ABACUS

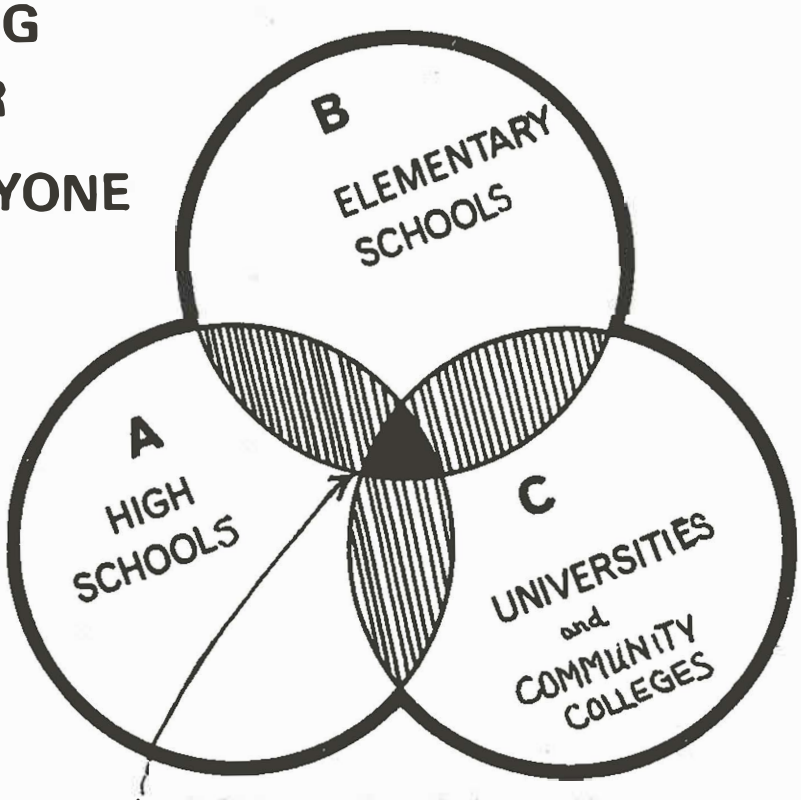
A PUBLICATION OF THE
ONTARIO ASSOCIATION for MATHEMATICS EDUCATION

OAME

Vol. 2 Number 2

November 1974

SOMETHING FOR EVERYONE



SECOND ANNUAL CONFERENCE

Ontario Association for Mathematics Education

U.W.O. LONDON

NEXT MAY

YOUR MEMBERSHIP IN OAME INCLUDES
THE ABACUS YOU CAN COUNT ON IT!
WRITE TO THE SECRETARY TREASURER

1974 - 75 EXECUTIVE

John C. Egsgard,
President,
Twin Lakes Secondary School,
Orillia, Ont. L3V 4Z2

Joan Routledge,
Vice President,
York County Board of Education,
Box 194,
Aurora, Ont. L4G 3H3

Morley MacGregor,
Secretary Treasurer,
247 Bright St.,
Sarnia, Ont. N7T 4E9



Letters To The Editor

from **Smiths Falls District Collegiate**

Dear ABACUS Editor

Mr. Evans, my math and computer science teacher, showed me the September, 1974 issue of the ABACUS. I tried to answer the proolem in the article "The Games of Two's".

Instead of trying it on a Mini-Calculator, I wrote a Fortran program to do it. The program and the answers are included.

I thought that you might be interested in the number of odd numbers produced for given odd numbers between 1 and 99.

```
    DAVID G GRIFFITH
    B=1
 50 B=B+2
    A=1
    C=B
    D=C
100 E=C *3+1
    E=E/2
200 N=F
    IF (E-N) 300,250,300
250 F=F/2
    GO TO 200
300 IF ((F*2)-1) 400,400,350
350 A=A+1
    C=F*2
    GO TO 100
400 WRITE (3,450) D
    WRITE (3,460) A
    IF (99-D) 500,500,50
450 FORMAT (/,' THE NUMBER IS ',F5.0)
460 FORMAT ('   IT TOOK',F6.0,' ODD NOS.')
500 CALL EXIT
    END
```

