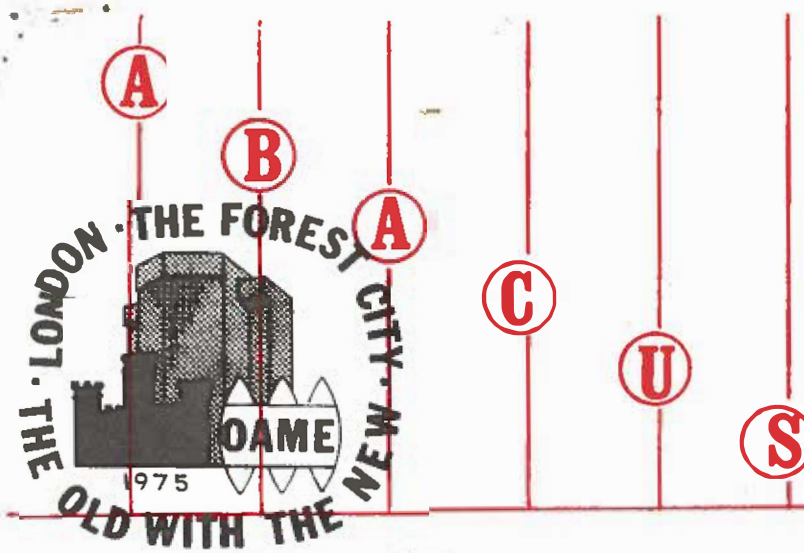


A PUBLICATION OF THE
ONTARIO ASSOCIATION for MATHEMATICS EDUCATION

VOLUME 2 NUMBER 3 FEBRUARY 1975

Take your pick from over 100 hours of interesting and practical workshops ranging from primary to college levels. Share the latest views of internationally renowned mathematics educators. Have a glimpse of some of the best things going on in Ontario math classes today. Join the hundreds going to LONDON in MAY. (23rd, 24th, 25th)

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YES! YOU CAN HELP MAKE IT A SUCCESS!
INVITE OTHER MATH PROFESSIONALS (MEMBERS OR NOT)
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YOU'LL BE GLAD YOU DID

YOUR MEMBERSHIP IN OAME INCLUDES
THE A B A C U S YOU CAN COUNT ON IT!
WRITE TO THE SECRETARY TREASURER

REMEMBER!....A CAR POOL IS A GREAT IDEA

1974 75 EXECUTIVE
John C Egsgard,
President,
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Vice President,
York County Board of Education,
Box 194,
Aurora, Ont. L4G 3H3
Morley MacGregor,
Secretary Treasurer,
247 Bright St.,
Sarnia, Ont N7T 4E9



Letters To The Editor

Lloyd (Rusty)McIntyre at Base Borden H.S. showed the 11X11 pattern illustrated in the Nov'74 ABACUS "Calendar of Challenges" to his grade nines. One of them found a pattern for 9.

9 X 9 = 81
99 X 99 = 9801
999 X 999 = 998001
9999 X 9999 = 99980001
etc.

Note: The digits 8 and 1 persist with the appropriate insertion of 9's before the 8 and zeros after it. Can you find patterns for other digits(5 say)? (Thanks to Jack LeSage for this letter)

Dear Sir:

I should like to respond to the article by M. Loring in the September Abacus concerning the teaching of inequalities in the early grades. The point that children are confused whether 3 dimes are greater than or less than 2 quarters is well taken. In my opinion, one solution to such dilemmas is to emphasize the concept of number line from the earliest stages simultaneously with the usual presentation of the cardinal numbers. In this way the children can also have the concept of number as distance. Then, only the symbol $<$ should be introduced, and only in the context of the number line. This is how it is generally used in higher mathematics anyway!! One could easily call it the "less than" symbol, or the "to the left of" symbol. One could refer to it as a bus going down a one-way street, or an arrow shooting from right to left, and $4 < 5$ means that 5 is picked up or shot first.

Only at a much later stage should one bother to note that this applies to cardinal numbers, i.e. two bears are less than three bears--which really implies that the set of three objects has a proper subset of two objects. This should be an intuitively obvious concept from the definition of cardinality. The notation $5 > 4$ should be introduced even later as a printer's error for $4 < 5$, and a suitable name and convention introduced. The important facts about "$<$" are that given any two different numbers a and b we can relate them, i.e. given a & b exactly one of $b < a$, $a = b$, or $a < b$ holds; and that given $a < b$ and $b < c$, then $a < c$. This teaches them how to make deductions!

