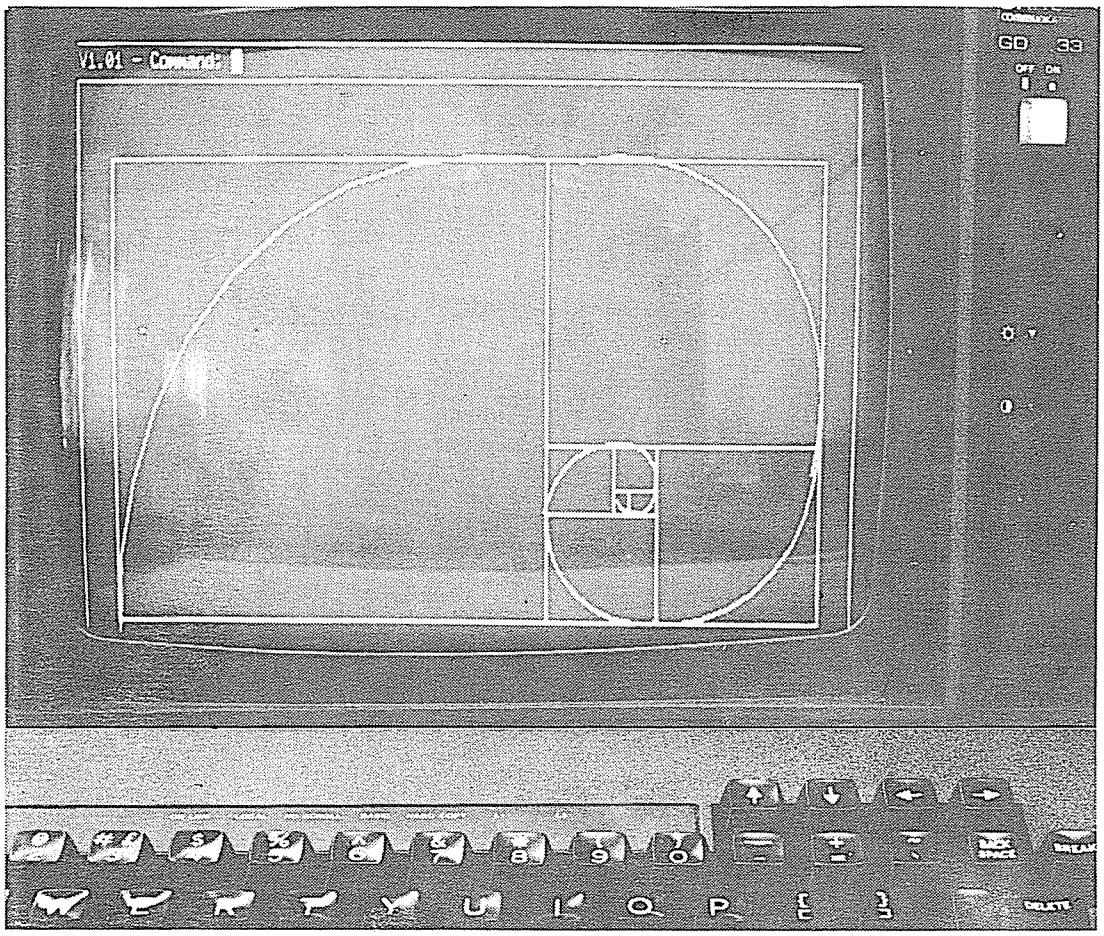
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THE ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION
L'ASSOCIATION ONTARIENNE POUR L'ENSEIGNEMENT DES MATHÉMATIQUES

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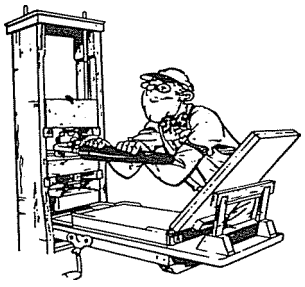
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Computers are a fad. They might make nice Christmas presents, and the games are O.K., but they certainly have no place in our schools.

Calculators should be prohibited from classrooms at all levels. Otherwise, not only will basic computational skills be eroded, but there will be high school graduates who don't know how to use a slide rule and who don't know how to use interpolation in a logarithm table.

Concrete materials should be eliminated. Primary children should be made to start at the abstract level. By eliminating all the "fiddling around" with manipulatives we could not only save Boards thousands of dollars, but we could have students taking Calculus by Grade 7.

Greetings, readers. I was just preparing a draft of a Policy statement I was going to send on to the Ministry of Education. If no-one minds, I'd like to indicate that these statements have the sanction of the membership of OAME. Oh, yes, if you **do** object to any of the above, let me know, otherwise of course I know it's a silent vote of support. What's that? Did I hear someone say (however faintly) that the statements above don't **really** reflect the opinion of the OAME membership? You may be right, but they **could!** All I'd have to do is put it as a set of recommendations from an OAME Committee and ask for a response from the membership. Since there would almost assuredly be none, the recommendation could then take on the strength of an OAME recommendation. Fortunately, this would never happen. Our Councillors, Directors and Council Committees take their job far too seriously to allow such frivolity as I indulged myself in above (aah, the power of an editor!). But it must be indescribably frustrating for them to try to get input from the membership to guide their deliberations and get almost no response. When the Computers in Education Committee surveyed the membership, the response was **very** low - I don't recall the exact number. In September the Publications Committee attempted to survey the membership regarding the **Gazette.** As I write this, **38** responses have been received. I can only assume the other 1000 are "in the mail".

To the members who did respond, a sincere thank you! The **quality** of the responses was superb. Virtually all contained excellent suggestions, comments, etc. to aid both the Publications Committee and the **Gazette** editors. Even though the **Gazette's** "track record" regarding articles of interest to elementary school teachers has not been good, several elementary teachers took the time to respond with not only criticism but ideas and suggestions. Although it's clearly dangerous to try to draw **any** conclusions from so sparse a base, it **appears** that the **Gazette** is fairly well regarded. But the **Gazette** should **not** just be an OAME "magazine" that one receives with membership, it **should** be an OAME "voice"! Where are your views for the "Open Letter" section; your reactions to OAME Committee reports, articles, etc.; your "neat" problems for the Problems Department? Is anyone out there? If so, please feel free to interrrupt me **anytime.** Meanwhile maybe I'll just keep working on that Policy statement.

Ralph Connelly
Sam Pravica
Editors, **OMG**

(note: in fairness to Sam, he didn't see this ahead of time, so don't blame him for its contents).

ORDERING/SERIATION

by
Bonnie Richardson

Editor's Note:

*This article comes from a booklet "**Hand's On**" Mathematics for the **Primary Division**, prepared by Bonnie Richardson and distributed by F.W.T.A.O. This excerpt is reprinted with the author's permission.*

Definition

All systems of numbers are of course based on the operation of seriation insofar as each number is meaningful and determined by its relative position within the sequential system.[16]

As early as one year of age...he can order by size three objects such as bricks if the size differences are easy to recognize, thus solving the problem on a perceptual level. However, he becomes incapable of seriating as the number of objects becomes larger, or as the differences in size become slight.[17]

Objectives

Through experiences in the primary division, the child:
1. will "match, compare, and order lengths, areas, volumes, masses...;"[18]
2. will understand and use the appropriate terms in comparing objects according to length, area, volume and mass.

Learning Outcomes	Teaching Strategies/Learning Activities
Understands the concepts involved in comparing objects according to length, width, height, weight, volume, and area.	- given two sticks, one longer than the other, selects the appropriate one on request: "long", "short" - given two blocks, one larger than the other, selects the appropriate one on request: "large", "small" - given two balls, one heavier than the other, selects the appropriate one on request: "heavy", "light" - given two attribute chips, one thicker then the other, selects the appropriate one on request: "thick", "thin"
Orders a group of objects from shortest to longest (ascending order).	- given a set of objects such as straws, sticks, strips of paper, etc., each approximately 1 cm longer than the preceding one, the child finds the shortest one and puts it first; then finds the one that is a little bit longer and puts it next, and so on to form a 'staircase' - begin with a set of 3, then 4, then 5, then 10 objects, proceeding to the next set after the preceding one has been mastered
Orders a group of objects from longest to shortest (descending order).	- orders a set of objects from shortest to longest, and then reverses the sequence (longest to shortest)

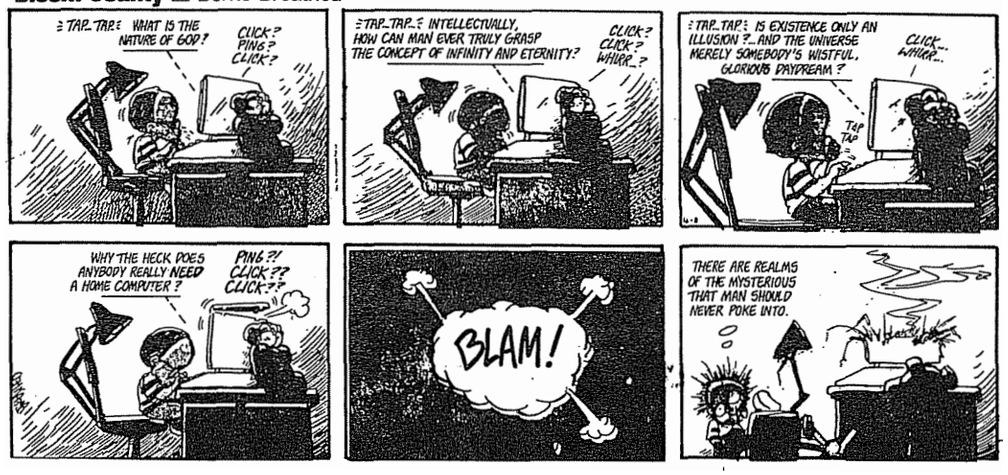
Learning Outcomes	Teaching Strategies/Learning Activities
Orders a group of objects according to dimensions other than length: area, mass (weight) volume (capacity). Reverses the order.	- arranges circles in order of size (area); reverses order - arranges rocks in order of mass (weight); reverses order - arranges balls in order of volume; reverses order
Inserts 'forgotten' objects into a series (interpolation).	- given seven objects in a series, orders them and then inserts the other three objects in their appropriate places in the series
Makes a double seriation: arranges two sets of objects according to two different dimensions side by side.	- given two sets of objects, such as sticks and balls, orders each set, matching the smallest ball to the shortest stick, through to the largest ball and the longest stick

16 Furth and Wachs, **Thinking Goes to School** p. 220.

17 Copeland, **How Children Learn Mathematics** p. 90

18 Ontario Ministry of Education, **The Formative Years,** p.6

Bloom County ☐ Berke Breathed



A man at the check-out counter of a grocery store was complaining that two of the eggs in the carton he just bought were broken. The clerk's only response was, "Think metric."

