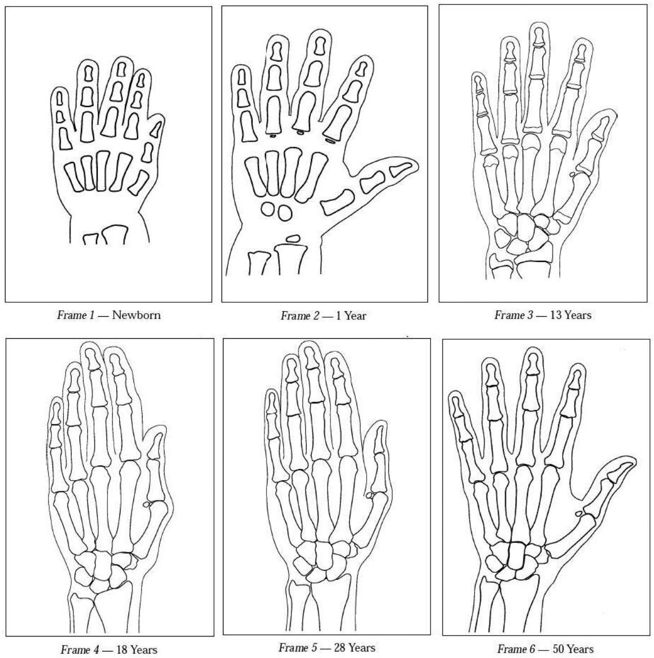
As we can see from the sequence of drawing of the hand skeleton, there are a group of bones that are completely absent at birth, but gradually appear as the hand ages.

▲ Call for Manuscripts

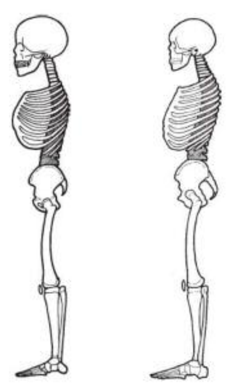
The *Ontario Mathematics Gazette* is inviting manuscripts for all grade levels.

Instructions for submission of manuscripts are found on page 1 of any *OMG*.

Contact the Editor for further details.



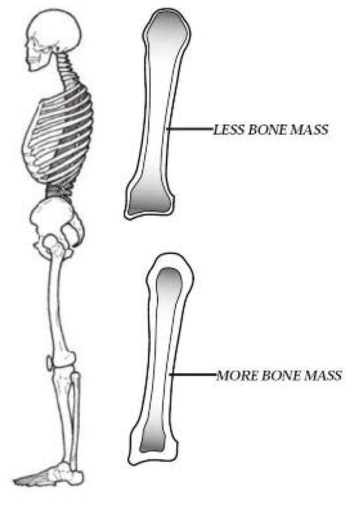
An examination of the progression in female hand skeleton X-rays above clearly illustrates how spaces (where growth plates of cartilage are found) gradually change to bone (a process called ossification). Over time, ossification centres appear in the spaces at the ends of the phalanges, carpals, and metacarpals, which will fuse or join with the shaft of long parts of the bone. This fusion will be complete by the age of 18, and growth in length of the bone will end. Can you determine the approximate age of the mystery hand by examining the other skeletons?



YOUTH (AGE 10 TO 13) ADOLESCENT/ YOUNG ADULT (AGE 14 TO 25)

Throughout childhood and adolescence, there is a dramatic growth spurt in the length of long bones. New bone is added to skeleton faster than old bone is removed. Therefore, bone grows larger, heavier, and denser. Although long bone length growth ends at about 18 years of age in females and 21 years of age in males, bones are still continuing to grow in width.

When we reach adulthood, we also reach a period of peak bone mass—a time when we enjoy maximum bone density and strength.



ADULT

After the age of 30, the height of a person begins to decrease at the rate of approximately 0.06 cm per year. Over time, the process of bone removal begins to overtake bone replacement; however, if the rate of bone removal is too quick, our bones become weaker and weaker—a situation that may cause progressive spinal deformity. In the case of the aged adult skeleton shown below, it is clear that extreme bone loss may cause the lower ribs eventually to rest on top of the pelvis. In fact, skeletal mass may be reduced to half what it was at age 30.

AGED ADULT WITH OSTEOPOROSIS

Now that you are so well informed about bones, perhaps the next time someone tells you that he or she may have dug up a skeleton, try running toward it, not away from it, and test out your new knowledge to determine its age and height. As you now know,

> *"... there is a brief but very useful and informative biography of an individual contained within the skeleton, if you know how to read it."*
> ~ Clyde Snow

In closing, I would like to suggest that there is another, very important reason for embarking on such a unit that is related to the use of mathematics for purposes of social justice. A study in The *Gerontologist* (2001) 41(3):322–332 showed that the use of materials such as those developed by the University of Texas Health Science Center moved middle school students toward a more positive view of elders. The team of authors suggests that Interdisciplinary teaching materials based on geriatrics and gerontology can be successfully developed and should be used in public school systems to affect attitudes about aging.

Answer to mystery hand: The X-ray drawing is from a six-year and ten-month-old female. You should have placed this photo between frame 2 and frame 3. ▲

▲ Mb4T (Mathematics by and for Teachers): Introducing Integers

ANN KAJANDER

Ann Kajander is an experienced classroom teacher currently teaching mathematics and mathematics education at Lakehead University. Her research interests relate to teachers' enhanced learning of mathematical concepts. She and her classroom-teacher colleague, Tom Boland, have recently co-authored a book for teachers called Math Without Memorization: Big Ideas in Modelling and Reasoning for Elementary Teachers and Other Learners.

As the old joke in mathematics education goes, the best "context" for integers is… algebra. Indeed, the study of integers, using concrete models, is impeded by the fact that historically, the need for, and development of, negative numbers came about because of algebra, rather than emerging from real-world contexts. For this reason, the classroom use of real-world contexts for learning about integers is challenging and sometimes feels contrived. Having said that, some contextual connections, as well as models, are still helpful.

Classroom Examples

When constructing classroom contexts for integers, it is important to be aware that sometimes contexts that might seem to require negative numbers can actually be represented by using a descriptor (such as "below," as in a temperature of 2 degrees *below* zero), or that idea that we *owe* a friend $2.00. It is important to note that both of these examples are actually making use of a positive quantity with a directional adjective. Students can be left confused as to why we suddenly refer to these ideas as -2, rather than a *subtraction* of positive 2, which is what they likely did in the past—and would still work fine. This is the case for many real-world examples discussed in everyday language… such as falling 3 metres or losing 2 pounds. Suddenly stating these ideas as "-3" and "-2" seems contrived—and in fact isn't needed in everyday language. The following examples provide other contexts to think about—some can be described in more than one way, and some can be written either with or without negative numbers.

- A person has $100 dollars in a bank account, and then writes a cheque for $115. Assuming that the bank allows the cheque to clear the account, what is the new bank balance (ignoring any bank charges or fees)?
- A machine starts to dig a well from ground level, which is 25 metres above sea level. The operator digs 40 metres before reaching water. How would you describe the point where water was reached?
- Liz has a mystery number for us to guess. The clue she gives is that the number added to 5 gives 3. What is Liz's number?

The bank account example can be represented with the whole-number subtraction 100 – 115, but the answer could be written -15. Alternately, however, we might say, "I have an *overdraft* of $15," which might be analogous to situations in which we *owe* someone money. The example of the well drilling is similar; in everyday use, we would likely state the answer as 15 metres *below* sea level, rather than using the negative integer -15. The third example, however, while less contextual, makes it clear that the only number we can use to add is -2. Interestingly, research suggests that children who have not yet been introduced to integers formally can still begin to construct the idea of such negative or "minus" numbers, as needed. Many teachers will have observed invented methods, such as the following, before formal introduction of integers:

53 – 27 = 50 – 20 + 3 – 7 = 30 – 4 = 26

Other children might write:

$$\begin{array}{r} 53 \\ -\ 27 \\ \hline 30 \\ -\ \ 4 \\ \hline 26 \end{array}$$

Such examples are interesting and important to celebrate, as they pave the way for more formal study of integers.

Models and Manipulatives

Temperature is one of the more meaningful contexts for the integer number line, and a thermometer is an excellent visual representation or model of temperatures above, below and at zero. The thermometer, which really is very much like a vertical number line, is a familiar context for integers—at least in northern climates! A more abstract parallel to the thermometer is the horizontal number line, extended in the negative direction. Students can enjoy drawing a chalk number line in the playground and "walking" out the problems. Some students really enjoy modelling the operations by movement, getting a strong sense of the negatives as either backing up, or reversing direction. This works especially well for kinesthetic learners.

Another useful manipulative is a set of counters or "chips," in each of two colours. When using coloured counters or chips to represent integers, we need to decide on one colour to represent a negative value, say red (here a shaded circle), and another colour to represent a positive value, say yellow or blue (here an unshaded circle).

However, as the mathematical ideas become more complex and abstract, the connection to a concrete manipulative requires more assumptions and understandings, and such is the case with integer chips. For example, the concept of *zero* pairs is necessary for some models of integer operations to be done with counters or chips. Those students who have already experienced alternate representations of numbers may be more willing to accept the idea of what happens when we add a "+1" and a "-1" quantity. Imagining a number line, we can see that +1 and -1 are equally far from zero, but in opposite directions—and in a sense, represent a balance of an equal positive and negative distance. Numerically, we see that +1 + (-1) = 0. When modelling, +1 + (-1), each of the +1 and -1 values can be represented by a counter, giving us a pair. Hence, one pair (or many pairs) of positive and negative counters is actually a way to represent zero. We call each such pair a *zero* pair.

So this is a model of zero: 

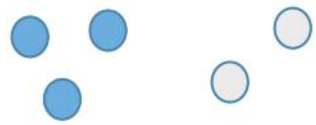
and so is this:

Adding Negative Numbers

Adding negatives with a concrete model can be done by simply using the negative counters. Using the temperature model, we can think of -5 + (-3) as starting at minus 5 degrees and adding more cold, so it results in getting colder. Similarly, we see -5 + (-3) as a combination of two groups of negative counters or chips.

In total, the 8 "negative" chips represent (-8).

But what if our problem involves adding both positives and negatives? Again, we can compare with the previous temperature examples for a meaningful context. Starting at -3 degrees and then warming up 2 degrees (-3 + 2) can be modelled like this:



But what number does this model represent in total? If we do a little rearranging, which is easier to see if we are working with the plastic chips and moving them around, we recognize zero pairs:

Taken literally, the representation is of -1 + 1 -1 + 1 -1, which becomes 0 + 0 + -1. So the answer is (-1), which can also be verified using the temperature model discussed previously.

Subtraction

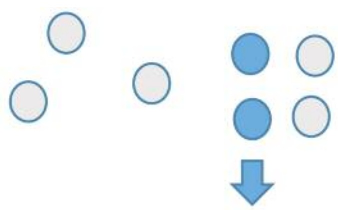
Subtracting a positive integer value is analogous to subtraction with whole numbers. From the temperature or number line model, we see that questions like 5 – 3 and (-4) – 2 can be understood as starting with the initial value (whether positive or negative), and then decreasing it (by moving down or left on either the thermometer or number line, respectively) by the units of the second number. However, more problematic is the question of subtracting a negative integer.

Students might be asked to pretend they are playing a board game similar to Monopoly. A scenario might be, you owe $2 to your opponent for a fine, but you have no money left. Now your opponent lands on your space and it turns out he owes you $2 in rent. So the two of you decide to cancel the previous $2 debt to effect this payment. This might be thought of as *removing* your previous $2 debt. In terms of operations, we can think of subtraction to express the action of *removing* an amount. So now the money you have in the end might be expressed numerically as (- $2) – (- $2). Students might be asked to model the story problem out (use play money, if necessary) to see the result. The "- (-2)" can be thought of as removing or eliminating the need to subtract or pay the $2 debt, giving a result of 0. To model this with the chips, we can start with two negative counters, and then *remove* two negative counters to effect the " – (-2). So we end up with no counters left, i.e. , zero.

Starting with a negative value, and then removing some of these negatives, such as in the case of (-3) – (-2), can simply be modelled by removing some of the negatives in the first value. This example results in surprisingly straightforward reasoning, such as “Start with 3 negatives. Then remove 2 negatives. So the answer is -1.” Subtracting negatives can be surprisingly easy to model, *as long as there are at least that many negatives in the first number*.

But what about 3 – (-2)? Here, there are no negatives in the first number, so the model requires an extra step. The key idea in modelling examples such as 3 – (-2) with integer chips when there are *no negatives to remove* (or not enough of them), is to re-represent the value 3 by incorporating zero pairs. The 3 can be modelled a number of ways. Here is a new model of 3 (think 3 + 0 + 0):

Now, removing the chips that show the (-2) will effect the “- (-2)”:



Based on this new model of 3, it is easy to imagine the “- (-2)” as *remove* (-2). The result with the model is:

So the answer is 5.

The principle of adding zero pairs to a model can be used anytime we want to model subtraction, if enough of the type of chip to be subtracted (removed) was not present in the initial model.

The remaining columns for this year will focus on integer multiplication and division. Stay tuned! ▲

▲ OAME/NCTM Report

TODD ROMIENS
E-MAIL: todd.romiens@oame.on.ca

Todd recently retired from teaching K–12 methodology in the Faculty of Education at the University of Windsor. He is a past president of OAME and a life member, as well as a previous editor of the Gazette*.*

What makes children want to learn? According to research, it is the joy of exploration, a hidden force that drives learning, critical thinking, and reasoning. We usually call this ability *curiosity*, and isn’t this a desire we have, to be able to instill curiosity in all of our students? Some questions arise: How much curiosity does a student have to start with as he or she enters our classroom? What impedes our ability to encourage this attribute in all of the students we teach? We would all like to have students come to our classrooms hungry for knowledge, anxious to learn, and willing to explore. Unfortunately, this is not who we generally see showing up. So when and where did curiosity fail to be a driving force in students’ minds, and what caused this? How do we change this situation for the better? As with all difficult scenarios, there are generally no easy answers or quick fixes.

Marilyn Price-Mitchell, a developmental psychologist, researcher, and writer, suggests some ways to stimulate student curiosity:

1. **Value and reward curiosity.**
   It is important to notice and reinforce curiosity when you see it in your classroom. Praise students for asking innovative questions and for their willingness to go beyond normal classroom discussions.
2. **Teach students to ask quality questions.**
   Good questions contain “why,” “what if,” and “how.”
3. **Notice when students feel puzzled and confused.**
   This is not a time to give students answers, but a time to encourage them to ask appropriate questions to allow them to understand. Convince them that problems are just mysteries to be solved.
4. **Spread the curiosity around.**
   Provide opportunities for more curious and less curious students to work together. Curiosity can be contagious.

5. **Use current events.**
   It is very important for students to see that the mathematics they do in school has relevance to them outside of school.
6. **Model curiosity.**
   You need to do this in your classroom. Explore student interests and engage them in meaningful dialogue about how mathematics impacts their lives (Tim Horton's contests, casino math earthquakes, etc.).
7. **Encourage curiosity at home.**
   Help parents and guardians to understand the importance of curiosity in their child's development, and suggest ways that they can foster it at home.

