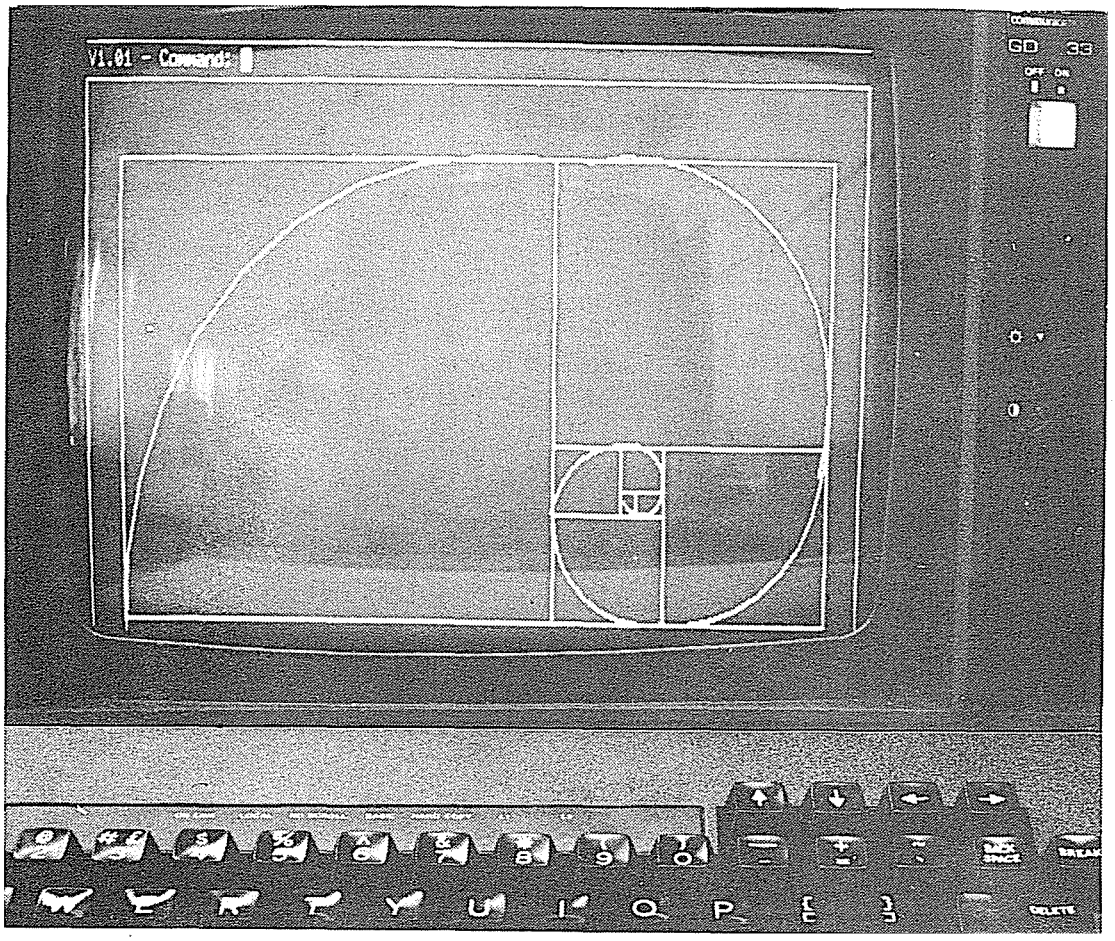
ontario mathematics gazette

VOLUME 23 **NUMBER 3** **MARCH, 1985**



THE ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION
L'ASSOCIATION ONTARIENNE POUR L'ENSEIGNEMENT DES MATHÉMATIQUES

EDITORIAL BOARD

Editors:
Ralph Connelly, Brock University, College of Education
Sam Pravica, Galt C.V.I., Cambridge, Ontario N1R 6V2

Associate Editors:
Trevor Thomson, Hamilton Board of Education
Andre Ladouceur, University of Ottawa
Wes Keener, University of Western Ontario
Walker Schofield, University of Western Ontario
Olive Fullerton, Faculty of Ed., University of Toronto
Monica Stevenson, Lincoln Co. Board of Education, St. Catherines, Ont.
Eric Muller, Brock University
Robert Smith, University of Western Ontario
Clifford Gravelle, Brock University, St. Catharines, Ont.
David Lenchyshyn, Niagara South Board of Education

PUBLICATIONS COMMITTEE

Trevor Thomson
Ralph Connelly
Robin Hood
Dave Lenchyshyn
Alex Norrie
Trevor Brown

O.A.M.E./A.O.E.M. EXECUTIVE

Past President: David Davidson
President: Paul Pogue
President Elect: Paul Lessard
Director: Ralph Connelly
Director: Tom Griffiths
Director: Elaine Harvey

MARCH, 1985

ISSN 0030-3011

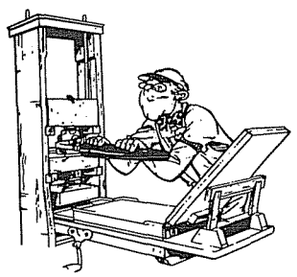
CONTENTS

Editorial 2

Math Games: To Use or Not to Use? *by Eldon Pipher* 3
Fear No More *by Bruce Shawyer* 6
Elementary School Curriculum Work - A Never Ending Process for Change *by R.S. Smith* 13
When Will Canada Become Fully Metric? Never! *by Alan Auerbach* 19
Applications in Secondary School Mathematics *by Hugh Allen* 24
Pinochle Probabilities: Chances of Matches *by Bonnie Litwiller & David Duncan* 31
Positive, Negative, Plus, Minus, Add, Subtract *by Bruce Shawyer* 34

DEPARTMENTS

Interview with Graham Ruddock *by Robert S. Smith* 36
Across the Editor's Desk 41
Problems 42
Book Review 43
Council Comment 44
Play It Again Sam. A Theme From The Trenches 46



The Ontario Mathematics Gazette is produced by:
POLE PRINTING
Box 69, Forest, Ontario
(519) 786-5112



Being a **Gazette** editor can sure give a person a warped sense of time. Back in November, I was frantically putting together the editorial and the finishing touches to the **December Gazette,** which you received in **January.** Now, I am again rushing to meet the mid-**January** deadline to finish the **March Gazette** which you'll be receiving in **February.** There actually is a logic to it all, related to Election procedures and getting the ballots to the membership, etc., but it sure can make for a harried existence at times.

Since you (and I) have just received the December **Gazette** it's too early to tell whether anything in that issue stirred any response.

The responses to the Publication Committee **Gazette** survey have been collated, and I would again like to thank all who took the time to respond to the survey. Although I know I probably shouldn't, I can't help but respond here to some observations made in one of the surveys. A reader noted that in the last few issues, except for articles by Bonnie Richardson, there were no articles by women, and felt this should be corrected. **I WHOLEHEARTEDLY AGREE!** However, the reader went on to suggest that this situation was one of design. In actual fact, over the past year **more** women than men have been approached with the request to submit material to the **Gazette,** and more women than men have promised to do so. However, more men than women have turned their promises into product. It is **not** my intent to draw dramatic inferences from this, but simply to clarify the facts. If it were proper to put an editor's note in an editorial, I would also point out that Bonnie Litwiller has been a consistent contributor to the **Gazette** but perhaps she wasn't counted because she and David Duncan co-author them (and David does all the work? Just kidding, Bonnie & Dave, **please** keep considering us for your articles!)

There are two avenues along which we as a membership must make advances. We must exert more influence on future curricula and we must have a greater input into how computers are to be implemented as a teaching tool in the classroom. We know a number of you have made inroads in this area-Let's hear from you! We need your experience.

In closing I encourage any and all readers of whatever sex to seriously consider taking those classroom ideas, those "neat little teaching tricks", those innovative programs, etc. put them down on paper and submit them to the **Gazette.** We'll look forward to receiving them before April or May (or is it June?) when we're working on next September's **Gazette** (which, by amazing coincidence will in all liklihood come out in September!), but of course we will gleefully receive them **any** time. Must go shovel out my driveway- Have a happy summer!

Ralph Connelly,
Sam Pravica
Editors, **OMG**

MATH GAMES: TO USE OR NOT TO USE?

by

Eldon Pipher

Many years ago, before the days and nights of television, a pair of twins invented a game which they and only they ever played. The boys were in early high school and WW II was in its infancy. The game had to do with countries attacking each other and the twins named the game Bellum, a word they had learned in Latin class.

It was a complicated game and in order to play it these students had to do considerable research: finding out the populations of each country, basic resources needed for war production and other items of information. To make a long story short, many years later, as a result of the game the twins could still cite from memory the populations of most of the world's countries, the location of each country and each of its bordering lands, the sources of coal, iron, bauxite, petroleum, geographical barriers (to armies on the move).

They learned a great deal of geography on their own initiative. No teacher was needed to motivate them or teach them. In fact their teachers didn't even know of the game or what they were learning. But why, you ask, are you writing about all this geography in a mathematics journal?

I do so because they also learned a lot about how to use mathematics. They learned how to use ratios (a country's population and size of army were in the ratio of 10:1); they did innumerable calculations in determining the number of tanks, battleships, or bombers which could be built, sold, or traded; they did many measurements as armies prepared for attack; they learned how to plot locations on a matrix, a word they had not heard of at that time. The examples could go on and on.

When I started to teach I began to make use of games and puzzles to help children develop a positive attitude toward mathematics, to learn problem-solving techniques, and to practise the skills they were learning. Perhaps it was my experience, along with my twin brother, in developing games for our own enjoyment, that influenced me to use such activities in my classroom. Perhaps the concept was reinforced by my father frequently giving us puzzles to solve or showing us fascinating things about numbers and by the school inspector who always had us do something interesting with mathematics whenever he visited our classroom.

Regardless of what actually influenced me, I made use of games and puzzles throughout my career in elementary schools. When I became involved in teacher education I started to show teachers in pre-service and in-service sessions some mathematical games and puzzles and attempt to convince them of their value.

I have often wondered why some teachers use games and puzzles with great effectiveness while other teachers rarely make use of them. When I meet my graduates some of them say things like, "I'm still using some of the games you showed us at teachers' college and the kids love them." Others show no evidence of using mathematical games and puzzles as they challenge children to think.

By the way, I distinguish between games that use mathematical thinking and skills in the actual activities and those games in which the mathematics is secondary and could just as easily be language or facts in environmental studies. The latter include such games as where the number facts are written on cut-out paper articles of clothing and the child who gets the correct answer hangs the shirt on the clothes-line. This type of game has value too, if played sensibly, but the proper use of these games is a different matter and not within the scope of this article.

Why do so many teachers not use valid mathematical games and puzzles? I'm not sure of the answer to this question; I can only speculate. May I suggest three major reasons. The first can be illustrated by these quotations, fictitious, I suppose:

Underlying belief of many people, "It can't be worthwhile if it's fun." (This applies to many areas in life besides education activities.)

Child at dinner table: "We really had fun in Math. class today."

Parent, in response to child's remark, "Didn't I tell you, Bertha, it's nothing but fun and games at school these days. Our teachers made us work hard when we went to school and we learned

something.''

Principal at staff meeting: ''Now I notice a lot of game-playing and laughter when I visit some classrooms. May I remind you that there's still a lot of public pressure on us for back-to-the-basics. I don't think I need to say anymore. Thank you.''

One teacher, and only one, in reaction to principal's remarks says to class, ''You've been learning a great deal of mathematics in our games and puzzles. From now on when anyone else is in our classroom keep doing them but look very serious, even sad. That way we won't have to discard them as part of our mathematics classes.''

Of course I'm being somewhat (but not entirely) tongue-in-cheek. I trust, however, that the point has been made without me articulating it. You see some of us believe that the enjoyment of mathematics and the learning of important and necessary concepts are not mutually exclusive processes. Some of our learning can be fun.

I believe another reason for overlooking the use of good mathematical puzzles and games is that some teachers really do not see the mathematical concepts that can be learned by such activities.

A third reason is that some teachers on hearing of or reading about a game or puzzle are too quick to discard it deciding that it is too difficult or easy for his/her class. My first response is that because the activity appears to be too easy or difficult doesn't mean that it is. Many teachers have found that what was perceived at first to be too difficult or easy did not turn out to be so. My second response is that even if a concept is considered unsuitable for a ''class'' it may be very appropriate for individual children. My third response is that many games and puzzles that are in fact too difficult (or too easy) for specific children can be adapted or modified and made very suitable and worthwhile.

I don't have many ideas to pass on to teachers who may neglect to use fascinating games and puzzles because of the first reason I mentioned. This belief that ''the medicine is no good unless it tastes terrible'' is difficult to change in people who apply it to teaching mathematics. What I want to do now, in view of the second and third reasons, is to suggest a few activities, briefly outline some of the mathematical concepts or skills involved, and suggest variations of the original game/puzzle.

FIRST GAME:

I have used this activity with children from Grade I to VIII and have found most of them eager to try it. (I have also shown it to hundreds of teachers and student teachers, yet rarely find a child who has tried it before. No explanatory comment needed here, I think.)