All this is meant as my personal opinion of one method of proceeding. Obviously one should not begin teaching "$<$" until the children truly know their left hand from their right. By the way, the question about which is greater, 3 dimes or 2 quarters, is the kind of semantic nonsense that may disaffect the good students, and is best avoided at all costs.

Yours very truly,
Irwin S. Pressman
Associate Professor of Mathematics
Carleton University

ABACUS ITEMS MAY BE REPRINTED WITHOUT FORMALITY EXCEPT CUSTOMARY ACKNOWLEDGEMENTS

.....

TRAVEL GAMES AND ACTIVITIES

ESTIMATING SPEED

Every mile along the 401 is marked with mile posts* that look like this:



While you watch this post go by, have your partner time how long it is between posts.

Record the number of seconds and calculate the speed in miles per hour. (Remember: 1 mile in 1 minute is 60 m.p.h.) *(We'll soon need a metric version)

How can you tell if our speed is steady, increasing or decreasing without actually calculating the speeds?

PENCIL AND PAPER GAMES

1. You have all played x's and o's.



The game takes on new interest if you win by forcing your opponent to get his three marks in line.
Try it!

2.

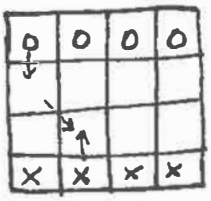


Starting with the above array each player in turn crosses out as many sticks in any one row as he wishes.

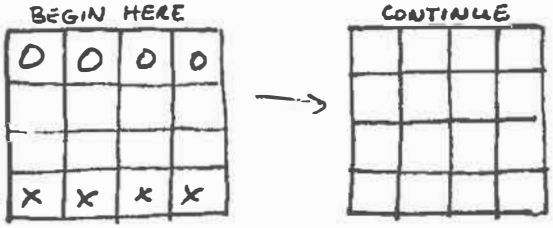
The object of the game is to force your opponent to cross out the very last stick.

3. Pencil Checkers

The Board with teams in play
(an o takes an x in 2 moves)



Object of Game: Get one piece to opposite side or make it so opponent has no move.
Moves: Pieces move one space directly ahead into empty space.
Pieces move one space diagonally ahead to remove an opponent in that space and take his place.
Technique of play: Indicate moves with arrows until a piece is captured then draw all new positions on a new board.
Try a Game!



SEND YOUR MATH GAME, puzzle book review, ideas to:

ABACUS EDITOR
c/o CITY MATH CENTRE
OAK PARK SCHOOL
40 HUNT CLUB DRIVE
LONDON N6H 3Y3

TRY IT YOURSELF CORNER

Originality not claimed. Solutions elsewhere in this issue. Submit appropriate problems to ABACUS Editor

Here's a chance 4 U 2 test your knowledge of geometrical terms, as well as your skill for making puns. A geometrical term will go with each of the following statements. For those who have left their "thinking caps" behind, a list of terms is included at the bottom of the page. Time is limited to fifteen minutes.

1. It has three feet, yet can't walk. .....Answer: _____
2. A dead parrot. .....Answer: _____
3. The way the poet wrote his love letters. .....Answer: _____
4. What a person should do when it rains. .....Answer: _____
5. What the husband did when his mother-in-law wanted to go home. .....Answer: _____
6. What the forgetful professor said about the letter he carried in his pocket a week before mailing. .....Answer: _____
7. Place where people are sent for committing crimes. .....Answer: _____
8. A beastly term. .....Answer: _____
9. A phonograph record that wasn't there. .....Answer: _____
10. A clever angle. .....Answer: _____
11. The one in charge. .....Answer: _____
12. What little acorns say when they grow up. .....Answer: _____
13. Sharp weapon. .....Answer: _____
14. What one old farmer sometimes says to another. .....Answer: _____
15. Triangle that's never wrong. .....Answer: _____
16. Boats that cross the ocean. .....Answer: _____
17. What is said about a man who can't talk clearly. .....Answer: _____
18. A food that is a dessert. .....Answer: _____
19. They voted YES for better tractors. .....Answer: _____
20. Gentlemen who go to the beach. .....Answer: _____

Some Geometrical Terms

Secant	Pi	Geometry	Chord	Yardstick	Right Triangle
Decagon	Prism	Sector	Center	Loci	Line
Transversal	Postulate	Protractor	Coincide	Acute Angle	Hexagon
Polygon	Sphere	Tangent	Hypotenuse	Rectangle	Ruler

Scores: 19-20 Atomic; 16-18 Volcanic; 12-15 Hot; 8-11 Warm; 4-7 Dying; 0-3 Dead.