NCTM has been very active in encouraging parents and guardians to become involved in their child's learning. Research shows that parents' attitude toward their child's education, and their involvement in it, have a significant impact on a child's success in school.

As you plan for the new approaching school year, remember to also make plans to engage parents and guardians in their child's learning. It would be helpful to provide them with answers to questions that you know they will ask, such as:

1. What math will my child be learning this year?
2. How will my child use a calculator?
3. How much math homework will my child be expected to do on a nightly basis?
4. How can my child make up work that he or she misses?
5. How often will a report on my child's progress in math learning be sent home?
6. Are you open to discussing my child's achievement in math class?

Clearly, providing answers to these questions and others they may ask is essential, if we want them to understand and accept your teaching practices. Support at home is vital to your students' overall learning. NCTM has published a book called *Principles to Action: Ensuring Mathematical Success for All*. This book will help update parental knowledge and beliefs about how mathematics is being taught in schools. The list price is $28.95 U.S., or $23.46 U.S. for members. Access the publication at **www.nctm.org/PtA**.

Suggestions to assist parents and guardians in supporting their child's learning include the following:

a) Practise basic facts. Immediate recall requires practice and understanding. They can be practised while in the car, shopping, walking, or waiting.
b) Play games. Children love to play games. Teach them Cribbage, UNO, or other number games. It is a great way to bond.
c) Continue to put math in contexts that are relevant to them. Use everyday situations that they experience.

On a final note, there are three regional conferences coming up:

Atlantic City, New Jersey – October 21–23, 2015
Minneapolis, Minnesota – November 11–13, 2015
Nashville, Tennessee – November 18–20, 2015

These are very worthwhile conferences, and if you are able to, please attend. ▲

▲ Mathematics Education Researcher Highlights

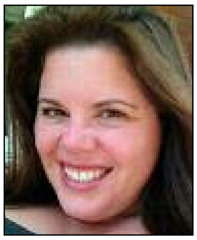
Math Education Researcher: Dr. Immaculate Kizito Namukasa, Western University

Projects: Immaculate Kizito Namukasa is currently involved in research on the use of manipulatives—concrete, virtual, and apps—in teaching. This interest has led her to research in private schools using 18th century tried-and-true philosophies of teaching using well-planned concrete materials. Previously she has produced video resources on math for teaching through problem solving, in which Ontario teachers are featured (**www.edu.uwo.ca/faculty_profiles/cssal/namukasa_immaculate/resources.html**). She has also worked with teams of teachers in Africa and Southern China, exploring how math for teaching may differ with geographical contexts. This summer, Immaculate (in collaboration with Dr. George Gadanidis and Dr. Donna Kotsopoulos) worked with teachers who are interested in teaching children to code as well as to learn computational skills. Information on the Coding and Math workshop may be found at **http://www.researchideas.ca/coding/**. From September 2012 to September 2014, Immaculate served as a journal editor for the *Ontario Mathematics Gazette*. ▲

▲ OMO 2015: Brain Puzzles

OMO 2015 CO-CHAIRS
MELISSA BLACK

LINDA LOFARO



Melissa has been teaching for 16 years, and for the last 10 years, has been teaching Grades 9–11 Math at All Saints High School. She has been past president of the COMA chapter, past writer for the Ontario Ministry of Education for the Financial Literacy Project, and is currently the OAME Rep for the Ottawa chapter. Linda has been teaching for 15 years and is currently teaching Hospitality & Tourism, Math, and Health & Physical Education at St Patrick's High School. She is the current COMO Co-chair, was an activity writer for OMO 2015, and has worked with the Ontario Ministry of Education on various projects including Math CAMPPP, Math CLIPS, and ELL Assessments in Mathematics.

On behalf of the entire OMO organizing committee and the many volunteers who helped out, we would like to sincerely thank all the participating teams at this year's Ontario Math Olympics. A special thank you goes to the coaches, who took the time to travel to Ottawa to make this year's event a true success. The host for the event was the Ottawa-Carleton chapter known as COMA. We were set up at Carleton University, who graciously accommodated everyone involved for this two-day event.

OMO 2015 got off to a great start with the mathletes, their coaches, and a few parents enjoying the Haunted Walk Tour of Ottawa on Friday evening. The light drizzle did not deter anyone from enjoying the Nation's Capital! Following the tour, the group explored the downtown area before returning to Carleton University.

The championship began Saturday morning. Teams participated in an individual activity as well as two paired activities. Upon their return from lunch, mathletes worked together in the full team activity. At the end of the day, top prizes were awarded for each event, which added a little more competitive spirit to the event.

The Ontario Math Olympics involves Grades 7 and 8 students grouped in teams of four—two girls and two boys, and two Grade 7's and two Grade 8's—an easy mix to put together! They compete in their local areas, and then take their math skills to the provincial level to vie for top honours in the province.

The many volunteers, from St. Patrick's High School in Ottawa, made the day run very smoothly. Under the leadership of Robert Cop, Ontario Catholic School Board (OCSB) Math Consultant, the Audio/Visual team took many pictures to capture the fierce competition taking place in the activity rooms. These pictures were then streamed as a slide show for the coaches to see in Porter Hall. This same team also used Google Hangout to showcase the students in action for their coaches. It brought another element to the competition for our coaches.

Upon completion of the activities, mathletes returned to Porter Hall for a wrap-up activity. Mathletes, coaches, parents, siblings, and volunteers were all part of the probability game of SKUNK. It got rowdy and competition was intense! It all came down to two students, with the highest score being 298.

Twenty-five teams from across Ontario participated in this year's OMO—from Thunder Bay, Kingston, Toronto, Muskoka, and everywhere in between. We are extremely proud of our local COMA teams, who finished 6th, 8th, and 10th.

OMO 2015 Activity Winners

- Top individual male and female: both from ISOMA 2 (coach Sandra Dellamaestra)
- Top in the Pairs activities: ISOMA 1 (coach Sandra Dellamaestra) and SAME 3 (coach Vivian Fu)
- Top in the Team event: SAME 2 (coach Andrzej Pienkowski)

OMO 2015 Top Finishers

- First Place team: ISOMA 2 (coach Sandra Dellamaestra)
- Second Place: ISOMA 1 (coach Sandra Dellamaestra)
- Third Place: SAME 2 (coach Andrzej Pienkowski)

We would like to thank our sponsors, whose support and contribution to this year's Ontario Math Olympics made it a huge success. Their generous donations will also help with future local events. The OMO trophy is currently being upgraded with the incorporation of another tier. Upon completion, the trophy will return to the University of Toronto Schools team, who were champions once again.

To next year's organizing committee from SAME, good luck!

To our entire organizing committee and the many student volunteers from St. Patrick's High School—THANK YOU! Thank you for all your time and effort in making this two-day event such an overwhelming success. For two newbies at taking on the OMO championships, you made it really easy for us!

OMO 2015 Committee Members

Activity Writers

Marc Cardinal
Cara Doxsee
Jimmy Pai
Valeria Pandeleiva
Lynne Villemaire

Room Coordinators

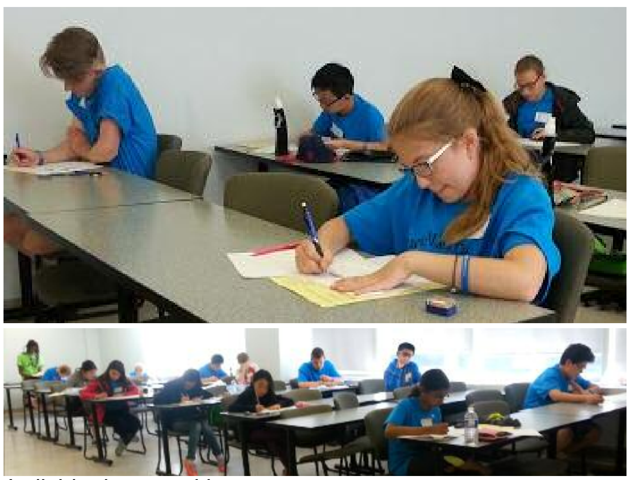
Ashley Boivin
Natalie Boucher
Lino Demasi
Debra Desjardins
Cathy Hall
Michael Lieff
Judy Loschmann

Audio/Visual Coordinator

Robert Cop (OCSB)

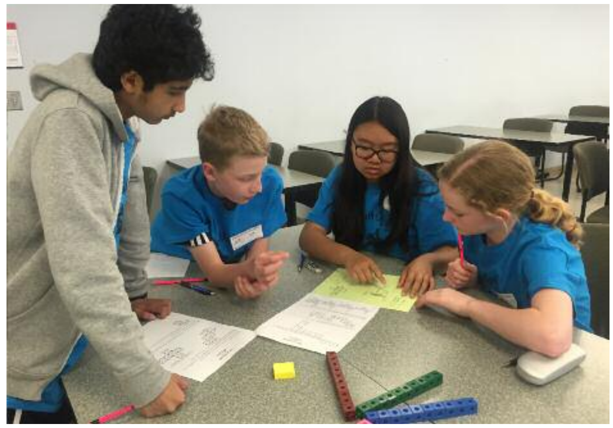
Scorekeeping Coordinators

Matthew Brash
Ian Taylor ▲



Individual competition

Pairs Activity



Final group activity

Volunteers and organizing committee

Participants from across Ontario

Winning Team: ISOMA 2

▲ Exponential Function and Tangents

METH DEVENDRA

Meth Devendra is an Ontario-certified teacher, working as the Program Chair of the Math & Science Department at Boren Sino Canadian School in Jiangmen City, China. He holds a Ph.D. in mathematics from the University of Dundee, Scotland.

This article focuses on equations of tangent lines of exponential functions. We use the property that when two curves touch at a point, as opposed to crossing, they have the same slope and intersection. In the first part of the article, we shall consider the exponential function and a straight line; in the second part, we will focus on exponential and trigonometric functions; and in final part, we shall examine the power function. This article will encourage teachers to provide opportunities for advanced students to make various connections between curves and tangent lines. In the Ontario Mathematics curriculum course, *Calculus and Vectors* (MCV4U), we find the equation of the tangent line to polynomial, trigonometric, exponential, and logarithmic functions. This article is an extension of this idea, where we seek a single common tangent line to two given functions.

Part I

In Part I, we consider the exponential function $f(x) = b^x$, $b > 1$ and a tangent line of the form $y = mx$.

Main Result:

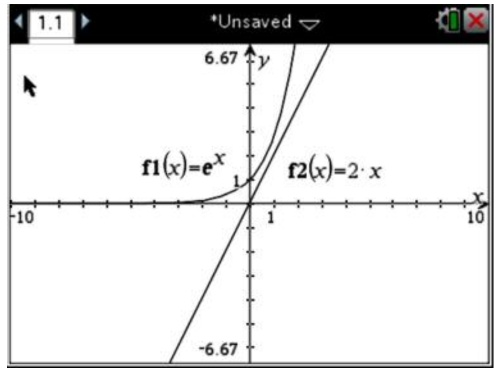
When $m = e \ln b$, the graph of $y = b^x$ has a tangent line $y = mx$, with tangent point $(\log_b e, e)$.

Let us first consider examples:

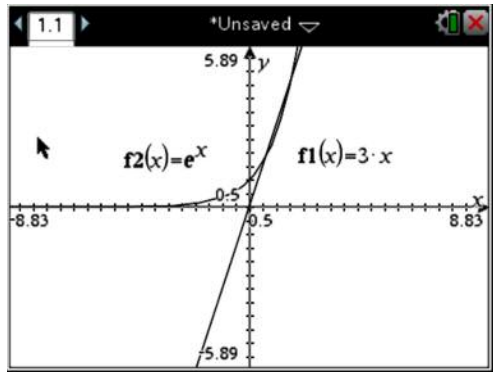
Case 1: Base number $b = e$

Find the equation of the tangent line to $f(x) = e^x$ of the form $y = mx$.

First choose $m = 2$ The graphs of $y = e^x$ and $y = 2x$ have no intersection.



Next let $m = 3$; then the graphs of $y = e^x$ and $y = 3x$ have two points of intersection.



Then we conjecture that, for some m, $2 < m < 3$, there is a tangent line of the form $y = mx$ to the graph of $y = e^x$.

To find the tangent line, we use the method of equating derivatives.

Consider $y = e^x$ and $y = mx$.

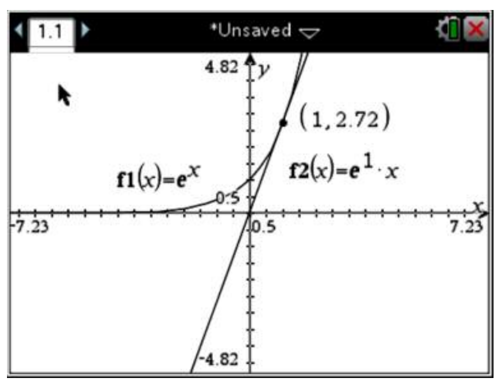
Since we seek the intersection, we get $e^x = mx$.

Next, consider the derivatives: $y' = e^x$ and $y' = m$.

Equating the derivatives, we get $e^x = m$.

Solving $e^x = mx$ and $e^x = m$, we get $x = 1$, $m = e$.

Now, when $m = e$, the graph of $y = e^x$ has the equation of a tangent line $y = ex$, with point of tangency $(1,e)$.



Case 2: Base number $b = 2$

Next we find the tangent line to $f(x) = 2^x$ of the form $y = mx$.

We use a similar method as before.

Consider $y = 2^x$ and $y = mx$ then we get $2^x = mx$.

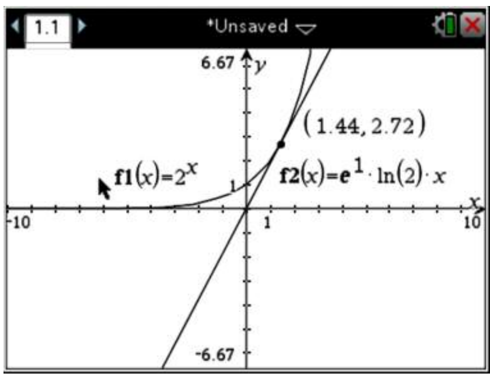
Next consider the derivatives: $y' = 2^x \ln 2$ and $y' = m$.

Equating the derivatives, we get $2^x \ln 2 = m$.

We solve $2^x = mx$ and $2^x \ln 2 = m$ by division to get $x = \frac{1}{\ln 2} \approx 1.442695$ and $m = 2^{\frac{1}{\ln 2}} \ln 2 \approx 1.884169$.

We can further show that $x = \frac{1}{\ln 2} = \frac{\ln e}{\ln 2} = \log_2 e$ and $m = 2^{\frac{1}{\ln 2}} \ln 2 = 2^{\frac{\ln e}{\ln 2}} \ln 2 = 2^{\log_2 e} \ln 2 = e \ln 2$.