Thank You

David G Griffith

```
THE NUMBER IS     3.
 IT TOOK     2. ODD NOS

THE NUMBER IS     5.
 IT TOOK     1. ODD NOS.

THE NUMBER IS     7.
 IT TOOK     5. ODD NOS.

THE NUMBER IS     9.
 IT TOOK     6. ODD NOS.

THE NUMBER IS    11.
 IT TOOK     4. ODD NOS.
```

etc.

```
THE NUMBER IS    91.
 IT TOOK    33. ODD NOS.

THE NUMBER IS    93.
 IT TOOK     4. ODD NOS.

THE NUMBER IS    95.
 IT TOOK    38. ODD NOS.

THE NUMBER IS    97.
 IT TOOK    43. ODD NOS.
```

IN REPLY:

* Mr. Thomas Ciancome (Bishop Ryan H.S.) pointed out our un-Canadian spelling of "metre". We too were upset when we saw that this had slipped by us. We enclose Meter or Metre?in this issue to help explain the problem.

Thanks for writing.

* Assoc. Prof. I.S.Pressman (Carleton U) wrote in response to M Loring's article. This must wait for the next issue owing to lack of space.

Math Games With a Purpose

- *Mary Beaton*

One of the goals we try to achieve with our mathematics program is to make students learn to think creatively about mathematics. To achieve this goal, students should have opportunities to play mathematical games. There are a number of games available that foster critical thinking.

Commercially made games can be used to greater advantage in a school if they are stored in the materials center rather than in individual classrooms. The games can be brought to each classroom to enrich specific topics. Games increase understanding of mathematical concepts and add variety to a unit of work.

Three different games are based on the formation of equations. In order of difficulty, these games are *Heads Up*, *TUF*, and *Equations*. All of these games can be played at various levels. They all require cubes marked with numerals or operational signs and one or two timers. *Heads Up* requires the players to form equations in a stated time. The game of *TUF* can be played at many levels of difficulty according to how many and what color of cubes are used. Cubes marked with square root signs and logarithmic signs may be used to increase difficulty level. Players compete to form the equation containing the greatest number of blocks or to use all of one's blocks. *Equations* is a game involving constant decision-making. It was invented by Layman Allen, a professor of law at the University of Michigan. This game is open-ended and changes rapidly as each player takes his turn. The level of difficulty is determined by the number of cubes used, the color of the cubes, and the skill of the players. Seven-year olds can play the game with the twelve red blocks. Twelve-year olds and older children can play a more difficult game with five blocks of each color. It is advisable to begin with the simplest possible version of the game as described in the book of instructions and gradually add the more sophisticated rules. This game can be challenging to students from Grade II level to adult level.

In every elementary mathematics classroom, practice of basic facts is essential. Two commercially made games, *Numo* and *Operations Bingo*, allow for practice which is fun. *Numo* contains twenty-four different games which give practice in the facts of addition, subtraction, multiplication, division. Two games are based on equivalent fractions and two games are based on factoring.

The new feature in *Operations Bingo* is the bingo sheet. It is made of eleven columns and fourteen rows of squares. Before a game begins, the players decide whether they wish to play on a 3 x 3 square, a 4 x 4 square, or a 5 x 5 square. Each player chooses his own square of the specified size within the master sheet. A number or fact may appear on a card more than once. The rules of each game are decided ahead of time. They may be any one or a combination of: any four squares in a row; any three squares in a row; the four corners; all the numbers; or center free.

Dr. Beaton is Associate Professor, Faculty of Education, University of Calgary, Calgary, Alberta.

Science Research Associates have produced sets of *Cross Number Puzzles* to give practice in all arithmetic operations with whole numbers, fractions, decimals, and also percent. The most recent box of cross-number puzzle cards provides practice with story problems. Humor and everyday situations add to the interest of the problems.

Operational Systems Games are challenging for junior high school students. Least common multiple, greatest common denominator, sets, clock arithmetic and modular arithmetic are the mathematical ideas on which the games are based.

One of the most popular mathematical games is *Numble*. This is a game which involves practice in addition, subtraction, multiplication, division, sequencing of numbers and in finding multiples of three. It is a game of strategy, for the playing board contains double and triple number squares as well as double sequence and triple sequence squares. *Numble* is a mathematical game which is analogous to the word game *Scrabble*. It can be enjoyed by all age levels.