A teacher was trying to impress her pupils with the fact that terms cannot be subtracted from one another unless they are like terms. "For example," she continued, "we cannot take five apples from six bananas." "Well," countered a pupil, "can't we take five apples from three trees?"

COMPUTING AND ESTIMATING DECIMAL PRODUCTS

by
James Vance

Editor's Note:

In addition to presenting some interesting data regarding pupils' abilities in operations on decimals, Dr. Vance also presents some ***excellent*** *suggestions for teaching estimation skills in the "Implications for Instruction" section.*

Decimal arithmetic is receiving greater emphasis in today's school mathematics curriculum. An understanding of decimal concepts and facility in operating with decimal numbers are of increased importance in our modern society. Calculators and computers input and output numbers in decimal form, and the metric system which is based on multiples and subdivisions of ten uses decimal representation.

The ability to multiply decimals, particularly powers and multiples of ten, is a key element in the successful application of decimal ideas. Results of mathematics assessments in British Columbia (Robitaille, 1981) and the United States (Carpenter and others, 1981) indicate that while students know the multiplication algorithm and the rule for placing the decimal point in a product, they do not use estimation to check that the answer produced is reasonable.

This article reports the results of a study designed to assess the ability of students in Grades 6, 7, and 8 to compute and estimate the product of two decimals (Vance, 1983). Some implications for instruction are also presented.

Procedure

A two-part paper and pencil test on multiplying decimals was administered to ten classes in a lower Vancouver Island middle school. Part 1 of the test consisted of twelve computational exercises presented in horizontal form. Students were instructed to show any written work in the space provided with each question. In seven of the questions one factor was a power of ten or both factors contained only a single non-zero digit. Part II consisted of ten multiple choice items. Seven of these were parallel to the computation questions described above and required only that the proper order of magnitude of the product be identified.

Using a stratified random sampling procedure the test papers of 150 students - 30 in Grade 6 and 60 each in Grades 7 and 8 - were selected for analysis. The Grade 6 and 7 samples consisted of ten boys and ten girls from each of three achievement levels - high, middle and low - as determined by report card grades in mathematics. At the Grade 6 level, several classes had not studied multiplication of decimals at the time of testing, and it was necessary to reduce the sample to five students in each category.

Results

Performance by students in Grades 6, 7 and 8 on three of the computation exercises is given in Table 1. While the Grade 6 students did as well as the Grade 7's and 8's on easier items, the older students scored higher on the more difficult exercises. Overall the students performed reasonably well on the computation section of the test.

Table 1
Performance on Selected Computational Items at Three Grade Levels

Item	Percent Correct		
	Grade 6	Grade 7	Grade 8
0.5 x 24	90	88	85
7.29 x 0.01	57	85	83
0.42 x 0.617	53	62	77

Table 2 summarizes the results obtained on the computation and estimation subtests by students grouped according to grade and achievement level. The data reveal that students did much better on the computation exercises than on the estimation items. As would be expected, performance on both subtests increased with grade level and achievement level.

Table 2
Performance on the Computation and Estimation Subtests by Grade and Achievement Levels

Achievement Level	Percent Correct							
	Grade 6		Grade 7		Grade 8		Total	
	Comp.	Est.	Comp.	Est.	Comp.	Est.	*Comp.	**Est.
High	80	44	91	58	90	61	88	56
Middle	61	25	79	38	88	50	79	40
Low	58	24	67	28	69	39	66	31
Total	67	31	79	41	82	50	78	43

* mean = 9.33; s.d. = 2.38; KR - 20 = 0.704

** mean = 4.25; s.d. = 2.23; KR - 20 = 0.656

Test results by sex and achievement level are given in Table 3. Overall the girls scored slightly higher than the boys in computation while the boys outperformed the girls in estimation. These differences were not significant, but statistically significant ($p < .05$) sex by achievement level interactions were indicated for both subtests. It can be observed that at the low achievement level girls tended to score higher than boys, but at the two higher achievement levels the boys did better than the girls, particularly in estimation.

Table 3
Performance on Computation and Estimation Subtests by Achievement Level and Sex

Achievement Level	Percent Correct			
	Girls		Boys	
	Comp.	Est.	Comp.	Est.
High	87	49	90	64
Middle	77	36	81	44
Low	72	34	60	28
Total	79	40	77	46

As noted earlier, students were much more successful in computing decimal products than in estimating them. For example, while 77% of the total group obtained the correct answer for 0.4 x 0.2, on the related estimation item, 0.378 x 0.24, only 37% chose the alternative with the decimal point correctly positioned relative to the digit 8 (Table 4).

Table 4
Estimation Item No. 5

5.		Percent of Students	
	0.00008	21	
0.378 x 0.24	0.008	20	
	0.08	37	*
	0.8	23	

The performance of students on the seven related pairs of computation and estimation items is provided in Table 5.

Table 5
Performance on Related Computation and Estimation Items

Computation Item	Percent Correct	Estimation Item	Percent Correct
6.14 x 10	87	6.138 x 10.35	86
50 x 600	85	48.92 x 623.8	64
7.29 x 0.01	79	6.825 x 0.012	48
7000 x 0.002	79	7189 x 0.0023	46
0.06 x 0.009	72	0.062 x 0.0089	44
1000 x 0.04	78	998.6 x 0.0428	39
0.4 x 0.2	77	0.378 x 0.24	37
Total	80		52

The estimation item which proved to be the most difficult for the students required them to place the decimal point in the answer to a multiplication exercise (Table 6). The number of students choosing the correct answer was less than what could be expected by chance, while 80% of the students selected the answer with three decimal places. Even though the item was part of the estimation section of the test, students simply applied the rule for placing the decimal point in a product and did not use estimation to check that the answer was reasonable. On the corresponding computation item, 360 x 0.25, 81% of the students obtained the correct answer. An examination of the 28 incorrect answers revealed that the decimal point had been misplaced by only seven of the students.

Table 6
Estimation Item No. 8

8. The decimal point has not yet been placed in the answer to the exercise below.

358.6 x 0.25 = 8 9 6 5

Which shows the correct placement?

	Percent of Students	
0.8965	2	
896.5	6	
8.965	80	
89.65	11	*

The last two estimation items were answered correctly by about only one-quarter of the students (Table 7).

Table 7
Estimation Items No. 9 and No. 10

9. The four exercises have approximately the same answer. One of the exercises has as its exact answer the number below.

2401.5473

Find which exercise has this answer by using estimation to eliminate the other three.

	Percent of Students	
59.77 x 38.59	19	
23.57 x 101.89	29	*
83.47 x 34.69	18	
240.87 x 10.09	33	

Table 7 con't pg. 9

10. Lisa multiplied to find the answers for the four exercises, and then rounded them to the nearest dollar. Looking back on her work she noticed that one of her answers seemed unreasonable. Use estimation to find which answer is wrong.

		Percent of Students	
59 x $205.82	= $12 143	13	
192 x $287.69	= $55 236	32	
85 x $ 75.24	= $ 6 395	31	
801 x $510.95	= $40 927	23	*

Computational Procedures

Analysis of work shown on the computation subtest revealed that students had used a variety of procedures and algorithms for multiplying two numbers. The different computational forms were organized into three main categories. A **short** procedure was assumed to have been used when the answer was written next to the question presented in horizontal format and the question had not been rewritten in a vertical format. Short algorithms for multiplying by a power of ten or finding the product of two multiples of products of ten with single non-zero digits involve "adding zeros" or "moving the decimal point". For example, in 1000 x 0.04, one moves the decimal point in 0.04 three places to the right, or in 4000 ÷ 100, two places to the left. An **intermediate** form was one in which the question had been rewritten in vertical format and an efficient algorithm used to obtain the answer. In algorithms in the **long** category, rows of zeros were shown as partial products or unnecessary zeros in one or both factors were multiplied. The three types of procedures are illustrated for one of the exercises in Table 8.

Table 8
Use of Three Types of Computational Forms of 1000 x 0.04

	Short Form				Total
response	40	40.0	40.00	other	
no. correct	19	5	4	2	30
no. of errors				10	10
	Intermediate Form				**Total**
response	1000 × 0.04 = 40.00	1000 × .04 = 40.00	0.04 × 1000 = 40.00		
no. correct	19	18	1		38
no. of errors	1	4	3		8

Table 8 con't pg. 10

MATH CAMP - A MATHEMATICS ENRICHMENT WEEK

by

Russ Garrett

At the N.C.T.M. meeting in Toronto in 1982, Robert Keller of Fox Lane High School in Bedford New York described a course for gifted students based on the Fibonacci sequence [1]. I decided to modify his idea by setting up a course held for 3 hours per day for five days in June 1984 for interested mathematics students in grades nine to eleven. Twenty three students from schools under the jurisdiction of the Lakehead Board of Education enrolled for this additional week of school. The applicants for the course had mathematics marks above 70. Classes were held at Fort William Collegiate in the computer lab from June 18th to June 22nd.

The goal of the course was to introduce students to the process and pleasure of discovering and proving mathematical results. The vehicle used was the Fibonacci sequence. Time was spent using the computer to explore relationships among the terms of Fibonacci type sequences and students were encouraged to generalize their discoveries. After this inductive approach students were encouraged to formally prove some of their ideas. Because of the diverse levels of mathematical maturity in the group it was assumed that students had little exposure to sequences or to computer programming.