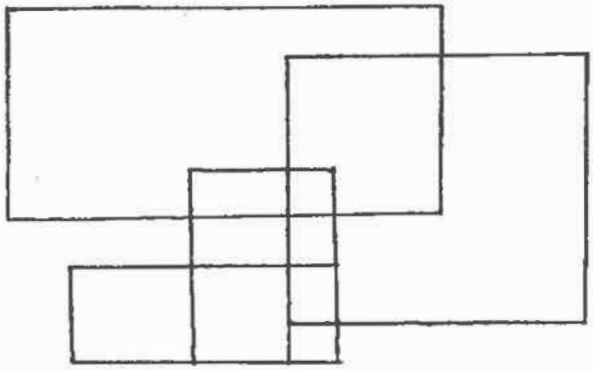
If Alice and Betty can complete their jobs in 2 hours and if Alice and Charlene can do the same work in 3 hours, while Betty and Charlene require six hours to do the same jobs, how long would it take each girl working alone?



"Wars, population explosion, pollution, and they still haven't figured out how many angels can dance on the head of a pin."

PHI DELTA KAPPAN

COUNT THE RECTANGLES



This (or a similar design) could be enlarged for a bulletin board, it could be placed on a transparency for the over-head projector, or it could be duplicated for individual distribution.

Have each child write his number on the chalk board or a piece of paper. When the numbers are given, there will no doubt be differences of opinion. In order to reconcile these differences, have various children trace the rectangles with their fingers to verify their answers. Hopefully there will be a square among the other rectangles which should provide a good opportunity to review the characteristics of a square and a rectangle to see if the square should be counted, emphasizing that the square is a special rectangle.

An art lesson might follow in which the child is to create a design using only rectangles of various dimensions.

This same idea could be used with other geometric designs, such as triangles.

Math Bloopers

(Thanks to Howard Eves, Mathematical Circles Revisited)

1. Trigonometry is when a man marries three wives at the same time.
2. Geometry teaches us how to bisex angels.
3. Algebraic symbols are used when you do not know what you are talking about.

Answers to Geometrical terms test P10

1. yardstick	2. polygon	3. inverse	4. coincide	5. center
6. postulate	7. prism	8. line	9. decagon	10. acute angle
11. ruler	12. geometry	13. sphere	14. loci	15. Right triangle
16. Transversals	17. Ellipse	18. Pi	19. Protractor	20. Tangents

THE LOST ART

SEND RESPONSES DIRECTLY TO
J. Del Grande

Comments and solutions to the old problems papers are still coming in - mostly from old timers.

Perhaps a clue to the fascination of doing these problems lies in the letter received from Fred Maskell.

"The variety of proofs available used to be one of the interesting features of the "old maths."

As an alternative to Frank Asbury's solution of the harmonic intersection of the sides of a square by a tangent to the inscribed circle, I suggest the following:

The unit circle may be used with no loss of generality, with $x = \pm 1$, $y = \pm 1$ for the sides of the square, and $x \cos\theta + y \sin\theta = 1$ for the tangent.

The abscissae of the points of intersection are (from left to right with $0 < \theta < \pi/2$) -1, $\frac{1 - \sin\theta}{\cos\theta}$, 1 and $\frac{1 + \sin\theta}{\cos\theta}$, with corresponding ordinates, $\frac{1 + \cos\theta}{\sin\theta}$, 1, $\frac{1 - \cos\theta}{\sin\theta}$, -1.

The projections of the intersections on the coordinate axes are both harmonically divided, since for the abscissae the internal and external ratios are:
$-\frac{1 + \cos\theta - \sin\theta}{1 - \cos\theta - \sin\theta}$ and $-\frac{1 + \cos\theta + \sin\theta}{1 - \cos\theta + \sin\theta}$, each of which reduces to

$\cot\theta/2$ with opposite signs. Similarly for the ordinators."

Ed Evans, although retired, is still dispensing good advice.