Now, when $m = e \ln 2$, the graph of $y = 2^x$ has the equation of a tangent line $y = (e \ln 2)x$, with point of tangency $(\log_2 e, e)$.



Case 3: Main result for any base value $b, b > 1$

We can now generalize our result to find the tangent line to $f(x) = b^x, b > 1$ of the form $y = mx$.

We use a similar method again.

Consider $y = b^x$ and $y = mx$ then we get $b^x = mx$.

Next consider the derivatives: $y' = b^x \ln b$ and $y' = m$.

Equating the derivatives, we get $b^x \ln b = m$.

Solving $b^x = mx$ and $b^x \ln b = m$ by division, we get $x = \frac{1}{\ln b} = \log_b e$ and $m = e \ln b$.

Now, when $m = e \ln b$, the graph of $y = b^x$ has a tangent line $y = mx$, with tangent point $(\log_b e, e)$.

Note that in general, the tangent line to $y = b^x$ at the point $(x_0, y_0) = (x_0, b^{x_0})$ is given by $y = (b^{x_0} \ln b)\, x + (1 - x_0 \ln b) b^{x_0}$.

Part II

In Part II, we consider tangential properties of the trigonometric function $f(x) = \sin x$ and the exponential function $f(x) = b^x$. To find common tangents, we horizontally translate the exponential function. Thus, we consider $f(x) = b^{x-m}$ for some m, and for $x > 0, b > 1$.

Main Result:

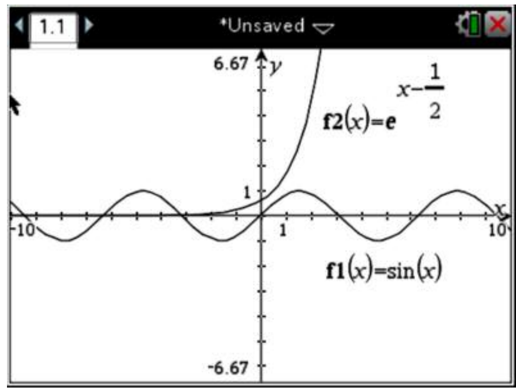
When $x > 0, b > 1$, the graphs of $f(x) = \sin x$ and $f(x) = b^{x-m}$ are tangential at the point $(\tan^{-1}(\log_b e), (\sin(\tan^{-1}(\log_b e)))$ and they share the common tangent line $y - \sin(\tan^{-1}(\log_b e)) = \cos(\tan^{-1}(\log_b e))(x - (\tan^{-1}(\log_b e)))$.

Let us first consider some examples.

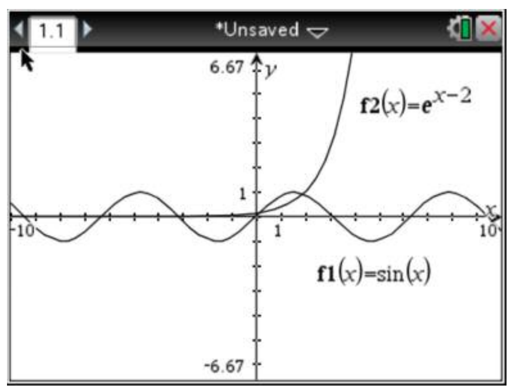
Case 1: Base number $b = e$

Let us consider when the graph of $f(x) = \sin x$ is tangential to the graph of $f(x) = e^{x-m}$ for $x > 0$.

First consider $m = \frac{1}{2}$. The graphs of $y = \sin x$ and $y = e^{x - \frac{1}{2}}$ have no intersection for $x > 0$.



Next let $m = 2$, then the graphs of $y = \sin x$ and $y = e^{x-2}$ have two intersection points for $x > 0$.



We now conjecture that for some $m, \frac{1}{2} < m < 2$, the graphs of $f(x) = \sin x$ and $f(x) = e^{x-m}$ will share a common tangent.

Consider $y = \sin x$ and $y = e^{x-m}$ and then we get $\sin x = e^{x-m}$.

Next consider the derivatives:

$y' = \cos x$ and $y' = e^{x-m}$

Equating the derivatives, we get $\cos x = e^{x-m}$.

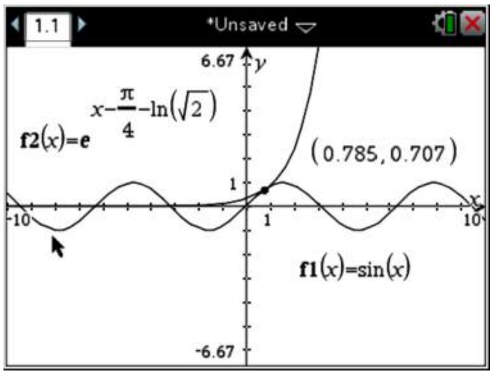
Solving $\sin x = e^{x-m}$ and $\cos x = e^{x-m}$, we get $\tan x = 1$.

So $x = \frac{\pi}{4}$.

Then $\sin\frac{\pi}{4} = e^{\frac{\pi}{4}}e^{-m}$, giving $\frac{1}{\sqrt{2}} = e^{\frac{\pi}{4}}e^{-m}$.

So $m = \ln\left(\sqrt{2}e^{\frac{\pi}{4}}\right) \approx 1.131971$.

Thus, when $m = \ln\left(\sqrt{2}e^{\frac{\pi}{4}}\right) = \frac{\pi}{4} + \ln\sqrt{2}$, the graphs of $f(x) = \sin x$ and $f(x) = e^{x-m}$ are tangential at the point $\left(\frac{\pi}{4}, \frac{1}{\sqrt{2}}\right)$, with tangent line $y - \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}\left(x - \frac{\pi}{4}\right)$.



Case 2: Base number $b = 2$

Let us next consider the case when the graph of $f(x) = \sin x$ is tangential to the graph of $f(x) = 2^{x-m}$ for $x > 0$.

Consider $y = \sin x$ and $y = 2^{x-m}$; then we get $\sin x = 2^{x-m}$.

Next consider the derivatives: $y' = \cos x$ and $y' = 2^{x-m}\ln 2$.

Equating the derivatives, we get $\cos x = 2^{x-m}\ln 2$.

Solving $\sin x = 2^{x-m}$ and $\cos x = \ln 2^{x-m}$, we get $\tan x = \frac{1}{\ln 2}$.

So $x = \tan^{-1}\left(\frac{1}{\ln 2}\right) = \tan^{-1}(\log_2 e) = 0.964684$.

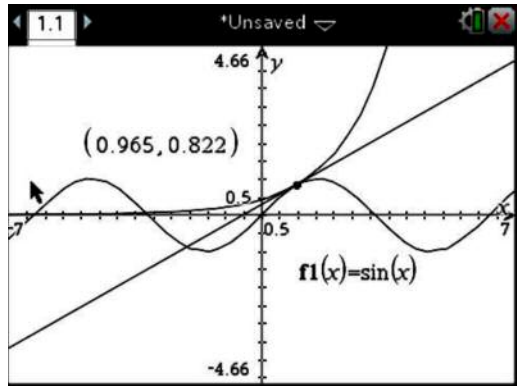
Next we solve for m.

Now, $\sin(\tan^{-1}(\log_2 e)) = 2^{\tan^{-1}(\log_2 e)-m}$.

Then $\log_2(\sin(\tan^{-1}(\log_2 e))) = \tan^{-1}(\log_2 e) - m$, giving $m = \tan^{-1}(\log_2 e) - \log_2(\sin(\tan^{-1}(\log_2 e))) \approx 1.247703$.

Thus, when $m = (\tan^{-1}(\log_2 e) - \log_2 \sin(\tan^{-1}(\log_2 e)))$, the graphs of $f(x) = \sin x$ and $f(x) = 2^{x-m}$, are tangential at the point $(\tan^{-1}(\log_2 e), \sin(\tan^{-1}(\log_2 e)))$ with tangent line $y - (\sin(\tan^{-1}(\log_2 e)) = \cos(\tan^{-1}(\log_2 e))\,(x - \tan^{-1}(\log_2 e))$.

We can simplify these expressions, using $\sin(\tan^{-1}x) = \frac{x}{1+x^2}$, $\cos(\tan^{-1}x) = \frac{x}{1+x^2}$. It is left to the reader to provide these simplifications.



Case 3: Main result for any base value b, $b > 1$

When $x > 0, b > 1$, $f(x) = \sin x$ and $f(x) = b^{x-m}$ are tangential at the point $(\tan^{-1}(\log_b e), \sin(\tan^{-1}(\log_b e)))$

Consider $y = \sin x$ and $y = b^{x-m}$; then we get $\sin x = b^{x-m}$.

Next consider the derivatives: $y' = \cos x$ and $y' = b^{x-m}\ln b$

Equating the derivatives, we get $\cos x = b^{x-m}\ln b$.

Solving $\sin x = b^{x-m}$ and $\cos x = b^{x-m}\ln b$, we get $\tan x = \frac{1}{\ln b}$.

So $x = \tan^{-1}\left(\frac{1}{\ln b}\right) = \tan^{-1}(\log_b e)$.

Next we solve for m.

Now, $\sin(\tan^{-1}(\log_b e)) = b^{\tan^{-1}(\log_b e)-m}$

Then $\log_b(\sin(\tan^{-1}(\log_b e))) = \tan^{-1}(\log_b e) - m$, giving $m = \tan^{-1}(\log_b e) - \log_b(\sin(\tan^{-1}(\log_b e)))$.

Thus, when $m = \tan^{-1}(\log_b e) - \log_b(\sin(\tan^{-1}(\log_b e)))$ and $x > 0, b > 1$, the graphs of $f(x) = \sin x$ and $f(x) = b^{x-m}$ are tangential at the point $(\tan^{-1}(\log_b e), \sin(\tan^{-1}(\log_b e)))$, and they share the common tangent line $y - \sin(\tan^{-1}(\log_b e)) = \cos(\tan^{-1}(\log_b e))\,(x - (\tan^{-1}(\log_b e)))$.

Part III

In Part III, we consider tangential properties of the power function $f(x) = ax^n$.

Result 1:

The power function $f(x) = ax^n$ shares a common tangent line with the exponential function $f(x) = b^x$, when $a = \left(\frac{e\ln b}{n}\right)^n$ at $(\log_b e^n, e^n)$. We prove the result for $b > 1$ and n is a positive integer.

Consider $y = ax^n$ and $y = b^x$; then we set $ax^n = b^x$.

Next consider the derivatives:

$y' = nax^{n-1}$ and $y' = b^x \ln b$.

Equating the derivatives, we get $nax^{n-1} = b^x \ln b$.

Solving $ax^n = b^x$ and $nax^{n-1} = b^x \ln b$ by division, we get

$$x = \frac{n}{\ln b} = \log_b e^n \text{ and } a = \frac{b^{n\log_b e}}{(n\log_b e)^n} = \frac{e^n}{(n\log_b e)^n} = \left(\frac{e\ln b}{n}\right)^n.$$

Now, when a = $\left(\frac{e\ln b}{n}\right)^n$ the graphs of $y = ax^n$ and $y = b^x$ touch at the point ($\log_b e^n$, e^n).

We can also seek the tangential property of the power function with the logarithmic function.

Result 2:

The power function $f(x) = ax^n$ shares a common tangent line with the logarithmic function $f(x) = \log_b x$, when $a = \frac{\log_b \sqrt[n]{e}}{e}$ at $\left(\sqrt[n]{e}, \log_b \sqrt[n]{e}\right)$. We prove the result again for $b > 1$ and n is a positive integer.

Consider $y = ax^n$ and $y = \log_b x$; then we set $ax^n = \log_b x$.

Next consider the derivatives: $y' = na\,x^{n-1}$ and $y' = \frac{1}{x\ln b}$

Equating the derivatives, we get $na\,x^{n-1} = \frac{1}{x\ln b}$.

Solving $a\,x^n = \log_b x$ and $nax^{n-1} = \frac{1}{x\ln b}$ by division, we get $x = b^{\frac{1}{n\ln b}} = \sqrt[n]{e}$ and $a = \frac{\log_b \sqrt[n]{e}}{e}$.

Now, when $a = \frac{\log_b \sqrt[n]{e}}{e}$, the graphs of $y = ax^n$ and $y = \log_b x$ touch at the point $\left(\sqrt[n]{e}, \log_b \sqrt[n]{e}\right)$.

We can easily compute common tangent lines to other classes of functions, using the technique adopted here. It is left to the reader to enjoy and make more connections between different classes of functions. ▲

▲ Mathematics Education Researcher Highlights

Math Education Researcher: Dr. Indigo Esmonde, Ontario Institute for Studies in Education, University of Toronto

Projects: Indigo's research considers issues of power and privilege as they relate to learning. Drawing from sociocultural theories of learning, and critical theories of race and gender, Indigo's research considers mathematics learning in many different venues: K–12 schools, families, community activism, and the workplace. Indigo is currently co-editing a book entitled *Power and Privilege in the Learning Sciences: Critical and Sociocultural Theories*, which will present theories of learning that combine attention to social and cultural practices, with systems of power, privilege, and oppression. Indigo is also working on articles addressing how activists develop mathematical expertise, and how mathematics classrooms can fight fatphobia in and out of schools. ▲

Encyclopedia of Mathematics Education (S. Lerman, 2014) Excerpts

"Since its introduction in schools—in the 1980s—the use of ICT (information and communication technology) in teaching mathematics has had two main functions: (a) as a support for the organization of the teacher's work (producing worksheets, keeping grades) and (b) as a supporter for new ways of doing and representing mathematics. The past decade has seen an evolution of technology itself with the introduction of new communication and representational infrastructures . . . The representational infrastructures used in mathematics education can involve specific software for teaching topics such as statistics, algebra, and modelling as well as graphical, numerical, symbolic, and geometric environments that are used to represent mathematical objects. Over time, teachers have moved from content-specific graphical and mathematical programs toward more generic and multi-representational environments."

Sinclair, N., & Robutti, O. (2014). Teaching practices in digital environments. In S. Lerman (Ed.), *Encyclopedia of mathematics education* (pp. 598–601). London, UK: Springer.

Rationale and Pedagogy

The origins of Foundation are to be found in the 1960s, when computer input devices were modified typewriters using only one dimension of the paper. For example, the expression $\sqrt{x^2 + y^2}$ was coded as something like SQRT (x*2) + y*2. This rendered Mathematical computing too tedious and confusing to be of much use for teaching purposes.

When these input problems were overcome with new technology, the capability was used for digitizing, editing, and printing Mathematics papers, but not for calculation. This surprising outcome may have originated in the fact that two dimensions were not necessary for Alan Turing's first machine design, 80 years ago. However, that doesn't stop two dimensions being valuable, particularly for school Mathematics pedagogy.

Using two dimensions adds simplicity and elegance to calculation. It has other benefits, including the solution of pedagogical problems identified in the literature on the teaching of Algebra.

