ABACUS

A PUBLICATION OF THE
ONTARIO ASSOCIATION for MATHEMATICS EDUCATION

VOLUME 3 NUMBER 2 November - December 1975

"... God rest ye merry gentlemen, let nothing ye dismay ..."

$\frac{1}{3}+\frac{1}{4}+\frac{1}{5}=\frac{3}{12}$
$=\frac{1}{4}$

$x^2=25$
$x=5$

$\cos\theta=30^\circ$

$48\div 0.16=3$

$2x+5=8=2x=3=x=\frac{3}{2}$

$\frac{3+8}{3}=8$

$\frac{2x+\frac{1}{6}}{3}=1$

$\therefore\ 4x+1=1$

$f(x)=\frac{2x-3}{3x+9}$

$f(x)=0$ for

$\frac{2x-3}{3x+9}=0$

$\therefore\ 2x-3=3x+9$

$\frac{\sin x}{x}=\sin$

$\cos 2x+\cos 7x=\cos 9x$

If $p=7$ and $k=3$
and $t=3p^2-4k^3$
$t=(3\cdot 7)^2-(4\cdot 3)^3$

$7(2x+3)=35x$

$\frac{\sin 2\theta}{\cos 2\theta}=\frac{\sin 1}{\cos 1}$
$=\tan 1$

$\frac{\cancel{4}^{1}}{\cancel{9}_{1}}\times\frac{\cancel{5}^{1}}{\cancel{8}_{2}}\div\frac{\cancel{10}^{2}}{\cancel{10}_{2}}=\frac{2}{4}$
$=\frac{1}{2}$

$\lim_{x\to 2}\frac{3x-6}{x-2}=\frac{0}{0}$
$=0$

YOUR MEMBERSHIP IN OAME INCLUDES THE ABACUS YOU CAN COUNT ON IT!

WRITE TO THE SECRETARY TREASURER

1975 - 76 EXECUTIVE

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Aurora, Ont., L4G 3H3

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From the Editor

Certainly there were some difficulties in getting this issue of Abacus ready for delivery, but finally it is in your hands. Postal difficulties were our main problem and I am sure that you will understand. This issue is, as you will see, under several impacts (one has already been pointed out.) : seasonal influence, Science Council of Canada Study No. 38: Mathematical Sciences in Canada, and your response to the last issue of Abacus.

During the last two decades, mathematics programs in the schools of Ontario, of Canada, as throughout the world, have undergone extensive reforms. These continuous innovations involved methods, conceptual material, and a variety of approaches for a variety of students. Emphasis on the understanding of mathematics, and the spiral approach to course content and/or the "new (approach to) mathematics" was to produce better mathematically oriented and better mathematically literate citizens. There is now a large body of informed opinion that this goal has not as yet been achieved, and that the teaching of mathematics in elementary and secondary schools in Canada, and many other places, is unsatisfactory. It is my firm belief that we, mathematics educators, are well aware of this criticism, and that there is an urgent need for improvement in mathematics curriculum and methodology. Yet the criticism is not altogether just; its main fault lies in the comparison of yesterday's students with those of today's school population (and this particularly applies to the secondary schools). The frequent promoters of the return to the old approaches to teaching - learning situations must recognize that the answer is not to return to the old system, but to search intensively for better ways of teaching and learning. We, the teachers, have a tendency toward "the closed door and leave me alone" policy, and the teachers of mathematics are inclined to what Whitehead calls the propagation of the "mystique of mathematics". Recent research indicates that unless we open our doors wide and improve our communication and rapport with our students and the public, we will face more and more difficulties. Better communication between mathematicians, mathematics educators, the public, and the students is urgently needed. As 1976 approaches I would like to suggest that all of us concerned with mathematics education improve our communication channels by:

(i) taking a more active part in our organization: All mathematics educators in Ontario to become members of O.A.M.E. and thus share our ideas ... and our problems ... and search for solutions.

(ii) make a concerted effort to let the public and our students know what mathematics is and what it is not.

Thus with these thoughts I and the editorial staff of Abacus wish you a Very Merry Christmas and a Happy New Year.