ACTIVITY:

In pairs children count alternately from 1 to 20. In each turn the child is allowed to say either one or two numbers. The first child may say 1 or 2; later if the opponent says 7, for example, the child may say 8, or 9 and so on. The object of the game is to be the one to say 20.

CONCEPTS/SKILLS:

Counting, looking for number patterns (children soon discover a pattern that permits them to win every time, if they go first.), practice in simple adding and subtracting.

VARIATIONS:

-object of the game is 25, 30, 40 or whatever

-object of the game is to avoid saying 20, 25, or whatever

-child is allowed to say 1, 2, or 3 numbers each turn

-similar rules as above except that the counting is done in multiples of 2, 3, 5, or whatever

-counting backwards, starting with 30, object is to say zero

-instead of counting you may add to your opponent's number from 1 to 10, the object is to be the one to say 100.

-counting using fractions with common denominators, or decimals in hundredths

-counting using Roman Numerals with one of the above set of rules

-have the children think of other variations

- have the more capable students see if they can find a rule or generalization to help them discover

the pattern for any of the variations above

See how the one simple game can be modified to have children work with many patterns, use addition and subtraction skills, (multiplication and division are also used in discovering a general rule for all such games), get practice in counting forward and backwards and in multiples, use fractions and decimals and Roman Numerals, and use several examples to form a generalization.

SECOND GAME
ACTIVITY:

Have the children (about Grade 3 level) draw three adjoining rectangles, each large enough for one numeral. Have them also draw two adjoining rectangles of the same size. You will need 4 sets of cards containing the numerals 0 to 9 inclusive. That is there will be 4 cards with 1, 4 with 2, etc. Shuffle the cards.



As each card is drawn and shown to the class each child writes the numeral in one of the rectangles. The object of the game is to have the greatest number displayed in the three rectanges and the least in the two rectangles. After the five cards have been drawn the children find out who the winner is. They can find out how much greater the winning number is than the "runner up" and make other comparisons.

CONCEPTS/SKILLS:

It can be seen that the children are using concepts of greater than, less than, how much greater or less, intuitive probability (e.g. if the second last numeral is 7 is the last numeral drawn more likely to be more or less?), they are practising reading and writing numerals, they are using subtraction, etc.

VARIATIONS:

-make the numbers greater or less by changing the number of rectangles

-draw seven cards allowing the children to discard two

-put a decimal point between two rectangles

-have the children draw two sets of two rectangles arranged vertically. Draw 4 cards and have them see who can have the greatest fraction on the left and the least on the right

-have them suggest variations

-have some of the children invent a game using letters instead of numerals in 5 adjoining rectangles. Suggest they decide on a number of discards and calculate how many of each letter of the alphabet should be included in the deck.

As you can see a simple game has several variations where children solve problems and practise many basic skills.

Many more games/puzzles could be used to illustrate their potential in exploring and discovering various concepts and to practise skills in an enjoyable way. I think that these two illustrations should suffice. For those of you who are not using this type of activity as you might, may I make the following suggestions:

1. If you have never tried the two games I described, try them with appropriate variations.
2. Be prepared to explain to parents the mathematical concepts and skills involved in any game you use.
3. Make a collection from the books available and mathematics magazines. List the concepts and skills involved and try to think of variations to make a game more suitable or to expand the concepts or skills.
4. Have the children suggest modifications in games, to bring in ones they know about, and to invent their own games.
5. You know you have reached the pinnacle of success when you have invented your own game or two and they work!

Good luck!

Eldon Pipher
Coll. of Ed.
Brock Univ.
St. Catharines, Ont. L2S 3A1

FEAR NO MORE

by
Bruce Shawyer
Department of Mathematics
University of Western Ontario
London, Ontario, Canada.

What I wish to discuss in this paper is a disease that is present in epidemic proportions in our present day society. It is a **social** disease, but yet a highly **anti-social** disease. The most common manifestation of this disease is in the form of a **fear** or **anxiety.** But before discussing the particular fear, I would like to spend a little time discussing **fear** in general.

Fear itself is a natural human characteristic. Although it appears to be absent in new-born babies, it is rapidly learned. But what does the baby learn to fear? And why? It appears that babies learn by experience. Their fears are learned from unpleasant experiences, such as falling or touching hot objects. And so the baby learns to **avoid** the circumstances that gave rise to the unpleasant experience.

The American psychoanalist, Karen Horney, has written:-

> "If a person is afraid whenever he stands on a high place, his reaction is called **anxiety,** but if he is afraid on a particular occasion when he loses his way on a mountain in a storm, his reaction is called **fear. Fear** is a reaction that is proportionate to the danger that one has to face, whereas **anxiety** is a disproportinate reaction to danger, or even a reaction to imaginary danger."

I would ask you to keep this distinction in mind as we proceed.

It appears then that **fear** is a learned characteristic and leads to **avoidance** of circumstances that are unpleasant.

We often use the expression "a fearless person", but we do not always use it as a compliment. For a fearless person is often a reckless person, or a fool-hardy person. And thus not always admired. Very few people have no fears at all. We all have our little fears. But some of them are not so little. Many of our fears are irrational. Perhaps they are not fears, but anxieties? And many can be overcome by "taking control". You are all familiar with the quotation: "you have nothing to fear but fear itself!"

To bring this discussion down to the personal level, I shall ask **you** the question: what do **you** fear? The unknown? Ghosts? Things that go bump in the night? People are sometimes afraid of certain other people. Such fears can be of the most irrational kind! So I ask **you** the question: **Are you afraid of me?**

Before you answer that question, let me tell you a story, and a true story at that.

You will have to admit that that is not an uncommon ploy in many social circumstances to enquire how a new acquaintance is gainfully employed. This is roughly how a conversation went:

Acquaintance: What do you do?
Me : Oh, I teach at Western.
Acquaintance: What do you teach?
Me : Mathematics.
End of conversation.
Why?
Because he was afraid of me!
Why??
Because I am a mathematician
He was afraid to talk to a mathematician!
Why???
Perhaps he was afraid that we might talk about mathematics?
Why????
Perhaps he was afraid of appearing ignorant?

C.P. Snow, in "The Two Cultures", identified that scientists and engineers were by and large, unread, while those who practiced the Arts were ignorant of science. You will recall he stated that the scientists must learn to communicate with the men of letters and vice versa. You would think

from this that he assumed that the various varieties of scientists actually spent some of their time talking with one another! But with mathematics, it seems that even the practical scientists and the engineers often take pride in proclaiming their avoidance of mathematics.

Why??
Why do scientists fear mathematics?
Why do people in general fear mathematics?
What makes them anxious?
Why do people say that they hate mathematics, or that they cannot cope with it.

Now, **I** am a mathematician.
Are **you** afraid of me **now**?

Perhaps you have good reason to be afraid of me! In fact there are excellent authorities to convince you that that is the correct attitude to take. No less august personage than St. Augustine himself said: "The good Christian should beware of mathematicians and all those who make empty prophecies. The danger already exists that the mathematicians have made a covenant with the devil to darken the spirit and to confine man in the bonds of hell."

Even earlier in history, the Roman jurists ruled that:
"To learn the art of geometry and to take part in public exercises, an art as damaging as mathematics, are **forbidden.**"

Now I will name the dread disease that is afflicting present day society. It is called **MATH ANXIETY.** Mathematics anxiety, or more properly, mathematics avoidance, is indeed one of the most serious malaises of our times. It has reached epidemic proportions. So much so that schools, colleges and universities all over this continent are setting up courses to combat math anxiety.

Here, yes, even here in London, in pleasant, nice, comfortable London, Ontario, at the University of Western Ontario, a university with such a high demand for its places that not all qualified students are accepted, here at Western, there is a popular and demanded course to assist those with mathematics anxiety.

We are living in the computer age. The computer revolution is invading all aspects of society. Those who are to survive in this new society must conquer the disease. Those who will benefit most are those who are numerate.

Will you survive?

What are the manifestations of math anxiety? They are many, but I think they are well explained in a "Funky Winkerbean" cartoon. It is headed:
"The Funky Winkerbean guide to taking tests!" Clip and save.

MATH ANXIETY
If, when faced with a math test, your heartbeat increases, your palms get sweaty, and your mind goes blank, you could have math anxiety!

Math anxiety often goes undiagnosed in the average student since its clinical symptoms are almost identical to those for stupidity! You probably have math anxiety if you exhibit any of the following signs:
(1) you have nighmares about being chased by a giant number 6!
(2) you tend to forget your locker combination during periods of stress!
(3) you frequently get confused when singing 'a hundred bottles of beer on the wall'!
(4) you can't figure out the square root of 1972481.911 in your head!

It all sounds very funny when it's in a cartoon, but in fact, it is serious. It is deadly serious.

Richard Skemp, writing in 1971 in his penguin book "The Psychology of Learning Mathematics", summed up the feeling of many students in a mathematics class:-
"Given an exposition, which, though not excellent, is nevertheless not altogether inadequate, some students will be able to understand it - some not. If those who do not understand feel overanxious at their failure, they will no doubt make a greater effort to comprehend. But, this over-anxiety can be self-defeating, in that it can actually diminish the effectiveness of their efforts.

Given several experiences of this kind, the situation itself, the mathematics lesson or lecture, becomes a learned stimulus for anxiety."

Thus, students can stop trying to work out problems assigned, since not trying eliminates the embarassment of not being able to perform well. It is better to say: "Oh, I haven't had time to try that yet!", than it is to say: "Oh, I can't do that question."

I am sure that each and every one of you, my readers, has many talents and abilities in many different areas. You are indeed a microcosm of the world at large. Since you are reading this journal, I presume that you feel confident with arithmetic. But there are many talented adults who shake and tremble when faced with a collection of figures. They experience frustration, tension, blanking out, and even panic.

I know of students in mathematical courses who will try to fake their way through the course work, obtaining assistance to do assigned work, but living in fear of the final examination which will no doubt expose their ignorance. And there are business executives who often reject the latest in calculators or computers for fear that they will not be able to understand the explanation of how it works, and then will appear to be ignorant! Sometimes attempts are made to make this manifestation of mathematics anxiety appear funny. If we all have a good laugh, perhaps the problem will go away.

There is a parallel to this in the television advertisements for long-distance:
Here we see the funny fat man being assailed by a rival company's agent. The rival is trying to sell **his** system for sending data. The rival, in loud suit and smoking an outsize cigar, produces a folding display of a flow chart to impress the funny fat man. Of course, since this is "long-distance"'s advert, the funny fat man's own service performs better. But at the end, the funny fat man points to a part of the rival's display, and says, "What does that do?" The answer is:
"We're not sure!"
We're not sure. Oh, isn't ignorance bliss!

There is a theory to explain all this. Just like Parkinson's Law explains why office workers are **always** busy. Here what we need to know is the **Yerkes Dodson** law, which simply states that:
"the optimal degree of motivation for a given task decreases with the complexity of the task."

We are led to the conclusion that the math anxious person is trapped in a vicious spiral.

So, having examined the symptoms of math anxiety, we must ask, what are the causes? And is there a cure?

The causes are many. But they can be grouped roughly into three categories:
(1)--the attitudes of society towards mathematics,
(2)--the methods of teaching mathematics,
(3)--the curriculum as presently taught.

The most difficult of these causes to deal with is the first: the attitudes of society towards mathematics.

Math anxiety is indeed a dreaded social disease. It is highly contagious and transmittable. And, believe it or not, society has been transmitting it for generations. Just remember the words of St. Augustine that I quoted earlier.

But what is there in present day society that causes math avoidance?

Many reasons are given. Some are evident from the symptoms that I have already described above. Some of the reasons are real. Some are based on folklore and history. But all the reasons are serious, whether they be real or imagined.

For example, it is often said that mathematics is not for girls. Why? What is there that is unfeminine about mathematics? Or for that matter, what is there that is particularly masculine?

Many years ago, in the dark ages of last century and before, education was essentially an all male club. The place of the woman was in the home. The German expression for this:

Kuechen, Kinder, Kirche,
cooking, children, church,
expresses this attitude well. The men had to be fed. The children, needed to prolong the line, had to be looked after. Someone had to pray for the well being of the men! Women were suppressed.

No, education was for gentlemen. Women's gentile brains were not to be given any difficult tasks. Science was difficult and dangerous. Mathematics was difficult and devilish. Therefore it was a male preserve. Women were to be discouraged. And when women did get access to education, their **male** teachers continued to propagate these myths. Thus, math anxiety is traditionally a female affliction.

One of the best known publicists of math anxiety is Sheila Tobias. She did a lot of the good work when she wrote her book, "Overcoming Math Anxiety". She was really galvanised into action by a 1981 report by two Johns Hopkins psychologists, Camilla Benbow and Julian Stanley. The Benbow-Stanley investigation centered on the thesis that mathematics was manly! They interviewed many professional **male** mathematicians and most could not (or perhaps even, would not) name more than **one** woman who had been an original thinker in mathematics. The one woman most commonly named was the world-re nowned German mathematician, Emmy Noether. However, the folklore here is that Emmy's colleagues refered to her as **der** Noether, because she exhibited "manlike" talents!

Benbow and Stanley also claimed that American boys tended to get higher scores in mathematics than did girls. They went on to hypothesise:

> "We favor the hypothesis that sex differences in achievement to and attitude towards mathematics result from superior male mathematical ability."