It has been fairly recently realized that it would be highly desirable to teach Mathematics through functions. Functions (*replacing abstract operators*) and numbers are the two building blocks of Foundation. Another major advantage is its ability to provide concrete introductions to concepts. Handwriting Mathematics on paper or a whiteboard is an abstract exercise; we do so in successive statements. For example:

x + 4 = 11
x = 11 – 4
x = 7

This is like a series of snapshots. The numbers, variables, operators, expressions, and equations in these snapshots are abstract objects. Each new snapshot is created from the last snapshot through abstract rules. By contrast, Foundation uses the computer to generate concrete, moving processes, more like a video and more like the real world. In the pilot project, it was a powerful introduction to the physical manipulatives students used and the paperwork they did.

Foundation is a common base for both Arithmetic and Algebra; it treats both in the same way. This provides a stepping stone that eases the transition from Arithmetic to Algebra and enables mathematical generalization to be better understood in earlier years.

Computer programming is not required. As a mathematical expression is built through function composition, the computer also builds its calculation program. This makes Foundation a model of school Mathematics, enabling the subject to be taught as scientific experimentation.

The focus on notation can help students perceive Mathematics as a developing science, rather than a series of disconnected algorithms. They can also be helped to understand what mathematical expressions are by watching them be both composed and calculated one step at a time.

Card Set C: Tables

T1

n	1	2	3	4
Ans	14	16	18	20

T2

n	1	2	3	4
Ans			81	144

T3

n	1	2	3	4
Ans		10	15	22

T4

n	1	2	3	4
Ans	3		27	48

T5

n	1	2	3	4
Ans			81	100

T6

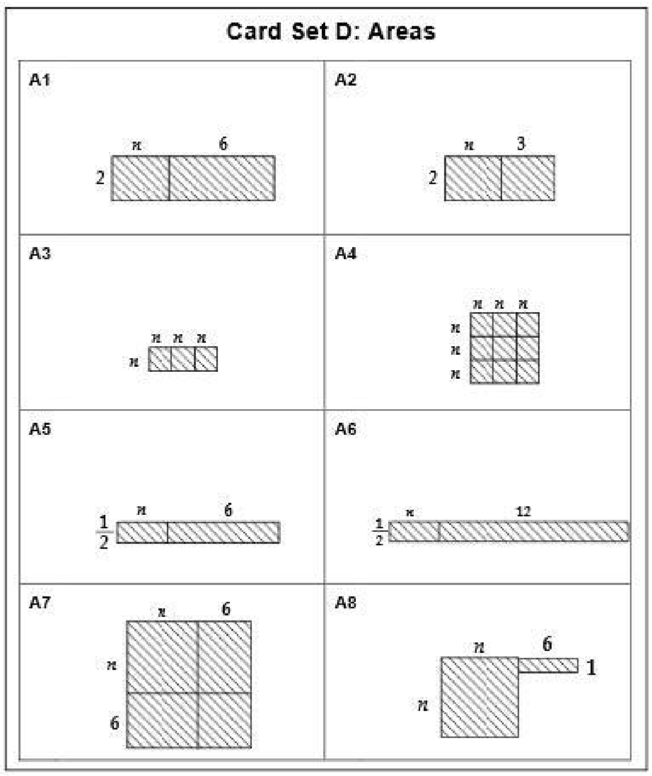
n	1	2	3	4
Ans		10	12	14

T7

n	1	2	3	4
Ans		4		5

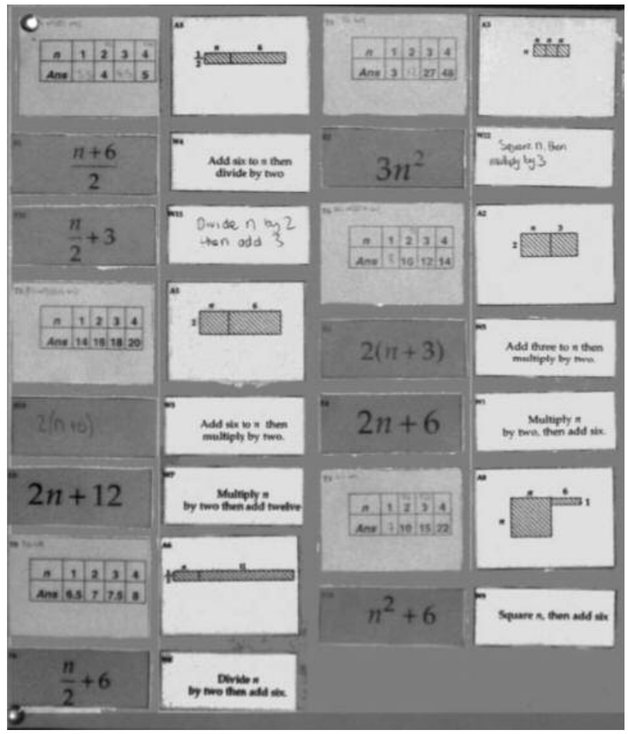
T8

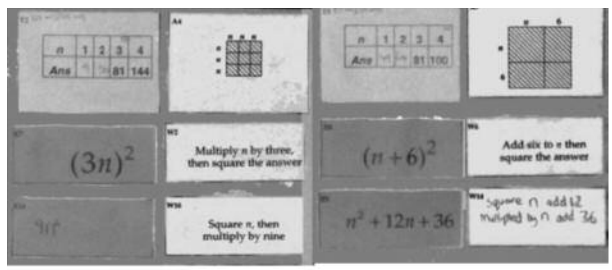
n	1	2	3	4
Ans	6.5	7	7.5	8



Not only does MAP provide this fantastic activity free of charge, but it also provides outstanding support. Each lesson comes with a “Before the Lesson” section, where students are given questions to complete individually, which will allow the teacher to determine what struggles students are having and then target the feedback appropriately. MAP walks you through suggestions of how to assess student work and provide common issues along with suggested questions and prompts. A “Suggested Lesson Outline” is also provided, which in this case includes an interactive whole-class introduction followed by the collaborative activity broken down into three parts. This is followed by a suggested whole-class discussion and a follow-up lesson.

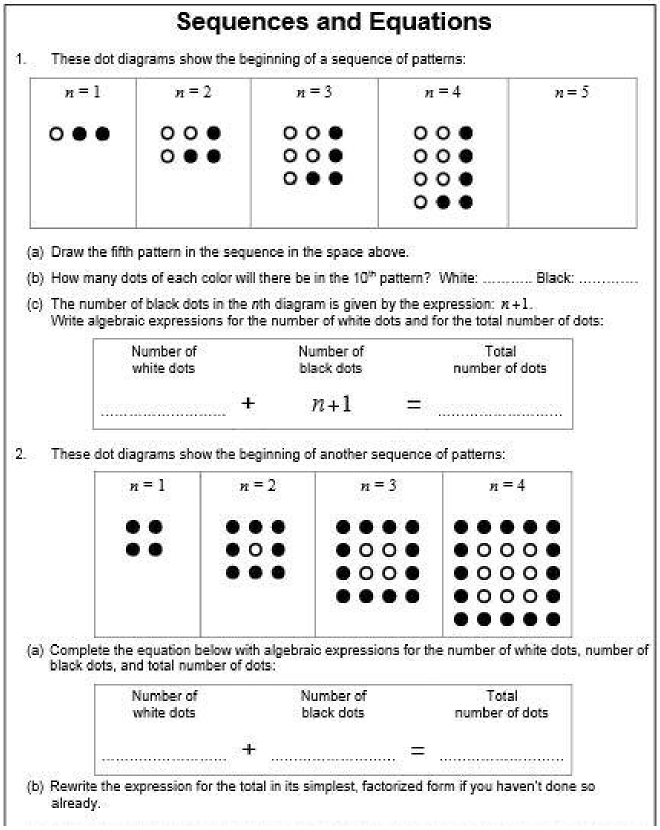
Here is a sample of completed work that MAP provides:





As you can see, this is truly a complete lesson. Experienced and inexperienced teachers alike can learn from the supporting materials and can adapt each lesson to best suit their needs.

Here is another lesson from MAP called Generating Polynomials from Patterns. Anyone who loves the visualpatterns.org website will appreciate this lesson. Here is the “Before the Lesson” task:



Sequences and Equations

1. These dot diagrams show the beginning of a sequence of patterns:

n = 1	n = 2	n = 3	n = 4	n = 5

(a) Draw the fifth pattern in the sequence in the space above.

(b) How many dots of each color will there be in the 10[th] pattern? White: .... Black: ....

(c) The number of black dots in the *n*th diagram is given by the expression: $n+1$.
Write algebraic expressions for the number of white dots and for the total number of dots:

Number of white dots		Number of black dots		Total number of dots
....	+	$n+1$	=	....

2. These dot diagrams show the beginning of another sequence of patterns:

n = 1	n = 2	n = 3	n = 4

(a) Complete the equation below with algebraic expressions for the number of white dots, number of black dots, and total number of dots:

Number of white dots		Number of black dots		Total number of dots
....	+	....	=	....

(b) Rewrite the expression for the total in its simplest, factorized form if you haven't done so already.

Here is one (of many) of the common issues in the "Before the Lesson" task that MAP addresses with the suggested questions and prompts.

Common issues:	Suggested questions and prompts:
Draws dot diagram incorrectly For example: The student draws the correct number of dots, but does not follow the pattern. Or: The student does not see the pattern at all and draws a dot diagram containing too many/few dots.	• Describe how the dot diagram changes as *n* increases. What is happening to the number of white dots? What is happening to the number of black dots? • What stays the same when you move from one diagram to the next? What changes? In what way does it change? • Can you see a pattern in the way the dots are arranged?

This is a really important piece, as it gives teachers who are doing the activity for the first time a way of anticipating student responses. Being prepared for these allows teachers to not have to improvise during the class, instead asking questions that will help address each particular challenge that a student encounters.

The "Suggested Lesson Outline" has an introduction, followed by collaborative small-group work that includes both a script, should you wish to use it, and strategies to help support group work. This is followed by sharing posters and a whole-class discussion with very good questions. There is also a follow-up lesson included and solutions are provided.

Some lessons have students analyze sample student work, which can be a really powerful tool. It is fantastic to have student work to look at that doesn't come from your own class and therefore allows students to express their mathematical opinions without judging their peers. Students can learn a lot from each other's work.

The site also has tasks for middle school and high school. These are broken down into novice, apprentice and expert tasks, and provide more or less scaffolding, depending on the level. Here is a sample expert level task that is wide open.

Best Buy Tickets

Susie is organizing the printing of tickets for a show her friends are producing.
She has collected prices from several printers and these two seem to be the best.

SURE PRINT
Ticket printing
25 tickets for $2

BEST PRINT
Tickets printed
$10 setting up
plus
$1 for 25 tickets

Susie wants to go for the best buy

She doesn't yet know how many people are going to come.

Show Susie a couple of ways in which she could make the right decision, whatever the number.

Illustrate your advice with a couple of examples.

All tasks include a scoring rubric, and some have sample unscored as well as scored student work. The latter is great if you want to display various solutions for your class so students better understand the difference between a poor and a good solution. There are also tests, PD modules, and more on the website. Investigating all the great content on the MAP site is well worth your time. ▲

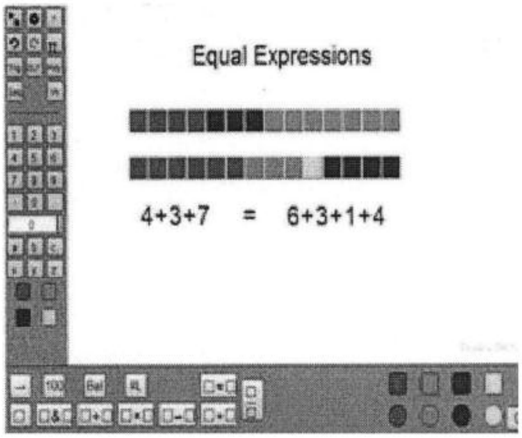
Encyclopedia of Mathematics Education (S. Lerman, 2014) Excerpts

"Virtual communities emerge in the early 2000s expanding affordances of the Internet technology while allowing for designing collaborative learning environments in mathematics. For example, a Math Forum community brings together teachers, students, parents, software developers, mathematicians, math educators, professionals, and tradespeople. While having different experience, expertise, and interest in mathematics by playing different roles, they all contribute in building sustainable learning space with a variety of educational resources that helps to scaffold each other's understanding of mathematics."

Freiman, V. (2014). Technology design in mathematics education. In S. Lerman (Ed.), *Encyclopedia of mathematics education* (pp. 605–610). London, UK: Springer.

Advertisement

Unlocking Mathematics
for **All** *Students*



Foundation is a unique and important mathematical discovery. It offers a fresh and concrete way of introducing Mathematics concepts that could make a considerable difference for all young children before they expand their capability in abstraction. Significant success was demonstrated in an 18-month Grades 1/2 pilot project (*see "Unlocking Mathematics for All Students" at www.FoundationNotation.com*).

In the pilot project, Foundation was motivating, engaging and easy to understand and learn. It worked with existing lesson plans and helped prepare students for Mathematical exercises on paper. It has been designed to achieve these results throughout the whole school Mathematics curriculum from Grade 1 addition to Grade 12 Calculus.

Until now, no way of calculating directly from Mathematics notation has been available. Foundation calculates any school mathematical expression directly, making the notation a computer language.

A good tool for learning by inquiry, Foundation takes the initiative away from the computer and hands it to the student or teacher. Students can think about what comes next, then try Foundation to see if their predictions are correct; this increased motivation in the pilot project.

Foundation uses a virtual manipulative that is a powerful and effective partner; many rules can be demonstrated simply by counting coloured squares (*this release of the app, 1.3, offers counting features to introduce addition, subtraction, division, multiplication, equality, variables, long calculation, and some algebraic rules. More will follow in future releases*). Release 2 will deal with number lines, negative numbers, and fractions.

Foundation is not intended to replace any other ways of teaching school Mathematics. It is an important tool that complements them.

The online Teacher Guide is intended both for documentation and self-teaching, with guidance for exercises in using Foundation. The difficulty of explaining the movement of Foundation on static paper is addressed with short videos.

Apps for Mac and Windows 8 are $9.99 Canadian (*plus tax*) in the Microsoft and Apple stores under the name:

"Foundation Notation".

An Ipad version will soon be available. Without purchasing the app, teachers can read the Teacher's Guide at FoundationNotation.com and watch the videos to see if they wish to use Foundation. In the Windows 8 app, a week's free trial is provided

Rationale and Pedagogy

The origins of Foundation are to be found in the 1960s, when computer input devices were modified typewriters using only one dimension of the paper. For example, the expression $\sqrt{x^2 + y^2}$ was coded as something like SQRT (x*2) + y*2. This rendered Mathematical computing too tedious and confusing to be of much use for teaching purposes.