A set of teacher references entitled *Let's Play Games in Mathematics* is available in seven volumes. The games are cross-referenced with content topics and with student behavioral objectives. This series and another single volume entitled *Math Games for Greater Achievement* would be useful references for elementary and junior high school teachers.

If an adequate supply of mathematical games is available in the Materials Center, teachers can use games as a valuable strategy to reinforce other methods of learning mathematical concepts. A corner of the Materials Center could be set aside for the playing of math games, and students could be encouraged to drop in for a game at recess or at noon. Layman Allen found that when children in Detroit schools became interested in the game of *Equations*, the rate of truancy dropped. Let's give students a chance to have fun with a purpose.

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_____ and Lowell D. Gunn. *Let's Play Games in Mathematics*, Volumes I-VII, National Textbook Co., Skokie, Illinois, 1971.

Numble, Selchow and Righter Co., Bay Shore, New York, 1968.

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Operations Bingo, Creative Publications Inc., Palo Alto, California, and Educational Tools Inc., Wichita, Kansas.

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(Reprinted from delta-K, Alberta Teachers' Assoc. Math Council Newsletter)

OUT OF THE MOUTHS OF BABES ---

A grade I teacher asked at a Mathematics in-service whether an activity being done by children might mislead them mathematically.

Using wooden squares, the children were asked to make rectangles (or squares) whose areas were 1, 2, 3, 4, etc. and place their answers on a chart.

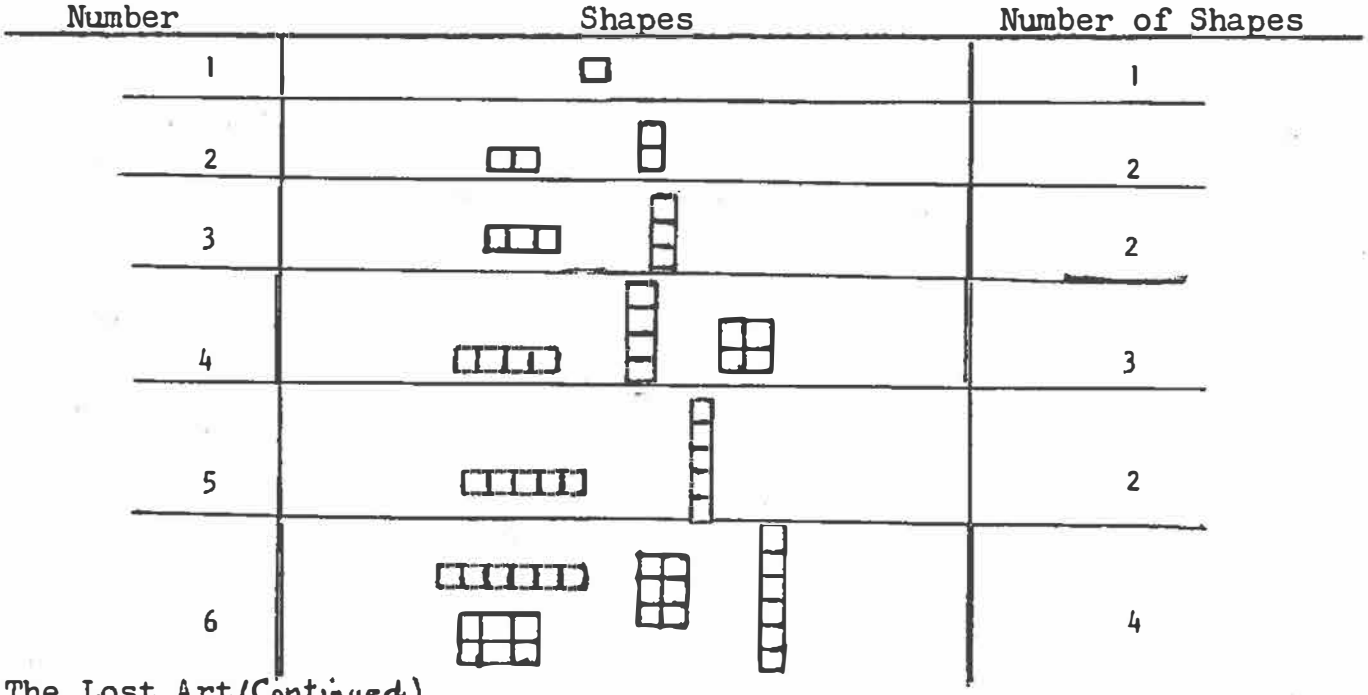
The chart was carried out to 10. The children wondered why some numbers gave just two shapes and others gave more.