Can you imagine the furor when this hit the popular press! There have been a multitude of criticisms of the report, both of its methodology and on how its conclusions were reached from its data. The methodology was suspect, so that the paper has engendered a host of rebuttals.

A real cause of math anxiety is parental influence. In general there appears to be an anti-intellectual attitude amongst those who have less education. Sometimes it is the "done thing" to be anti-learning. Many people with little formal learning decry any intellectual activity in others. They often use this as a defence mechanism for a feeling of inadequacy or ignorance.

School students suffer terribly from peer pressure. No-one seems to enjoy, or is allowed to enjoy, being the one who does well at school. There are all sorts of derogatory terms for such people. Too much prestige is given to the star football player. His name will be known throughout his school, and he may even make the headlines of the local paper. Take, for example, Richard Thompson, quarterback at South Collegiate in London. His name was headlined last fall in the London Free Press during the football season. But which reader of the London Free Press knew the name of the person at South Collegiate who did best in the national mathematics examination run annually by the Canadian Mathematical Society? Even if they had been told that the person who was placed **first** in the 1983 competition lives and went to school in London, how many of the readers could have named either him or the school he attended?

No, society has no love affair with learning.

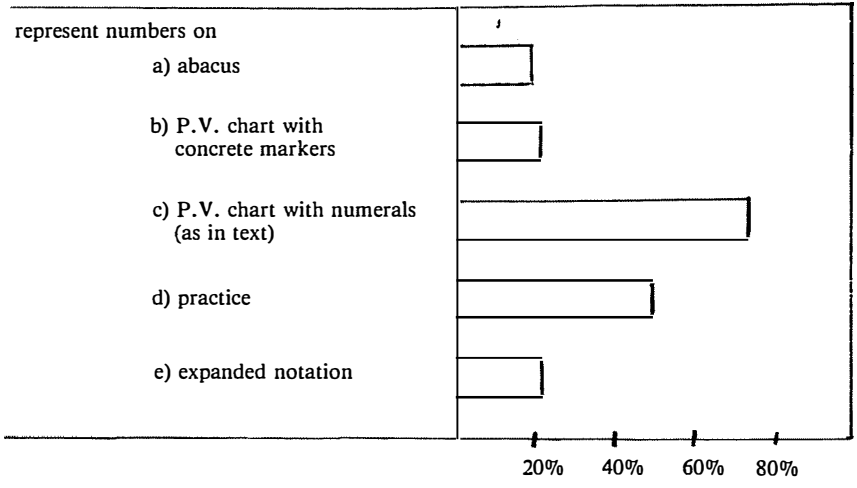
But a far more important influence is the attitude of elementary school teachers. I suggest to you that a prime place for the transmission of math anxiety is in the very place where it should be being countered - in the classrooms of our elementary schools, particularly in the first three grades. At that age, children are like sponges. They absorb new ideas and concepts almost by osmosis. They are still uninhibited in their attitudes to most things. But if a teacher suggests to them that math is difficult, or to be disliked, or to be avoided, she (and I say **she** since most of the teachers of these children are female), she is doing them an enormous disservice. She is influencing their future. She is influencing societies' future. It is indeed a disaster for the future of our society.

Unfortunately, the teacher can cause math anxiety when she thinks that she is countering it. By saying things like
"I know that this is difficult, but..."
or giving a page of math examples as a **punishment** for mis-behaviour. This comment has, of course, much wider significance than only to the learning of mathematics.

Strategies used by Teachers to Extend Place Value

Approximately 73% of teachers lean toward abstract place value representation in teaching the extension of decimals to thousandths compared to 22% who opt for a more concrete representation. Both the literature and attained curriculum items suggest teachers might well reconsider the timing of the move from concrete place value representation.

Teacher Preferred Strategies for Extending Place Value
(teachers checked 2 choices)

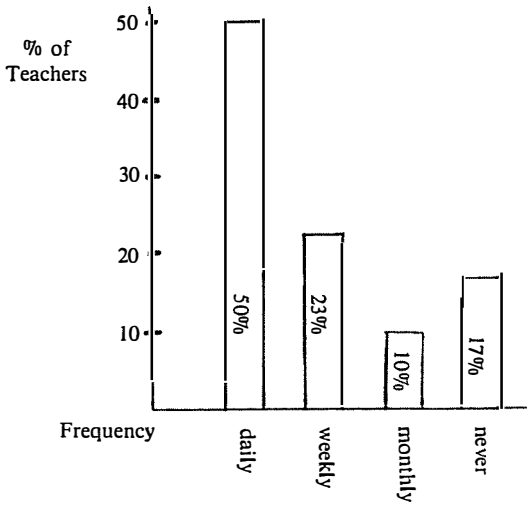


Recognizing that available time, texts, philosophical attitudes to use of devices such as the calculator and other issues affect learning, here are some other findings.

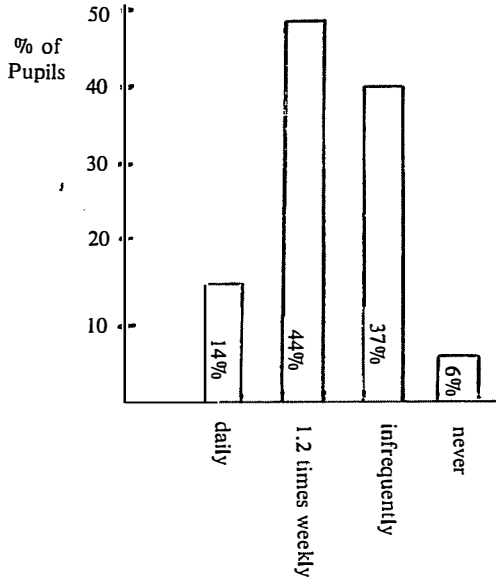
Time

While the time for math ranged from 30 to 60 minutes daily, the average time allotted was 46 minutes daily. Based on 45 minutes daily, teachers felt that 90.2% of their students could successfully complete the present regular grade 5 program.

Teachers Assign Homework



Pupils Do Homework



Oral Mathematics

There was some confusion between oral math and straight fact drills but nearly 20% of the pupils and slightly more than 20% of the teachers found time for daily oral math with 60% and 66% taking time once a week. Students seemed to like the oral math but said they usually did not have enough time to verbalize or explain how answers were obtained. If we recognize the importance of work 'done in the head' as well as the resultant explanations, perhaps more carefully prepared oral work will be found as well as more time.

Calculators

About 36% of the students remembered making some use of calculators in last year's grade 5. About 50% of this year's grade five indicated that some use of calculators would be made. As more familiarity develops with the strategies available with a calculator and as a better balance is achieved between calculations carried out mentally, on paper or with a calculator, there is bound to be a change in curriculum. It was interesting to note that students essentially agreed with their teachers that calculators were alright if basic facts were mastered, but some looked on calculators as a kind of cheating.

Sources of student exercises and drill work

Over half of the student seatwork comes from pupil tests, 18% are work sheets designed by the teacher, 15% are designed by the teacher and presented as blackboard exercises.

Objectives and Tests for Units

With respect to student involvement in long term (more than 1 day) math objectives, 56% of the students thought it was a good idea to know about the upcoming topics and what was to be learned. 79% favoured writing a test at the end of each unit, particularly if it points out the sorts of errors being made and allows them to upgrade their own levels of topic mastery.

The teacher's questionnaire showed that unit tests seemed to contribute most in their evaluation and for mark assignment. Daily work short quizzes and more formal exams ranked about equally, but only about half as important as term tests. Marks for notebooks and/or projects run a distant third in importance. Pretests are generally somewhat informal but do receive some attention as a step in the teaching/learning process.

It seems we are at cross purposes. Pupils look on unit tests as part of formative evaluation whereas often teachers use the unit test as part of summative evaluation for reporting achievement.

Attitudes towards Mathematics

When asked how important mathematics is to them, to their parents, to their future work, more

referring to multiple texts or varying text use to suit group or individual needs. Research also indicates that there appears to be a rather firm adherence to covering the material in the text (unless it is outside the specific curriculum requirements).

Attained Curriculum

From the results of both the pre- and post-test results, it is clear that students do not learn all of the implemented curriculum. Nor did the pupils measure up to the suggested level of performance for some individual items as judged by a panel of mathematics educators. (As part of the process, those involved with the Gr. 5 study made a prejudgement about the expected achievement for each item on both the pre- and post-test items. These expected achievement levels were then compared to the actual achievement.) The following is an example. For the pretest, the panel expected better than 85 percent attainment in questions dealing with the reading and writing of decimal fractions. The goal was not achieved. Either expectations are too high or the topic was more difficult than the panel believed. In fact, the panel scaled down its estimate for the June tests.

Intended Curriculum

The main result of this project is a greater awareness on the part of the Gr. 5 teachers of the intended curriculum and an ongoing exchange of ideas as to how to implement it. As well, the Mathematics Coordinator is attempting some revisions of the curriculum - a pulling together of topics into units as well as preparing a set of unit tests to match the units to be used as a resource material.

Yes, curriculum work is a neverending process. Change will come about in the classroom and hopefully in the results attained by children as these monitoring programs continue. However, major changes in direction await external influences. Among unresolved curriculum concerns are:

1. The scope of curriculum content.
   How much work centered around graphs, charts, and mathematical investigations should be presented in the elementary school?

2. The role of the computing technology.
   How much longer will children spend time practicing unrealistic long division? No one except students in school uses it. We adults use a calculator or our own short cuts. Outside the school the move from pencil and paper has already happened.

3. The impact of competency testing.
   Indeed, can it be justified when the main thrust is toward problem solving?

Curriculum change, in fact, must continue!

WHEN WILL CANADA BECOME FULLY METRIC? NEVER!

by

Alan Auerbach

Our language is not the product of a committee; it is the result of an evolutionary process which started long before recorded history. Over scores of generations the maladaptive becomes extinct and only the most useful survives. Our measurement system has obeyed the same evolutionary principle as our language.

What happens when a government ignores the evolutionary history of measurement conventions and unilaterally legislates a change from one system to another? If the change were in the service of converting chaos to order, there would, presumably, be a short period of grumbling followed by acceptance and gratitude. But if the change disregarded the locked-in, permanent, physical adherence to the original system, order becomes chaos, citizens rebel, and governments fall.

The decision of the Canadian government in the early 1970's to convert one measurement system to another disregarded the intimate link between numbers and everyday life. Next to language, numbers constitute our main source of logic, cognition, communication, thinking. The farmer's day is as number-filled as the aerospace engineer's. The contractor, the do-it-yourself repairer, the car driver, the traveller, in fact anyone who deals with money, time, shopping, cooking, trading, party-giving, jogging, rope-skipping, relies upon numbers. The government seemed to disregard the ubiquity of numbers in everyday life. Ever since, we have been deluged with metric propaganda, or metprop, and it's time we looked at some of it.

Metprop 1: metric measurements are more precise. Fact: they're not. Metprop 2: metrics are more scientific. Fact: they're not. In accuracy and in value to science, 0.8763 cm has no advantage whatsoever over 0.345 in.

Metprop 3: metrics are more widely used throughout the world. Fact: depends how you determine "widely". In the world of printing, aviation, railways, or wherever the U.S. dominates, metrics are rare.

Metprop 4: metrics are easier to use. Fact: depends on what they're being used for; ask a carpenter or architect.

Metprop 5: The conversion is voluntary. Fact: some of those who disagree have to undergo huge legal costs and face relentless government prosecution. And when the metric commissioners met with industry representatives, the question was not "Do you wish to convert?" but "When and how do you wish to join the inevitable conversion to metric?"

Metprop 6: Canada needs metrics in order to export effecively. Fact: 70% of our exports are to the United States, where metrics are about as common as Iranian diplomats.

Metprop 7: The number 10 and its multiples are more convenient to deal with than the number 12 and its relatives. Fact: the twelves form our most basic measurements -- inches per foot, degrees in a circle, measurements of time, measurements of the most common wholesale and retail multiple --because 12 is divisible by twice as many whole numbers (not counting itself and 1) as 10 is.

Metprop 8: metrics are more natural. Fact: no measures are more natural than the human body parts on which the inch, foot and yard are based. And it is hard to think of a derivation more artificial than one ten-millionth of the distance of a meridian of the earth from the equator to the pole, which is the basis of the metre.

Metprop 9: metrics were adopted in Europe because of its superiority. Fact: there were so many systems and so many interpretations of the original metric system, that an international standard was essential for them. We may be troubled by American and British gallons; Europeans were saddled with a chaos of competing standards for their metric measurements.

Metprop 10: with some effort, lots of government help, and plenty of time, Canada will eventually become fully metric. This is the point that most needs to be explored. The fact is, Canada will not and cannot become fully metric; here's why.

The changes that the government has effected up to now have been the easiest, simplest, and most

Quebec largely excepted, Canada used to be an English-language country, which has become essentially bilingual. It is local custom, personal preference, and business consideration which determine whether a specific interchange takes place in English, French, or, for that matter, sign language. Canada is surviving bilingualism, and it will survive bimetrology, in which there are two official measurement systems, imperial and metric.