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▲ What's the Problem? Totalling the Tilings

SHAWN GODIN

Shawn is the head of Mathematics, Business Studies, Law, and Computer Science at Cairine Wilson Secondary School in Orleans, Ontario. He is an advocate of mathematical problem solving. He currently helps create and mark math contests for the Centre for Education in Mathematics and Computing at the University of Waterloo. He currently works on math contests for the Canadian Mathematical Society and has worked as editor of their problem-solving journal, Crux Mathematicorum*. His Google Drive folder for supplementary material can be accessed at* **https://drive.google.com/folderview?id=0ByDlaUaj..8StpanhnUWo2bEV6ZE0&usp=sharing**.

Welcome back to another school year, problem solvers. I hope you had a restful summer. Feel free to check out my *What's the Problem?* Google Drive folder. It will occasionally contain supplementary material related to a particular column. Specifically for this column, there will be some extended explorations and listing of cases.

Last time, I left you with the following problem:

In figure 1, a rectangular ceiling, PQRS, measures 6 m by 4 m and is to be completely covered using 12 rectangular tiles, each measuring 1 m by 2 m. There is a beam, TU, that is positioned so that PT = SU = 2 m and that cannot be crossed by any tile. Determine the number of possible arrangements of tiles.

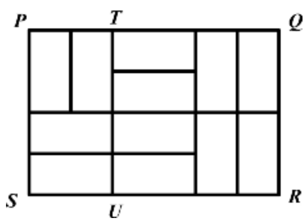


Figure 1

This problem appeared as question 24 from the 2015 Cayley contest for Grade 10 students from the Centre for Education in Mathematics and Computing at the University of Waterloo. You can check out the many resources offered by the CEMC, including copies of past contests, at their website, **www.cemc.uwaterloo.ca**.

This is a great problem to emphasize the idea of looking at all cases systematically so that none are missed. First note that since no tile can cross the beam TU, the ceiling is broken into two pieces, PTUS and TQRU, as shown in figure 2. Thus, the total number of tilings of PQRS is the product of the number of tilings of PTUS and the number of tilings of TQRU.

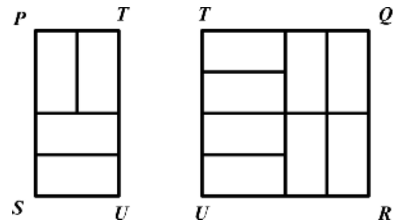


Figure 2

Let's work with the smaller rectangle first. This is a perfect problem that can be solved with manipulatives, if we are careful. If we start at P, then the tile that contains P can either be vertical or horizontal (in which case, it would also contain T). If it is vertical, it must contain another vertical tile beside it that contains T. Thus, we are left with a square at the bottom that we can only tile in two ways: two horizontal tiles or two vertical tiles, which are shown in figure 3.

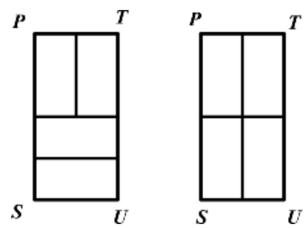


Figure 3

If the tile containing P is horizontal, we need to be careful. It would be easy to put another horizontal tile underneath it, in which case, we are left with a square, as in the previous case, as shown by the two left rectangles in figure 4. It turns out that we could also put a vertical tile underneath the first one, as shown in the rightmost rectangle in figure 4. The vertical tile forces another vertical tile to be beside it, and a horizontal tile to be underneath it. As such, there are three possible tilings, with the first one horizontal; thus, there are five ways to tile PTUS.

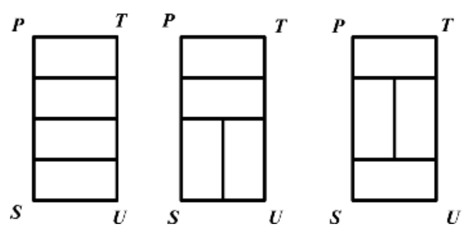


Figure 4

For the larger rectangle, TQRU, we will use some mathematical properties as we systematically look at all cases. First we note that the rectangle that contains T must be either horizontal or vertical. Notice that if we take a tiling that has a horizontal tile at T and we reflect the whole configuration in diagonal TR, we get a new tiling that has a vertical tile at T, as shown in figure 5. Similarly, if we took a tiling with a vertical tile at T and reflected it in TR, we would get a new tiling with a horizontal tile at T. This means that the tilings can be paired up using symmetry about TR. Thus, we only need to consider the case where T is contained in a horizontal tile.

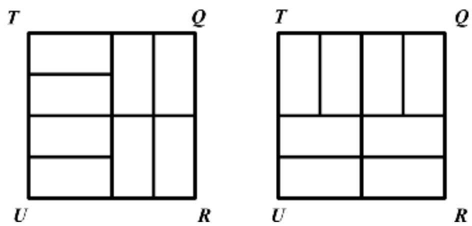


Figure 5

Now we will proceed by breaking the tilings down into cases and subcases and enumerating as we go. Since a horizontal tile contains T, we have two main cases (see figure 6): a vertical tile contains Q (case 1) and a horizontal tile contains Q (case 2).

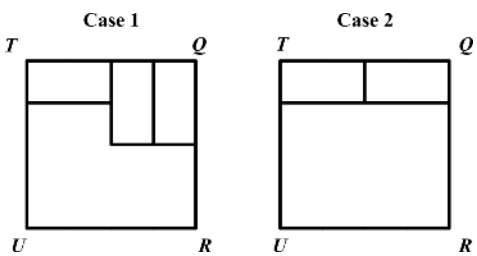


Figure 6

Case 1 breaks down into two subcases. If a horizontal rectangle is placed under the original horizontal rectangle that contains T, then we are left with the 2 m by 4 m rectangle shown as case 1a in figure 7. From our earlier work, we know that there are five ways to tile such a rectangle. If we place a vertical rectangle under the original horizontal rectangle, it must have another vertical rectangle beside it, which forces a horizontal rectangle under them, as shown as case 1b in figure 7. In this case, we are left with a square in the bottom right corner that can be tiled in only two ways. Thus, case 1 leads to seven possible tilings.

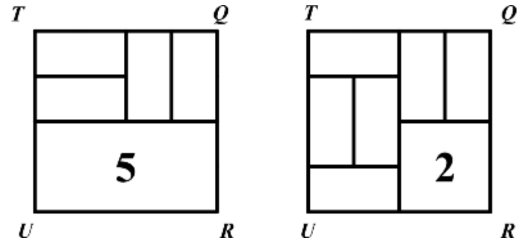


Figure 7

Next we tackle the more interesting case 2. Proceeding systematically, we see that case 2 can be further broken down into two subcases, depending on the placement of the tile under the original tile that contained T. Case 2a will be the case of a horizontal tile, and case 2b will be the case of a vertical tile (see figure 8).

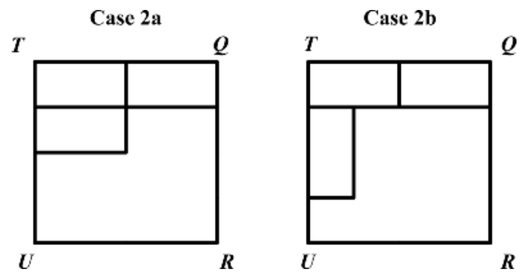


Figure 8

Case 2a further breaks into two subcases, depending on the configuration of the “next” tile. If we place a horizontal tile beside the last tile (case 2ai), we have a 2 m by 4 m rectangle left, which can be tiled in five ways. If we place a vertical tile beside the last tile, it must be paired with another vertical tile and a horizontal tile underneath (case 2aii). This leaves a 2 m by 2 m square that can be tiled in two ways. Hence, case 2a yields seven more tilings, similar to case 1.

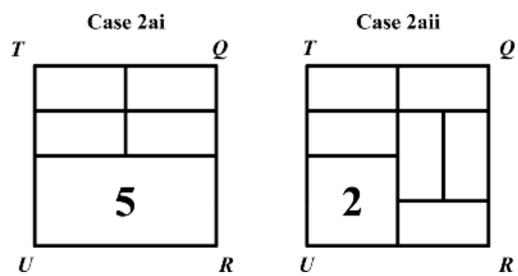


Figure 9

Finally, we are left with case 2b, which breaks into thee subcases: two horizontal tiles beside the last tile (case 2bi), two vertical tiles beside the last tile (case 2bii) and a vertical tile, and horizontal tile beside the last tile (case 2biii). The first two cases are easy, as when we fill in the next tiles, the remaining tiles are forced into place, so they each yield only a single tiling. For last case, the bottom left tile is forced into place, leaving a 2 m by 2 m square that can be tiled in two ways.

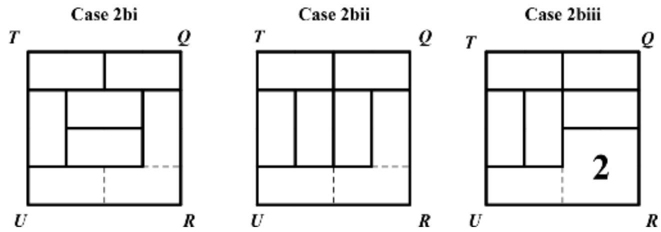


Figure 10

Thus, case 2b yields four tilings, so the number of ways to tile TQRU with a horizontal tile at T is 7 + 7 + 4 = 18 ways. Hence, the total number of ways to tile TQRU is 18 x 2 = 36, and so the total number of tilings for the problem is 5 x 36 = 180.

Let's look at a compact quick solution. This solution may not have been obvious at first, but as we start looking through cases, as in the previous solution, some patterns begin to emerge. First, in cases 1a and 2ai, we were left with a horizontal 2 m by 4 m rectangle. Note that when this happens, the top part is also a horizontal 2 m by 4 m rectangle. The number of tilings that can be broken into two horizontal 2 m by 4 m rectangles is 5 x 5 = 25. Similarly, we could have broken TQRU into two vertical 2 m by 4 m rectangles, also in 25 ways.

Unfortunately, if we do this, some tilings will fall into both categories. Which tilings would be "double counted"? It would have to be the tilings that can be broken down into four 2 m by 2 m squares, like both examples in figure 5. Each square can be tiled in two ways, so there are 2 x 2 x 2 x 2 = 16 ways that have been "double counted."

We still have some cases unaccounted for. All of our cases from the first solution can be broken down into two horizontal or vertical rectangles, except case 2bi. The only other case would be its mirror image in TR, hence the number of ways to tile TQRU is 2 x (5 x 5) – 2 x 2 x 2 x 2 + 2 = 36, as we saw, eventually, in the first solution.

Nice counting problems usually have several possible ways to be solved. We have seen how we can solve a problem exhaustively by listing all the possible solutions, that is, rectangle PTUS, but in most cases, this in not practical. We also saw how we could carefully break the problem down into cases, and used some enumeration techniques to simplify our counting. Finally, we saw how we could use a combination of techniques to significantly reduce our work, but this requires some insight and careful consideration. For another nice solution, check out the official solution on the CEMC website. As with many problems, if you are unsure how to deal with it, playing around with it is always a good strategy. Now for your homework:

A pirate leaves the following instructions: "The island where I buried my treasure contains a single palm tree. Find the tree. From the palm tree, walk directly to the falcon-shaped rock, counting your paces as you go. Turn a quarter-circle to the right, walk the same number of paces as you just counted, and plant a stick into the ground. Return to the palm tree. From the palm tree, walk directly to the owl-shaped rock, counting your paces as you go. Turn a quarter-circle to the left, walk the same number of paces as you just counted, and plant a stick into the ground. Connect the two sticks with a rope and dig beneath its midpoint to find the treasure." By the time the pirate's instructions are found, the palm tree has long since died, but the two rocks are still identifiable. Where should you look for the treasure?

Until next time, happy problem solving! ▲

Encyclopedia of Mathematics Education (S. Lerman, 2014) Excerpts

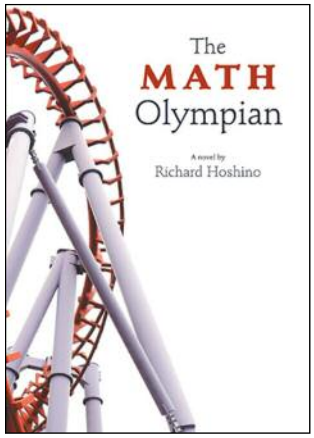
"The zone of proximal development is a category that emerged from the work of L. S. Vygotsky, the father of activity theory. . . . [The ZPD] denotes 'the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers (Vygotsky, 1978). . . . In summary, the zone of proximal development is a powerful category for understanding learning that arises when people enter relations with others. Aphoristically we may state: What these relations are today will be psychological functions of the participants tomorrow."

Roth, W.-M. (2014). Zone of proximal development in mathematics education. In S. Lerman (Ed.), *Encyclopedia of mathematics education* (pp. 647–650). London, UK: Springer.

▲ An Olympian Speaks Up!

REVIEWED BY TIM SABBALD

Book review of:
Hoshino, R. (2015). *The math olympian*. Victoria, BC: Friesen Press. For details, see **www.friesenpress.com/bookstore/title/119734000016976420/Richard-Hoshino-The-Math-Olympian**.



This is an inspired book that reads easily, while presenting a first-hand account of some formidable math challenges. The book begins with a Grade 12 student being dropped off by her mother at the school board office, where a superintendent supervises her participation in the Canadian Math Olympiad.

The five problems that constitute the Olympiad are presented near the start of the 475-page book, and the reader is made aware that there is a three-hour time limit. Much as the mathematics is rich and accessible: there are many flashback snippets of school experiences and lessons learned along the way for Bethany to have reached the competition. In many respects, the experiences are what makes the book compelling.

As a teacher, I was quite struck by some of the classroom recollections and vivid portrayal of bullying experiences. Accounts are presented of teachers who attempted to draw the shy student out, but inadvertently added to the potential for the student to feel awkward because her shy demeanor was being challenged. Some sections are excellent food for thought, especially for less experienced professional teachers and for teacher candidates. The portrayal of a vice-principal, who meets the student where the students' needs are, is illuminating.

The mathematics is presented as a flow of thoughts, and this is wonderful for anyone who has wondered about the process used by high-performance problem solvers. The strategies are provided in relatively small steps, and many strategies are evident. Often the strategies are presented through flashbacks to tutors, who used simpler examples that help the reader understand the strategy. The approach is very well suited to high school readers, and many teachers will enjoy it as well. In terms of more detailed resources, there is a mention of **www.artofproblemsolving.com** that provides an opportunity to go beyond the book itself.

Within the pages, many smaller issues are addressed. Consider, for instance, that question #3 is presented as there being 25 men sitting around a table. "I roll my eyes, twenty-five men" (p. 184). While the gender issue is not as prevalent as it once was, it continues to be an area that is in need of attention. What is more, at high levels, there are gender issues, and the approach used to address it is effective. The protagonist does not give up in the face of an unnecessarily gender-bias question; rather, as she works through the problem, she uses all female names.