Out of the mouths of these babes came perhaps the easiest definition of prime and composite numbers ever formulated.

Prime numbers give just 2 shapes

Composite numbers give more than 2 shapes

The number 1 is neither prime nor composite.



The Lost Art (Continued)

Several letters were received after the publication of the first article. One in particular made my effort worthwhile. It was from a retired math teacher - 85 years old. That proves that if you want to live a long and fruitful life - do problems. Here's a quote from his letter:

"I recall one question involving a pentagon and the use of Menelaus and Ceva, which I despaired of, and finally asked Jenkins of Jarvis if he had tried it. He gave it to young Jimmy, his son, who was then in grade 12, and Jimmy sent me his solution the next day."

I almost wept when I read this because J.T. Jenkins was my teacher at Jarvis and Jim junior and I were in the same grade 12 class. Jim is a topologist at Notre Dame University and I'm just a tired school teacher with a lot of memories.

I was delighted to hear from Frank Asbury, now retired but formerly with the Ministry of Education. Frank is still one of the most highly respected teachers of mathematics and I somehow knew that Frank would try #12 on that 1923 problems paper - He not only tried, but he sent in a solution! ONE UP FOR THE OLDTIMERS.

Here's a solution by a not so old teacher - Jim Swift of A.Y. Jackson.

2. Given $x^2 + y^2 + z^2 = 1$

$a^2 + b^2 + c^2 = 1$

$ax + by + cz = 1$

Prove $x = a$, $y = b$, $z = c$

Let $\vec{R} = x\vec{i} + y\vec{j} + z\vec{k}$ and $\vec{r} = a\vec{i} + b\vec{j} + c\vec{k}$

The given equations are equivalent to

$\vec{R}\cdot\vec{R} = 1$, $\vec{r}\cdot\vec{r} = 1$, $\vec{r}\cdot\vec{R} = 1$

$(\vec{R}-\vec{r})\cdot(\vec{R}-\vec{r}) = \vec{R}\cdot\vec{R} - 2\vec{r}\cdot\vec{R} + \vec{r}\cdot\vec{r} = 0$

$\therefore \vec{R}-\vec{r}$ is not perpendicular to $\vec{R}-\vec{r}$

$\therefore \vec{R}=\vec{r}$ and $x = a$, $y = b$, $z = c$

Here's my old fashioned solution. (It's really the same as the vector proof.)