Peter Taylor

A Primary Aim for Mathematics Education *

"... the chief and overriding aim for the teaching of mathematics in the schools (to be) the widest possible dissemination of an understanding of what mathematics is and what it is not. As far as we know, this has never been adopted as a conscious aim of any school system. When it is achieved, the average citizen will understand that mathematics is (a) a way of thinking which provides a powerful tool for analyzing subtle and unobvious aspects of our experience, and (b) a cultural resource which can add interest and enjoyment to life. Further, (c) it will be apparent that the symbolism of mathematics - algebraic and graphical - constitute an important language which is essential for communication of ideas and for formation of societal goals."

* Science Council of Canada Background Study No. 38: Mathematical Sciences in Canada, A.J. Coleman, G.D. Edwards, K.P. Belpzner.

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Challenging Problems: Expand your mind. "EUREKA" is a publication of Algonquin College of Applied Arts and Technology. "Eureka" is for those who enjoy mathematics - who want to expand their knowledge of it - develop their mathematical skills - feel a professional obligation to do mathematics with others. - Interested ?? For the present, there is no charge, contact Mr. F.G.B. Maskell, Algonquin College

Samples from Eureka, Volume 1, Number 8.

1. SUGAR and SUMAC are the only two words in the English language which begin with SU and are pronounced SHU -

   SURE, everybody knows that!

2. The smallest positive integer is 1.

   Wrong! 1 is the largest positive integer. For if $N \neq 1$ is the largest positive integer, then $N + 1 > N$, which contradicts the hypothesis.

3. Problem # 71. Proposed by Leo Sauve, Algonquin College.

   If ten sheep jump over a fence in ten seconds, how many would jump over the fence in ten minutes?

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Mathophobia: Symptoms: fear and dislike of mathematics. Infection occurs early in childhood, and becomes chronic, incurable disease if no preventive action taken. Best prevention: an enthusiastic and well qualified elementary school teacher of mathematics. Research on the cure urgently needed. Forward suggestions etc. to "Abacus".

Solutions to Problems in Volume 3, Number 1, Abacus

Page 3

Before I left, I wound the wall clock. When I returned, the change in time it showed equaled the time I took to go to my friend's place and return plus the time I spent there. I knew the latter because I looked at my friend's watch both when I arrived and when I left. Subtracting the time of the visit from the time I was absent from my home, and dividing by 2, I obtained the time it took me to return home. I added this time to what my friend's watch showed when I left, and thus set the sum on my wall clock. (Editor)

Page 4 #1

Let $mb^2 = n$, it follows that we have to prove:

"If $a^2 + 2n$ ($a \epsilon^+ I$, $n \epsilon^+ I$) is the square of a positive integer, then $a^2 + n$ is the sum of squares of two positive integers."

Proof: $a^2 + 2n = k^2 \quad k \epsilon^+ I$

$$\therefore n = \frac{k^2 - a^2}{2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$\therefore a^2 + n = a^2 + \frac{k^2 - a^2}{2}$$

$$= \frac{k^2 + a^2}{2}$$

$$= \left(\frac{k + a}{2}\right)^2 + \left(\frac{k - a}{2}\right)^2 \ldots\ldots\ldots (2)$$

Equation (1) indicates that $k^2 - a^2$ is an even number, that is either k and a are both even or both odd.
Therefore $\frac{k + a}{2}$ and $\frac{k - a}{2}$ are positive integers, and from equation (2) it follows that $a^2 + n$ is the sum of the squares of two positive integers. Q.E.D.

Page 4 #2

How many digits in $2^{2^{16}} + 1$

Use of 5 place logarithmic tables makes this problem relatively easy.
log 2 = 0.30103, i.e. the error in approximation is less than 0.00001.

$$0.30102 < \log 2 < 0.30104$$

$$2^{16} \times 0.30102 < \log 2^{2^{16}} < 2^{16} \times 0.30104$$

but $2^{16} = 65536$

thus $19727 < \log 2^{2^{16}} < 19729$

It follows that $2^{2^{16}}$ has either 19727 or 19729 digits. This however does not give the final answer. To establish it we must note that $2^{10} = 1024$, and $\log 1024 \approx 3.01030$ with an error less than 0.000001.

$$10 \log 2 > 3.010329$$

and $$\log 2 > 0.3010329$$

thus $$\log 2^{2^{16}} > 0.301029 \times 65536 > 19728$$

that is the number $2^{2^{16}}$ has 19729 digits and it is an even number, i.e. the last digit cannot be 9, therefore the number $2^{2^{16}}$ has also 19729 digits. Q.E.D.

Page 6 "1 2 3 4 5 6 7 8 9 = 100"

Again there is only one way:

$$123 - 45 - 67 + 89 = 100$$

A good problem for Computer Science students!!!