The story goes beyond math problems and broaches the challenge of choosing universities. It also presents tension between a father and daughter regarding the role of spirituality, specifically Christianity, and how one might reconcile faith with mathematics. On both accounts, I wondered about the suitability for students and the extent they were potentially biased by the author's point of view. However, I concluded that they were suitable and handled significant issues quite well. In the case of selecting universities, there is a push and pull between two specific universities that the author has experience with, but the issue for readers will be seen to be broader because the two specific universities are at opposite ends of Canada.

In terms of spirituality, it represents a turning point in the maturation process of the characters, where they have to address personal psychological elements in order to continue making progress. While this is developed through a spiritual component with one friend, it is also supported through the mother of the main character studying psychology. It is not overdone for students, but is minimal for teachers. For example, the mention of the "self-esteem movement" (p. 430) that suggests that students are not generally sufficiently challenged today is debatable.

While the book is an invaluable resource for mathematics teachers, it has one significant limitation: it only presents competitive enrichment. The issue of co-operative versus competitive involvement is addressed in a few spots, such as remarks made about teammates on a co-operative math challenge: "They were all keen, but none were competitive" (p. 219). However, the co-operative element is couched in a competitive environment, such as a team relay competition. Similarly: "I'm not interested in how many people I beat, or what place I finish. It is me versus the clock" (p. 74). However, this vision is not consistent when an episode of playing the rules is explained. In the account, the protagonist constrains the possible solutions and repeatedly hands in answers (allowed by the rules) until she reaches the correct one.

As a teacher, I have always wrestled with the dominance of contest-oriented enrichment. This book, in spite of its strength of providing a valuable resource in math education, does not address non-competitive enrichment. Given the Canadian math landscape, I was not surprised by this. However, teachers should be aware that the book provides an in-depth view of a very specific form of enrichment.

Overall, Richard Hoshino has provided an important contribution to math educators and should be applauded for this book. The spirit it brings makes it an excellent enrichment prospect for high school students. It is well suited to school libraries, a classroom bookshelf, and as a potential motivational gift for students who deserve special treatment. ▲

▲ Thinking About Teaching, Learning, and Mathematical Mindsets Leads Me to Learning Skills

JAMIE PYPER

Jamie is an Assistant Professor of Mathematics Education, and the Coordinator of a research and curriculum development team, the Mathematics, Science, Technology Education Group (MSTE), at the Faculty of Education, Queen's University. He spent 20 years teaching high school mathematics in two school boards in Ontario and mathematics education at Western University. He has been a member of OAME since 1995 and is often a workshop presenter at OAME and local chapter conferences.

> "All learning begins when our comfortable ideas turn out to be inadequate." ~ John Dewey

The Ontario math curriculum states that assessment and instruction should be seamless (OME, 2010); others say that classroom management and instruction should be seamless (e.g., Jones, Jones, & Jones, 2000); and some state that curriculum and assessment should be seamless (e.g., Drake, Reid, & Kolohon, 2014). Each of these statements comes from a particular perspective that is theoretical, practical, or a combination of both. I would like to focus these perspectives into a mathematics classroom context—the space teachers live and work in for hours a day with hundreds of students. It is what has happened in my mathematics classroom that has helped me realize, over time, when my comfortable ideas of teaching mathematics became inadequate, when student learning was as uninspired as the routine of my teaching, and like a badly designed spiral curriculum, I was going around in circles and never really feeling like everything came together at the same time. Don't get me wrong, I saw regular successes in my students' achievement, and felt I had many good teaching moments, but a growing sense of inadequacy prodded me to broaden my knowledge base and then my professional practice.

I began to conceptualize my professional practice with the metaphor of the centre of gravity, or centroid, of a triangle. Imagine a piece of cardboard, cut into the shape of a triangle, balanced on the end of a pencil placed at its centroid. Consider the vertices to represent classroom management, instruction, and assessment (see Figure 1), and this balanced triangle represents a teacher's professional practice. Knowing a single metaphor may not be able to capture everything, and for the sake of argument and simplification, I imagine that these three vertices represent the sum of all the wisdom and understanding of what teaching is all about—everything we know about teaching can be captured within the metaphor of this triangle. The challenge for the classroom teacher is to maintain that balance in professional practice—do not let the triangle get too heavy on any one side or vertex and slip off the pencil. However, this can be difficult because we continue to learn so much about the teaching and learning of mathematics as we develop knowledge and skills in these areas of professional practice, and we risk upsetting the balance if we focus too much on any one aspect. So, as I learn, I continuously reflect on how classroom management, assessment, and instruction are being affected, and how their interaction (rather than just the sum of their parts) must be considered when I implement any new learning in my classroom teaching. This is my perspective on the lived experience of the classroom teacher—that context matters, and this balance is achievable, and desirable.

Figure 1. *A centroid metaphor for classroom practice*

When I think of the students in my classes, I appreciate that they are more than just students of a classroom mathematics course. They are also citizens of our current society, and twenty-first-century skills are becoming an important outcome from schooling and formal education. An overview of these twenty-first-century skills identifies them as critical thinking, problem solving, reasoning, creativity, imagination, innovation, perseverance, self-direction, self-discipline, initiative, collaboration, leadership, and active multiple literacies. These skills are appearing to be valuable to my students' expressions of achievement and mathematical performance, but more importantly, these skills are becoming increasingly visible in the policies, documents, and curriculum for mathematics courses and schools in which I teach. The current vehicle to achieve something like facility in twenty-first-century skills is our schools and our classrooms. I wonder how a student can learn these twenty-first-century skills, as well as the mathematics of the curriculum, without the skills of learning, or Learning Skills. When I ask my students, they agree that it is hard to learn something without Learning Skills.

As teachers, we also develop twenty-first-century skills, such as our own multiple literacies, or professional literacies (Pyper, Chin, & Reeve, 2015). Current thinking about learning is giving us some new language about teaching and learning. Professional literacies of learning and teaching are expanding with ideas like "fixed" and "growth mindsets" (see Dweck, 2006). These ideas in particular represent concepts of learning and thinking that are providing insight into learners' motivations, efforts, and identities. These ideas and issues are important to teachers and students. We wrestle with them daily in our lessons, tasks, and classrooms.

Growth mindsets are important, and they develop habits of mind. "A habit of mind is a pattern of intellectual behaviours that leads to productive actions" (Costa & Kallick, 2008, p. 16). To explain the basic nature of this pattern of intellectual behaviours, Costa and Kallick describe six dimensions: i. value – choosing to employ one pattern of intellectual behaviours over others; ii. inclination – feeling a tendency to employ this pattern; iii. sensitivity – perceiving opportunities to employ this pattern; iv. capability – possessing the skills and capacities to carry through with this pattern; v. commitment – constantly reflecting on and improving performance; and vi. policy – making it a personal policy to embed this pattern into one's thinking and actions. Some of these dimensions sound strikingly similar to the Ontario curriculum Learning Skills; for example, inclination is like initiative, sensitivity points to organization, and commitment appears under self-regulation as persistence.

But what are the specific habits of mind? Costa and Kallick (2008) list 16 habits of mind (see Table 1). Of the 16 they identify, a number of them feel like they align with other constructs and ideas about learning, which lends support to the premise that these habits of mind are authentic to our experiences of learning. Table 1 contains a partial list of some of these other constructs I can see fitting in. As you read the list, what ideas and constructs of learning are you thinking about? (For further details about these habits of mind, see Costa and Kallick's book.)

Habits of Mind	Connections
Persisting	→ Resilience, self-efficacy
Managing impulsivity	→ Self-regulation
Listening with understanding and empathy	
Thinking flexibly	→ Mathematical processes, problem solving
Thinking about thinking (metacognition)	
Striving for accuracy	
Questioning and posing problems	→ Assessment as learning
Applying past knowledge to new situations	→ "Thinking" Achievement Category
Thinking and communicating with clarity and precision	→ "Communication" Achievement Category
Gathering data through all senses	
Creating, imagining, innovating	
Responding with wonderment and awe	
Taking responsible risks	→ Learning Skill
Finding humour	
Thinking interdependently	
Remaining open to continuous learning	

Table 1. *The 16 Habits of Mind (Costa & Kallick, 2008) and some associated connections to other teaching and learning constructs*

It will be beneficial for one's learning to make these habits of mind automatic or natural states of thinking because "habituation (Mason & Davis, 1989) is a fundamental process through which people come to know" (Mason & Spence, 1999, p. 152). Knowing, now, is likely the crux of the issue. What do we know, and how do we know it? It may be nice and wonderful to develop and have habits of mind at our disposal, but what is the epistemological sense of the knowledge that occurs because of these habits? What is the nature of knowing that we are habitually acting upon?

Here are three ways of "knowing" from a mathematics perspective. Cuoco, Goldenberg, and Mark (2010) list eight mathematical habits of mind: pattern sniffing, experimenting, describing, tinkering, inventing, visualizing, conjecturing, and guessing. These habits of mind are described through a geometrical approach and an algebraic approach to thinking (see Cuoco, Goldenberg, & Mark, 1996 for details on these two approaches). In the process of exploring and explicating the assessment of mathematics, the Harvard School of Education (1995) described mathematical inquiry (or thinking) as five phases using eight verbs: hypothesising, modelling and formulating, transforming and manipulating, inferring and concluding, and communicating. These may look familiar to many who taught or experienced the 1999 version of the Grades 9 and 10 mathematics curriculum—these verbs formed a foundation to the pedagogical approach to that curriculum. Mason and Spence (1999) described two ways of knowing mathematics, knowing-about and knowing-to. In essence, knowing-about consists of knowing-that (the facts), knowing-how (the procedures), and knowing-why (the reasons). Knowing-to, though, is the ultimate goal, and this kind of knowing is all about knowing to act when presented with a problem, a situation, or question. (See Figure 2 for a visual graphic of "knowing.")

"Once the moment of knowing-to takes place, knowing-how takes over to exploit the fresh idea; knowing-that forms the ground, the base energy upon which all else depends and on which actions depend; knowing-why provides an overview and sense of direction that supports connection and modification if difficulties arise en route" (p. 146).

As they described these two ways of knowing, Mason & Spence linked them to assessment. The easy kind of assessment uses indicators of knowing-about, and the more challenging kind of assessment uses indicators of knowing-to. Their contention is that it is the more challenging kind of assessment and mathematical performance (knowing-to) that we, as teachers, need to emphasise for the benefit of our students' learning.

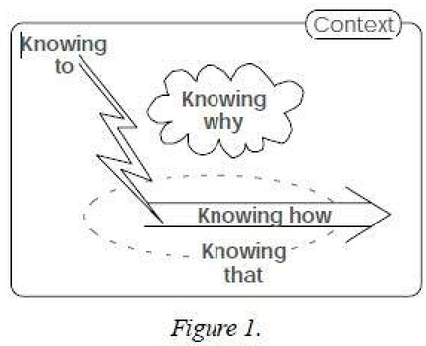


Figure 2. *Figure 1 from Mason & Spence (1999), page 145, illustrating the relationship between knowing-about and knowing-to*

		LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4
Responsibility	*Coursework (at home and in class)*	Completes ½ the work, submits consistently late.	Completes ¾ of the work, submits occasionally late.	**Completes all work, submits on time every time.**	Completes all the work, submits on time and sometimes earlier.
	Choices	Avoids, ignores, resists ownership for choices.	Accepts responsibility for choices after it is brought to one's attention.	**Takes ownership and responsibility for choices.**	Takes ownership and proactively makes necessary choices.
Independent Work	*Accomplishing tasks*	Does not start work until the teacher makes a personal request, does not complete task.	Starts work after a couple of requests by the teacher, uses about half the class period effectively.	**Starts when told and uses the class period to complete task.**	Starts right away and uses the full class period to complete the task.
	Following instructions	Misses most instructions from the teacher, follows others' lead, copies work.	Misses some of the instructions from the teacher.	**Listens to instructions and can get to work.**	Listens to all instructions and provides leadership to others.
Initiative	*Academic risk taking*	Not willing to take risks, needs continuous prompting from others to try something new.	Hesitant, needs many prompts from others to try something new.	**Willing, interested, needs few prompts from others to try something new.**	Willing, curious, innovative, does not need prompts from anyone.
	Attitude	Focuses on own wants and needs. Consistently negative outlook on the course.	Focuses on own interests and "rights." Often pessimistic outlook on the course.	**Works for own and others' rights and responsibilities.** **Positive outlook on the course.**	Works and advocates for own and others' rights and responsibilities. Optimistic and upbeat.
Organization	*Prioritizing*	Makes no plans. Does not prioritize.	Makes a plan. Does things in the order they are written.	**Establishes a plan and prioritizes.**	Establishes, prioritizes, and periodically updates the plan.
	Managing resources	Comes to class with nothing.	Brings some things, remembers once he/she gets to the classroom.	**Before coming to class, identifies, gathers, then uses all resources.**	Proactively identifies, gathers, uses, and evaluates the use of all resources.
Collaboration	*Peer relations/ Working together*	Dependent. Doesn't participate, just sits there.	Independent. Doesn't share with group.	**Interdependent.** **Shares ideas, encourages others' participation.**	Effective, positive, interdependence. Encourages critical thinking, elaborates on anyone's ideas. Everyone feels valued.
Self-Regulation	*Goals*	Has no individual goals.	Sets unrealistic or vague goals.	**Sets specific and achievable goals.**	Sets, monitors, and re-evaluates measurable goals.
	Persistence	First, and often the only, reaction is to give up.	First attempt is considered acceptable, works for a "pass."	**Sticks to it and doesn't give up.**	Will improve work, even after it has been assessed.

Figure 3. *Learning Skills & Work Habits (2010)*

To consider the growth and development of knowing-to, what might we need to attend, to help our students develop these habits of mind? Costa and Kallick (2008) suggest we explore meanings, expand capacities, extend values, build commitment, and increase alertness. Again, I notice an alignment of these four goals with the Ontario curricular Learning Skills and mathematics processes. I feel embedding Learning Skills into our daily classroom learning will meet each of these goals, which leads me to the questions, how would one teach *for* the habits of mind, how would one teach of the habits of mind, and how would one teach *with* the habits of mind?

The centroid and triangle metaphor is my framework for teaching—thinking of the seamlessness of teaching/instruction, assessment *for* and *of* learning, and classroom management of students' learning, and considering mathematical mindsets in the context of a math classroom. So that teaching turns into students' learning, the potential lynchpin to all of this is Learning Skills. Learning Skills are described in *Growing Success*, initially in the 1999 version, and now in the revised 2010 document. As a Grade 9 math teacher back in 1999, I appreciated the value of Learning Skills, but found it difficult to imagine how to incorporate them into my classroom practice. During a staff meeting before report card time in 2000, some colleagues presented a way to capture Learning Skills in a one-off opportunity just in time to gather some evidence and be able to add something to the report card under Learning Skills. I remember listening to their presentation and thinking that this is much too small a sample of Learning Skills evidence for me to accurately describe and evaluate my students' Learning Skill levels for a report card. Walking away from that presentation, I thought of the tracking sheet I was using for monitoring aspects of student classroom learning (see Pyper, 2005 for details about the tracking sheet). This was a somewhat teacher-centric and teacher-managed process, but wondering if I could add Learning Skills to the tracking sheet inspired me to imagine a more student-centric and student-managed process. I developed a Learning Skills rubric from the then four Learning Skills of the curriculum. I used the official document's titles and descriptions and transformed the rubric criteria into language students would accept and understand. I then wrote the four levels of quality of the criteria, again in language students would understand. I implemented and tested in my classrooms, and responding to students questions and input over the next two semesters, improved the wording in the rubric. Over the years, this was used by members of the math department and the summer school math teachers. When I moved to a new school in 2009, three other teachers in the new school (teaching English, History, and Science) started using this Learning Skills rubric and tracking sheet. When the new *Growing Success* document was published with six Learning Skills and included modifications to the descriptions, the four of us tweaked the Learning Skills rubric for the 2010 school year. This is the rubric you see in Figure 3.