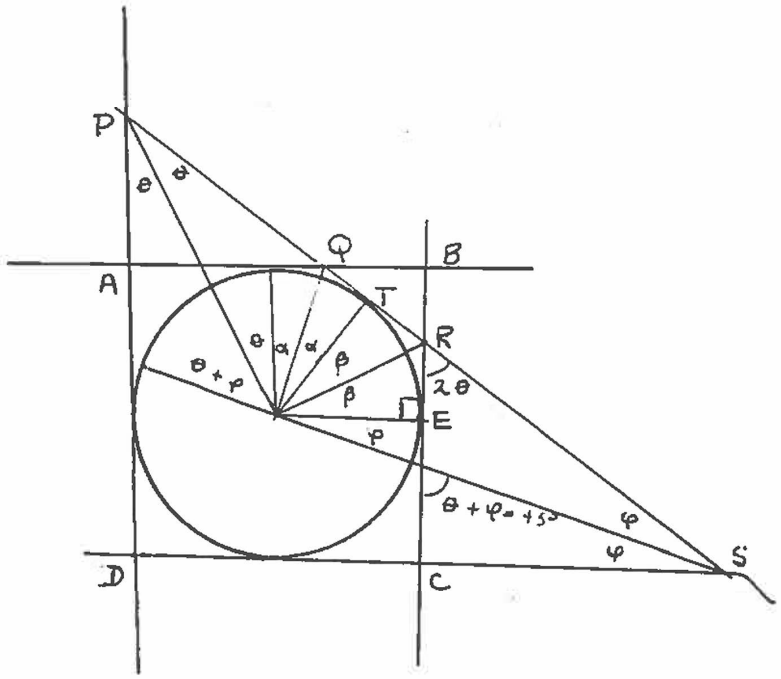
"I am impressed with the attainments of today's high school students. Their familiarity with computer programming, etc., and their ingenuity in the maths exhibits."

Of course Ed had to make his contribution.

"Your comment on Frank Asbury's rising to your bait of #12 on the 1923 paper, caused me to look at my own solution.

If P,Q,R, and S on the tangent are to form a harmonic range, then, taking O as the centre of the circle, OR and OP should bisect the interior and exterior angles at O of the triangle QOS. This was readily proved to be the case. (See my diagram)

Of course, I was just lucky to hit on this simple solution on the first try. It was interesting to see the right angles at O."



Ross Mackintosh (U.C. 44) wrote to say that he's glad to see that the "lost art" is being revived and submitted another solution to #2 of the 1923 paper.

"The problem of finding the solution set for the system of equations $x^2 + y^2 + z^2 = 1$, $a^2 + b^2 + c^2 = 1$ and $ax + by + cz = 1$ can alone be handled by the methods of 3-dimensional coordinate geometry, with a bit of vectors thrown in $x^2 + y^2 + z^2 = 1$ is the equation of a sphere, centre (0,0,0), radius 1. Since $a^2 + b^2 + c^2 = 1$, then (a,b,c) is a point on the sphere. $ax + by + cz = 1$ is the scalar equation of a plane which alone contains (a,b,c) since $a^2 + b2 + c^2 = 1$. Hence (a,b,c) is a common point.

Because the direction vector of the straight line (radius) joining (a,b,c) to (0,0,0) is (a,b,c) which is also the normal vector to the plane, the plane is tangent to the sphere at (a,b,c) and therefore there are no other common points.

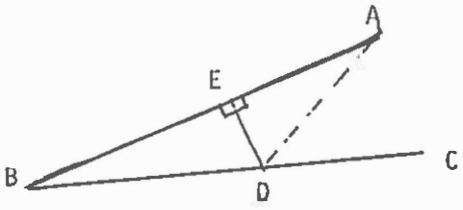
Thus the solution set for the given system contains only one member (a,b,c); in other words x = a, y = b, z = c is the only solution."

Frank Asbury wrote giving us some interesting notes about the "old OCE".

"Problem #10 in the 1961 Problems paper recalled old memories. The first time I ran into this was in Prof. J. T. Crawford's Specialist Seminars at O.C.E. - that was just about 60 years ago! Imagine my surprise to see it again.

Prof. Crawford, I believe, gave it regularly to his seminar groups and collected all the different solutions received. How many he had, I don't recall. I myself,have half a dozen - most of them originating with Sarnia pupils.