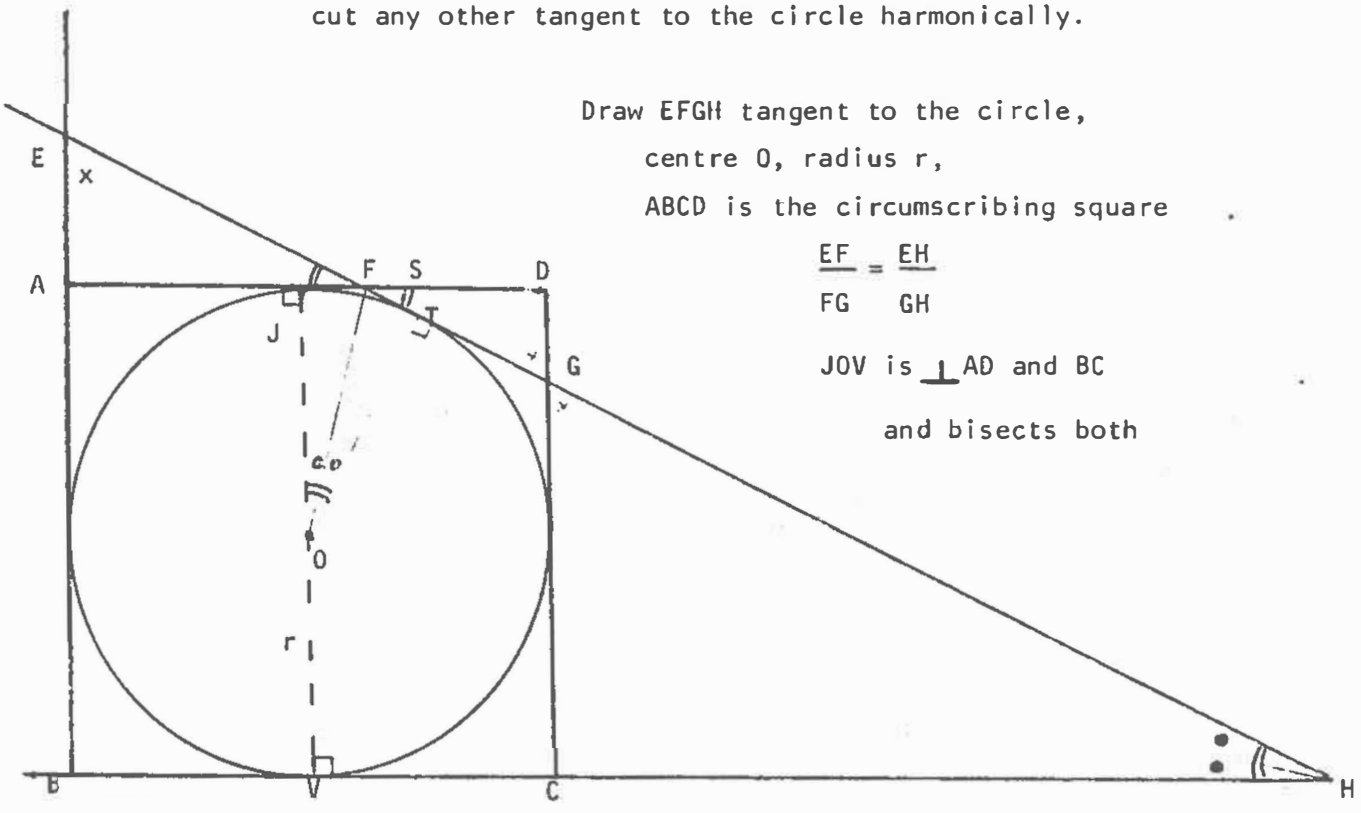
$(x^2 - 2ax + a^2) + (y^2 - 2by + b^2) + (z^2 - 2cz + c^2) = 0$

$(x - a)^2 + (y - b)^2 + (z - c)^2 = 0$

$\therefore x = a$, $y = b$ and $z = c$

And now for the Frank Asbury solution.

12. The four sides of a square circumscribing a circle cut any other tangent to the circle harmonically.



Draw EFGH tangent to the circle,
centre O, radius r,
ABCD is the circumscribing square

$\frac{EF}{FG} = \frac{EH}{GH}$

JOV is $\perp$ AD and BC
and bisects both

$\Delta JOF \sim \Delta TOF$

$\angle JOF = \frac{1}{2} \angle JOS = \frac{1}{2} \angle SFT = \frac{1}{2} \angle FHB = \angle OHV$

and $\Delta JOF \equiv \Delta VHO$

$AF - FD = (AJ + JF) - (JD - JF) = 2JF$

$BH - CH = BC = 2r$

$BH + CH = (BV + VH) + (VH - VC) = 2VH$

$$\therefore \frac{AF - FD}{AF + FD} = \frac{2JF}{2r} = \frac{JF}{r}$$

$$\frac{BH - CH}{BH + CH} = \frac{2r}{2VH} = \frac{r}{VH}$$

But in similar Δ's JOF and VHO

$$\frac{JF}{OJ} = \frac{OV}{VH} \quad \text{or} \quad \frac{JF}{r} = \frac{r}{VH}$$

$$\therefore \frac{AF - FD}{AF + FD} = \frac{BH - CH}{BH + CH}$$

$$\therefore \frac{AF}{FD} = \frac{BH}{CH}$$

But $\frac{AF}{FD} = \frac{EF}{FG}$ and $\frac{BH}{CH} = \frac{EH}{HG}$

$$\therefore \frac{EF}{EG} = \frac{EH}{HG}$$ (Gotcha!)

Frank said he went up quite a few dead ends before he got the solution.

The following is the 1961 problems paper that may have a more interesting range of questions. I'd be pleased to hear from you if you are trying these papers and your suggestions or solutions would be welcomed.