The first day was spent getting the background required to begin a detailed study of the Fibonacci sequence ($F(1) = F(2) = 1$, $F(N) = F(N-1) + F(N-2)$, $n > 2$). Students were exposed to words such as sequence, term, finite, infinite, recursive, and explicit. To give students practice in guessing and expressing patterns in sequences, time was spent looking at the first few terms in some sequences. From these terms they tried to generate possible subsequent terms. Some of the sequences used in this game were:

A, B, C, D,....
1234, 4123, 3412,....
1/2, 2/3, 3/4, 4/5,....
1, 11, 101, 111,....
O, T, T, F,....

Students were introduced to notations for expressing sequences both explicitly and recursively. Students were then given some computer programs that generated sequences. Two of the programs were:

```
10 for i = 1 to 100
20 print 2*i-1;
30 next i
40 end
```

```
10 f1 = 1
20 print f1;
30 for i = 1 to 99
40 f1 = f1 + 2
50 print f1;
60 next i
70 end
```

Students with a great deal of computer experience assisted those who had little knowledge of computers. After some time for computer experimentation students were asked to write programs to generate some new sequences. To provide a break from the computer the morning was completed with an introduction to the Method of Differences. After learning how this method works students were asked to develop explicit formulae for a number of sequences that were generated using polynomials. This first day was the only day where there was an extensive amount of time spent on formal instruction. Students who wished to stay for the afternoon to experiment on their own were allowed to do so.

On the second day the traditional rabbit breeding problem was introduced. Students were asked to produce a table representing the situation for each of the first 12 months. In addition they wrote a program to generate the Fibonacci sequence and other sequences at the same time so that some comparisons of growth could be made. A Lucas sequence ($L(1) = 1, L(2) = 3$, $L(N) = L(N-1) + L(N-2)$, $N > 2$) was introduced and they wrote a program to generate this sequence. They were also asked to investigate the ratio of adjacent terms for the Fibonacci sequence. This value, the Golden Ratio, was discussed in detail which lead to the necessity of introducing and using the quadratic formula. Students were then given free reign to look for patterns in the Fibonacci and Lucas sequences. As

soon as an individual discovered a relationship she/he recorded the idea on a piece of paper. At a convenient time this result was shared with the class. Some examples of student results are presented below in an unedited form.

If you cross out every second number in the Fibonacci sequence starting with the first one, the resulting numbers are the product of Lucas numbers multiplied by Fibonacci numbers. (Kirsten Nelson)

The sum of a Lucas number and a Fibonacci number at the same level always gives you an even number. (Rob Fanti)

If you subtract the Lucas sequence from the Fibonacci sequence you get a number from the Fibonacci sequence but in a negative form. (John Mah)

If you add the Lucas sequence to the Fibonacci sequence you get another sequence that follows a Fibonacci pattern. (Geoff Batstone)

The second day did not provide a lot of time for individual work, however their curiosity was aroused.

On the third day other problems such as the crawling bee and light reflection from Martin Gardner's Mathematical Games in Scientific American[2] were investigated. Students were given lots of time for discovery and the results were exciting. Some examples are presented below in unedited form.

F(N)*F(N+2) is either 1 more or 1 less than F(N+1)*F(N+1). (Leigh Berrenger)

F(X)*F(X) + F(X-1)*F(X-1) = F(2*X-1). (Brooks Rapley)

When you add the rows of Pascal's triangle diagonally you create a Fibonacci sequence. (Anne Scofich)

2*F(N) = L(N+1) - F(N+1). (Tom Moffat)

L(N) - F(N+1) = F(N-1). (Rob Karachuk)

If N is divisible by 5 so is F(N). (Tim Lunn)

Every third Fibonacci number is even. (Susan Mayes)

F(N) * L(N) = F(2*N). (James Geils)

On the fourth day more discoveries were made and the idea of proof was introduced. Arithmetic and geometric sequences were discussed and the traditional sum formulae were developed for the corresponding series. This was used as an introduction to a proof for one of our discoveries that the sum of the first N Fibonacci numbers equals the F(N+2) - 1. A general discussion of sequences and series developed and we looked at some practical applications of these concepts. The process of mathematical induction was introduced and it was used to prove one of our results.

On the final day copies of all our results were distributed and students worked individually on proofs for their favourite discoveries. Also we began a search for an explicit formula for the Fibonacci sequence. The only hint I gave was that it had something to do with the roots (r and s) of the equation x*x - x - 1 = 0 which we had looked at when studying the golden ratio. Three students independently discovered the Fibonacci formula. This is the result expressed by one individual.

To generate a generalized Fibonacci sequence with first terms p and q use the formula:

$$\frac{p(r^n - s^n)}{\sqrt{5}} + \frac{(q-p)(r^{n-1} - s^{n-1})}{\sqrt{5}}$$

(Carl de Boer)

CREATIVE THINKING TECHNIQUES

by

Gary Flewelling

In the December '82 issue of this journal Don Woods defined creativity as, "the ability to think up alternatives." This article presents and then describes, with an example, a seven step recipe for generating these alternatives, that is, a seven step recipe for creativity.

A RECIPE FOR CREATING SOMETHING

Step 1. **List** the **attributes** of the seed crystal.

Step 2. Identify attributes amenable to **reversal(s).** Carry out reversals on these attributes noting those that hold promise.

Step 3. Identify attributes amenable to **extension(s).** Carry out extensions on these attributes noting those that hold promise.

Step 4. Identify attributes amenable to **disguising.** Carry out disguises on these attributes noting those that hold promise.

Step 5. **Combine** selected **attributes** (or their transformations). Stop when you have something that is pleasing, novel, useful, important, or interesting.

Step 6. Put creation into **deliverable** form.

Step 7. Cycle back through the above steps **looking for** other promising **variants.**

A CLOSER LOOK AT THE RECIPE

Step 1 - Describing the Seed Crystal

Before we can consider alternatives, we must first consider that which we would wish to alter. I say this in the belief that creations do not spring forth out of a vacuum or, as Julie Andrews sang in the Sound of Music, "Nothing comes from nothing, nothing ever could." I will call the thing which we wish to alter the 'seed crystal'. I like using this term because it conjures up the idea of something upon which a new idea, solution, alternative or creation might grow. This seed crystal can be anything that sparks our interest, grabs our attention, piques our curiosity, demands attention or solution, excites our imagination and so on. It could be a theory, fact, puzzle, game, piece of art work, process, mechanism, etc.

One way to understand or come to grips with the seed crystal is to list its properties or attributes. Some say, list its 'critical' attributes, that is, those things that really makes it tick. The problem with this is that you often can't be sure of the significance of each attribute listed. This is especially true when you consider that alternatives are going to contain transformations of some of the seed crystal's attributes (see steps 2, 3 and 4 of the recipe). Often, some seemingly insignificant attribute can, when transformed, have a profound influence on the generating of an alternative. The best bet is to list as many attributes as possible and worry about their significance later.

This first step in the recipe puts a premium on observation skills, knowledge, experience, and an ability to analyze. If new ideas or creations are going to spring out of new combinations of transformations of these attributes, it is important that the seed crystal's attribute list be set down as comprehensively and insightfully as possible.

Steps 2, 3, and 4 of the recipe transform attributes and in so doing provide "a different arrangement of information which will provoke a different way of looking at a situation" (de Bono, **Teaching Thinking**), and give us the building blocks for creating alternatives.

Step 2 - Transform Attributes by Reversal

Some attributes of a seed crystal are usually amenable to reversal. For example, the game of Checkers is a game in which you "play to win your opponent's pieces". An alternative game results

when this attribute is transformed to read a game in which you "play to lose your pieces".

Below are examples of a variety of reversals.

Reversing **Order**

before ⟷ after
first ⟷ last
A,B,C ⟷ C,B,A
A,B,C,D ⟷ A,C,B,D

Reverse **Direction**

right ⟷ left
up ⟷ down
in ⟷ out
forward ⟷ backward

Reverse **Function**

walls support building ⟷ building supports walls
tap turns on water ⟷ water turns on tap
A causes B ⟷ B causes A

Reverse **Action**

take ⟷ give
cover ⟷ uncover
combine ⟷ separate
slide ⟷ rotate

Reverse by **Negation**

yes ⟷ no
is ⟷ isn't
true ⟷ false
finite ⟷ infinite

Step 3 - Transforming Attributes by Extension

Some attributes of a seed crystal are usually amenable to extension. By extension, I mean an increase or decrease in number or dimension. For example, in the game of Checkers, the attribute, "a game played on a two dimensional board" could by extension be transformed to read, "a game played on a three dimensional board". (This hints at an alternative game played on several layers.) Another attribute, "a game played on sixty-four squares" could be extended to read, "a game played on thirty hexagons".

Below are examples chosen from a limitless pool of extensions.

more ⟷ less
n-gon ⟷ p-gon
square ⟷ cube

polygon ⟷ polyhedron
surface ⟷ edge
line ⟷ point

Step 4 - Transforming Attributes by Disguising

Some attributes of a seed crystal can usually be disguised. By disguise, I mean carrying out some form of replacement, trade or translation. For example, Lincoln's Gettysburg address could be transformed by turning each letter into a corresponding number and then, by modular arithmetic, each number could be transformed into its corresponding musical note. This would generate a musical equivalent/disguise of Lincoln's address.

Below are examples of disguises.

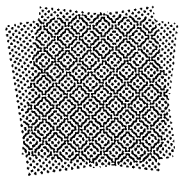
number ⟷ other numbers
number ⟷ colour
number ⟷ letter
verbal ⟷ symbolic
real ⟷ mathematical model

Step 5 - Combining Attributes

In this step, we carry out a synthesis by bringing together some of the original seed crystal's attributes (or their transformations). Not all of the seed crystal's attributes need be used in the

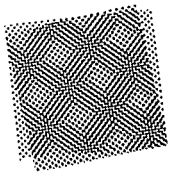
Alternative #3

Extend attribute #6 of original seed crystal to read, "a lattice of points" and combine with the remainder of original attributes.



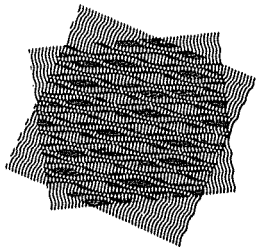
Alternative #4

Combine attributes of alternative #3 with its first attribute reversed to read, "two similar pictures".