The one I turned in at "Faculty" was quite straight forward:



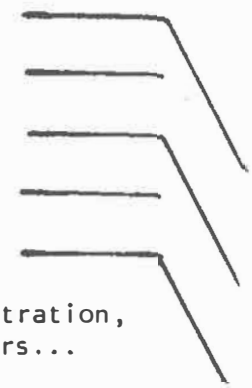
with ED the right bisector of AB,
ADC = 2 B

Since AD(hyp) > AE or AC
∴ C > 2 B"

To illustrate how much enjoyment mathematics can be, I recall a problem posed by Clark MacDonald, a superintendent from the Lakehead Board of Education. Clark wrote about it - here is his version.

"On the Bloor subway from Islington the man in the grey striped suit contemplated passing stations in anticipation of a transfer at Yonge.

And it came to pass that the square-cut tail of his jacket fell obliquely across his knee so that the space between two stripes at the jacket's edge merged with one stripe of the trousers.



And he beheld the juxtaposition and pondered the pattern so created.

And after some manipulation of the grey woollen material, he wondered if the angle formed by the intersection of pin stripes might be constant--and if so, what could the angle be?

Then having almost missed his stop because of idle concentration, he dismissed the problem temporarily in the rush of Yonge Street cars...

On that same evening, midst wine and cheese, he did meet up with John under crystal chandeliers, And behold, John was an honoured guest, recognized for his writings and for his acquaintanceship with Galton, Euclid, and Venn.

And after some refreshments the man in the grey striped suit and John did discourse on many things touching curriculum and pedagogy. In due course the problem of the stripes was raised and hands manipulated cloth in that crowded hall to see how two bands merged into one.

And John was heard to mutter, "Mon Dieu, it's a new theorem!" In so speaking, he revealed both brilliant insight as well as respect for his country's bilingual complexion.

The MATHEMATICS STUDENT is published four times a year, in October, December, February, and April, at 1906 Association Drive, Reston, Virginia 22091, by the National Council of Teachers of Mathematics.

In a flash of Trig he pronounced the angle to be sixty degrees. And there may, at that instant, have been great rejoicing. But lo! the man in the striped suit knew how legs can be pulled by mathematicians, especially under crystal chandeliers where Sauterne flows with Danish Blue.

And being a cautious man, did not allow himself to cry Eureka!

But he still wears the grey striped suit and, in quiet moments, superimposes coat on thigh wondering, "If one into two does in fact result in a sixty degree deflection, then will one into three give seventy, and one into four, seventy-five - - - - ?"

Clark has since found many applications for the geometrical property he discovered. More in later issues.

We're looking for more solutions to the 1961 problems paper. Send your solution in - it may be a novel way to do the problem.

A CRITIQUE OF MATH PROBE

Authors: Bye and Elliot

Published by Holt, Rinehart, & Winston of Canada, Limited

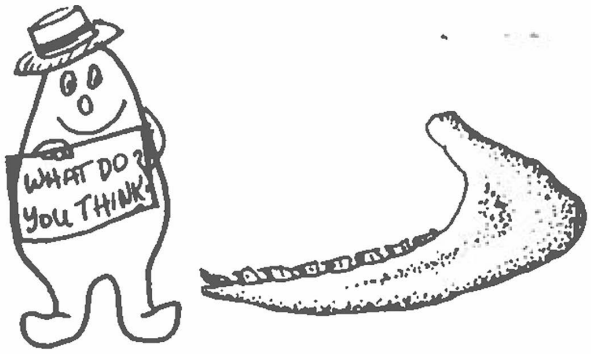
Submitted by Darlene Kidd, Teacher, Macklin, Sask.

Macklin High School has been asked to pilot the alternate Division IV Mathematics course--Probe 2 for Grade 10 and Probe 3 for Grade 11. After a careful consideration of the material in the text and a rather intensive screening program, we decided to introduce several students who had had and were having extreme difficulties with the traditional Algebra to the Probe series. Of the eight students that began the course in January, all had been consistantly failing. They began a completely individualized program without any instruction or scheduled classroom assistance. Two students will complete the course by the end of this term, one student discontinued enrollment in school, one chose to rejoin regular classes, and the other four continue to work at their own pace, not for academic credit but because they are actually finding some enjoyment, purpose, and satisfaction in the exercises and experience.

Student evaluation of the course include these comments:

* It is much easier to understand and do than traditional Algebra.
* I feel that this Mathematics course is more interesting to the student, even though it teaches many of the same concepts as the traditional Algebra.
* I liked the Probe series when I was taking it.
* This course seems less complicated and confused.
* I think the Math Probe course could overcome a lot of problems that students have with Algebra.
* I liked the course and I think that every student with problems in mathematics should take this course.