Bimetrology will no more lead to market-place anarchy than bilingualism has. With bimetrology, the publishing industry can decide for itself whether the next cookbook uses imperial, metric, or both, and whether the page width is 6 inches or 16.5 cm. Industries that trade with the United States will not have to feel anti-Canadian when they manufacture in imperial. Roof-shingle companies will no longer have to balance pleasing the feds against frustrating re-roofing contractors. If the welded-chain industry decided that the millimetre and centimetre are the ideal units for chain-links, let that industry enjoy all the benefits of metrics.

There would be nothing inconsistent with a fisherman ordering 60 metres of 10-millimetre chain for his 200 horsepower, 40-foot boat; both measurement systems can work together. Bimetrology has proven its value in many spheres. Every photographer who orders a 4 X 5 print from a 35 mm negative, and every doctor who wants to lower your blood pressure by 10 mm and your weight by 10 pounds shows that bimetrology holds no problems.

Bimetrology is not new. For decades cars all over the world have specified metric threads on spark plugs and even inches for wheel rims. For a century hunters have carried inch-based shells in one pocket and millimetre-based rounds in the other. Let's help the Metric Commission to retire gracefully, having at least established bimetrology, if not full metrication.

One model of bimetrology is the Canadian Armed Forces. If there is one advantage of their being saddled with major equipment from past decades, and with being forced to purchase their new combat jet, the F18 Hornet, from the U.S., it is that the service is locked into imperial measurements. But when it suits their purposes, as in the adoption of NATO ordinance standards, they use metric as well.

Lest readers worry that side-by-side use of metric and imperial measures will saddle Canada with a measurement system from the dark ages, let's end with a look to the stars. James Michener, in his recent book SPACE, describes the assembly of the Apollo-Saturn moon craft thus.

Inside the largest building in the world were to be mated for the first time six extremely complex devices, each built separately, by different industries. This incredible assembly would comprise some 22,000 flawless matings of wires, bolts and connectors. The paperwork consisted of 30,000 complex documents. How could everything be expected to fit perfectly? These were the massive

APPLICATIONS IN SECONDARY SCHOOL MATHEMATICS

by
Hugh Allen
Faculty of Education
Queen's University
Kingston, Ontario

The purpose of this paper is to discuss the role of applications in the teaching of secondary school mathematics; to distinguish between suitable and unsuitable applications; to identify some sources of applications; and to make suggestions as to how teachers of mathematics can develop their own applications.

Why are Applications Needed? Which Applications Best Serve these Needs?

It is instructive to ask teachers of mathematics why they want to have applications, and what kinds of applications they use for these purposes. Some teachers admit that they don't bother with applications. "Takes too much time", they say, "We have barely enough time to get through the work as it is!" Closer scrutiny of these teachers' programs usually reveals a great deal of time spent on repeated drill of basic arithmetic or algebraic operations. (One grade 8 teacher at the end of November had not progressed beyond the multiplication and division of whole numbers. "And we're not going on until they can do it; even if it takes all year!", she said.) Fortunately, these situations are rare.

"Well", says another teacher, "I want to have something to say to the kid who says to me, 'what do we have to do this stuff for?'". A student who asks a question like this is usually doing so due to frustration and lack of success with the topic. He is really saying "I don't like this topic. It's difficult (or boring) and seems pointless. Can't we do something else?"

Teachers tend to respond to these questions in two ways. The first is to point out that the topic will be needed in later mathematical study; thus, "If you don't learn how to factor trinomials, you won't be able to do the later work on solving quadratic equations." That is, you need to learn this if you want to pass this course (or subsequent ones). This is hardly an application (although some teachers believe it to be); nor is it a satisfactory response. The other response to the question "Why do we have to learn this stuff?" is to attempt to provide a genuine situation in some other discipline (physics, chemistry, economics, etc.) where the particular topic is used; e.g., "Physicists (economists etc.) use this topic where they're studying the properties of electromagnetic waves (considering supply and demand curves). "Such comments,[1] even if perfectly true, do little to help student with his frustration. One might serve the student better by responding "It sounds as if you're having trouble (or bored with) this work", and through active listening determine the sincerity of his request for an example of the use of the material.

"Well," continues the teacher, "Perhaps he is seriously interested in knowing about an application. I want to be able to show him how mathematics is applied to everyday life."

This is an excellent idea. Unfortunately, most of the mathematics taught in secondary schools (particularly in the advanced courses) has no application in the day-to-day existence of the majority of people - except those who work in mathematically related fields. Let's face it; there is no time in one's daily routines that one has occasion to solve a pair of simultaneous equations by the comparison method, or multiply two negative numbers together, or find the slope of the line joining P(2, 5) to Q(-3,4), or find out how far Kathy lives from school given that:

> **Example 1**[2]
> Kathy hates school! In the morning, she walks from her house to school at 2 km/h and is invariably late; in the afternoon she jogs back home at 6 km/h and saves 15 min.

Teachers of elementary school mathematics and of general and basic courses in secondary school have much less trouble providing illustrations of the role played in everyday life by the mathematics they teach. In our day-to-day life we are constantly confronted with problems whose solution involves a knowledge of various areas of mathematics including arithmetic, geometry, algebra, probability, and statistics. Calculating the amount of carpeting or wallpaper used when redecorating; converting a metric plan to imperial units; working out the amount of various ingredients when cooking for nine using a recipe that serves two; and whether or not it is too early to plant the peas are examples of everyday problems. Here are two examples.

Example 2
At the supermarket 'Brite Smile' toothpaste is sold as follows: 25 ml, $0.79; 50 ml, $1.05; 100 ml, $1.79; and 150 ml, $2.59.
(i) Find the unit cost per ml for each tube of toothpaste above.
(ii) Which container is the best buy? Suggest some reasons why shoppers might **not** choose this one.

Example 3 (modified slightly from original)
The square based tent shown is advertised as a "three-man tent".
(a) If the side of the square base is 275 cm, how much floor space does each person have?
(b) If each person's sleeping bag is 193 cm by 86 cm, how much space is left in the tent for gear when the campers are sleeping?
(c) Draw a diagram to indicate where the space for the gear would be. How much of that space is really useable?

These examples represent reasonable and realistic calculations that could be expected in analogous situations in everyday life. Not only are the requested calculations sensible, but the problems involve an appropriate non-mathematical component dealing with the reasonableness or utility of the mathematical answer. Good problems like these - and we need lots of them - usually involve mathematics taught, if not learned, in elementary school, and rarely require secondary school mathematics.

One should be on guard when looking for examples that illustrate the role of mathematics in everyday life, to avoid those problems which **appear** to involve reasonable calculations in situations using **language** from everyday life, and **pretending** to be applications. The objection to these could be due to their use of unrealistic values (gasoline that costs 20 cents per litre); or to their gross simplification of the real-life situation they purport to represent; or to their implicit use of unidentified assumptions.

Example 4
Find the number of 1000 W electric heaters required to heat a room 4 m by 6 m by 2.5 m, if 35 W will heat 1 m^3 of air.

Example 5
A concrete mix has 3 parts cement to 5 parts sand to 12 parts gravel. How many shovelfuls of cement and gravel are needed if 80 shovelfuls of sand are used?

Example 4 ignores the effects of openings in the room and of air circulation (the problem refers to a recreation room illustrated as a rectangular parallelepiped devoid of openings). Example 5 describes a very poor recipe for concrete, and the size of the batch produced would not fit into any mixer that would be loaded with a shovel.

The following example - although hardly an application - like the previous two examples, attempts to make the mathematics 'relevant' by using familiar words. It has a serious flaw.

Example 6
In the James Bond movie "Goldfinger" they loaded one million dollars worth of gold in the trunk of a Lincoln Continental. Prove that the Lincoln would have broken springs. (The price of gold is $35 an ounce).
*[*In the metric edition of the text the price of gold is quoted as $1.23/g*]*.

Now to do this problem one would need to know what a Lincoln Continental can carry in its trunk (assuming, of course, that the vehicle was not specially reinforced - which I believe it was). But that's not the main problem. The metric version quotes the price of gold as $1.23/g, which can be obtained by calculation from the non-metric value of $35 an ounce, as follows:

$$\frac{\$35/\text{ounce} \times 16 \text{ ounces/pound} \times 2.2 \text{ pounds/kg}}{1000 \text{ g/kg}} = \$1.23/\text{g}$$

However, gold is still sold by the ounce[3] (and should be reported as such), and this ounce is a **Troy** ounce which is **not** 1/16 of a pound that is 1/2.2 of a kilogram. (An ounce in Troy weight is 1/12 of a pound in Troy weight which in turn is less than a pound (avoirdupois).)

Examples 4, 5, and 6 involve mathematical calculations that could, in fact, pertain to reasonable real-life problems. Students aren't fooled by attempts to make calculations topical (Example 6), and

they are deceived by examples that misrepresent real life. There are lots of good examples that could be used to frame these calculations.

"All right," you say, "I'll be careful that my applications to everyday life are reasonable. I also need applications that will show how the secondary school mathematics I teach is used in other disciplines. Fortunately, there are some in the textbook."

Sure there are; but are they really good applications? Many problems and examples purport to be applications of mathematics to other disciplines through their use of terminology from these disciplines. The following are examples.

Example 7
The energy of a moving automobile is given by

$E = \frac{1}{2} ms^2$ m = mass in kilograms
s = speed in metres per second

An automobile has a mass of 500 kg.
(a) What is its energy at 50 km/h?
(b) What is its energy at 100 km/h?
(c) The speed has doubled from Part (a) to (b); What has happened to the energy?

Example 8
In an electrical circuit, the impedance z, in ohms, is found by the formula $z = \sqrt{R^2 + x^2}$, where R is resistance in ohms and x is the reactance in ohms. If R is 14.8 ohms and x is 11.1 ohms, find z.

Example 9
The cost C, in dollars per hour, of operating a certain type of aircraft is given by the formula

$$C = 900 + \frac{m}{200} + \frac{20\,000\,000}{m}$$

where m is the cruise altitude in metres. Find the hourly cost of operating the aircraft at 8000 m; at 10 000 m.

These examples use the language of other disciplines, or allude to practical application; but fail to explain the significance of the variables (example 8), and fail to include the units (example 7), or fail to explain where the formula comes from. In fact, these problems simply present a formula as a basis for a computational exercise. It might be educationally more honest to ask

If $a = \frac{b}{2} \times c^2$, find a when b = 500 and c = 50

or

Evaluate $900 + \frac{m}{200} + \frac{20\,000\,000}{m}$ when

m = 8000 and when m = 10 000.

If examples that use the language of other disciplines, or involve variables relating to real-world problems are to be considered as suitable applications, then these examples must be presented in a manner that allows students to understand fully the significance of the variables, and concepts associated with other disciplines, and the basis of the development of the formula. There are lots of excellent examples of this type.

"They're not in my textbook", says the mathematics teacher, "and even if they were, we don't have time to go into all the details. (Frankly, I don't know what impedance and reactance are)."

Textbooks writers have to decide whether to write for the student, or for the teacher. Writing for the student - and seriously trying to help the students understand the concepts involved - would result in tomes suitable for blocking the opposite wheel while changing the flat tire on an automobile. Also, there is no guarantee that the high school student can, or would, read the text. To write for the teacher would result in a text containing teaching strategies and examples. Textbook writers aim somewhere in the middle, and consequently cannot be expected to provide an extensive background associated with a particular application. Nevertheless, there are some excellent examples in the texts.

Example 10

In winter, you will have noticed that the stronger the wind the colder it feels. Winter weather forecasts often give "wind-chill" temperatures. For wind speeds of 10 km/h and higher, the following formula gives an approximate value for the wind-chill temperature:

$w = 33 - (0.23\sqrt{v} + 0.45 - 0.01v)(33 - T)$

where w is the wind-chill temperature, in degrees Celsius,
v is the wind speed in kilometres per hour, and
T is the still-air temperature in degrees Celsius.

For a still-air temperature of -18 °C, calculate the wind-chill temperatures at these wind speeds.

a) 100 km/h b) 80 km/h c) 30 km/h d) 15 km/h

Example 11

The unit of measurement for gold is the Karat(K). Pure gold measures 24K and can be mixed with other metals to make jewellery and coins.

The amount of pure gold in an item is marked with the K symbol.

A 10K gold coin means that the ratio of the mass of gold in the coin to the total mass of the coin is 10:24.

If the mass of the coin was 36g, you could easily find out the mass of gold in the coin. The gold content is 10K or 10:24. That is 10 twenty-fourths of the coin is pure gold. Since $10/24 = 5/12$ in lowest terms, then $5/12 \times 36$ is the amount of pure gold in the coin.

1. Write the gold content of the following as a ratio in the lowest terms.
   a) a 14K gold bracelet b) a 22K gold coin
   c) a 16K gold ring d) a 24K gold bar
   e) a 9K gold pin f) a 18K gold charm

2. a 14K gold medal has a mass of 21.8 g.
   a) Find the mass of pure gold in the medal.
   b) What is the mass of the other materials in the medal?
   c) If gold is valued at $27.42 per gram, find the value of the gold in the medal.
   d) Try to find out how much pure gold must be contained in an item for it to be legally labelled as gold.