Alternative #5

Combine original attributes with attribute #6 transformed to read, "a family of parallel sinusoids".



Alternative #6

Combine attributes of any alternative above with attribute #1 transformed to read, "one picture only". Here the transparency is held before a reflective bendable sheet of mylar. The reflection provides the second picture and interference patterns again arise.

You can cycle back again and again bringing colour, more complex geometric patterns, playing with 3D, progressively changing widths of lines and/or spaces between them, getting into the area of camera filters and so on.

SUMMARY

This recipe of describe, transform, combine, deliver and do it again is simplistic and mechanistic. It does, however, work. I have seen it used to effect in such diverse areas as art, music, writing, LOGO, puzzle/game design and mathematics (particularly problem solving). In mathematics, for example, Polya's problem solving technique of relaxing a condition or solving a simpler problem is related to transforming by extension. The drawing of diagrams or flow charts, when problem solving, is related to transforming by disguising.

Creativity is an art but it is not black magic. The creation might be magical but the process leading to the creation is often quite deliberate. This recipe has been presented in an attempt to make the creative process a more deliberate one.

Gary Flewelling, Mathematics Consultant
Wellington County Board of Education

AN UNUSUAL METHOD TO FIND $\sqrt[p]{n}$

by

Garnet J. Greenbury

To find the square root of 2 start with any ratio $\frac{a}{b}$ and change to

$\frac{a + 2b}{a + b}$ Repeat this procedure as long as desired.

$\frac{1}{2}$ becomes $\frac{1 + 2(2)}{1 + 2} = \frac{5}{3} = 1.6666666$

$\frac{5}{3}$ becomes $\frac{11}{8} = 1.375$

$\frac{11}{8}$ becomes $\frac{27}{19} = 1.4210526$

$\frac{27}{19}$ becomes $\frac{65}{46} = 1.4130435$

$\frac{65}{46}$ becomes $\frac{157}{111} = 1.4144144$

$\frac{157}{111}$ becomes $\frac{379}{268} = 1.4141791$

$\frac{379}{268}$ becomes $\frac{915}{647}$ © 1.4142195

$\frac{915}{647}$ becomes $\frac{2209}{1562} = 1.4142126$

The correct answer is 1.4142135

To find the square root of 7,

$\frac{a}{b}$ becomes $\frac{a + 7b}{a + b}$

$\frac{1}{2}$ becomes $\frac{1 + 7(2)}{1 + 2} = \frac{15}{3} = 5$

$\frac{15}{3}$ becomes $\frac{36}{18} = 2$

$\frac{36}{18}$ becomes $\frac{162}{54} = 3$

$\frac{162}{54}$ becomes $\frac{530}{216} = 2.4537037$

SOLVING MAX/MIN PROBLEMS USING IMPLICIT DIFFERENTIATION

by
Norman M. Rice

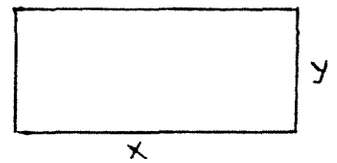
Here is a calculus method for solving max/min problems that is a little different from the usual method. As well as often being easier and more natural, it also provides another opportunity for pointing out to students the usefulness and importance of implicit differentiation. (They will love you for that!)

The usual method for solving max/min problems is to express the quantity S to be maximized or minimized as a function of one variable x, and then check the critical points where $\frac{dS}{dx}=0$. Very often, however, the quantity S is given most naturally as a function of two variables x, y, and there is some constraint relating x and y. This constraint is solved for y in terms of x, and then used to eliminate y from the expression for S.

Students need to become thoroughly familiar with this standard procedure, but once they are comfortable with it, it is worth pointing out that alternative procedures can sometimes be advantageous. In particular, it may often be advantageous **not** to explicitly solve the constraint equation and eliminate variables before differentiating. Rather, a given problem may be easier to solve, and the solution may appear in a better form, if we differentiate **before** eliminating variables.

Here is how this works in a couple of familiar examples.

Example 1 A farmer with 100m. of fencing wishes to make a rectangular enclosure of maximum area A. What dimensions x, y, should be chosen?



Solution Quantity to be maximized: $A = xy$ (1)
Constraint on x and y: $2x + 2y = 100$ (2)

Differentiate (2) and (1) (implicitly, thinking of y as a function of x):

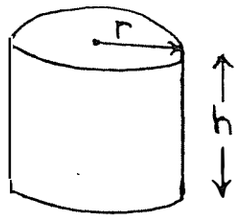
from (2): $2 + 2\frac{dy}{dx} = 0$; so $\frac{dy}{dx} = -1$

from (1): $\frac{dA}{dx} = x\frac{dy}{dx} + y$ (3)

$= x(-1) + y$ (from (3))
$= y - x$
$= 0$ when $x = y$

Thus the field has a maximum area when it is square. This is the information we really want. To find the actual dimensions, substitute $x = y$ into the constraint equation (2) to get $x = y = 25$. The method involves no less calculation here than the usual method, but the fact that the optimal shape is a square comes in a much more natural way.

Example 2 What dimensions should a tin can have if its volume is to be some fixed constant V_0 and its surface area S is to be as small as possible?



Solution Quantity to be minimized: $S = 2(\pi r^2) + (2\pi r)h$ (1)
Constraint on r and h: $V_0 = \pi r^2 h$ (2)

Differentiate (2) and (1) impicitly with respect to r:

from (2): $0 = \pi r^2 \frac{dh}{dr} + 2\pi rh$; so $\frac{dh}{dr} = -2h/r$ (3)

from (1): $\frac{dS}{dr} = 4\pi r + 2\pi r \frac{dh}{dr} + 2\pi h$

$= 4\pi r + 2\pi r(-2h/r) + 2\pi h$ (from (3))

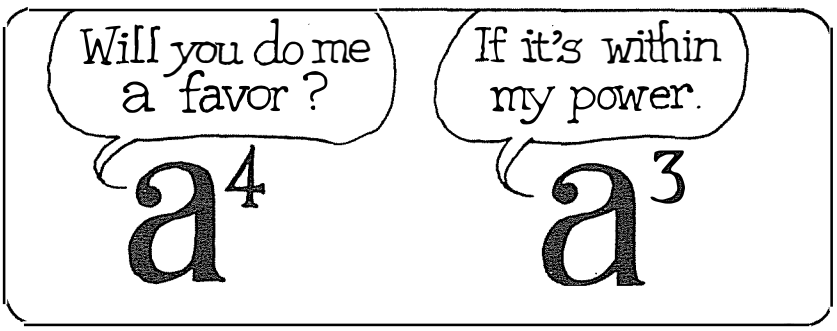
$= 4\pi r - 4\pi h + 2\pi h = 2\pi(2r-h)$

$= 0$ when $h = 2r$.

Thus the optimal shape is to have the height equal to the diameter (twice the radius). Again, this is the information we really want, and it comes with much simpler calculation than the usual method. To find the actual dimensions, substitue $h = 2r$ into the constraint equation (2) to get $r = (V_0/2\pi)^{1/3}$ and $h = 2(V_0/2\pi)^{1/3}$.

Not every max/min problem is appropriate for this method, but when the problem is really asking for an optimal relationship between variables (as in the area and volume examples above), it is well worth trying. Try it on open tin cans, semi-closed fields, fields with fences down the middle, rectangles enclosed in semi-circles, etc.

Norman Rice
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Jeffrey Hall
Queen's University
Kingston, Ontario



Questions For 25th International Mathematical Olympiad

1. Prove that

$$0 \leq yz + zx + xy - 2xyz \leq \frac{7}{27}$$

where x, y, z are non-negative real numbers for which $x + y + z = 1$.

2. Find one pair of positive integers a,b such that:
(1) ab (a + b) is not divisible by 7,

(2) $(a + b)^7 - a^7 - b^7$ is divisible by 7^7.
Justify your answer.

3. In the plane two different points 0,A are given. For each point X of the plane, other than 0, denote by a(X) the measure of the angle between 0A and 0X in radians, counterclockwise from 0A (0 a(X) 2π). Let C(X) be the circle with centre 0 and radius of length $0X + \frac{a(X)}{0X}$. Each point of the plane is colored by one of a finite number of colors. Prove that there exists a point Y for which a(Y) 0 such that its color appears on the circumference of the circle C(Y).

4. Let ABCD be a convex quadrilateral such that the line CD is a tangent to the circle on AB as diameter. Prove that the line AB is a tangent to the circle on CD as diameter if and only if the lines BC and AD are parallel.

5. Let d be the sum of the lengths of all the diagonals of a plane convex polygon with n vertices (n 3), and let p be its perimeter
Prove that

$$n - 3 < \frac{2d}{p} \left[\frac{n}{2}\right] \left[\frac{n+1}{2}\right] - 2.$$

($[x]$ denotes the greatest integer not exceeding x.)

6. Let a, b, c, d be odd integers such that $0 < a < b < c < d$ and $ad = bc$.
Prove that if $a + d = 2^k$, $b + c = 2^m$ for some integers k and m, then $a = 1$.