As a result of the limited experience that I have as the assistant to the students taking the Math Probe course this year, I tend to rate it very highly as a tremendous mathematical learning experience for those students with good basic skills but low ability to abstract. I feel that it would not challenge students of good or above average ability in mathematics.



"With the jawbone of an ass. . ."

Some two thousand years ago, the "Good Book" made it clear that with a jawbone of an ass one could slay thousands. And for the last two thousand years teachers have used the same technique to slay the interest of thousands of students.

There are always problems in education. One problem, like taxes, seems constant. This problem, the inadequacy of teacher preparation for the teaching of elementary school mathematics, is deserving of a high rating on any listing of priority items in need of concentrated correctional efforts. At a time when the results produced by "modern matematics" after several years of implementation in the classrooms are being scrutinized and criticized, it is essential that the area of the degree of preparation and competence of the teacher be examined anew. Observations and research related to this topic produce some rather significant findings.

We tried that before. Our system won't allow it. We're too small for that. It costs too much. They won't let us. We're too big for that. You can't do that with kids like ours. It may work in your place, but not here. Let's sleep on it. Write it up and we'll put a committee on it. It may work in industry but it'll never work in education. We should wait awhile. It may work later. We have too many people to try it. We don't have enough equipment. We don't have enough room. The kids are not ready for that

* * *

Children working with apparatus are doing precisely what good research workers do.

They have a problem.
*They understand what the problem is.
*They really want to solve the problem.
They guess an answer.
They test the answer for themselves.
If it does not work, they try something else.

In the rhythm of rote learning all the emphasis is on the answer. In the rhythm of research, the emphasis is on the two items marked with a star.

INDIVIDUALIZING MATHEMATICS THROUGH ACTIVITY CARDS ?

HERE IS A SAMPLE CARD FOR EQUIVALENT FRACTIONS

FRACTION DISC KIT

1. PLACE THE $\frac{1}{2}$ PIECE IN FRONT OF YOU.
2. USE YOUR $\frac{1}{4}$ PIECES TO MAKE A FRACTION THAT IS THE SAME AS $\frac{1}{2}$.
3. DO YOUR $\frac{1}{4}$ PIECES LOOK LIKE THIS?



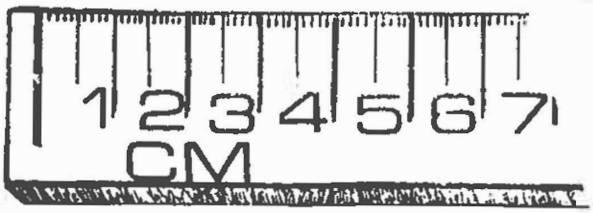
4. IF SO, VERY GOOD. HOW MANY $\frac{1}{4}$'S ARE THE SAME AS $\frac{1}{2}$?_____
5. DID YOU SAY 2?_____
6. PLACE YOUR $\frac{2}{4}$ ON TOP OF $\frac{1}{2}$. ARE THEY THE SAME SIZE?_____
7. YES, THEY ARE. WE SAY THAT $\frac{1}{2} = \frac{2}{4}$.

from J. Easgard

A man has a map that gives the following instructions."On the island, go to the gallows. From there walk to the pine tree measuring the distance. At the pine tree turn right and walk that same distance to A. Again from the gallows walk to the oak, measuring the distance. At the oak turn left and walk that same distance to B. Join AB. The treasure is at mid-pt. of AB."When you arrive at the island the trees are there,but the gallows is gone. How can you find the treasure?

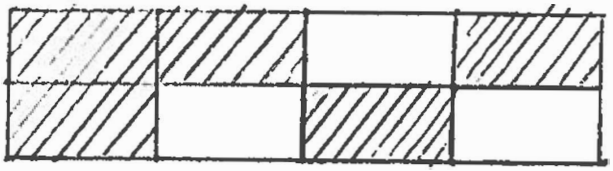
Take any spot P as position of Gallows. Follow instructions. No matter what spot you chose for P you always end at treasure





Do you remember when there were HIGH SCHOOL ENTRANCE EXAMINATIONS?