These examples are presented in a way that attempts to explain the meaning of the terms used, or to set the stage so that the situation makes some sense to the reader. More examples of this type are sorely needed, and fortunately they exist in sources other than the textbook. Chapter 4 of the 1979 NCTM yearbook **Applications in School Mathematics** is entitled "Applications through Direct Quote Word Problems" and contains some excellent examples relating to various disciplines. Each example involves a background description that sets the stage, a quotation from a stated reference, a glossary of terms, and some questions pertaining to the situation. These applications show that mathematics is directly involved, through formulae and calculation, in other disciplines, and they are presented in a manner that would assist the student to **understand** the significance of the application. More good examples of this type are needed.[4].

There is no doubt that presenting an application in this way takes more time than it would if one merely presented the students with a formula and ask for specific calculations. But it's time well spent! Whatever you do, do it well, or not at all.

Genuine Applications of Mathematics

The preceding section suggests some of the common reasons why teachers of secondary mathematics say they need applications. These reasons and the types of applications employed have a common feature; namely that the applications are used to attempt to justify the teaching of certain mathematical skills/concepts, and they all derive from the viewpoint of the mathematics. **Genuine applications** of mathematics, either to real life or to other disciplines, are quite different. In the first place they originate **outside** the field of mathematics in fact they may not appear to involve any mathematics at all; and in the second place, the situation is rarely directly translatable into mathematical terms without involving the introduction of several **assumptions.**

If a teacher really wants his students to see what mathematics has to do with the real world (that is, to see genuine applications) then he must remember that the mathematics came **after,** or grew out of the consideration of a real-world problem. Historically, this is the way in which applied mathematics develops. First, one has a problem. The solution of this problem involves the identification of principles and patterns **later** organized into the discipline of mathematics as we know it. Mathematics arises from attempts to solve problems. One must start with a problem - not with the mathematics pretending that it came first, and that it can be 'applied', like a coat of paint, to problems that arise in life.

Our secondary school mathematics courses are riddled with pedagogically unsound approaches to topics - approaches that derive from a misconception concerning the development of mathematics. If functions are defined to be "sets of ordered pairs in which no two different ordered pairs have the same first component", that is, as special relations, then you may be assured that you will have problems finding applications of functions, since you will rarely see a set of ordered pairs in real life. We use the 'ordered pair' definition in order to be mathematically correct (even though some treatises in axiomatic set theory consider a function to be a ordered **triple)** and in so doing we have abstracted our concept away from the real-world problems that gave birth to the idea of a function. Our formal approach leads to ridiculous extremes. Does an increasing function look like this.

$$\{(3,4), (7,8), (9,10), (15,34)\}$$

or like this

$$\{(2,1), (5,7), (18,20), (20,22)\}$$

After all what is meant by an increasing set of ordered pairs? As a consequence, we end up devising formal definitions in order to do productive work with functions - work that could have been developed on an intuitive basis.

Originally, a function was a concise way of describing the way in which one variable depends upon another, and the variables in question represented real and usually measurable physical quantities. Herein lies one of the main reasons why secondary school teachers of mathematics have difficulty finding and using genuine applications. The mathematics taught in secondary school has very little to do with the real world. We present some kind of distillate of mathematics abstracted away from its real world origins. This is a terrible situation.

There is only one reason for teaching anything in schools and that is to help students better understand the world in which they live. The mathematician - philosopher A. N. Whitehead had it right over 60 years ago when he wrote "There is only one subject matter for education, and that is Life in all of its manifestations"[5].

To teach the applications of mathematics would involve finding a problem from the real world that wants to be understood, making whatever assumptions are appropriate to make sense of the problem, accepting that mathematics may not be involved in the solution of the problem until the problem has been discussed and understood from other points of view, and finally, that there may be no satisfactory solution. Here's such a problem:

Which is the best local store to shop at for groceries?

Here is another:

What is the strongest beam that can be cut from a circular log having a minimum diameter of 40 cm?

and another:

What should the maximum number of packages be that would be permitted at the express checkout of a supermarket?

What is Involved in Generating and Using Genuine Applications?

Teachers of mathematics must train themselves to examine the world around them in such a way that it becomes natural for them to see problems that might yield suitable applications. Television commercials are riddled with meaningless statements, contradictory statements, and unsupported claims. Everyday the newspaper presents situations suitable for study. Observing a boy on a 10

speed bicycle, or a line-up of cars at a traffic light, or people lined up to reserve squash courts suggests a number of suitable investigations.

Teachers may also have to change the way they view certain mathematical topics - perhaps even the way they view mathematics as a whole. For example, here are several standard forms for the equation of a straight line:

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\frac{y - y_1}{x - x_1} = \frac{x_2 - x_1}{y_2 - y_1}$$

$$\frac{y - y_1}{x - x_1} = m$$

$$y = m(x-a)$$

$$y = mx + b$$

$$Ax + By + C = 0$$

It would be sufficient to teach only

$$\frac{y - y_1}{x - x_1} = m$$

since all others are derivable from it. So, why teach the form $y = mx + b$? There are many possible answers to this question; but the one that relates most clearly to an interest in applications is that $y = mx + b$ is the **statement of partial variation.** Perhaps the entire treatment of straight line equations could evolve from a consideration of actual situations of partial variation.

The teacher who wants to use genuine applications must learn about things other than mathematics and be prepared to discuss topics with which he is not familiar, and in which his opinions may not be viewed as authoritative. Many teachers avoid genuine applications precisely because they lie outside the teacher's field of expertise. The teacher is not comfortable in an area in which he is no longer the 'authority' and in which he may in fact know less than one of his students. This understandable discomfort may manifest itself in statements such as "I don't know anything about orienteering" or "We don't have time to do that sort of thing"or "That won't work. The kids don't know any astronomy!" Perhaps it would help to realize that the teacher might better **not** function as an authority, but rather function as model **learner** and an orchestrator of his student's learning.

The teacher who uses genuine applications must learn to ask questions rather than provide answers - not only of his students, but also of the situations in life he encounters. "What would happen if...." is one of the best questions to ask in order to understand the world in which we live. It's also a very good way to generate genuine applications.

Better Applications and their Use

In 1980 the Canadian Mathematics Education Study Group held its annual meeting at Universite Laval. The topic for Working Group B at that meeting was "Applications of Mathematics for High School Students". The recommendations of that Working Group might help teachers decide which applications to use in schools. According to the group[6], better applications are those that can involve and challenge the student at his level of mathematical maturity. These applications present a situation that can be understood quickly, thereby allowing for a fairly rapid formulation of a problem whose solution is not immediately obvious. Better applications have an element of surprise ("Oh, I wouldn't have expected that!" and/or provide the student with a new understanding of the way the world functions. Good applications lend themselves to "what if..." questions; that is give rise to other questions and in doing so illustrate the cancerous nature of mathematical inquiry. Good applications do not deal solely with abstraction, but rather, allow for the use of concrete materials in helping the student understand and solve the problem. Better applications do **not** define

a problem so clearly that they do not allow for a sorting out of relevant information, nor for different assumptions and approaches both in the problem formulation and in the solution.

Even if a teacher is able to identify a first-rate application, care must be taken in the way in which it is presented in the classroom. For the student, the applications will be considered as simply another exercise in mathematics if it is presented by the teacher as a **mathematical** problem with a specific solution or involving a specific skill. The student must be allowed whatever time **he** needs to work with the problem and not be **expected** to have a solution by Thursday's class. Students should be permitted to work together; that's the way problems are solved. The teacher must permit the students to follow **their** own path to a solution - even if the teacher is not comfortable with the path, or believes it to be a dead end. The use of concrete materials and representations to solve problems, must be encouraged where appropriate.

Summary

Mathematics teachers should give serious consideration to the reasons why they use applications in the classroom, and to the suitability of the applications themselves. There are genuine applications of mathematics all around us in the world. If the teacher is inquisitive, eager to learn and not afraid to be a learner, they are not that difficult to find. But they won't be found 'frozen' in print. One has to seek them out or invent them.

Footnotes

1 Pamela Ames in her article "A Classroom Teacher Looks at Applications" calls these "level zero applications." **A Sourcebook of Applications of School Mathematics.** NCTM, 1980.

2 All examples in this paper are taken from textbooks currently in use in Intermediate and Senior Division mathematics courses. In selecting these examples there is no attempt to classify the texts as good, bad or indifferent.

3 Some things don't change under metric conversion. For example, a piece of lumber known as a 2 × 4 is still called a 2 × 4 (even if it isn't 2" by 4").

4 Another excellent source is the NCTM's **Sourcebook of Applications of School Mathematics.**

5 Whitehead, A. N., **The Aims of Education and other essays.** Mentor Books. New York, 1949.

6 Canadian Mathematics Education Study Group. **Proceedings of the 1980 Meeting.** Laval University, Quebec.

PINOCHLE PROBABILITIES: CHANCES OF MATCHES

Bonnie H. Litwiller and David R. Duncan
Professors of Mathematics
University of Northern Iowa
Cedar Falls, Iowa

Situations giving rise to matches are a good source of probability problems. In two previous articles we discussed some "match" probabilities concerning the standard deck of 52 playing cards. (See references at the end of the article.) We now consider the pinochle deck. This deck contains 48 cards. Six face values are involved (9, 10, J, Q, K, A); for each of these face values, there are two identical cards in each of the four standard suits.

We now describe two problems using the pinochle deck of cards.

1. Suppose that each of n persons has a full pinochle deck. If each person draws a single card at random from his/her deck, what is the probability that there will be at least one match; that is what is the probability that at least two people will draw identical cards?

To solve this problem suppose that the n people are ordered and that they draw their cards consecutively. Each of the n persons can draw any of the 48 cards. By the Fundamental Principle of Counting, the number of ways that n cards can be draw is $\underbrace{48 \cdot 48 \cdot 48 \cdot 48 \cdot \cdots \cdot 48}_{\text{n factors}}$, or 48^n. Since in each draw all 48 cards have an equally likely chance to be drawn, and since the n draws are independent events, all 48^n possible outcomes are equally likely.

To determine the probability of a match directly would be very difficult, since all possible ways in which the match could occur would have to be distinguished. Consequently, we employ a method commonly used in probability—to determine the probability that the event does **not** occur and then subtract from 1.

To find the probability that a match does not occur, consider the n cards being drawn in order and determine the number of ways that **no** matches occur. The first person can draw any of 48 cards. The second person must avoid the two cards identical to the one already drawn; 46 cards are then available on the second draw. The third person must avoid both of the first two cards drawn; 44 cards are thus available for this draw. Similarly the fourth person can draw any of 42 cards, the fifth person can draw any of 40 cards, and so forth until the n^{th} person can draw any of 48 - 2(n - 1) or 50 -2n cards.

The number of ways in which no match occurs is thus

$$\frac{48 \cdot 46 \cdot 44 \cdot 40 \cdot \cdots \cdot 2(25 - n)}{48^n} = \frac{24 \cdot 23 \cdot 22 \cdot 21 \cdot \cdots \cdot (25 - n)}{24^n}$$

The probability that at least one match occurs is thus

$$1 - \frac{24 \cdot 23 \cdot 22 \cdot 21 \cdot \cdots \cdot (25 - n)}{24^n}$$

It should be noted that this outcome is identical to the one that would occur if the "duplicate" set of 24 pinochle cards was discarded from the deck, leaving only 24 distinguishable cards.

Table 1 displays the probability of at least one match for various values of n.

Table 1

No. of People Drawing Cards (n)	Probability of at Least One Match
1	0
2	4.2%
3	12.2%
4	23.1%
5	35.9%
6	49.3%
7	62.0%
8	73.1%
9	82.0%
10	88.8%
11	93.5%
12	96.5%
13	98.2%
14	99.2%
15	99.7%
16	99.9%
17	100.0%

(Rounded to one decimal place)

Observe that only 7 persons need to be drawing cards for the probability of at least one match to exceed ½.

2. Suppose that we introduce an additional person called the leader. At given time each of the n original persons and the leader draw a card from his/her deck. In this problem, however, we calculate the probability that at least one of the original group of n people matches the card of the **leader.** It is not important whether or not the n people match each other.

As before, the n people may draw their cards in 48^n ways. Now determine the number of ways in which no matches with the leader result. Suppose that the leader draws his/her card first. Each of the n people may then draw any of 46 cards to avoid matching the leader. The number of ways in which these non-matching choices can be made is $\underbrace{46 \cdot 46 \cdot 46 \cdot 46 \cdot \cdots \cdot 46}_{\text{n factor}}$ or 46^n. The probability that no one matches the leader is then $\frac{46^n}{48^n} = \left(\frac{23}{24}\right)^n$. The probability that at least one person **does** match the leader is then $1 - \left(\frac{23}{24}\right)^n$

Table 2 displays these probabilities for various values of n.