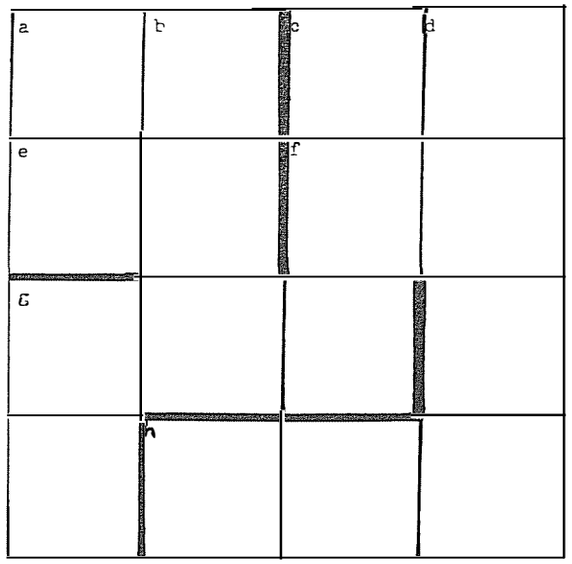
Ed Barbeau
Math Department
University of Toronto

CROSS-FIGURE PUZZLE 8

by
Gail Hess

Editor's Note:

In the March, 1984 ***Gazette*** *a short article appeared explaining cross-figure puzzles and how they work. The puzzles were continued in the Sept. 1984 Gazette. This issue contains the remaining cross-figure puzzles and their solutions.*



ACROSS

a a palindromic number

e square of a prime

f (e across) - 2

g a square number which is also a square number with its digits reversed

h (c across) x 7

DOWN

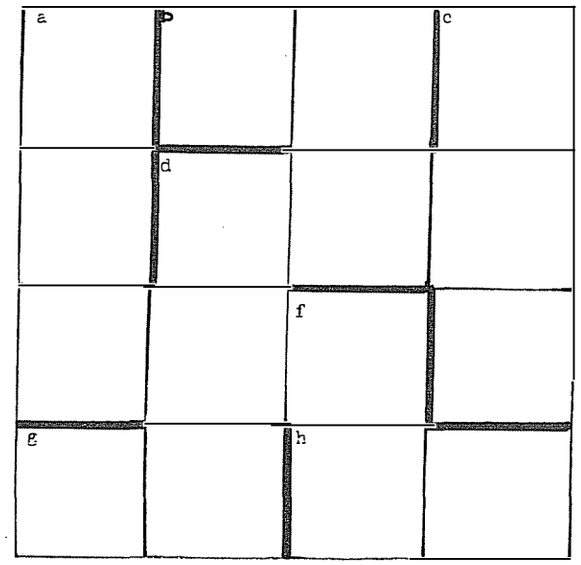
b (a down)^2

d (a down)^3

g square root of (g across)

CROSS-FIGURE PUZZLE 9

by
Chris Lymberopoulos



ACROSS

b square of a prime

d (b across) x 8

g multiple of 14, and is 99 less than (d down)

h (c down) ÷ 4

DOWN

a digital total of ten, and is also the square of a prime

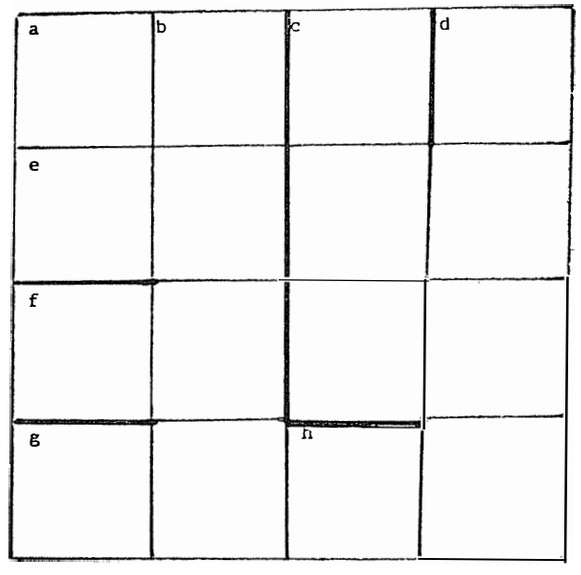
c half of (d across)

d (g across)^2

f twice a prime, and if reversed, is also twice a prime

CROSS-FIGURE PUZZLE 10

by
Diana Paul



ACROSS

a (a down)

c ($\sqrt{}$ e across)

f (c down: 1st digit + 2nd digit + 3rd digit x 8)

g A prime and also prime when digits are reversed. Its digital total is (f across) ÷ 22

h (f across) ÷ 4

DOWN

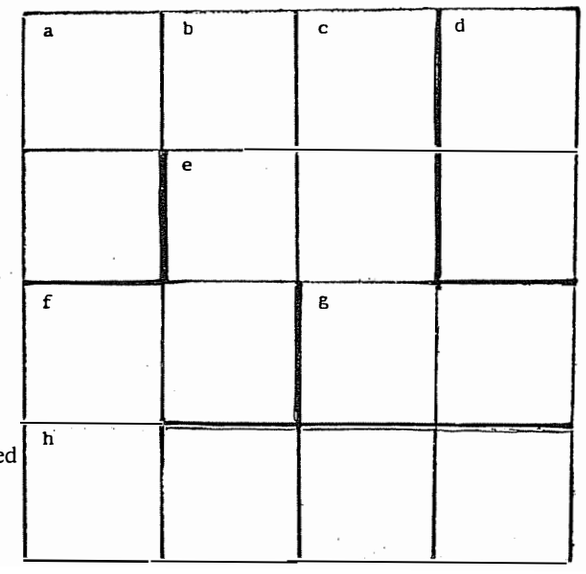
b (a down)^2

c (g across) x 14

d (b down) x 2

CROSS-FIGURE PUZZLE 11

by
Vicki Pierce



ACROSS

a (a down)^2

f (a down) x 8

g (d down) x 2

h (d down) x 50

DOWN

b (c down) x 2

c a square number, which is also a square number with its digits reversed

d (e across) ÷ 4

f fourth power of a prime

CROSS-FIGURE PUZZLE 12

by
Penny Smith

ACROSS

a (h down)^2

c (h down) + (h down)

e (a across) + (h down)

f same as (a across)

g same as (b down)

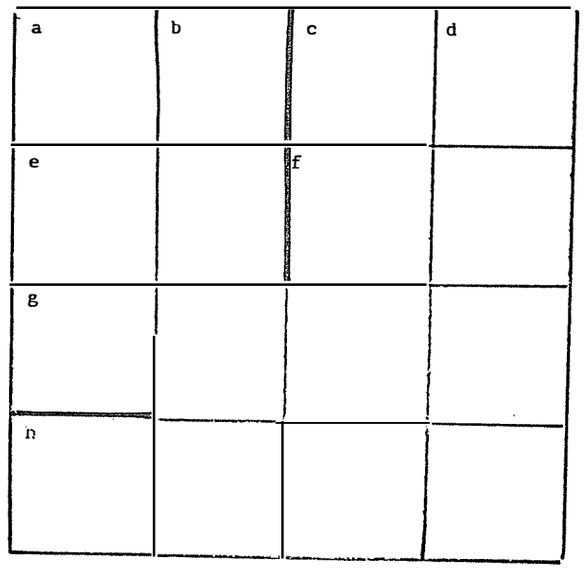
DOWN

a (a across) x 11

b (h across) ÷ (h down)

c (e across) x 20

d (e across) x (e across)



CROSS-FIGURE PUZZLE 13

by
Sandie Soderquist

ACROSS

a (b across)^2

c (b across) + 6

f (g across) + (h across)

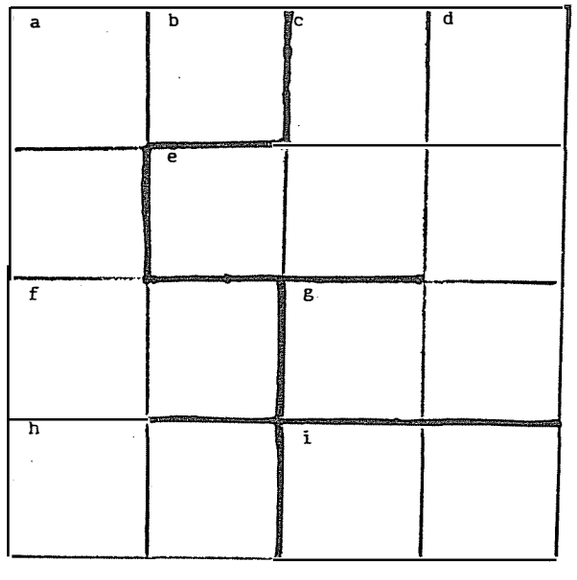
g (c across) x 4

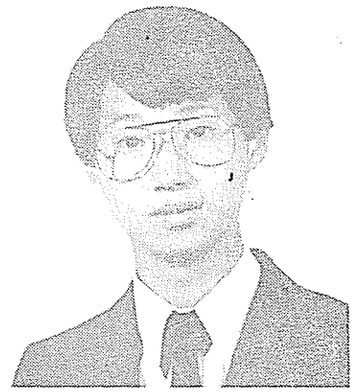
h (c across) + (g across)

DOWN

a (c down) x (e across)

d (f across) + (i across)





THE JOHN G. MCCARTNEY SCHOLARSHIP

The John G. McCartney Scholarship is awarded annually by the Ontario Association for Mathematics Education in conjunction with the Science Research Associates (Canada) Limited. John McCartney died in 1977 after a short but distinguished career in the field of Mathematics.

This scholarship is administered by Morley & Mona MacGregor and in the year 1984 was judged by Paul Pogue of Barrie, and Douglas Henderson of Waterloo.

On August 24, we had the pleasure of presenting this $400 scholarship to Alexander James Pak, Scarborough. Alexander Pak is the son of Mr. & Mrs. Ron Pak. When he won this scholarship he was 15 years of age. He has completed his grade XIII education at Woburn Collegiate. Alexander was born in Seoul, Korea and moved with his family to Canada at the age of four months. His entire education was in the Scarborough schools. In grade 4 he entered the Scarborough Board's gifted program. His father is a supervisor for a variety store chain. Writing tests and achieving high marks is not new to Alex. Among the tests he has scored high marks are in, Junior Math Contest, Fermat Contest, Euclid test, Mathematics Association of America High School Mathematics Examination, and Korean University Students' Association Sr. Math Contest. This is just to mention but a few.

Alex told us that his parents believed that no student should ever be idle. He rises early in the morning and has trained himself to be an accomplished pianist. He enjoys physical activity as long as it is not competitive. One of his chief relaxations is playing Structured Games.

In 1983 he spent his summer at Waterloo University. In 1984 he attended Ohio State University studying Pure Mathematics - Number Theory & Mathematics of Choice. The week after he received this scholarship he was leaving to attend Harvard University. Alex knows that he will be one of the youngest students there. He intends to study in the field of Mathematics and all fields associated with Mathematics. His ultimate goal at the present time is to become a professor at a renowned institution.

We know that all readers of the **Gazette** wish Alex the very best.