Figure 4. *Learning Skills rubric on a classroom wall*

There are a number of features to the design and implementation of this Learning Skills rubric. Size is important—it needs to be large and wall mounted for everyone to see easily. I use regular 8.5x11 sheets of paper in plastic page protectors for each cell of the rubric. Colour is important—the Level 3 column is the "standard," as defined by the Ontario curriculum, so I wanted a colour that students' eyes would be drawn to first. Level 4 is a goal, not a standard, and so I wanted students to feel purposeful if they were trying to achieve a Level 4, hence the darker colour red. This is also a colour that requires one to sometimes go up to the wall itself to read the words—in a small sense, the physical demand of going to the wall increases the motivation to know what it says and then act accordingly. The rubric's Levels 1 and 2 columns are the colour of the wall, designed to blend in and not look appealing—I don't want students to select Level 1 or 2 as their goal, so I want these pages to either be missed or ignored. The criteria for each Learning Skill is chosen to reflect the behaviour or learning skill I have seen most often in need and which might benefit students' learning most. The descriptors in each level are purposefully worded to make Levels 1 and 2 unappealing to students, so they strive for Level 3 as a standard level of achievement and behaviour. There is a tension between an official pass being 50% or Level 1, and the standard level of achievement being 75% or Level 3. I have chosen to emphasise the standard level of achievement as a beneficial focus for students' success. See Figure 4 for an example of what this rubric could look like on a classroom wall.

For the first six weeks of class, I emphasize a Learning Skill each week. Starting from the top of the rubric, and each week, moving down to the bottom, we discuss the Learning Skill and focus on Level 3, self-assess with it, and learn what it means to embed Learning Skills in a metacognitive way while learning math. For the next two months, I select a different Learning Skill each week to align with what I have planned pedagogically. Soon students are selecting the Learning Skill for the week, and then by the last third of the course, we pick Learning Skills as necessary for individual days or periods of time. For the first six weeks, we are assessing Learning Skills together frequently, but by the middle of the course, we are assessing Learning Skills about three times a week.

I have seen students experience great success with this Learning Skills rubric. Students start to use the language of the Learning Skills (think learning skills literacy) and apply it to themselves and their peers (think assessment for and as learning). For example, at the end of one class and during the time needed for the class to self-assess and write on their tracking sheets, one student spoke out loud in class to say that another student (and named the student) should get a Level 4 for initiative because that student had been going around and helping people. The value of this story is that both were at-risk students, one much older in age, and both had not expressed much value for school in their previous years as a student. (The speaker, unfortunately, dropped out a month later, but the recipient of the praise stayed, finished this second attempt at the Essentials 10 course, and then completed the Applied 9 and 10 courses the next school year. In a personal communication at the end of the Grade 10 Applied course exam, the student attributed this to the use of Learning Skills in his classes—as soon as he started to figure out how to learn, the math got easier and easier.)

The implementation of this Learning Skills rubric is successful for me in my math classroom because it aligns with and supports so many aspects of instruction, assessment, and classroom management (think of my triangle metaphor.) This Learning Skills rubric is also successful for students in my math classroom because of its alignment with, and support for developing twenty-first-century skills, growth mindsets, habits of mind, and mathematical mindsets. Students feel successful in their learning because of the continuous application, its authentic and natural integration into mathematics lessons, and the ease with which students find that their improvement of the skills of learning invariably improve their achievement. They see this improvement in increased attendance, greater content learning and retention, and improved course marks. Student ownership over their learning becomes natural, and this is a most wonderful feeling as a teacher.

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▲ Provincial Digital Learning Resources – What's New? E-mail List, Images in Notepad, and New Resources




GREG CLARKE

AGNES GRAFTON

ROSS ISENEGGER

MARKUS WOLSKI

Greg is from the Simcoe Muskoka Catholic District School Board and Ross is from the Near North District School Board. Both are currently on assignment with the Ontario Ministry of Education as Provincial Math Leads, working on the CLIPS project, in conjunction with their colleague, Agnes Grafton, from the Brant Haldimand Norfolk Catholic District School Board. Markus Wolski is a teacher with the Bluewater District School Board and has been working on the CLIPS project for the past two years.

What's New E-mail List

Do you want to be among the first to know about developments with digital learning resources? If so, sign up for the e-mail list at **http://mathclips.ca/WhatsNewEmailList.html**. Not only will you find out about newly released resources, you will be given the opportunity to test and provide feedback on materials in development.

Import Images into the Notepad by Mathies

In the last issue's column, we announced the availability of the Notepad app at **http://mathies.ca/learningTools.php**. As it was being featured in the Ministry's K–12 Professional Development regional sessions, it became apparent that having the ability to import an image would greatly increase its functionality.

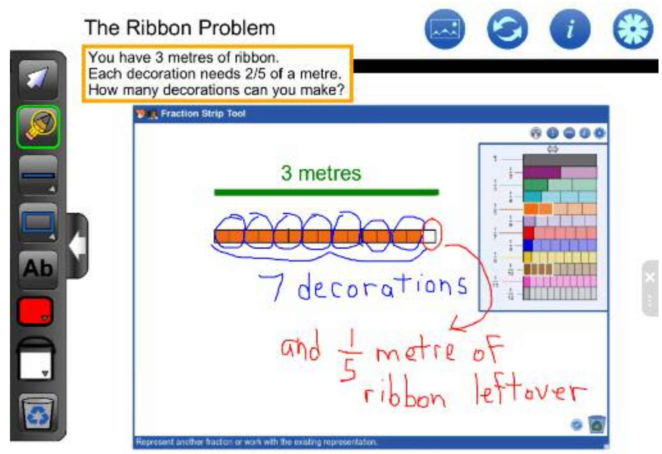
To import an image on a mobile device or computer, simply click the image icon at the top right. You can then select an image from the device and it will be placed as a background on the screen.

During the most recent Ministry K–12 sessions, this allowed participants to annotate representations gleaned from tools that do not have the annotation feature built right in.

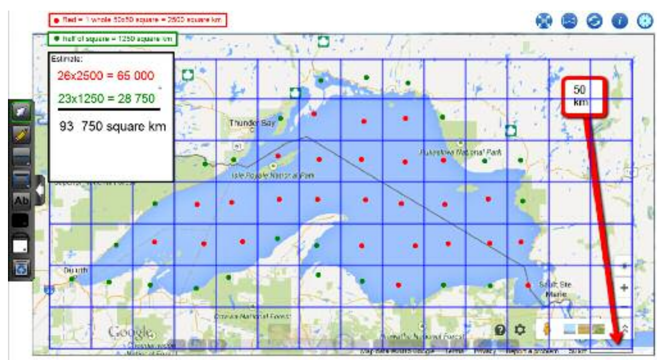
For example,

1) on a mobile device, use Puffin Academy to display the Flash-based Fraction Strips tool
2) use the magic button sequence on the device to take a screenshot that is automatically added to the Camera Roll
3) import the image into Notepad, where the solution and thinking is explained

See some of the participants' work and reflections on Twitter at #k12litnum, including some related to the Ribbon Problem, which is annotated below.



However, there are a lot of situations where a student-generated photo or screen capture would benefit from being imported. In the example below, the windowed grid feature of the Notepad tool is used to match the scale of an image from Google Maps to come up with an estimate of the surface area of a Great Lake. That is a superior use of the tool, don't you think?

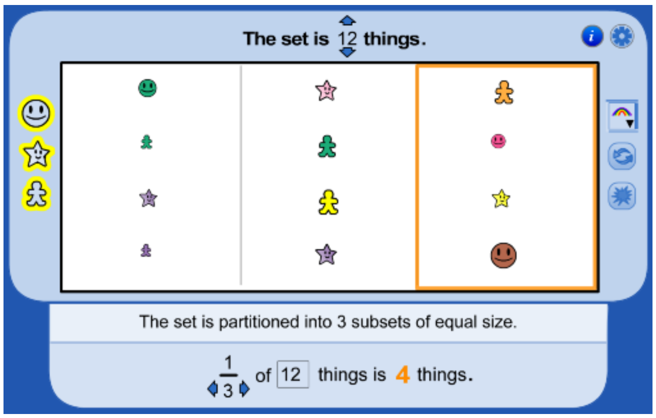


The team has a long list of other improvements that we would like to make to the annotation tool. If you want your idea to float higher in the priority list, give us a shout at **WhatsNew@oame.on.ca**. In the meantime, make sure you update your Notepad version so that the image import is available to you.

Partitioning the Partitioning Sets Tool

The Partitioning Sets Tool (**http://mathies.ca/learningTools.php#Pp0**) allows for the exploration of questions like:

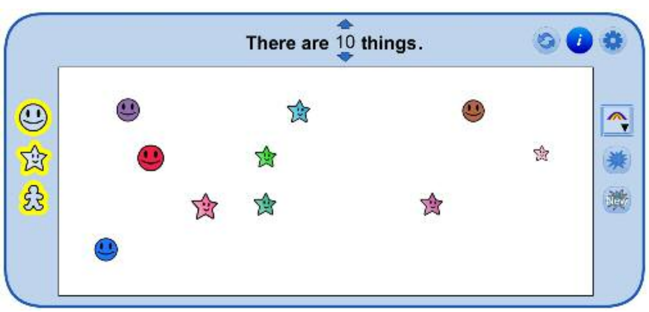
- Is $\frac{1}{6}$ of 12 a whole number?
- Can 10 objects be partitioned into 4 subsets of equal size?



The screen starts with a random arrangement of the things on the screen, and allows students to arrange the objects in their own way. It was clear early on that this starting screen would be a helpful tool for younger students to explore number as they sorted and classified the set of objects.

So the bottom of the screen was partitioned and discarded, and several tweaks were made to come up with a first version of the Set Tool (**http://mathies.ca/learningTools.php#Ss1**).

The Set Tool opens up with a random arrangement of 10 things.



The things have different colours, sizes, and shapes, providing attributes to sort, compare, and count. To have fewer attributes, change the colour picker at the right from multi-coloured to a single colour, deselect some of the shapes at the left, or adjust the Size Difference setting. You can scramble the existing things, using the starburst icon, or create an entirely new set of things, using the button underneath it.

As a challenge to older students, have them conduct probability experiments to see if they think the sets generated are truly random.

There are lots of good ideas for increasing the Set Tool's functionality, including allowing students to create their own interesting sets, to move more than one object around at a time, and to organize the things using built-in organizers, like ten frames, grids, and Venn diagrams. We would also like to make it available as a mobile app.

If you want to have your two cents worth, we are listening to feature requests at WhatsNew@oame.on.ca.

Wiki Supports

There are a variety of supports housed at **http://mathclips.wikispaces.com/**, many of which are linked to from the various tools, using the "i" button.

There is now a "Good to Know" section at the Notepad tool page (**http://mathclips.wikispaces.com/Notepad+Tool**), which explains some of the more arcane features of the tool.

A new introductory video for the Money Tool is now available as one of its supports (see **http://mathclips.wikispaces.com/Money+Tool**). It will even automatically play for you! You may find that there are some helpful features of the tool that you were unaware of.

If you have created some helpful resources for using mathies resources, or have some student work to share, please send it to **WhatsNew@oame.on.ca**.

Conclusion

Digital tools can reveal important mathematics and assist learners with a variety of learning styles and needs. The annotation/notepad tool allows a vehicle for students to communicate their thinking. We look forward to sharing additional developments to the e-mail list and in this column—maybe something wholly linear next time or something integral to numbers? ▲

▲ In the Middle: Love With and Without Locks

CARLY ZINIUK

I am always surprised that after years of trying to get middle school students to see math all around them, so often they still think that "real math" is done in isolation. For over a decade, we have been including Application and Inquiry formally in our assessments, students still find "word problems" contrived, overly simplified, or involving only single application of a math concept. As new adolescents exploring their world, they recognize that the world isn't divided into such rigid subject learning.

Part of the problem here is that math-specific teachers worry that working with larger, more meaty problems in a problem-based context will lead to loss of curriculum instruction time and less depth of math comprehension. This is a genuine concern if the math learning in the problem is relegated to budget calculations or substitution into measurement formulae. It happens too often when the problems originate in science or social sciences.

One way around this is to begin to look for problems with math at the core, like this problem of the weight of locks on Paris's Pont des Arts. Look at how the questions fall naturally from the context and that building a newspaper article/blog can be an assessment of the learning. I look forward to hearing how you adapt this with your students and what kind of questions and extensions all of you discovered!

When in Paris, along with seeing the Mona Lisa in the Louvre or photographing the Eiffel Tower, devoted couples have attached a padlock to the Pont des Arts pedestrian bridge, and as a symbol of their everlasting love, tossed the key into the Seine below. That was a typical tourist experience recently. Let's use some math to figure out why the Pont des Arts had to remove their locks.

> Officials from Paris indicated that a 3-metre panel on the bridge weighed an extra 500 kg with the locks on it. There are 112 such panels. Reports indicate that each lock weighs between 50 g and 100 g.

1. Estimate how much all of the locks on the bridge would weigh.
2. How long is that bridge?
3. Using an approximate average value for weight, determine how many locks are on the bridge?
4. If the average tourist weighs 70 kg, how many people are equivalent to the weight of the locks?
5. African elephants range in mass from 4000–7000 kg, while Asian elephants are smaller, ranging from 3000–5000 kg.
   a. How many of each of these elephants would match the weight of the locks?
   b. What are the assumptions in your calculation?

> Interestingly, the lock "tradition" is a recent one and began in Europe with a Young Adult novel (later turned film) by Italian Frederico Moccia in 2006.

6. Use your estimate as the value for 2014, and assuming a linear relationship, how many locks were put on per year?
7. However, as with many fads, the locks probably did not go up at a steady rate (linearly), but instead, multiplied year by year. Create a table that indicates increases at a multiplying rate per year and produce a graph that matches.

> As part of the Parisian riverfront of the Seine, the bridge is a UNESCO World Heritage site. The extra weight of the locks caused concern for structural reasons. In June 2014, a section on the railing of the Pont des Arts collapsed, and the city began cutting off the locks.