Table 2

No. of People Drawing Cards (n)	Probability of at Least One Match
1	4.2%
2	8.2%
3	12.0%
4	15.7%
5	19.2%
6	22.5%
7	25.8%
8	28.9%
9	31.8%

Table 2 con't pg. 35

10	34.7%
11	37.4%
12	40.0%
13	42.5%
14	44.9%
15	47.2%
16	49.4%
17	51.5%

Note that it takes 17 persons drawing cards before the probability of at least one match with the leader exceeds ½. This "break even" mark could have been calculated in another way. The problem is to find the value of n so that

$$1 - \left(\frac{23}{24}\right)^n > \frac{1}{2}$$

Then: $\left(\frac{23}{24}\right)^n < \frac{1}{2}$

$$n \log_e\left(\frac{23}{24}\right) < \log_e 0.5$$

$$n > \frac{\log_e 0.5}{\log_e \left(\frac{23}{24}\right)}$$ (Recall that the $\log_e \left(\frac{23}{24}\right)$ is negative.)

$$n > \frac{-0.6931}{-0.04256}$$

$$n > 16.3$$

Since n must be an integer, 17 persons are needed.

On Table 1, it may be noted that at least 17 persons were needed to yield a probability greater than 99.95% (rounded to 100.0%) for a match.

For which value of n in Table 2 would the probability first round to 100.0%?

Thus: $1 - \left(\frac{23}{24}\right)^n > .9995$

$$\left(\frac{23}{24}\right)^n < 0.0005$$

$$n \log_e \left(\frac{23}{24}\right) < \log_e 0.0005$$

$$n > \frac{\log_e 0.0005}{\log_e \left(\frac{23}{24}\right)}$$

$$n > \frac{-7.6009}{-0.042560}$$

Since n must be an integer, 179 persons must be drawing cards.

Challenge for the reader:
Suppose that each of n persons has a deck of r distinguishable cards. For various values of r, calculate the smallest value of n needed so that the probability of at least one match exceeds ½ for each of the two problems discussed in this article.

References by the authors:
1. "Birthdays, Cards, and Matches: An Experience in Probability", **Ontario Mathematics Gazette** 11(4): 279-282, September, 1973.

2. "The Fixed Element Match Problem: An Illustration With Cards", Delta-K (Alberta) 19(2): 7-10, November, 1979.

POSITIVE, NEGATIVE, PLUS, MINUS, ADD, SUBTRACT

by
Bruce Shawyer
Department of Mathematics
University of Western Ontario
London, Ontario, Canada

With the advent of the "new math" in the 1960's, an attempt was made to explain to younger students the concepts of the integers, both positive and negative. This attempt was based on the distinction between:

-5 -- a negative number
-5 -- subtract a number

and to a lesser degree

-5 -- the additive inverse of a number

The first texts of the "new math" expounded at length on these distinctions. In the early grades, particular attention was paid to writing things out in full after the symbol. For example:

-5 (negative five)

and

-5 (subtract five)

In order to prevent confusion, the first of these was often written as:

$^{(-)}5$ or $^{-}5$.

That led to notations such as:

$^{(+)}3$ or $^{+}3$

for positive numbers. The **property** of being positive or negative was indicated by a superscripted sign whereas the arithmetic instruction remained in its usual position. Typical exercises involved calculating values of expressions such as:

$^{+}5 + {}^{-}3$; $^{-}5 + {}^{-}3$;
$^{+}5 - {}^{-}3$; $^{+}5 - {}^{+}3$;

and so on.

But as the texts progressed, the authors reduced the number of written out statements until they considered it no longer necessary to follow

$^{-}5 - {}^{-}3$

with

negative five subtract negative three.

Eventually the superscripted signs were returned to their usual level with brackets being inserted if necessary to make the meaning clear. For example:

-5 - (-3).

It was assumed that everyone now understood what was going on.

Now all of this is quite defensible if the distinction between positive and negative integers has been understood and the arithmetic of them has been explained and understood.

But there is a real problem because the symbol "-5" is now almost universally read as "negative five" no matter where it occurs. The disaster is that all this has happened before the student is introduced to any algebraic technique.

The great power of algebra lies in the use of symbols (letters) to denote unknown quantities. I shall illustrate what goes wrong by a very simple example:

Chris has 7 apples. Some of them are given to Pat. Chris has 2 apples now. How many were given to Pat?

Although this is a very simple question, consider this possible algebraic solution.

Let x denote the number of apples given to Pat.
Then $7 - x = 2$
Subtracting 7 from each side of this equation, we get
$-x = 2 - 7 = -5$.

Stop here, because the vast majority of student will read this line as:

"negative x equals ...".

Although the case can be made that this is short for

"the negative of x equals ..."

meaning, really

"the additive inverse of x equals ...".

I have found that most students do not interpret it in the latter ways. However, they are quite convinced that

"negative x" is a negative number!

This misconception leads to terrible confusion later, particularly when the student meets the absolute value function:

$|x| =$ x if x is positive

$|x| =$ -x if x is negative.

On being told that $|x|$ is a positive quantity, they tend to reply:

"but negative x is negative!"

In order to remove this confusion from students' minds, it is important that teachers in the senior elementary grades and in high schools make sure that their students understand the difference between:

-5 -- a negative number
-5 -- subtract a number
-5 -- the additive inverse of a number

and their corresponding algebraic quantities:

-x -- subtract a number
-x -- the additive inverse of a number

and particularly that -x may be either positive or negative. -x is positive whenever x is negative, and -x is negative whenever x is positive.

Add to this the fact that the expression $+x$ is never read as

positive x,

and the fact that $+x$ may be either positive or negative according as x is positive or negative.

Whereas the name of a mathematical object is not of great importance, the precise meaning of it is of the utmost importance. I strongly recommend that all teachers herewith abandon the expression:

negative x

and return to the expression previously universally used and understood:

minus x.



INTERVIEW WITH GRAHAM RUDDOCK

by
Robert S. Smith
Faculty of Education,
The University of Western Ontario

RS: It's Wednesday, May 17th. I'm in London, England, well, just out of London England at Slough where the National Foundation of Educational Research (NFER) is. A very interesting building. In fact, Graham, you were telling me that the older parts were built in the last century.

GR: Yes, the cool offices in the summer are in the older part. The newer bits suffer from a greenhouse effect.

RS: Graham, I met you first as a graduate student working with Margaret Brown and Kathy Hart at CSMS. I believe Dr. Matthews was your supervisor.

GR: Yes, he supervised my Ph.D. and was a great influence at that time, not only on me I think but on a lot of things that are going on in mathematics education in Britain.

RS: I would agree with you there. You are now working with NFER on a project called Mathematics Monitoring for the Assessment of Performance Unit (APU). I suppose we should start at the beginning for our Canadian audience. What is NFER?

GR: The National Foundation for Educational Research is an independent research organization funded partly by money from the various local government authorities in Britain, but also conducting research on behalf of a variety of sponsors. One of our biggest sponsors is the Department of Education and Science, and we also do research for the Welsh Office Education Department, the Department of Education in Northern Ireland, and a range of other organizations. So, really, we look at educational research issues for England, Wales and Northern Ireland.

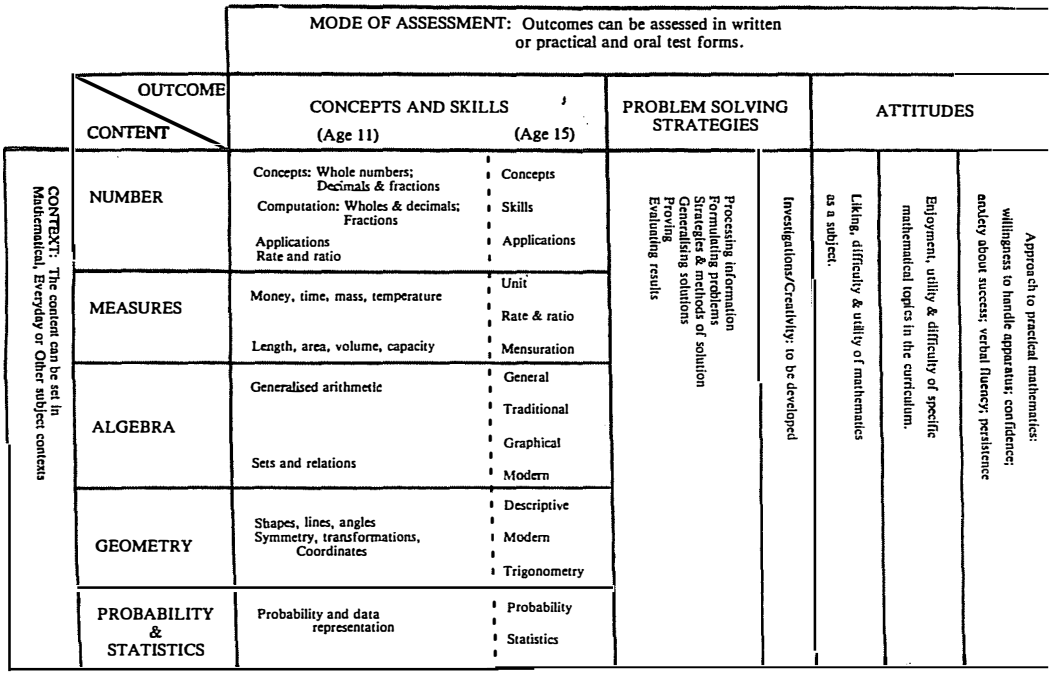
RS: What subject areas do you study?

GR: Basically we have a very wide-ranging role. We have departments which deal with mathematics, with English language, with foreign languages as taught in our schools here and also we look at science as part of the IEA project (the International Evaluation of Achievement). We also look generally at the structure of education, further education, handicapped pupils, and a whole range of educational issues.

RS: Well, I'd like to zero in on mathematics and, in particular, what is known as Assessment of Performance Unit (APU). Please give us some of the background of the APU.

GR: The APU was set up by the Department of Education and Science back in the mid-70's. There was a great debate on the nature of education and where it was going in Great Britain at that time and the DES decided after consultations, to set up the APU. The Assessment of Performance Unit aims to provide information about general levels of performance of children and young people at school and how they change over the years. The terms of reference include: to promote the development of methods of assessing and monitoring the achievement of children at school and to seek to identify the incidence of under-achievement; and to sponsor the creation of new instruments and techniques for assessment, having due regard to statistical and sampling methods. The APU commissioned projects to look at performance over a number of lines of development, mathematics being one of these, scientific thinking being another, language being a further one. There were six of these lines of development of which four are currently being looked at by projects funded by the Department. Among the current ones are mathematics, english, and foreign language as taught in our schools. These three are carried out at NFER here and a further one on science is carried out jointly by the Centre for Science Education at Chelsea and Leeds University. So, a broad spectrum of lines of educational development is being looked at. The mathematics project started in 1977 with a brief to look at changes in performance over time and was designed to lay down a baseline so that changes in mathematical performance could be isolated and looked at. It was also to provide answers to questions concerning the basic attainment levels of pupils in England, Wales and Northern Ireland. In mathematics a pilot study was set up in the middle 1970s called TAMS (Tests of Attainment in Mathematics) and they looked at the possibility of developing items for national use. Following the work of this project, TAMS, a monitoring team within NFER was set up to carry out full scale survey work in mathematics.

RS: It seems then that you looked at a number of models and came forth with the assessment model which follows.



There was also a concern to look at what children are thinking as they are doing the mathematics. What kinds of practical interview testing were developed to do that?

GR: Mathematics is not really just a pencil and paper subject. It's not just a subject in which pupils produce right answers or non-right answers. This led to a variety of testing techniques. We've got the standard short item tests, round about 50 items, which look at a whole range of mathematical content but we've also had from the beginning other tests, particularly what we call practical tests. Perhaps it's a misnomer. They're more properly called oral interactive tests. One of their purposes is, indeed, to look at whether children can use particular pieces of apparatus, for example, a balance, and whether they can weigh, measure and so on. But that's only one aspect of it. The other side of it is to find out how children do things, what justifications they give, whether they can explain and indeed whether they can take on a task and carry it through to a conclusion. For example, we now have tasks in which the pupil has to organize a day out. In the case of the eleven year olds it's for their class, in the case of the fifteen year olds it is organizing a day for themselves and a friend. From information that's given to them about what activities are available, timetables, menus, etc., they must decide when to travel, when and where to eat, and what activities to undertake. This looks at what we would call everyday problem solving. We look at whether the pupils not only show they possess the component skills, such as reading a timetable or performing calculations with money to keep within a cost constraint, but, more importantly, whether they can do this not in isolation but in the context of a larger task where that is merely one component. We believe strongly that while looking at these component tasks is indeed a valuable activity, things should go further than that. It's not enough just to look at whether a child can read a timetable. What one really needs to know is whether the child can read a timetable in a larger situation where that's just one of the requirements. So I think we've moved a long way from single item testing. For practical testing and oral interactive tests, we have developed techniques of briefing testers to ensure a standardized presentation to children. But on the other hand the children have a chance to use their own initiative and to take the things to their own conclusion.

RS: In most tests it appears problem solving is a worded problem in which children must come up

with some statement of work to be done and arrive at an answer. You seem to have gone beyond that in your definition of problem solving. What is problem solving to you, Graham?