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Some further problems sent to us by John Leightell,
Pickering College,
Newmarket, Ontario.

2-1 If $\frac{1}{a} + \frac{1}{b} = \frac{1}{2}$, a,b N

(a) find solutions for the equation

(b) prove there are only two solutions

2.2 What is the smallest product of 3 <u>consecutive</u> integers that is <u>not</u> divisible by 6?

2-3 In the world there are an <u>even</u> number of people who have shaken hands an <u>odd</u> number of times.

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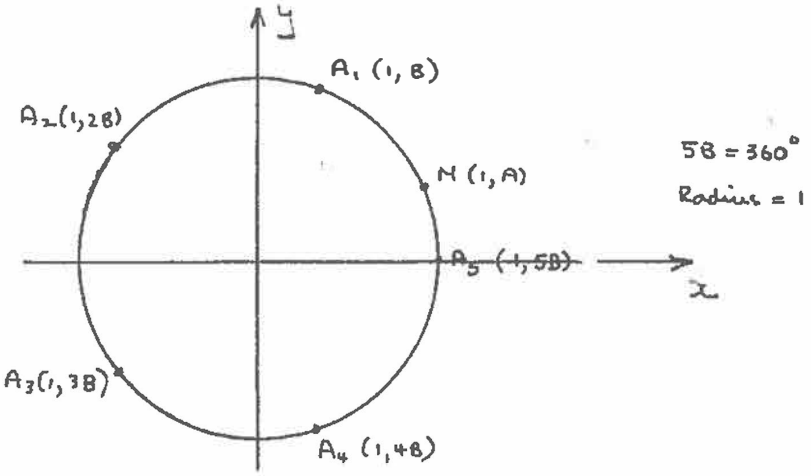
N.C.T.M. Convention : Detroit, March 4 - 6, 1976.

Elementary Sessions - Secondary Sessions - Plan ahead and attend

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Past Event: A successful "Seventh Annual N.O.M.A. Conference" in Sudbury, Ontario, October 24,25, 1975. Next conference in North Bay, October 1976.

Alternative solution to Problem Page 7, Submitted by Peter Maygeng
Levack H.S.



Since the property does not depend upon the orientation of the axes with respect to the vertices of the pentagon, I will take the axes in a convenient position. The coordinates of the vertices are thus (1,B); (1,2B);(1,3B);(1,4B), and (1,5B). The last vertex can also be expressed as (1,0).

The angles have been written as "A" and "B" for typographic reasons.

By Pythagoras;

$$MA_1 = \sqrt{(\sin B - \sin A)^2 + (\cos A - \cos B)^2}$$

$$= \sqrt{\sin^2 B + \cos^2 B + \sin^2 A + \cos^2 A - 2\sin A \sin B - 2\cos A \cos B}$$

$$= \sqrt{2 - 2\cos(B-A)}$$

$$= \sqrt{2}\sqrt{1 - \cos(B-A)}$$

$$= \sqrt{2}\sqrt{2\,\sin^2\frac{(B-A)}{2}} \qquad \text{note: } \cos(B-A) = 1 - 2\sin^2\frac{(B-A)}{2}$$

$$= 2\sin\frac{(B-A)}{2}$$

The similar other expressions for the distances are:

$$MA_1 = 2\sin\frac{(B-A)}{2} \qquad MA_2 = 2\sin\frac{(2B-A)}{2}$$

$$MA_3 = 2\sin\frac{(3B-A)}{2} \qquad MA_4 = 2\sin\frac{(4B-A)}{2} \qquad MA_5 = 2\sin\frac{(5B-A)}{2}$$

The problem may be rephrased as follows:

To Prove:

$$MA_1 - MA_2 + MA_3 - MA_4 + MA_5 = 0$$

We may divide this expression by 2 and equate the trigonometric equivalents.

$$+ \quad \sin\left(\frac{B-A}{2}\right) = \sin\frac{B}{2}\cos\frac{A}{2} - \cos\frac{B}{2}\sin\frac{A}{2}$$

$$- \quad \sin\left(\frac{2B-A}{2}\right) = -\sin\frac{2B}{2}\cos\frac{A}{2} + \cos\frac{2B}{2}\sin\frac{A}{2}$$

$$+ \quad \sin\left(\frac{3B-A}{2}\right) = \sin\frac{3B}{2}\cos\frac{A}{2} - \cos\frac{3B}{2}\sin\frac{A}{2}$$