DEPARTMENT OF EDUCATION, ONTARIO

GRADE 13 ANNUAL EXAMINATIONS, 1961

PROBLEMS

NOTE 1. *Ten questions constitute a complete paper.*
NOTE 2. *A supply of squared paper and a book of mathematical tables should be obtained from the Presiding Officer.*

1. (*a*) What condition must be satisfied by the constants a, b, c for the quadratic function

$$f(x, y)=x^2-y^2+2ax+2by+c$$

to be the product of two linear factors?

(*b*) Indicate the nature of the graph of $f(x, y)=0$ in the general case, and also when the condition referred to in part (*a*) is satisfied.

2. To what extent are x, y, z, which satisfy the equations

$$x+y+z=2,$$
$$xy+yz+zx=1,$$

determined by the condition that they be real?

3. If s_n denotes the sum of the first n natural numbers, find the sum of the infinite series

$$\frac{s_1}{1}+\frac{s_2}{2}+\frac{s_3}{4}+\frac{s_4}{8}+\cdots.$$

4. Prove that

$$\frac{m}{(m+1)(2m+1)}<\sum_{r=m+1}^{2m}\frac{1}{r^2}<\frac{1}{2m},$$

for all integers m, where

$$\sum_{r=m+1}^{2m}\frac{1}{r^2}=\frac{1}{(m+1)^2}+\frac{1}{(m+2)^2}+\cdots+\frac{1}{(2m)^2}.$$

5. (*a*) In the triangle with vertices $(a, 0)$, $(-a, 0)$, $(0, b)$, prove that

(i) the medians concur in a point G (the centroid),
(ii) the right-bisectors of the sides concur in a point H (the circumcentre),
(iii) the altitudes concur in a point K (the orthocentre).

(*b*) Find the ratio $HG:GK$.

(*c*) Draw a figure to show the positions of these three points when $a=b$.

[OVER]

6. If the circle $x^2+y^2=r^2$ meets the x-axis in A_1 and A_2 and any chord parallel to the y-axis in P_1 and P_2, prove that the locus of the point of intersection of A_1P_1 and A_2P_2 is the rectangular hyperbola $x^2-y^2=r^2$.

7. A chord PQ of a parabola cuts the axis in R; tangents at P, Q intersect in T. Prove that RT is bisected by the tangent at the vertex.

8. Tangents are drawn to the circle $x^2+y^2=1$ from any point (h, k) on the fixed line $x=h$, where $h>1$, and also from the point $(-h, -k)$. Prove that the other two vertices of the parallelogram formed by these tangents lie on the ellipse

$$x^2+y^2\left(1-\frac{1}{h^2}\right)=1$$

for all values of k.

9. The inscribed circle of the triangle ABC touches the sides BC, CA, AB in the points P, Q, R respectively. Show that

$$\cos\frac{A}{2}:\cos\frac{B}{2}:\cos\frac{C}{2}=QR:RP:PQ.$$

10. Given that one side of a triangle is twice as long as another side, show that the angle opposite the first-named side is more than twice the angle opposite the other side.

11. Two equal rectangles, both inscribed in the circle $x^2+y^2=1$, with their axes of symmetry along the x-axis and y-axis, cross each other, forming a square $ABCD$ which is common to both rectangles.

(*a*) If θ is the acute angle between the diagonal of one rectangle and its major axis of symmetry, find, in terms of θ, the total area of the four rectilinear figures exterior to $ABCD$.

(*b*) Find the value of $\tan\theta$ when this total area is a maximum.

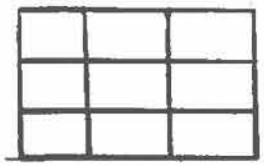
12. A cylinder of weight w pounds rests with its axis horizontal in a trough made of two planes which are perpendicular to each other and equally inclined to the vertical. A horizontal force of F pounds, applied at the top of the cylinder and perpendicular to its axis, is just on the point of turning it. Assuming that the cylinder in turning remains in contact with both faces of the trough, and that the coefficient of friction between the cylinder and each of the two planes is μ, show that

$$F=\frac{w\mu\sqrt{2}}{\mu^2(1+\sqrt{2})+1}.$$

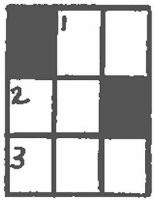
A CALENDAR OF CHALLENGES

Magic Square

1. Make a magic square using the numerals 3, 4, 5, 6, 7, 8, 9, 10, 11
2. The sums must be the same in all directions



Cross Number Puzzle



Across
1. 9 x 2
2. 9 x 10
3. 9 x 109

Down
1. 9 x 12
2. 9 x 11

Make your own cross number puzzle.