GR: I think perhaps problem solving is one of the most widely used terms and has a great capacity to mean all things to all men or, indeed, all women. We tend to take the view that problem solving implies a degree of novelty to the would be solver, that is, it is not the application of a well learned routine to a familiar situation. It involves the pupils in applying not only their mathematical skills and concepts, but their more general thinking skills and procedures to a particular context or situation which may not be that familiar to them but gives them the chance to bring these to bear and to explore their ability to apply their mathematics. I think many of us would agree this is what mathematics is about. It's more than being able to recall facts and perform skills when needed. We use written problem solving tests to look at some of the component processes, but we don't go as far there in the way of whole problems as we do in the interactive set up, where we can introduce the child to all the information required in a gentler manner. We have to be careful about introducing children to too much data too quickly.

RS: APU also looked at children's attitudes toward mathematics. What were some of your findings?

GR: In nine of our ten surveys we have had an attitude questionnaire. We've had the standard sort of attitude questionnaire where children have to agree or disagree with statements about mathematics in the school, in everyday life and so on but we've also asked children to look at mathematical topic names and rate them for utility, difficulty and enjoyment. Again, we've asked them to do particular items from our written tests and to rate them for enjoyment, usefulness and difficulty. In the later surveys there has been a free response section where children get the chance to give their views on mathematics in a way where they don't just have to react to what we give them, but where they can tell us what they feel about the subject without being particularly prompted. The children have been extremely frank in this and we've got some very interesting feedback from this approach.

RS: Would you share a bit of it with us. I know from some surveys done in America, children like math. It is one of their favourite subject disciplines. On the other hand, adults often strongly state their dislike and avoidance of mathematics.

GR: Overwhelmingly children first of all find mathematics useful. They think it's a good thing to study. They think it will have a bearing on their future employment prospects. They do enjoy it although we find that as they get older, they tend to enjoy it rather less, which is somewhat sad. We also find some interesting sex differences in attitudes. One particular finding that I'd like to share with you here is on differences in response between boys and girls when we ask them to do an item from our written tests and then rate it for difficulty. The boys tend to regard the items as easier than their performance would suggest it actually is for them, while girls, on the other hand, say it's harder than their performance actually suggests it is for them. This sort of finding is really telling us rather a lot about why we are getting sex differences in performance in mathematics. Also, with regard to attitudes, we get some cries from the heart - the child who really says the teacher goes too fast (a common cry for help) and also feelings of isolation that come across. One child, in the free response section said "I don't think anyone can feel like I do about mathematics". Not true, there were lots of other children saying the same sort of thing. I think we are beginning to make a start on finding out what it is that makes children have particular attitudes to mathematics but we have still a long way to go in finding out what the relationship is between attitudes and performance.

RS: So further examination of your data.

GR: Yes, that's one of the things we're engaged in right now. We've completed a series of annual surveys of eleven and fifteen year olds. For each of five years we ran a survey of eleven year olds in May and a survey of fifteen year olds in November. We don't now do another survey until 1987 and in the intervening time one of the things we will be doing is to follow up the absolute mountain of data we now have and try and get some further leads on what exactly is making children tick in mathematics.

RS: So as well as the six survey reports(1) that are available through Her Majesty's Stationery Office there is to be a cumulative report of the first years of your existence.

GR: Yes, the separate reports were issued on each of the first three primary surveys and the first three secondary surveys - that took us from 1978 up to 1980. The policy now is that we will have an overall report - what we tend to call the retrospective report as a working title - which will look back over our findings. We hope this will be published late in 1984. It will contain a review of what we found about the general performance levels of pupils in mathematics across a wide range of topics. It will also contain comparisons on the performance of eleven and fifteen year olds on the same topic. One feature that we think is extremely enlightening is to look at the results of items, and groups of items, after we have broken down our samples into five attainment bands of 20% each. We look not only at the success rate of each of these bands on a particular task, but we can also look at their likelihood of making particular errors. We find that errors are not made randomly very often. There are errors that are made by lower attaining pupils, but there are also errors that are made more commonly by pupils in the middle of the attainment spectrum, and there are a few errors that are made mainly by higher attaining pupils. It's this sort of relationship between pupil attainment and error making that I think will prove to be an interesting line to follow up. It is also I think, telling us a lot about the way pupils tackle items. For example, one of the errors we notice very frequently amongst the pupils at the bottom of the attainment range is one that could be described as replacing the question they've been given with a simpler one. For instance, when asked to order decimals, the pupil orders them as if they were whole numbers. If asked to reflect something say in a sloping mirror, they reflect it in a vertical mirror. That's one of the characteristic errors we note from lower attainers. From the middle of the attainment range we get a different type of error which suggests that the pupil knows, for example, that decimals don't behave exactly like whole numbers do, but the pupil has not yet reached the stage where they can carry out the action correctly. The reaction of the lower attainers is often to treat the item as if it were about whole numbers, perhaps in a computation item, no decimal point will be inserted in the answer. In the middle of the attainment range a decimal point will be inserted but it won't be in the correct place, and decimals ordered not as whole numbers but not correctly either. I think these data should tell us a lot about the way children are actually thinking in mathematics.

RS: That sounds as though you are talking more as a researcher than you are simply an evaluator.

GR: I think so. Traditionally, lots of mathematics results have been scored on a one/nought basis. Either it's correct or it isn't. We've never done this, and we are finding that there's a wealth of information about pupil performance, and indeed about why they don't get things right, when one looks at the errors that are made. But we are not the first to do this, and I'm sure we won't be the last.

RS: The interview testing in practical form with interviewers who have been trained was a departure for NFER. Do you see any other developments of this type perhaps taking place before the '87 assessment?

GR: Yes, that was one of our briefs. The original brief included looking at existing assessment techniques and instruments and making use of them where suitable, but our brief also said that where we didn't find something suitable we should develop something new. I think practical testing was probably one of the best examples of our having done so, and having done so successfully. We've also looked at problem solving, where we've structured the task to varying degrees for the pupil. We haven't so far looked at the pupils attainment levels in carrying out an investigation - the sort of task where the child is given a starting point and told, more or less, to find out what happens. We try to access not only the traditional activities but to make strides towards assessing that which is more difficult to assess. It is very important that we should not shy away from that.

RS: I attended David Fielker's workshop yesterday called Investigations. he dealt with a third year mixed ability class of students, probably fourteen, fifteen years of age and asked them to draw a line around nine squares on a square grid, then to put three circles in any three of the nine squares. After getting that sorted out he asked them in how many different ways you could put those three circles in the nine squares. Would you consider that an investigation?

GR: Yes, very much so. The starting point has been given to the child. One very clear end point has been given - to find out how many ways. What has not been said is what the rules are for drawing a circle, how big they may be, whether they may overlap or not. And I think part of an investigation would be for the child to first of all sort out rules. Which rules the child adopts for this, and how far children go along the route of varying the rules and seeing what happens, is all of worth.

RS: In music piano examinations a part is performance of standard numbers or exercises. Another

part is to play pieces of your choice. In the math investigation you can choose a very simple route to do it well or you can choose a difficult route and perhaps stumble. You are giving the child an opportunity to choose his own investigation and by even the very choosing he is showing some feeling as to what he believes mathematics to be.

GR: Yes, I think often in that situation **we** know the set of rules to adopt which produces the situations that interest **us** mathematically. What we don't often do is give the child the chance to find out which sets of rules lead to interesting situations, which sets of rules lead to chaos and which sets of rules lead to nothing in particular. I think that's part of what the child should be able to investigate -the implications of adopting a particular starting point for an investigation.

RS: Well, as a pragmatic teacher I'm going to come back of course and ask now, what kind of rules are you going to place to the marking of such an open investigation? It certainly doesn't fit my idea of a computer marked standard deviation situation.

GR: No. Our marking setup varies to reflect the assessment we are using. In the written items we code different answers and we will obviously classify some of those as correct. When we move over to our existing problem solving items, the marking scheme is really more of a classification scheme. It classifies answers and methods into different types which we think are coherent, then moves on to deciding which of those are better than others. It's a far more fluid situation. I think one has to accept that when children are given license to explore, then the problems of assessment and coding are much greater. But I don't think that we should let the tail wag the dog. Our problem becomes how we will assess it afterwards. We shouldn't refuse to give children such opportunities merely on the grounds that it makes life hard for us.

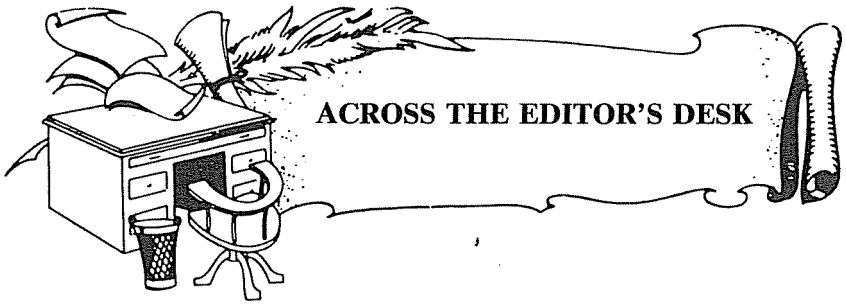
RS: Very well, I'll sit back and be an assessor of your ability to come up with some way to compare these students.

GR: Well, it will not be easy. One wouldn't pretend that it is. It's fascinating and I suppose it's one of the advantages of being a researcher that we do have a certain amount of time to be reflective and try to classify things. It is one of the joys of the job.

RS: Agreed. Thank you very much, Graham, it's been a pleasure chatting with you.

Footnotes

1. Primary Survey Report # 1, H.M.S.O., £5.60 net
   Primary Survey Report # 2, H.M.S.O., £5.80 net
   Primary Survey Report # 3, H.M.S.O., £6.60 net
   Secondary Survey Report # 1, H.M.S.O., £6.60 net
   Secondary Survey Report # 2, H.M.S.O., £6.70 net
   Secondary Survey Report # 3, H.M.S.O., £6.75 net
   Available from H.M.S.O., 49 High Holborn, London, England WC1V 6HB



Dear Editor:

I'm sending you a couple of mathematical poems written by one of my colleagues, Michael Whelan.

I thought your readers might find them creative and amusing, as I did.

Sincerely,
T.H. Fitzsimmons

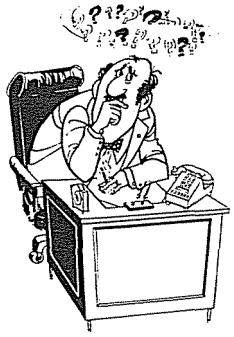
The gnome began with but a hut,
And ended with a mansion.
This was rather easily done
By gnomial expansion.
For when he found
The first floor stood
This master of construction
Proved that any number would,
By mathematical induction.
First he just assumed for K
Then proved for K + 1
That any number could be built
Before the final story's done.

Appolonius began with a point and a line
And embarked on some curious trips.
And when he returned, he announced to the world
"I shall name this new path an ellipse."
For each place that I've been is quite different you see
If I've measured the distance correctly,
The path to the point, and that to the line
Must surely have varied directly!"

But alas, the poor man was carried away
By this new found source of felicity
That his voyage appeared to take longer each day
Was no more than increased eccentricity.
But still he continued to follow new curves
His trips were extended to months,
'Though this statement appears hyperbolic in form
He went two different directions at once.

And so if you're tempted to over indulge,
To be carefree, to play, and to frolic
Recalling how Apollonius left us that day
This story is quite parabolic.
What Apollonius failed to observe,
His focus being somewhat disjoint,
All of his effort at blazing new trails
Were merely just stretching a point.

Poems by: Mike Whelan
Denis O'Connor High School
Ajax, Ontario



Problems

Editor's Note: *A very high percentage of those who returned the Reader Surveys indicated that they always read the "Problems" section. One reader commented that he would prefer the answers in the same issue for convenience in using them with his classes. No one else stated a preference either way, so this month, in addition to including the answers to December's problems, we'll include the answers for the problems given below. For this month's Problems we go back* ***40*** *years, and draw our examples from a problem-solving master, Polya, and his book,* ***How to Solve It,*** *where the simple 4-step process for solving problems in such common use today was first proposed.*

1. A bear, starting from point P walked 1 km due south, then changed direction and walked 1 km due east. He turned again to the left and walked 1 km due north, arriving at exactly the point P where he started. What colour was the bear?

2. Bob has 10 pockets and 44 silver dollars. He wants to put his dollars into his pockets so distributed that each pocket contains a different number of dollars. Can he do so?

3. To number the pages of a bulky volume, the printer used 2989 digits. How many pages has the volume?

4. Among Grandfather's papers a bill was found:
   72 turkeys $___67.9___$

   The first and last digit of the number that represented the total price are replaced here by blanks for they have faded and are now illegible. What are the two faded digits and what was the price of one turkey?

Answers on page 45

ANSWERS TO DECEMBER PROBLEMS

1.
$$\begin{array}{r} AT \\ +\ A \\ \hline TEE \end{array}$$
A = 9 T = 1 E = 0

2.
$$\begin{array}{r} IN \\ +RID \\ \hline DOOR \end{array}$$
O = 0 D = 1 I = 5 N = 8 R = 9

3.
$$\begin{array}{r} SAG \\ \times\ AS \\ \hline USES \\ SAG \\ \hline BEDS \end{array}$$
A = 1 U = 2 G = 3 S = 5 E = 6 B = 7 D = 9

4. $\sqrt{MADAM} = MAM$ M = 1 A = 0 D = 2

BOOK REVIEW

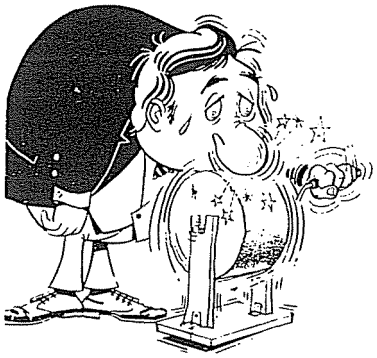
Children Reading Mathematics, edited by Hilary Shuard & Andrew Rother, 1984, John Murray Lts., 50 Albemarle St., London, W1X 4BD, England.