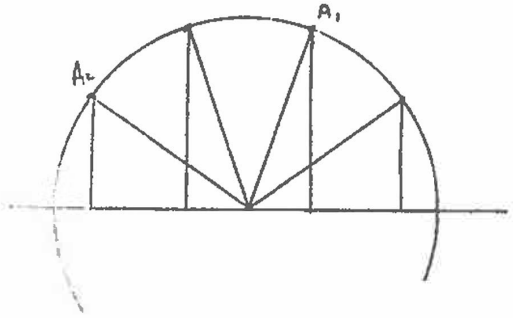
$$- \quad \sin\left(\frac{4B-A}{2}\right) = -\sin\frac{4B}{2}\cos\frac{B}{2} + \cos\frac{4B}{2}\sin\frac{A}{2}$$

$$+ \quad \sin\left(\frac{5B-A}{2}\right) = \sin\frac{5B}{2}\cos\frac{B}{2} - \cos\frac{5B}{2}\sin\frac{A}{2}$$

These expressions are summed and the common factors taken out. We thus need to prove that:

$$\cos\frac{A}{2}\left\{\sin\frac{B}{2} - \sin\frac{2B}{2} + \sin\frac{3B}{2} - \sin\frac{4B}{2} + \sin\frac{5B}{2}\right\}$$

$$- \sin\frac{A}{2}\left\{\cos\frac{B}{2} - \cos\frac{2B}{2} + \cos\frac{3B}{2} - \cos\frac{4B}{2} + \cos\frac{5B}{2}\right\} = 0$$



From symmetry:

$$\sin\frac{B}{2} = \sin\frac{4B}{2}$$

$$\sin\frac{2B}{2} = \sin\frac{3B}{2}$$

$$\sin\frac{5B}{2} = \sin(180^\circ) = 0$$

Thus the first $\{\}$ = 0

Now rotate the diagram by 90°. This will transform the sines into cosines, and thus the second is also 0.

Q.E.D.

For a more formal trigonometric solution we proceed as follows.

Consider:

$$\sin\frac{B}{2} - \sin\frac{2B}{2} + \sin\frac{3B}{2} - \sin\frac{4B}{2} + \sin\frac{5B}{2}$$

Now $\sin\frac{3B}{2} = \sin\left(\frac{5B-2B}{2}\right) = \sin\left(180^{\circ}-\frac{2B}{2}\right) = \sin\frac{2B}{2}$

and $\sin\frac{4B}{2} = \sin\left(\frac{5B-B}{2}\right) = \sin\left(180^{\circ}-\frac{B}{2}\right) = \sin\frac{B}{2}$

and $\sin\frac{5B}{2} = \sin(180^{\circ}) = 0$

Thus we have:

$$\sin\frac{B}{2} - \sin\frac{2B}{2} + \sin\frac{2B}{2} - \sin\frac{B}{2} + 0$$

$$= 0$$

A similar expression can be derived for the cosines.

Peter Waygang
Lewark High

Magic Squares: A Little Review

(Submitted by Frank Ebos, Faculty of Education, University of Toronto)

It is easy to incorporate Magic Squares into the mathematics program as a useful device for computational practice.

2	7	6
9	5	1
4	3	8

Fig. 1

By just knowing the numbers of a basic magic square as shown to the left, the teacher or student can generate an endless number of "seemingly new"magic squares. The square to the left is magic because the sum of the numbers in any row, any column, or any diagonal is constant, i.e. 15.

Another property of a magic square is that if each member is multiplied by, added to, subtracted from, or divided by the same number, the result is still a magic square.

For example, use the above basic magic square. Multiply each entry by 2 to obtain Fig. 1, then subtract 1, from each entry. The final result is Fig. 2.

4	14	12
18	10	2
8	6	16

Fig. 1

3	13	11
17	9	1
7	5	15

Fig. 2

A simple check: adding rows, columns, or diagonals shows that the square is still magic, except that this time each sum is 27.

By removing entries from the squares of Fig. 1 and Fig. 2, we have a number of "seemingly new" magic squares by providing 4 entries as shown in Fig. 3 and Fig. 4.

3	13	11
	5	

Fig. 3

		11
17	9	
7		

Fig. 4

As a variation for another type of magic square, the fourth number provided is the "magic number as indicated in Fig. 5

	13	
		1
		15

Fig. 5

The magic number is 27.
Find the missing numbers.