A Calendar of Challenges

Send in comments or your own challenges

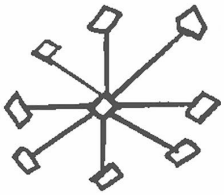
Magic

1. Ask a friend to choose any number.
2. Have him add 9.
3. Multiply the sum by 2.
4. Subtract 4.
5. Divide by 2.
6. Tell him to subtract his first number.
7. Now you can tell him his answer.
   It is always the same one.

A Relationship

1. Multiply any 2 factors
2. Multiply any 3 factors
3. Multiply any 4 factors
4. Multiply any 6 factors
5. What relationship is there between the number of factors and the products each time?
6. Does the same pattern hold for all sets of factors?

Place the numerals 30, 40, 50, 60, 70, 80, 90, 100, 110, in the diamonds below so that the sum is the same in all directions.
Use each numeral once.



Patterns

Multiply 11 x 11
Multiply 111 x 111
Multiply 1111 x 1111

What do you observe about the products?

Continue the pattern.

Bases

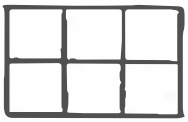
Rule a calendar for this month.
Write in the numerals using Base 5.
Try a more difficult base

Break the Code

There is an important message in English waiting for you below.
Can you discover what it says?
13, 1, 20, 8, 5, 13, 1, 20, 9, 3, 19...9, 19... 6; 21, 14
Do you agree with what it says?

Teachers: (Mathematics is fun)

1. Arrange 17 toothpicks to form 6 squares like those below.



2. Remove 6 toothpicks to make 2 squares.
3. Remove 5 toothpicks to make 3 squares.

Bases

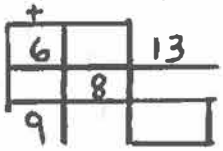
Write the age and weight of yourself and a good friend.
Change these to
a) Base 5
b) Base 8
c) Base 2
Which base do you prefer? Why?

Patterns

1. Choose any 3 different numbers less than 10.
2. Make as many two-digit numerals as possible (six)
3. Add the six different numerals.
4. Add the three original numbers.
5. Divide the sum of the 6 numbers by the sum of the 3 original numbers.
6. Choose three different numbers between 0 and 10 and repeat.
7. What surprise do you get?

Number Puzzles

1. Make a lattice as shown.
2. Put the operation sign on top.
3. Fill in some of the addends or sums.
4. Tell the students to add up, down, or across.
5. Complete the puzzle.



Magic Square

Complete the following magic square.

9		
2		
7		

The Toothpick Game

1. Take 12 toothpicks
2. Divide them into 3 piles
3. Find a partner to play with you.
4. Have him draw any number of toothpicks from any one pile. He may take them all if he wishes.
5. Now you draw.
6. If you can make your partner take the last toothpick, you win.

Remarkable Nine

1. Think of any number. Make it as long as you wish. 763298164750281
2. Rearrange to make another number. 128074651982376
3. Subtract the smaller from the greater.
4. The answer can always be divided by 9.

Magic Square

Complete the following magic square.

		6
		13
		8

Patterns

Multiply 6 x 7
Multiply 66 x 67
Multiply 666 x 667
Write the next three questions and their products.

Film

Order the film "Donald in Math magic Land" from the AV centre.
- Gr. 6 & up would find it beneficial
- view the film - have the pupils list the different uses of mathematics.

next day discuss Galileo's statement, "Mathematics is the alphabet with which God has written the universe".
- this statement comes in the film.

LEADERSHIP SEMINAR NIAGARA FALLS 1974



Angela Walsh of London signs up with Mr. & Mrs. H. at this year's fall Seminar.

From all accounts it was a very successful session.

THINK METRIC - METER OR METRE?

As you may know, the Board of Directors of the National Council of Teachers of Mathematics recently recommended that for ordinary use in the United States the spellings METER and LITER be used rather than METRE and LITRE. To most people, any argument over the spelling of the words is a tempest in a teapot. Nevertheless, a decision must be made by publishers and film producers, as well as many others, as they write about these units. Actually, the spelling of liter (or litre) is of less importance than the spelling of meter (or metre). This is because the liter is not a base unit in SI (Systeme International).