Compare some example questions to those from C. T. B. S. (1967)

THE OLD	THE NEW.
(1945) Express 5/8 as a decimal.	Which of these is closest to .6 ? 1) 2/3 2) 5/10 3) .7 4) 1/6
(1934) Multiply 2758 by 6308	Which line of the following shows the product of 20 and 389? 1) 2723 2) 389 3) 7780 4) 10503 389 x27 2723 7780 10503
(1945) In a school of 750 pupils, 27,000 War Savings Stamps costing 25¢ each were sold. What was the average amount invested per pupil?	The average number of hens in the flock for eleven months, Sept. through July was 66. The total number of eggs Alice gathered during that time was 11,616. What was the average egg production for the period? 1) 186 3) 176 2) 177 4) not given
(1945) From the sum of 3 5/9 and 4 2/3, subtract 2 5/6	From the picture, what correct number sentence can you make?  1) 1/4 + 3/8 = 5/8 2) 1/4 + 1/2 = 3/4 3) 3/4 + 1/2 = 5/4 4) 3/4 + 1/4 = 5/8

SEVENTH CANADIAN MATHEMATICAL OLYMPIAD, 1975

Patron: The Governor-General of Canada

Date of Examination

The Seventh Olympiad will be written on May 7, 1975 from 9 a.m. to noon at schools throughout Canada. The examination papers will be marked in May and the results will be circulated to schools as soon as possible thereafter.

Conditions of Examination

All examination materials, whether submitted for marking or otherwise, are the property of the Canadian Mathematical Congress. Decisions of the examiners are final. Cash prizes and scholarships may be awarded at the discretion of the examiners, who may choose to subdivide or otherwise alter the terms of awards.

Eligibility of Candidates

Candidates must be Canadian citizens or landed immigrants, and must be registered in a high school or its equivalent, as follows:

in Ontario the competition is open to students in Grade XIII or earlier;

Nothing in the foregoing is intended to preclude the student's enrolment in advanced mathematics courses, at his own institution or elsewhere.

Nomination of Candidates

All candidates of the Olympiad must be nominated by the chairman * of a provincial competition or by a school principal. Copies of nomination forms can be obtained from:

Professor G.H.M. Thomas,
Chairman, Organizing Committee,
Department of Mathematics,
University of Saskatchewan,
SASKATOON, Saskatchewan S7N 0W0.

* ONTARIO - Prof. K.D. Fryer, Department of Mathematics, University of Waterloo, WATERLOO, Ontario N2L 3G1

First Prize	--	$1500			
Second Prize	--	$1000	Fourth Prizes	--	5 prizes of $200
Third Prize	--	$500	Fifth Prizes	--	10 prizes of $100

The First Five Olympiads

A booklet containing the questions of the first five Olympiads and solutions to the questions and the names of the prizewinners has been prepared and may be obtained from:

Secretariat,
Canadian Mathematical Congress

at a cost of $2. including postage.

INSTRUMENTS FOR EVALUATION

The O.A.M.E. Publications Committee have made arrangements for the circulation of some materials from the Grand Valley Math Association. These are:

(i) Grade 9 Preparedness Test
(ii) Grade 13 Sample Examinations.

The publications are available at no cost to O.A.M.E. members and for a fee of $2.00 to non members of O.A.M.E. To obtain your copy, simply write

The Grand Valley Mathematics Assoc.,
161 William Street W.,
Waterloo, Ontario.

HELP WANTED

Are you a member of O.A.M.E. ?

Who would you choose to be Vice - President (President elect) ?

Who do you know who would be a good Executive member ?

Who would you like to represent you as a Councillor ?

The Nominating Committee needs your suggestions

Your O.A.M.E. Executive and Council will only be as good as your nominations make it.

If possible this nomination should be accompanied by information which supports the nomination.

Return to: D.W. Alexander

Please use the following form or its duplicate.

I nominate _____ (name)

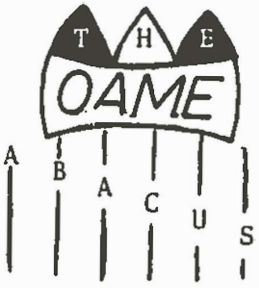
_____ (professional address)

for the position of _____ (Vice President, Executive, or Councillor)

Name of Nominator _____

Professional Address _____





"MATHEMATICS EDUCATORS
SHARE THEIR IDEAS"



Mr. Robert A. Tuck
247 Lee Dr.,
North Bay, Ont. P1A 3J2