Morley & Mona MacGregor



INTERVIEW WITH MARGARET BROWN

by
Robert S. Smith

Faculty of Education, The University of Western Ontario

RS: Margaret Brown was the chief researcher for Concepts in Secondary Mathematics and Science in 1974 when it began. I met her in 1975-76 at the Lillie Road Building, London, England where CSMS had quarters. Now her headquarters are in a disused school adjacent to Chelsea College. Margaret, what is your connection with Chelsea?

MB: Well, I'm actually employed by Chelsea College, I'm paid by them which is the important thing. I'm Senior Lecturer in Maths Education at Chelsea at the Centre for Science and Mathematics Education. At the moment I'm paid officially for 10% of my time by Chelsea and I'm still on the staff there but the other 90% of my time is funded by the Nuffield Foundation to work on the Graded Assessment In Mathematics Project (GAIM).

RS: Margaret, what is GAIM's aim?

MB: First of all I think it is perhaps worthwhile saying "Whose project is it?" because that tells us a bit about the aim of it as well. It's a joint project involving two examination boards: the University of London GLE Board and the London Regional CSE Board, so that straddles most of the ability range. The Boards are cooperating with the Inner London Educational Authority (ILEA) and with Chelsea College, with the Nuffield Foundation putting in some of the funds. (ILEA are also putting in funds in the form of seconded teachers.) The aim of the project is to produce a scheme and materials for graded assessment in mathematics. This will run hopefully across the age range from 11 to 16, across the whole ability range as well, in other words, from the weakest 11 year old to the brightest 16 year old. The aim is to produce a mathematical structure in terms of levels so children will progress through these levels at their own rate. It will be able to provide not only diagnostic information for the teacher in terms of what children should be taught but also information to the parents and to any eventual employers as well as to the children themselves.

RS: The funding I understand has been for two years initially, 1984-86.

MB: It's three years initially from Nuffield. Nuffield actually only funds two people, myself and one research fellow. The others are seconded teachers - four from the Inner London Educational Authority. In fact, we also have two teachers from other Authorities now being seconded to the project so it's wider than just one, single, educational authority. We hope to run a five year project but the Nuffield Foundation have only agreed to the final two years providing that an evaluation committee finds it satisfactory after the initial three year period.

RS: Since the Local Authorities are involved what is it they hope to get out of it?

MB: First, they hope to get some sort of motivation for children in mathematics classes. This problem is more noticeable at the lower ability end in the secondary schools. At present, the only aims of our mathematics curriculum appear to be an examination at the end of the fifth year. It's very difficult to sustain children's motivation when this is such a long way off. In many cases the children, unless they're in the top 60%, are unlikely to get any sort of grade out of it. The aim for GAIM is to try and recognize what children can do at any particular time and so give them a target which they can see an end to, one which they can hopefully achieve.

RS: Since the Authorities are seconding teachers to the project it is hoped that these teachers will be able to work with their colleagues to implant your work in the Authorities when they re-enter them.

MB: That's true. I think when I said the aim was to provide short term goals, that was one of the aims. Another is to actually change the sort of mathematics that goes on in the classroom. This has been tried many times but teachers have always complained that the exam at the end of the fifth year inhibited any real change of mathematical activity. Therefore, by changing the type of assessment, we would also be hoping to enlarge the type of mathematical activity that goes on in ordinary classrooms.

RS: That brings us to the interesting part. It appears you are going to look at assessment in a much broader way than simply the pencil and paper testing. What are some of these other ways you are thinking about?

MB: Well, at the moment we are not really writing anything out, we are trying to think of all the different ways we could assess children and we are trying to keep as many of those going as we can. We think, particularly at the lower achievement levels, it's important to assess what children say, as well as what they write. It's also important to assess group work as well as individual work. It's important to assess not just short items or questions, in fact, maybe not that at all, but to assess larger pieces of work. This also perhaps, harks back to motivation because at present it appears that children are doing far too many repetitive exercises and examples which have no particular purpose, to the children. The only purpose to the teacher is probably to rehearse for an examination. One of our aims is to involve children in longer term project work and in investigation work although we probably will also have some single skill assessments.

A POSSIBLE STRUCTURE FOR GAIM ASSESSMENT METHODS

		Mental Tasks	Specific Practical Tasks	Practical Problem Solving	Investigations	Projects
CONTENT	Logic					
	Measurement					
	Number					
	Space					
	Statistics					

An alternative would be some sort of profiling system so that if teachers want to do all project work, then they could actually decree that children had exhibited the skills tested by the mental or specific practical tasks in the course of their project work.

RS: Sounds very interesting. I'm wondering if we could go back to look at some specifics. If you are going to assess what children say as well as what they write, this suggests an interview format and out of that has to come some kind of checklist to arrive at what might more likely be thought of as an assessment, mark, grade, or anecdotal report. What is your thinking here?

MB: Well, we are not aiming at any kind of mark, that's the first point. We are actually aiming at mastery of a particular set of content and process skills at a particular level, so we are not interested in a kind of marking scheme. We are only interested in the criteria for satisfactory attainment of a skill of some sort. To go back to the oral assessment, I think we have thought very hard about interviewing children and I think we have decided not to build this in as one of our five types of assessment. This is largely from the feedback we've had from teachers which has suggested that they might be able to do it for one class but they couldn't do it for all the classes they teach. We've ruled it out as essential, we haven't ruled it out as an option. In fact, if some children perhaps find it difficult to communicate on paper, we would hope that teacher would actually employ the one-one interview and talk to those children individually. On the profile form might be, "Can interpret a pie chart". Now, there are various ways of doing that. We would hope to provide some mental assessment perhaps that would relate to interpreting pie charts. We'd hope to provide ideas for project work which would involve children making up their own pie charts and interpreting them. And we would also perhaps expect, in some cases, that teachers might prefer to do this by talking to a particular child about a particular task. Certainly, the project work would allow the teacher to observe what general thinking processes are used in solving mathematical problems.

RS: Give me an example of the kind of question you could put before a child in an attempt to assess how far he is along in the general thinking process?

MB: I should say that perhaps we are working at both ends on this project. We are trying to work from the theoretical end of trying to decide what the framework should be and we are also trying to work at the classroom end. We've been trying out a lot of ideas that we felt were basically good things to do in classrooms and we are trying to marry them in the middle somewhere. We haven't got the final answers to any of these things yet. But let me take an example on the process side. In how many ways can you shade three eights of a 2x4 rectangle drawn on squared paper? Almost all the children who tackled this were able to understand the question. There were one or two children who appeared to perhaps have understood the question for a while and then went off and did something different which wasn't related to it. There were only one or two children who hadn't really understood what the task was about. So that bit of the problem solving process was generally satisfactorily completed. But being systematic was at very different levels. There were some children who just shaded their rectangle in random ways whilst there were others who found a particular way of shading it in, perhaps to shade three squares in a row, then rotated them, and translated them and tried them out in different ways. Some low ability children later on thought of ways of dividing the squares in halves. At a higher cognitive level there were children who hadn't even shaded them in. They'd just said to themselves 'I can think about this in my head'. It was in mixed ability groups where there were children who were able to think about it as a theoretical problem and come to some solution that way. But even in the very lowest ability levels there were different degrees of being systematic and that's the sort of process that we think we will probably be assessing but we're not clear yet as to whether that builds into one of our content frameworks. We're thinking at the moment that it may actually build into what we call the logic framework. We still haven't settled some very basic questions.

RS: Interesting. I visited Graham Ruddock yesterday at NFER and in their APU (Assessment Performance Units) one of the items that they've looked at are attitudes. Are you thinking of doing anything in this area at all?

MB: That's a difficult question. I don't think we will be in the terms of mathematical attainment. I don't think we are going to have different levels of attitudes. I'm sure we're not going to say to a child, in order to obtain a level 1 mathematics you must actually enjoy mathematics. However, the whole of the graded assessment movement in the London Authority is part of a wider movement which is alternative methods of assessment. Certainly as part of a record of personal achievement one might well be thinking about some sort of assessment by the teacher or, probably even better, self assessment by the child of what their attitude is. That would certainly be part of some sort of record but we wouldn't expect it to be a part of a graded assessment.

RS: But you might well hope that attitude changes would come about. Has some thought been given in terms of your scheme to some assessment of the calculator and the child's ability to use it in a constructive way?

MB: Yes, well, I suppose the calculators we regard as so much as integral part of mathematics we haven't really thought about it except as part of our number framework. It will be listed in the number framework alongside other skills like mental calculations, estimations, and so on.

RS: Basically, then, this project is different to the CSMS project. I would call it more a development project than perhaps a research project. Am I correct there?

MB: Yes, that's right. I mean, we actually have a deadline for materials to be available in classrooms. We call ourselves a 'research-informed development project'. We are, I think, producing research evidence at the same time. For instance, we're looking at what kinds of mathematical skills will be in each of our levels, our grades. CSMS actually produced some evidence on this. But the picture isn't complete, so we will be producing, if you like, research as part of the development.

RS: Do you feel that your project has been a direct result of the Cockcroft Committee and its feeling that mathematics was failing the student body and, indeed, not giving a large number of them any chance to even be tested or write any exams?

MB: Yes, I think the project is an outcome of that because the money is dependent on Cockcroft. I think the Cockcroft Committee report was very helpful in mathematics education in bringing these

problems to the awareness of politicians and funding agencies. I think everyone in maths education was aware of these problems before Cockcroft and certainly one of our reasons for doing the original CSMS project was a feeling that this was so.

RS: Margaret, the graded nature of assessment is an idea whose time has come. Would you say a little bit more about what you mean by graded assessment?