8. If it takes one worker 2 minutes to cut off one lock, determine the one worker rate expressed in
   a. locks per minute?
   b. locks per hour?
   c. locks per day?
9. If the goal were to have all of the locks removed in one year, how many workers would be needed?

> Unfortunately, while the workers cut off the locks in one panel, couples were replacing them in other sections. In June 2015, they removed the lockable panels and began replacing them with plywood and glass panels that could not have locks attached.

10. How long do you think it would take one worker to remove all of the locks? What assumptions and simplifications would you need to make to answer this question?

This is not just a problem for the Parisians. City authorities around the world have taken to asking couples not to do the same and leave padlocks on their bridges either, both for structural and aesthetic reasons. Similar problems with locks have occurred in Corktown Footbridge in Ottawa and Humber Bay Arch Bridge in Toronto. On the other hand, many artists and cities have created standalone pieces, including one in the Distillery District in downtown Toronto.

11. This installation is about 9 m wide and 2.5 m wide and has the letters LOVE with a heart. How many locks do you think it holds and how heavy would it be (Figures 1–3)?

Figures 1–3. *Toronto Distillery District lock installation poster, installation, and close-up of locks.*

Similarly, couple-sent #lovewithoutlocks to Twitter and photos to Instagram became a Parisian alternative once the Pont des Arts was closed to locks. In the past year since the City established **http://lovewithoutlocks.paris.fr/**, there were less than 700 photos.

12. How many years will it take to match the actual locks that were on the Paris bridge?

Iron Trees were placed in Moscow as an alternative to locks on the bridge sides. These trees hold over a thousand locks each and are placed on the middle of pedestrian bridges, to match the Russian custom of having newlyweds kiss on a bridge. The Luzkhov Bridge is often called the Bridge of Kisses, with tourism reporting that divorce rates in the city have gone down with the installation of the trees. However, the city does have to regularly dredge the bottom of the canal to collect up the tossed keys.

13. What style of tree would allow you to hold the most locks?

14. What dimensions would be the most practical for couples to place the locks on all parts of the tree?

Sample Answers

1. 112 * 500 kg = 56 000 kg
2. 3 * 112 = 336 m or approximately 1/3 of a kilometre
3. If average lock is 75 g (halfway point from 50 and 100), and using the 500 kg of extra weight on one panel,

   500 * 1000 = 500 000 g is the total extra weight in grams
   500 000 / 75 = 6667 locks in one panel
   6667 * 112 panels = 746 704 locks
4. Total lock weight is 56 000 (from above).
   56000 / 70 = 800 people
5. a. Average African elephant assumption 5500 kg (halfway point between 4000 and 7000)

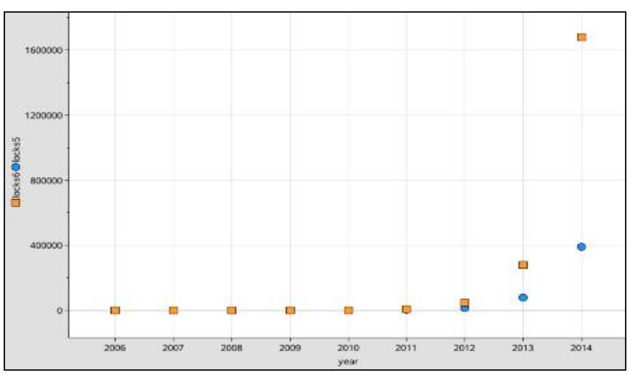
   Average Asian elephant assumption 4000 kg

   So the lock weight 56 000/5500 about 10 African elephants

   And similarly 56 000/4000 about 14 Asian elephants

   b. Elephants are evenly distributed in that weight range (not more heavy ones than light ones) and the average weight is in the middle of the weight range.
6. 8-year span for 746 703 locks, which would mean approximately 93 338 locks per year
7.

Year	Number of Locks (5x each year)	Number of Locks (5x each year)
2006	1	1
2007	5	6
2008	25	36
2009	125	216
2010	625	1296
2011	3125	7776
2012	15 625	46 656
2013	78 125	279 936
2014	390 625	1 679 616



8. a. Half a lock per minute (does this make any sense?! Maybe cutting the first side of the clip?)

   b. 30 locks per hour

   c. So, we can assume one worker works 8 hours and then goes home and goes to bed… or we can assume they have them working shifts into the night so three people covering the entire day of lock cutting…

   One worker would be 8 * 30 = 240

   Three workers (all day) would be 240 * 3 = 720
9. 746 704 divided by one worker day of locks would be 746 704/240 ~ 3111 worker days

   Now assuming each worker works about 250 days per year (that's a round number approximation with 365 days in a year with 52 * 2 = for the weekends and a reasonable amount of vacation and sick days).

   That is cutting locks for 8 hours a day, in the snow, in the lightning storms,…

   Each worker then has 250 days per year to cut off 240 locks per day.

   That would mean at least 12 (probably 13) workers doing this every day all year.
10. Even if we just looked at the linear model, which is probably not the case, it was adding 93 338 locks per year, which was about 255 per day. This is more than the worker would be able to remove per day, 240 locks. It definitely takes longer to remove a lock than it does to lock it. And there are lots of people putting them up and only one work removing them.

    Oh No!
11. Assume that the width is the same, but that the length is three times longer. A 3 m panel had 500 kg, so a 9 m wide installation could be 1500 kg.

    6667 locks in one panel, so about 20 000 locks in the installation (this is way too much – check out the pictures?!) The problem here is that that bridge is solid locks and the letters have spaces and gaps between them.
12. 746 704 locks total with 700 photos per year

    It will take about 1066 – at least over 1000 years! Yikes!
13. Something with lots of surface area and that allowed you to go up and inside to put locks in them. Something like a double helix would be great, although not tree like.
14. So the problem is that it probably shouldn't be more than about 2 m high because most people couldn't

reach more than that and you wouldn't want to have heavy things like locks too far above your head (ouch!).

You also only want the base to be larger than the top because the weight above will crush the bottom or topple it over.

Love with and without Locks Evaluation

Write a 300- to 600-word article/blog entry that uses your research, calculations, and understanding to explain the following:

Why did the Pont des Arts bridge have its locks removed?

- Incorporate at least one graph in your article.
- Include at least two separate calculations in your analysis. One must include a rate in your explanation.
- Draw at least two additional conclusions other than the prompt above as justification/additional points of interest and connect them to data/statistics specifically. ▲

	LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4
KNOWLEDGE	• Few calculations are performed correctly, including rates, direct proportion, graphing, and unit conversions. • Work shows a limited understanding of proportional thinking and terminology.	• Some calculations are performed correctly, including some of rates, direct proportion, graphing, and unit conversions. • Work shows some understanding of proportional thinking and terminology.	• Most calculations are performed correctly, including some of rates, direct proportion, graphing, and unit conversions. • Work shows an accurate understanding of proportional thinking and terminology.	• All calculations are performed correctly, including all of rates, direct proportion, graphing, and unit conversions. • Work shows a comprehensive, accurate understanding of proportional thinking and terminology.
APPLICATION	• Data is included with limited regard to usefulness and context described, and changes explained in a limited manner. • Assumptions and estimations are not addressed.	• Data shows some degree of usefulness, with contexts described. • Assumptions and estimations are addressed briefly.	• Data is appropriate and useful, with some contexts described. • Assumptions and estimations are addressed for only obviously problematic answers.	• Data is appropriate and useful, with all contexts described. • Assumptions and estimations explain how to deal with real-life variability, and problematic answers justified.
THINKING	• Graphs are not constructed correctly or displays limited appropriate information. • Conclusion shows limited evidence of careful thought and consideration of mathematics.	• Some graphs are included and constructed correctly, displaying some appropriate information. • Conclusion shows some evidence of careful thought and consideration of mathematics.	• Most graphs are included and constructed correctly, displaying mostly appropriate information. • Conclusion shows evidence of careful thought and consideration of mathematics.	• Relevant, appropriate graphs are included and constructed correctly. • Conclusion shows much evidence of careful thought and consideration of mathematics.
COMMUNICATION	• Introduction outlines mathematics to be used in a limited manner. • Evidence is provided in a limited manner.	• Introduction outlines some mathematics that will build into logical article. • Evidence is provided to some extent.	• Introduction outlines most mathematics that will build into a logical article. • Evidence is appropriately provided for the most part.	• Introduction clearly outlines all mathematics that will build into a logical article. • Evidence is appropriately provided.

▲ Ask Assessment Abby: Connecting with Parents/Guardians

ASSESSMENT ABBY
E-MAIL: assessmentabby@oame.on.ca

Ask Assessment Abby A^3 is a regular column in the OAME *Gazette*, where teachers can share concerns and best practices about assessment, evaluation, and reporting of mathematics. Please send your questions to Ask Abby at **assessmentabby @oame.on.ca**.

Dear Assessment Abby,

This year, I am working on improving my communication with parents/guardians. What I want to know is, how can I help parents/guardians understand my assessment practices?

Connecting with Parents/Guardians

Dear Connecting with Parents/Guardians,

First of all, you can remind parents and guardians that assessment is more than tests, quizzes, and percentage grades. Assessment is "... the process of gathering information that accurately reflects how well a student is achieving the curriculum expectations in a subject or course. The primary purpose of assessment is to improve student learning" (*Growing Success*, 2010, p. 28). According to *Growing Success*, it should include triangulating a range of evidence from observations, conversations, and products. It may also be worth clarifying that assessment and evaluation focus on curriculum expectations and incorporate the four Achievement Chart categories: Knowledge and Understanding, Thinking, Communication, and Application. We can help students and parents/guardians understand our assessment practices by sharing rubrics, learning goals, success criteria, and descriptive feedback. It is important to share with parents/guardians that assessment is connected to the ongoing collection of student data.

Keep Math Rich with Students in Mind,
Assessment Abby

Assessment Abby would like your thoughts on the following questions from our readers:

Dear Assessment Abby,

My son came home with a group mark on his progress update. He got 80% on his individual part, but the group got 25% on the group aspect. The final mark that was assigned to my son was 65%.

Please advise.

From a Concerned Parent in the PRMA chapter.

Dear Assessment Abby,

Please clarify the use of the four categories as they are used for assignments and tests. For instance, when marking an assignment or test, how do you decide under which category each question or section should be reported?

From a Concerned Teacher in the PRMA chapter.

Please e-mail your suggestions, questions, or comments to Assessment Abby at **assessmentabby @oame.on.ca**. ▲

Encyclopedia of Mathematics Education (S. Lerman, 2014) Excerpts

"Historically, technology and mathematics go alongside by mutually influencing each other's development . . . History does provide us with many technologies that enhance people to count (stones, pebbles, bones, fingers), to calculate (abacus, mechanic devices, electronic devices), to measure (ruler, weights, calendar, clock), to construct (compass, ruler), and to record statistical data (cards with holes, spreadsheets). . . . The learning model that can be extracted from our examples features three major educational trends related to the web 2.0 technology: knowledge building/co-constructing, knowledge sharing, and socialization by interaction with other people. Moreover, further development towards semantic web (web 3.0) technology has a potential to enhance self-learning, critical thinking, and collaborative and exploratory learning."

Freiman, V. (2014). Types of technology in mathematics education. In S. Lerman (Ed.), *Encyclopedia of mathematics education* (pp. 623–629). London, UK: Springer.

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eds@oame.on.ca

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paul.alves@oame.on.ca

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Bill Otto
Stillwater Lake, NS
bill.otto@oame.on.ca

Sandra Jean Price
Oshawa, ON
sandra.jean.price@oame.on.ca

CHAPTERS & REPRESENTATIVES

CHAMP: Claudio Attanasio
Brampton, ON
claudio.attanasio@oame.on.ca

COMA: Melissa Black
Kanata, ON
melissa.black@oame.on.ca

GOLDEN: Pamela Chun
Grimsby, ON
pamela.chun@oame.on.ca

GVMA: Vincent Glavine
Guelph, ON
vincent.glavine@oame.on.ca

ISOMA: Charlotte Aust
Toronto, ON
charlotte.aust@oame.on.ca

MAC^2: Paulene Washington
Barrie, ON
paulene.washington@oame.on.ca

NOMA: Denise Filipovic
Sudbury, ON
denise.filipovic@oame.on.ca

NWOAME: Corrine Russell-Pritoula
Thunder Bay, ON
corrine.russell-pritoula@oame.on.ca

PRMA: Lori Yee
Pickering, ON
lori.yee@oame.on.ca

QSLMA: Kathy Pilon
Smith Falls, ON
kathy.pilon@oame.on.ca

SAME: Freda Liu
Toronto, ON
freda.liu@oame.on.ca

SWOAME: Christine Sasso
Windsor, ON
christine.sasso@oame.on.ca

TEAMS: Bart Vanslack
Toronto, ON
bart.vanslack@oame.on.ca

WOMA: Ann Michele Stenning
London, ON
annmichele.stenning@oame.on.ca

Y^{4}MA: Chi Hang
East York, ON
chi.hang@oame.on.ca

DIRECTORS – TERM EXPIRING 2016

Jill Lazarus
Pembroke, ON
jill.lazarus@oame.on.ca

Bart Vanslack
Toronto, ON
Bart.vanslack@oame.on.ca

DIRECTORS – TERM EXPIRING 2017

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jung.choi-perkins@oame.on.ca

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anne.yeager@oame.on.ca

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patricia.kehoe@oame.on.ca

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bill.lundy@oame.on.ca

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Todd Romiens
Windsor, ON
todd.romiens@oame.on.ca

WEBSITE CO-ORDINATOR

Greg Clarke
Orillia, ON
web@oame.on.ca

GAZETTE EDITOR

Dan Jarvis
North Bay, ON
dan.jarvis@oame.on.ca

ABACUS CO-EDITORS

Mary Lou Kestell
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marylou.kestell@oame.on.ca

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NOMINATIONS FOR OAME/AOEM BOARD OF DIRECTORS 2016–2017

Each year, elections are held for a President-Elect, a Vice President (3-year term), and two Directors (up to a 3-year term) of OAME/AOEM. OAME/AOEM members running for Director will declare their intent to work on either Grades JK–6 or Grades 7–12 issues based on their personal experience, expertise, and interests.

Nominations for these positions are to be submitted
by 4:00 p.m., January 31, 2016, to:
Paul Alves, Past President
paul.alves@oame.on.ca

Include a paragraph of the nominee's qualifications with the Nomination Form. A nominee must be a member of OAME. By March 2016, the Nomination Committee will publish a slate of candidates, including candidate bios, for consideration by the membership. Voting will be online and is open to all OAME/AOEM members. The election results will be released at the Annual General Meeting of the OAME/AOEM in May, 2016.

NOMINATION FORM

I nominate:____

for the position(s) of:

☐ President-Elect ☐ Vice-President ☐ Director

Nominee's Address: ____

____

E-mail: ____

Telephone: Home (________)____________________________

Work (__________) ____

Nominator's Name : ____

Nominator's Address: ______________________________________

____

E-mail: ____

Telephone: Home (________)____________________________

Work (__________) ____