At a higher level, practice with adding and subtracting two or three digit numerals can be obtained by creating a new magic square.

48	168	144
216	120	24
96	72	192

Fig. 6

48		
	120	
96		192

Fig. 7

Namely, multiply the numbers of the basic magic square by 24 to obtain Fig. 6

Again by omitting all entries except 4 we generate a "seemingly new" magic square in Fig. 7.

106	371	318
477	265	53
212	159	424

Fig. 8

$\frac{1}{3}$	$1\frac{1}{6}$	1
$1\frac{1}{2}$	$\frac{5}{6}$	$\frac{1}{6}$
$\frac{2}{3}$	$\frac{1}{2}$	$1\frac{1}{3}$

Fig. 9

To make "more complex" magic squares, we need only multiply by large numbers; for example, each entry could be multiplied by 53, 77, etc. See Fig. 8.

By varying the entries as shown in the earlier "magic" examples, we obtain again "seemingly new" magic squares.

$\frac{1}{3}$		
	$\frac{5}{6}$	$\frac{1}{6}$

Fig. 10

-3	+2	+1
+4	0	-4
-1	-2	+3

Fig. 11

Dividing each entry by 6 in the basic magic square, we obtain a magic square for reviewing some skills with fractions as shown in Fig. 9 and 10. For Fig. 10: The magic number is $2\frac{1}{2}$. Find the missing numbers.

For some simple magic squares that provide practice with integers, we need to subtract the same number, say 5, from each entry of our basic square and the result shown in Fig. 11 is obtained.

-3		
+4		-4
-1		

Fig. 12

-3		
	0	
		+3

Fig. 13

By varying the entries that are left out, magic squares with integers are obtained. Two examples based on the above square are shown in Fig. 12 and Fig. 13.

$-\frac{2}{3}$	$\frac{1}{6}$	0
$+\frac{1}{2}$	$-\frac{1}{6}$	$-\frac{5}{6}$
$-\frac{1}{3}$	$-\frac{1}{2}$	$\frac{1}{3}$

Fig. 14

		0
	$-\frac{1}{6}$	
$-\frac{1}{3}$	$-\frac{1}{2}$	

Fig. 15

Using the earlier magic square with fractions, all we need to do is to subtract 1 and we obtain a magic square that provides practice with rationals, as shown in Fig. 14 and Fig. 15. For Fig. 15, find the missing rationals.

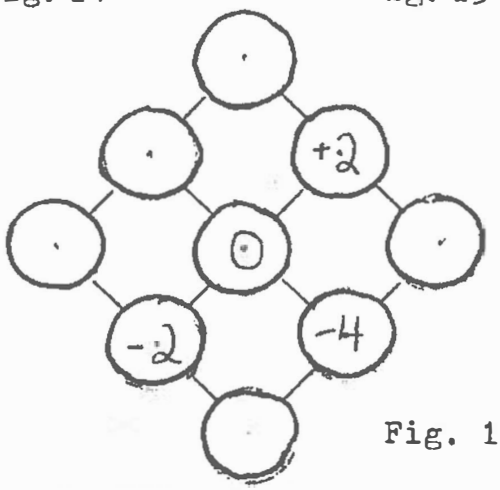


Fig. 16

With all the above examples, the students might appreciate variety if occasionally we would introduce an array of magic circles. as shown in Fig. 16. For Fig. 16, find the missing integers.

19	16	13	28
14	27	20	15
26	11	18	21
17	22	25	12

Fig. 17

If you feel ambitious, all of the above comments apply to the larger 4 x 4 magic square, as your basic magic square, and thus to create other magic magic squares to review basic skills at almost any level.

As an interesting activity, aome students like to create their own magic squares. If a class were given time to create new magic squares, at the end of the activity the teacher would have at least ten different magic squares that can be "stored" for next year or, better still, used as a review throughout the year. By labelling them as "Harry's Magic Square" or"Susan's Magic Circles" the class might be better disposed to practicing what might otherwise be known as Ho-Hum Drill (some kids call it dull-drill).

More from "Mathematical Sciences in Canada"

"... poet, W.H. Auden, asserted that well into their teens, children should study only two subjects: their native language and mathematics. These two are essential for communication between humans. -... at all levels of the Canadian educational system we should make every possible effort to ensure that the youth of our society gain mastery of French or English and a confident control of mathematical thinking." (p. 100)

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What every young MATHLETE should know.