First, we should all understand that either spelling is correct and acceptable. The only question is, which is preferable?

In the United States, such words as center and theater are usually spelled with the "er" ending. This is phonetically defensible. Thus, for the sake of consistency with our way of spelling, we recommend the "er" ending for meter and liter. Canada and Great Britain, on the other hand, have traditionally spelled those words using the "re" ending. Thus, for the sake of consistency, we would expect those countries to favor (favour) the "re" ending in metre and litre.

For whatever value it may have for those who must make a decision as the write, it is the recommendation of NCTM that the "er" ending be used in ordinary day-to-day use, such as in textbooks, films, posters, charts, and displays. On the other hand, if the principal audience is the technical community or an international assembly of scientists, then the "re" ending would be preferable. This policy is consistent with that of the National Bureau of Standards.

We should all realize that no group, not even the National Bureau of Standards, has any authority to make the decision. Only public usage will determine the preferred spelling.

P.S. note:
Let's try to avoid ' ' '

From NCTM Bulletin for Leaders, June 1974

SEND YOUR MATH GAME, puzzle, activity, book review, ideas to:

ABACUS EDITOR
c/o CITY MATH CENTRE
OAK PARK SCHOOL
40 HUNT CLUB DRIVE
LONDON N6H 3Y3

FINANCIAL STATEMENT OAME YEAR ENDING AUGUST 31/74

Receipts

Balance on Hand	$ 3263.50
North Bay Annual Meeting Repayment of Advance	500.00
Excess of income over Expense North Bay	4314.25
Membership fees	7114.67
Gazette Advance OMC	700.00
Gazette Subscriptions	3355.97
Interest	253.38
Le Comite de langue francaise	375.00
Samuel Beatty Fund	500.00
Sales NCTM material	134.62
	20511.39

Trust monies

Western Region $729.05
St. Lawrence Region 306.15
1035.20

Audited by S. Wallace
D. Christensen
at Sarnia, Sept. 13/74

Expenditures

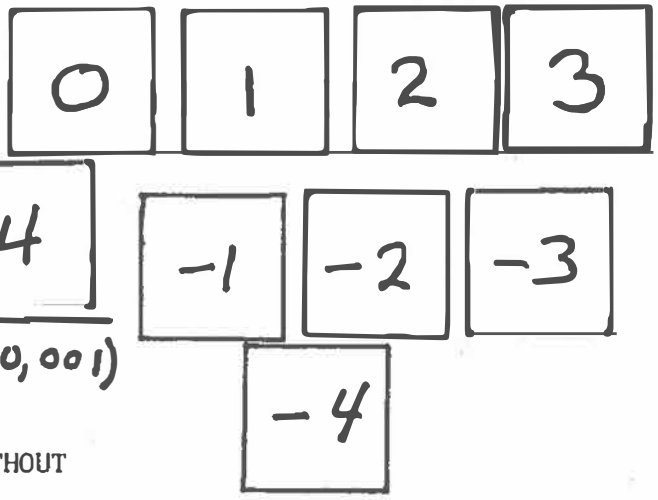
OEA Registration	$ 500.00
Telephone	120.71
Postage	680.51
Gazette Printing & Postage	2784.16
Abacus Printing & Postage	578.05
Office expenses	1695.99
Council travel	1416.34
Secretary travel	405.50
Kingston Annual Meeting Advance	500.00
Gazette refund	26.00
North Bay Annual Meeting Expense	60.30
NCTM affiliated Group fee	30.87
Publicity Committee	187.23
Secretary Honorarium	1000.00
COMA refund	123.00
NOMA refund	36.00
Expense for Nov. NCTM meeting	99.21
Gift for President	27.71
Senior Cyclic Review Committee	36.19
Balance on hand	10203.62
	20511.39

TRY IT YOURSELF CORNER

Originality not claimed.

Submit appropriate problems to ABACUS Editor

Have some square fun - that is, magic square fun. Cut out the 9 squares below and arrange them in a 3x3 matrix so that each row, each column, and each diagonal adds up to 0.



Simplify the following.
Time Limit: 2 minutes (100 seconds to think, 20 seconds to figure.)

$$\frac{999,999}{1,000,000^2-(999,999\times1,000,001)}$$

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