MB: I think it really stems out of a desire to know what children have attained and therefore it relates to the criterion referenced assessment movement and this has obviously come not only in mathematics but in other subjects. We have the idea that children are building up some sort of record of what they have achieved. In some subjects more than in others the nature of this attainment is to some extent, but certainly not completely, hierarchical and that therefore you can actually arrange these skills and contexts into levels through which children move. This provides a framework for the curriculum as well as for the assessment. It doesn't necessarily mean that the child has to complete the whole of one grade before they move on to the next one. We hope that it would mean that children could be working at different levels in different mathematical aspects. For instance, a child might be doing grade 3 work in shape and grade 1 work in number, so that it shouldn't, I hope, be seen as a complete straitjacket for the curriculum. But it does mean that there will be much more system to the sort of curriculum that children meet. Many teachers state to me that they are very much aware that they don't actually know where children are in mathematical attainment. Teachers decide what they are going to teach, they teach it, and they try and check immediately whether children have got hold of that idea, but they never really check it at any later stage as to whether it has been retained or understood. If someone were to walk in and say "Take this particular child, or this particular class, what mathematics do they know", then most teachers would be really hardput to be sure of what any particular child knew. I think we are trying to provide a means of having this sort of information available.

RS: Even when teachers feel they have a fairly good knowledge of the child's understandings, they've been very much surprised that researchers have often found that concepts are not quite understood.

MB: Yes, that's right.

RS: Well, I'm sure then this has brought up a number of difficulties in terms of assessing and I wonder if you could speak a bit about those.

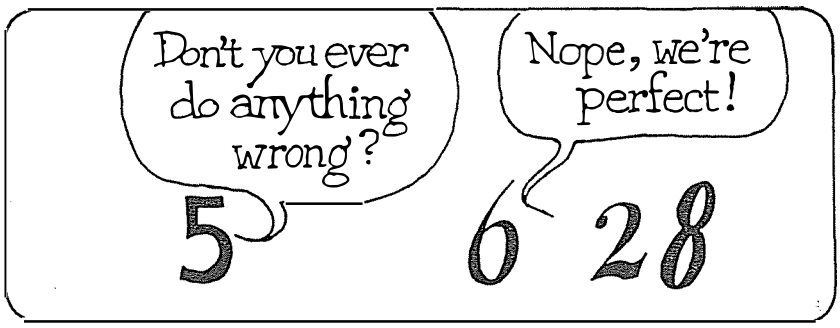
MB: There are difficulties really all the way through because, essentially, it has enormous implications for the way mathematics is taught right through the secondary school. The first point is that if we are going to take seriously the outcomes of such graded assessments, then it almost certainly follows that there will have to be some sort of individualization because if you actually find out that your particular child is on grade 1 number and grade 4 space and the child next to it is on grade 3 number and grade 1 space then it's very difficult to justify continuing to give both the same work at any particular time. This implies that it would be very difficult to continue giving the whole class the same sort of treatment all the time. It doesn't necessarily mean a stop to whole class activity. This is where some of our investigations and projects come in. We hope that it shall be possible to set a class certain tasks which they can then go about doing in the way they feel able to do them. We have to find some way of assessing these tasks to say what level children have reached on them. So we hope that we are not altogether going to an individualized method of working. The second difficulty, I think, is really in introducing a much wider range of classroom activities instead of just teaching the way to do a particular type of question and then practising doing it. We are suggesting opening up into project work and investigations but also into problem solving in all sorts of different ways both very open ended problems with groups of children tackling real life problems and also perhaps children doing more closed problems on an individual basis as well. So we are hoping to spread the range of activity. This obviously has enormous implications for inservice training and we can't actually just put materials on the market and say this is our assessment scheme, now you can do it. We obviously have to build in working with teachers. Already at the beginning of the project we have groups of teachers working with us. Those on the whole have been volunteer teachers but before it can be taken up in any wide way we will have to have a much higher degree of inservice training.

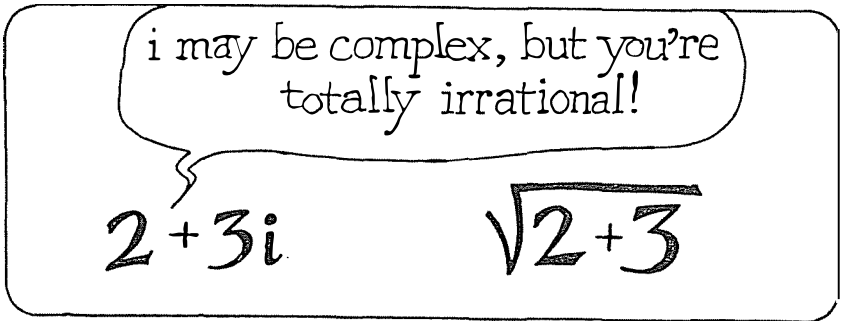
RS: It sounds to me, Margaret, as though we are going to leave arithmetic behind and all those hours on teaching long division algorithms and get on to what mathematics is about.

MB: Well, that's certainly the hope. I hope we won't leave mathematical skills as a whole behind. I

would be a bit doubtful about the long division one anyway. We are certainly still saying children need to be able to calculate not in abstraction, but rather in the real situation where they know what they are calculating and they have to make the decision themselves about how they are going to calculate and what sort of answer they are looking for.

RS: This Graded Assessment In Mathematics certainly sounds like an exciting project. I'll look forward to hearing more about it.







Problems

Editor's Note:

Cryptarithms, where a given letter stands for a specific digit, are always good for sharpening problem-solving skills. December's problems give a cross-section of cryptarithms from the exceedingly simple to the very complex.

1.
```
  AT
+  A
-----
 TEE
```

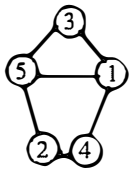
2.
```
  IN
+RID
-----
DOOR
```

3.
```
  SAG
x AS
-----
 USES
 SAG
-----
 BEDS
```

4. $\sqrt{\text{MADAM}} = \text{MAM}$

ANSWERS TO THE SEPTEMBER PROBLEMS

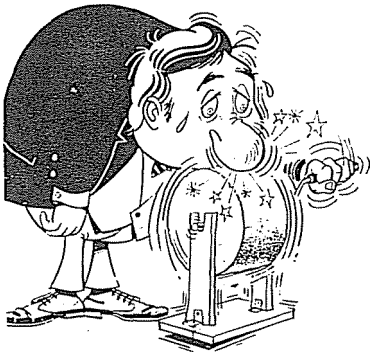
1.



2. (a) 5 pennies in each roll
   (b) 12 pennies in each roll

3. Fill the 3L container, and empty it into the 5L container. Fill the 3L container again, empty into the 5L container until it is full. Empty the 5L container. Pour the remaining 1L from the 3L container into the 5L container. Fill the 3L container once more and empty it into the 5L container - end result, 4L.

4. (a) 1 (b) 6 (c) 12 (d) 8 (e) 0



COUNCIL COMMENT

by
Bob Robinson

Editor's Note:

Coucil Comment appears in an issue of the Gazette following OAME/AOEM Council meetings. Your comments are most welcome and should be addressed to:

The Editor
Ontario Mathematics Gazette
College of Education
Brock University
St. Catharines, Ontario
L2S 3A1

The October Council meeting was held on Saturday, 1984 10 13 at the Ramada Inn (Airport West).

Future meetings of the Executive and Council are:

EXECUTIVE	COUNCIL
1984 11 16	1985 02 09
1985 04 05	

Annual General Meeting - Sudbury
1985 05 09-11

CHAPTER REPORTS

Note: For a complete listing of chapter representatives of Council, consult the September, 1984 issue of the **Gazette.**

• COMA

No report was presented.

• GOLDEN SECTION

A seminar is being planned to be held at Brock University in February. The central theme for the evening will be the new proposed Ministry guideline. The format will be speaker, dinner and workshops.

• GVMA

The Grade 11 resource project held last July was a great success. The Grade 11 Basic and Basic-Basic resources packages have been completed and will soon be going to print. These materials will be presented at a workshop at OAME '85 in Sudbury.

You are reminded that a wide variety of mathematics resources publications are available from

GVMA. For a list of those materials contact:

RON SCOINS
Faculty of Mathematics
University of Waterloo
Waterloo, Ontario
N2L 3G1

• NOMA

No fall meeting was scheduled this year as all efforts are being directed toward the organization of the **Annual '85** to be held in Sudbury.

• PINE RIDGE

During 1983-84, scholarships were presented to 10 teachers. Applications are being accepted for 1984-85. This scholarship fund is used to help teachers who are upgrading their qualifications in Mathematics Education.

• QSLMA

On 1985 04 10, QSLMA is returning to RMC for its spring meeting where it is hoped that their record attendance of 2 years ago will be surpassed.

• S.A.M.E.

The major project is the K-13 P.A. Day Program on 1985 02 15. This full day program, at the Holiday Inn - Don Valley, will be a joint project between S.A.M.E. and the Science Department of the Scarborough Board. Twenty workshops and a controversial keynote speaker are being planned.

• TORNY

The chapter is working on ways to have the group organize more efficiently. Announcements will be forthcoming in the next issue of the **Gazette.**

• OTF

The OAME Council has rejected an OTF proposal concerning the Curriculum Forum. This decision will be communicated to OTF through the OAME executive.

• LEADERSHIP SEMINAR

An impressive program for this year's Leadership Seminar was created by Dave Davidson, Paul Pogue and Morley MacGregor. This leadership seminar in mathematics has proven, over the years, to be one of the best values for prospective leaders in mathematics education.

• 1984 ANNUAL KINGSTON

Elaine Harvey presented her report and once again thanked Council and all those who attended for their support of all the efforts of the many volunteers who helped to make the Kingston-Annual such a success.

• 1985 ANNUAL SUDBURY

Mickey Sandblom presented the progress report regarding the **Sudbury-Annual.** An outstanding program is being put together and many interesting educational and social activities are being planned. Science North tours and Mine tours are being organized and the "the northerners" are enthusiastic to welcome us all to Sudbury on 1985 05 09-11.

Make your plans now to attend the **'85 Annual** where 120 sessions, make-and-take workshops, and a swap shop are being planned.

• 1986 ANNUAL LONDON

Tom Griffiths reported that plans are progressing well and the **'86 Annual** will be held on 1986 05 09-11 in London.

• 1987 ANNUAL ?

The site of the 1987 Annual has not been selected. If your area is interested in hosting the **'87 Annual,** contact any of the following members of the site committee: Paul Lessard, Dino Dottori, Bob Robinson.

• FATHER FAUGHT AWARD

The award was won by University of Toronto schools. Councillor Chester Carlow will present the award on behalf of OAME.