This book examines the relationship between the pupil and the written materials commonly used in mathematics lessons. We have all felt that children's difficulties in mathematics are often related to difficulties in understanding what is written and what is being asked of them, i.e. their ability to read, their ability to get meaning from the page. We have also recognized that the style of mathematical writing is quite different from the styles found in other texts. Thus, the introduction and definition of the terms Mathematical English (ME) and Ordinary English (OE) I found quite useful. These terms allowed me to focus on the vocabulary, syntax, non-verbal elements, graphic language, pictures, and symbols in the following sections and to realize to what an extent ME is a hybrid language. The differences between ME and OE also point up the difficulty, in fact the impossibility, of applying standard readability formulae in attempting to arrive at an informed judgement as to the readability of a particular mathematics text. But this text does not leave us with a discussion of the difficulties. The latter part of it does offer some help to meet the problems as well as to meet the challenge presented in the Cockcroft Report, paragraph 311:

> The policy of trying to avoid reading difficulties by preparing work cards in which the use of language is minimised or avoided altogether should not be adopted. Instead, the necessary language skills should be developed through discussion and explanation and by encouraging children to talk and write about the investigations which they have undertaken.

In particular, Chapter 12, with its specific suggestions to help children improve their skills in reading ME, I found to be most useful. I also compared these to the suggestions given in Chapter 9 "Teaching Reading Skills and Using Text Books" in **Mathematics Instruction: In the Elementary Grades** by McKillop and Davis, 1980, to see the different views, on British, the other American. Both are useful in making us more aware of the ways in which the printed mathematical page communicates with the reader and we are indebted to both sets of authors.

R. S. Smith
Faculty of Education
The University of Western Ontario



COUNCIL COMMENT

by
Ralph Connelly for
Bob Robinson

Editor's Note: *Council Comment appears in an issue of the Gazette following OAME/AOEM Council meetings. Your comments are welcome and should be addressed to the Gazette editor. This month's comments are briefer than usual due to time constraints in their preparation, but reflect the main points of business dealt with by Council.*

Future meetings: **Executive** 1985 04 26

Annual General Meeting-Sudbury 1985 05 09-11

Chapter Reports:

Coma - has a new Executive consisting of Duff Butterill, President; Barry Nicoll, Secretary; John LeBrun, Treasurer, and the following Executive Representatives - Ken Williams, Paul Beesack, Marcel Deruaz, Ralph Tross, Jerry Lunney, and Don Fox.
The next general COMA meeting will be held at the Ottawa Curling Club on Thursday, March 7, 1985 from 5-7:30 p.m.

GVMA - Spring Mini-Conference being held April 11, 1986 at Eastwood C.l., Kitchener at 4:30 p.m. For details contact Tom Rajnovich at Bluevale C.l., Bluevale Ave. N., Waterloo N2J 3R5 Ph. (519) 885-4620.

PINE RIDGE - Mini conference planned for April 16, 1985 at General Vanier School, Oshawa.

FINAL REPORT-OAME '84 - Elaine Harvey reported. OAME '84 was a success in every way, educational, social, and financial. All who worked so hard to make OAME '84 so memorable are to be commended.

1985 Annual - Sudbury - Mickey Sandblom reported. An outstanding program is planned for the 1985 05 09-11 OAME '85 Conference. 120 Sessions, workshops, etc. are being offered, along with an exciting and varied social program.

1986 Annual - London - Tom Griffiths reported that plans are progressing well for OAME '86 for 1986 05 09-11.

1987 Annual - no site as yet

Final reports of the year's activities were received from all OAME Council Committees.

Council approved the following recommendations regarding Life Membership in OAME/AOEM:

Life Members In OAME/AOEM
Life Membership shall be granted as follows:

1. The Life Membership Committee shall consist of three members, one of whom shall be a past president. The Committee shall be struck by the executive and approved by Council.

2. The name of any candidate of Life Membership shall be:
   (i) Submitted to the Life Membership Committee for consideration,
   (ii) After approval by the Committee, the name and credentials of the candidate shall be submit-

ted to the executive for recommendation to council,

(iii) Upon the recommendation of the Executive, the candidate's name shall be submitted to council to be elected by ballot at the February council meeting,

(iv) Presentation of the Life Membership would occur at the annual meeting in May, in the form of a certificate.

3. To qualify for Life Membership in OAME/AOEM a candidate shall:
(i) Have ten or more accumulated years of membership in OAME/AOEM,
(ii) Have demonstrated outstanding leadership in Mathematics Education locally and provincially, and/or have contributed in a significant way to Mathematics Education in general and in OAME/AOEM in particular.

4. The maximum number that may be elected in any one year shall be two. There need not be a recipient each year.

5. A Life Member shall not be required to pay annual membership fees and shall enjoy all the privileges of a regular OAME/AOEM member.

ANSWERS TO THIS MONTH'S (MARCH) PROBLEMS

1. White - **But, P could** be the North Pole **or** a point on a parallel circle very close to the South Pole.

2. No. To do so would require a minimum of 45 dollars.

3. 1024 pages - **Hint:** a volume of 999 pages needs $9 + 2 \times 90 + 3 \times 900 = 2889$ digits.

4. The total is $367.92, The cost of one turkey would be $5.11 - **Hint:** Use divisibility rules. For a number to be divisible by 72, it must be divisible by both 8 and 9.

PLAY IT AGAIN SAM. A THEME FROM THE TRENCHES

"Money isn't everything!" sayeth the millionaire.
"What's wrong with being bald?" sayeth the pate full of hair".

"Curriculum guidelines are just that, guidelines; What's the big deal?", sayeth the Ministry official.

Do you ever wish that you had a way of instantly trading places with these people so that they could experience your frustrations? Unfortunately what would happen is that you would find yourself sounding like the people you switched with. I know... I've been there. The problem is that both parties are right from their respective positions.

In the eyes of the Ministry, we in the trenches are in the best position to translate guidelines into meaningful classroom activity. It is one of the areas where the Ministry and teachers agree. Where we seem to be polarized is in the fundamental question of where the ministry's responsibilities in curriculum development and implementation end and where our's begin? Certainly the answer is not in the extremes. Our integrity as a profession would be in serious jeopardy if we received a document that lacked flexibility in implementation. On the other hand it would be equally damning to receive a guideline that lacked the foundation of good curriculum development. The answer is somewhere in the middle.

It would seem to me that guidelines should be detailed enough to take the guesswork out of program planning. It should be a detailed "road map." Flexibility is in the implementation leaving the teacher with the task of selecting an appropriate adventure for his/her students. The teacher's responsibility should be mainly that of translating curriculum into meaningful experiences.

During the past few months I, as I know many of you have, have been translating guideline content into courses of study. I find frustrating road blocks. I find that important concept threads are not clearly defined. For example, a fundamental concept underpinning the nations of calculus is the slope of a line! Yet the only time it is recognized in the guidelines is at the grade 10 advanced level geometry (2). If this isolated concept is not threaded through to the O.A.C. level the students may not receive the necessary reinforcement over the formative pre-calculus years. Granted there are a number of places where the guideline hints at reinforcing this idea...relate rates of change to slope of lines; slopes of lines to rates of change.. etc. If this thread is not picked up then the parts will be taught in isolation and the very important concept may be missed.

The Ministry may well counter with, "But that is one of the roles that the school should adopt, hence, we have achieved one of our goals." Nay, I say! It is the Ministry's role to adequately define these threads. If this added burden falls on the shoulders of the teachers then the activity of orchestrating appropriate classroom activities will, by default, end up being less of a priority; a trade-off that, unfortunately, would affect the students. The Ministry might then just as well give me its broad goals and then give me the freedom to create my own curriculum. This, we all know, the Ministry cannot do. The alternative then, is to provide the teacher with a comprehensive curriculum guide that has been given an adequate preparation time and that has been properly field tested.

Rush a curriculum through seems to be the Ministry's new pastime. It breeds frustration and apathy. Proper feedback gets lost in the shuffle of papers, an adequate implementation phase is overlooked and important new developments get relegated to the back burners. One of the new developments I refer to is the issue of computer technology.

We cannot ignore what is impacting society more than any technology of the past. We are in the middle of the computer revolution, not creeping up to it. Classroom practice must begin to reflect this changing environment. We needed a sylabus that **has** made room for the implementation of computer technology in the classroom, not one that hopes teachers will somehow adopt it. Some teachers have tried to juggle content around to try to be current with varying degrees of success. The problem of trying to make room for this new content is like trying to squeeze a pregnant lady into an over crowded elevator. Yes we can squeeze in a bit more, but in so doing the ripple effect reduces the space around everything else.

I sympathize with the guideline committee and their difficult task. Given the parameters of their mandate they have done the impossible. Your problems are not unlike the ones we faced while putting together the 1980 intermediate mathematics guideline. I remember particularly, the pleas from

the audiences during the many meetings saying "Why doesn't the Ministry listen." And do you know that's exactly the way it appears now as I try to peer above the trench line.

We are expected to teach with understanding. To do this adequately it requires more time then is provided.

We are expected to enrich the students' experiences with field trips and practical materials. Love to - give me the time.

I would like to begin implementing the computer as a tool in my class - I need more time. Sounds familiar? It should. It's been voiced enough.

In conclusion I would like to make a proposal for future curriculum development activities.

The Ministry is committed to a cyclic curriculum development process. Why don't we do it right this time.

1. Let's provide an adequate time line for curriculum development - lets start right now.
2. Let's profile what we would like the students to have acquired by the end of the O.A.C.'s.
3. Let's follow a thread of development of these concepts from the O.A.C. right down to the primary grades.
4. Let's include activities in the program that require the computer as an integrated teaching tool.
5. Let's incorporate the knowledge we have acquired about the adolescent stages of development into the curriculum and provide students with what they are mentally capable of handling.
6. Let's create a curriculum that brings the joy of teaching back to the classroom and eliminate the current hectic pace that makes us feel "good grief we've just finished a set of exams and here comes another."

In closing, this article is intended to stir your emotions. You may or may not agree with what I have put to pen. That's great! Then I challenge you. What are your feelings. Are you happy with the current state of the art? If you are give us your insights. If not I would like to hear about your experiences. Above all I would like to get a dialogue going. That's what this section is about in the **Gazette.** Or are you just afraid that someone might challenge your point of view. Any reaction to this article would be greatly appreciated.

★ – WANTED: GAZETTE EDITOR – ★

The term of the current editors of the Ontario Mathematics Gazette will end with the March, **1986** issue. The new editor(s) would begin with the September, **1986** issue. The Publications Committee of OAME wishes to begin the search for a new editor early so that continuity might be assured.

Any person(s) wishing to become editor should write to:
Trevor Thomson, Chairman
OAME Publications Committee

Please include any information which would be relevant in helping the Committee to make its selection.

Applications should be received by May 31, 1985.

DESCARTES MEDALISTS

This year's Inductees: **GORD NICHOLS**
MOE POIRIER

This semi-autonomous society sets high standards in its selection criteria. Inductees must have shown outstanding leadership in mathematics education inside and outside the classroom.

GORD NICHOLS of Preston High School, Cambridge, is recognized for his excellence in teaching; his ability to motivate students to perform consistently above average in the Descartes competitions, and his outstanding contributions on various county and provincial mathematics curriculum committees.

MOE POIRIER of Ecole Secondaire, Garneau, is recognized for his excellence in teaching; his contributions to the development of materials for French language classrooms; his outstanding contributions on various county and provincially-appointed committees which include mathematics and computer-related curricula.

Excellent selections - Well deserved. Nominations for future medalists should be sent to: Ron Dunkley, University of Waterloo.

MEMBERSHIP RENEWAL NOTICE

All 1984/1985 Memberships expire on April 30, 1985

MEMBERSHIP FEES:
(a) One year (to April 30, 1986) $18.00
(b) **Five** years (to April 30, 1990) $80.00
(c) **Canadian Mathematics Teacher** - you may order this annual publication (1986 issue) for an additional $3.00

OAME offers you **FOUR** ways to renew your membership:

1. Include your membership fee with your registration when registering for the annual conference in Sudbury.

2. Purchase you membership **PERSONALLY** at the OAME desk in Sudbury.

3. If you are not attending the conference, please send your cheque to OAME, 247 Bright Street, Sarnia, Ontario N7T 4E9

4. All members who have not renewed by May 14, 1985 will be sent a renewal notice. This is **not** the preferred method due to costs involved. We **hope** everyone will use one of the first **three** methods.

M. Sandblom, Membership Chairman
M. MacGregor, OAME Secretary-Treasurer