1. In $(a + b)^n$, if n is integral, then there are $n + 1$ terms in the expansion, and the sum of the coefficients is 2^n.

2. Whenever a pattern is discernible in a set of equations, some general technique may yield the solutions easily.

   (i) Solve $x + y = 10$, $y + z = 23$, $z + x = 19$.
   Add all equations to get $x + y + z = 26$, then the individual unknowns are obtained by subtracting each of the given equations in order.

   (ii) Solve $xy = 72$, $yz = 9$, $zx = 8$.- Multiply the equations to get $xyz = \pm 72$, and find the individual unknowns by dividing.

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A bit of challenge: Find the next ten dates after 1961 which read the same upside down as upright.

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Postal difficulties and subsequent delays prevented inclusion of some of your responses in this issue. What is missing now will not be forgotten. The next issue of ABACUS will be out in March: please submit your ideas etc. on or before February 15, 1976.

Address all correspondence to:

Andy Czempinski

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"Mathematize with joy" (A.J. Coleman).

THIRD ANNUAL CONFERENCE

Ontario Association for Mathematics Education

MAY 14, 15 and 16, 1976

St. Catharines, Ontario

A THEME APPROACH TO CONFERENCE PLANNING

OBJECTIVES: This format presents an opportunity for theme participants to share, as a common experience, their concerns, ideas, and reactions in more depth than is possible in sectional meetings.

Each theme starts with a keynote person of outstanding reputation and then develops into a number of Workshops, Expository Lectures, etc., at the various levels of education, K to University.

The Themes are:

1. Where Have We Been and Where Are We Going?
2. Standards, Achievement and Evaluation
3. Geometry: Approaches and Influence
4. Meaningful Arithmetic/Algebra
5. Remedial Mathematics
6. Mathematical Models Representing Reality
7. Calculating Devices, A Help or a Hindrance?
8. What about Statistics?
9. Computers
10. Metric Measure and Mathematics
11. Beautiful Mathematics
12. Motivating Mathematics

AN EARLYBIRD PROGRAM

Starting Friday Morning - May 14, 1976

Friday a.m. -
- (i) Mathletics Competition Finals: (a) Secondary (b) Elementary
- (ii) School Visitations in Region 6, Lincoln, Niagara South and Hamilton

Friday p.m. -
- (i) 1:00 p.m. - Curriculum Development Presentations by Superintendents of Curriculum
- (ii) 3:00 p.m. - Informal Discussions with our Guest Speakers
- (iii) 4:30 p.m. - "Warm-up Session" - an Informal Collection of Mathematical Demonstrations and Humour
- (iv) Displays, Exhibits and Trading Post
- (v) Mathematical Games Room

TRADING POST

Keep us in mind now, run off 20 pages of your best handouts for trading purposes, which entitles you to 20 pages of handouts from someone else.

Look for the pre-registration package and complete program information next month.

Register Early:
1. To obtain sufficient budget funds.
2. Because accommodations should be in demand during the week of Blossom Sunday, particularly in this St. Catharines Centennial Year. (Accommodations at the Brock Campus are limited to 350 beds and are on a first come, first served basis.)
3. Get your name on the list of attendees if your pre-registration form and fees are received by February 27, 1976.

ST. CATHARINES CENTENNIAL

OAME

BROCK UNIVERSITY REGION NIAGARA

LOOKING AHEAD

1977 ANNUAL OAME CONVENTION

MAY 12, 13, 14, 15, 1977

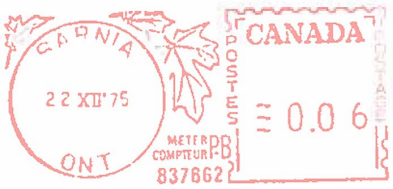
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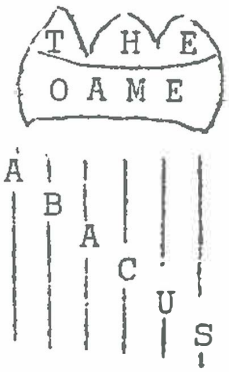
Ottawa University Carleton University

for further information contact the Co-ordinators

Mr. Bruce Topping or Mr. Duff Butterill

D Aubrey Moodie Intermediate School





"MATHEMATICS EDUCATORS
SHARE THEIR IDEAS"

Mr. Robert Tuck