• MCCARTNEY AWARD

In August, Morley and Mona MacGregor presented the award on behalf of Council to Alexander Pak of Woburn Collegiate.

• HONORARY LIFE MEMBERSHIPS

Dave Davidson and his committee are putting together a proposal for Council consideration.

• NCTM

Council was reminded that in order for OAME to receive a commission on sales of NCTM publications, orders should be sent prepaid in U.S. funds through Ralph Connelly (NCTM Representative) at Brock University.

The special elementary school service to elementary schools where a year's subscription to the **Arithmetic Teacher** can be obtained for $13.00 (less than the cost of one textbook) is now available. Every school district should consider this service. For more information contact:

BOB ROBINSON
Director-NCTM
The Hamilton Board of Education

Upcoming NCTM meetings are:

Winnipeg 1984 10 18-20

Montreal 1985 10 3-5

Columbus, Ohio 1985 12 4-6

63rd Annual
San Antonio, Texas 1985 04 17-20

64th Annual
Washington, D.C. 1986 04 2-5

65th Annual
Anaheim, California 1987 04 8-11

66th Annual
Chicago, Illinois 1988 04 6-9

67th Annual
Orlando, Florida 1989 04 12-15

Note: NCTM will rebate to OAME $5.00 for every new NCTM membership we process. Any individual (or institution) that has not been an NCTM member for at least 13 months will be classified as a new member. Request a membership form from Ralph Connelly or Bob Robinson and return it to Bob Robinson (see address on previous page) so that OAME can take advantage of this offer.

• PUBLICATION COMMITTEE

A notice is placed in this issue of the **Gazette** stating that anyone wishing a copy of the **Canadian Mathematics Teacher** send $2.50 to the Secretary-Treasurer. Further, the 1985-86 OAME membership form will offer an opportunity for a member to order the **Canadian Mathematics Teacher** for $3.00

• MEMBERSHIP/PUBLICITY

The committee will continue correspondence to the mathematics department and librarians of the Community Colleges in order to improve the membership from this sector of the mathematics education community.

All members of OAME are asked to seek out former members and encourage them to renew their membership.

OAME will institute an optional five year OAME membership fee of $80.00 in the 1985-86 membership year.

WATCH FOR IT!

• FORMATIVE YEARS

It appears, at this time, that no session is planned for the Formative Years. Our committee will continue to investigate possibilities to provide impact to the Early Years Committee of the Ministry.

• COMPUTERS

Your attention is directed toward the questionnaire appearing in the November **Abacus.**

• SENIOR GUIDELINES

There is some concern regarding inservice training of the Intermediate level teacher. The difficulty levels of the grade 11 general courses was discussed with specific reference made to the technical components being to difficult and the business too easy.

• NOMINATIONS

If you or a colleague wish to serve on Council, please contact Dave Davidson for more information as to how you may responsibly serve this valuable mathematics association.

A POSITION PAPER ON PROVINCIAL EXAMS

by
Bruce Mullen

Dear Colleagues:

It has been increasingly apparent that the abolition of Provincial high school examinations in Ontario twenty years ago was a mistake. Marks inflation, back to basics movements, unhappy university teachers, lack of uniform Provincial standards, a sceptical public, disillusioned high school teachers and band-aid and bandwagon educational policies all indicate an organization in need of help. Clearly the Provincial examinations should never have been abolished; they should have been modernized. There is no doubt that the old examination system, which had existed in Ontario in one form or another before even Confederation had outlived its full effectiveness: it stressed too much memorization - type questions, at the cost of true understanding; it caused the curriculum to shrink to the extent that many high school teachers stopped teaching as early as February in order to review for the June examinations; it was unfair to the students of the average teachers because they had to compete against the students of the gifted teachers; it provided little recognition of the brilliant students and too much recognition of the memorizers; and worst of all, it reduced one of the most noble, highly creative vocations to simply that of a professional crammer. Nevertheless, the system of external examinations has some merit - it can provide the students and teachers with a focus; it can give them performance goals and feedback; it can provide uniformity of Provincial standards; and it can guarantee that the high school graduates have at least memorized the fundamentals of their school courses. Therefore it is astonishing to think that Ontario would abandon one form of Province-wide teacher and student supervision and not replace it with another. It is also difficult to conceive of any organization anywhere functioning in such a manner without disaster. However, hindsight is always 20-20. Moreover, we were living in naive times in the early 1960's. Flower power was in full bloom: everyone, it appeared, including teachers, wanted to do his or her own thing. Unfortunately, this philosophy has been followed to such an extent that it is difficult to evaluate our students' marks and our teachers' abilities.

For example, is a mark of 75% in Calculus still a good mark? Did the course contain much theory? integration? power series? who knows? The time has come to put some structure back into the educational system of Ontario; the time has come to help teachers to do a better job; the time has come to finally replace the old exam system with a new one. It is with all the above in mind that I make the following statements and corresponding recommendations.

STATEMENT 1:
A system of management which is perceived by the teachers as threatening will almost immediately cause the teachers to protect themselves and their students. They will almost immediately stop taking risks, stop experimenting and stop growing professionally. Furthermore, there are few things more frightening to a teacher and his students than an old-style Provincial exam in which the students' futures are measured by one exam. Furthermore, in my opinion it matters little if this examination is worth 20%, 30% or 100% of the students' final marks because the universities would in all likelihood only look at the results of this exam and all but ignore the individual high school marks.

RECOMMENDATION 1(a):
That the students of Ontario write a compulsory objective Mathematics examination for selected grades throughout K-13 in order to provide a goal and feedback to the teachers and students.

RECOMMENDATION 1(b):
That these examination results not be part of the students' high school marks in order to reduce the threatening component of external exams.

RECOMMENDATION 1 (c):
That the results of these objective examinations not be made available to the media or the universities, but that they be used by the teacher, department head and Principal only for professional growth.

RECOMMENDATION 1 (d):
That these tests be based on the entire curriculum and not just the core material in order to avoid the shrinking curriculum phenomenon.

STATEMENT 2:
Most teachers like to know how they rate as teachers and would enjoy some feedback or proof of their abilities. Unfortunately, most of us tend to over-rate ourselves as teachers: it is part of being human. Consequently, a supervision program should allow the average teacher to see himself as a success; it should tell him or her what she/he could be rather than what she/he is. This is the dilemma in management: how do you show a teacher the results of his teaching without demoralizing him or her? I suggest that you do so by not being too specific, by allowing the teacher to take credit for his success and by allowing him to avoid taking too much blame for his failures.

RECOMMENDATION 2 (a):
The examinations should provide each school or Mathematics Department with an analysis of the examination results. However, the over-all ranking of the school or teachers should not be done. In other words, a teacher should be made aware how his students did on individual questions and topics but the teacher or school should not be ranked provincially.

RECOMMENDATION 2 (b):
That a subjective exam be made available for our best students in order to recognize and reward them and to provide a challenge, a goal and feedback to our teachers.

RECOMMENDATION 2 (c):
That the marking of such exams in 2(b) be done in central locations in order to provide the teachers with the opportunity to grow professionally by associating with other teachers throughout Ontario.

STATEMENT 3:
All adults like to feel that they have some control over their environment. They do not long for independence as an adolescent might, but nevertheless, they cherish a feeling of interdependence. They want to be part of the decision making process; they want to feel that they are special individuals working within a special group of people. Consequently, teachers want to be largely in control of their students' evaluation process; they do not want to be treated as just another number, nor subjected to a supervision process over which they have little control.

RECOMMENDATION 3 (a):
That the Ministry of Education inform the teachers of the standards expected of them by supplying each school in Ontario with a set of tests, examinations, contests and assignments reflecting these standards.

RECOMMENDATION 3 (b):
That the Ministry have enough faith in teachers to permit them to try to meet or surpass these standards in an atmosphere of challenge and pride.

STATEMENT 4:
Most teachers are smart enough to defeat any form of supervision perceived to be not of their liking. Consequently, any new form of supervision of teachers must be very carefully thought out and explained before implementation. The teachers cannot be forced to do a better job, they must be taught and encouraged to do a better job.

RECOMMENDATION 4:
That any changes to the supervision of teachers be designed and implemented very carefully.

Bruce Mullen
Mathematics Head
J.S. Woodsworth S.S.

NOTICE

An Invitation to Join in the Study of Problem Solving

The Ontario Association of Mathematics Education (OAME/AOEM) is promoting a long-term, broadly-based study of problem solving and invites all mathematics educators in Ontario to become involved (at their own pace).

The issues being focused upon this year are listed below.

1. Sex differences in problem solving - against a background of similarities.

2. The motivational dilemma in problem solving - if the problems are too difficult the students become frustrated and lose interest; if the problems become too easy, they are not genuine problems. What is the optimal balance and how is it achieved?

3. Teacher training and problem solving. What are the most promising approaches?

4. Learning mathematical content through problem solving.

5. The selection and sharing of up to 10 "best" problems encountered during the year at a given grade level. (They may be old standbys, variations of old standbys changed to fit a particular situation, or original problems created by teachers or their students.)

The committee is positive that many teachers will have had various experiences with some or all of these issues and/or will have strong concerns that they wish to express. If you have any information to share on any of these issues or if you wish to join in the study of one or more of these important issues please contact any of the following:

Lorna Morrow, Chairman

Ed Barbeau, University College, 15 Kings College Circle, University of Toronto
Bill Bisset

Chester Carlow, Curriculum Department, O.I.S.E.

Peter Crippin

Bill Proctor

Ron Sauer

THE CANADIAN MATHEMATICS TEACHER

The Council of OAME has again agreed to arrange for delivery and distribution of the **Canadian Mathematics Teacher** (CMT) to interested OAME members. Expected publication date for the CMT is March/April, 1985. Lorna Morrow is the CMT editor for this issue.

If you would like to obtain a copy of the (1985) **CMT**, please complete the form below (or write your name and address on a sheet of paper and return it with a payment of **$2.50** (cheques payable to OAME to:

Morley MacGregor
247 Bright Street
Sarnia, Ontario N7T 4E9

Orders should be received by Feb. 15, 1985.

Name: _____

Address **(include postal code)**:

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