8 PAGE INSERT ELEMENTARY MATH

Ontario Mathematics Gazette

OAME – ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION

AOEM – ASSOCIATION ONTARIENNE POUR L'ENSEIGNEMENT DES MATHÉMATIQUES

Vol. 54 #2
December 2015

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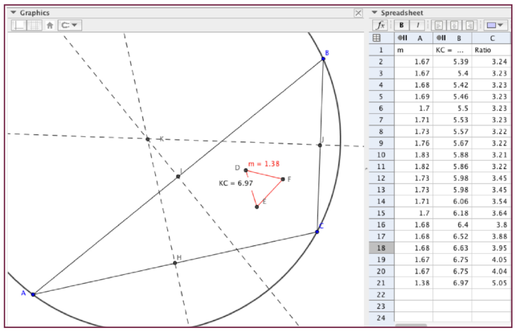

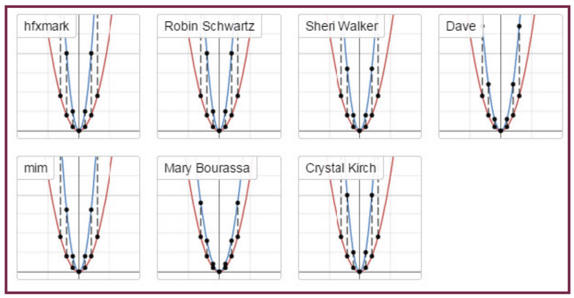

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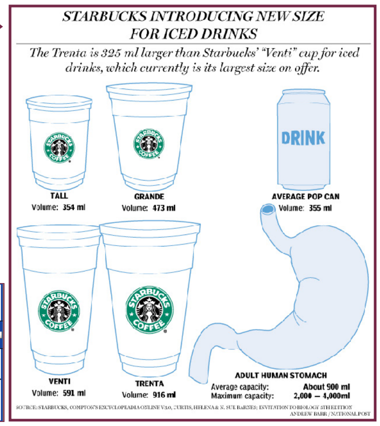

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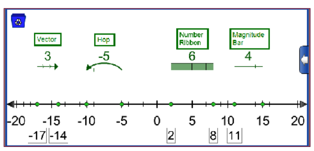

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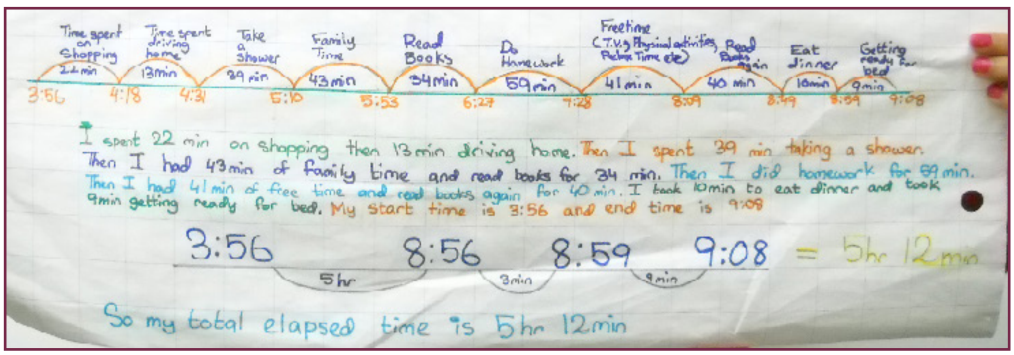

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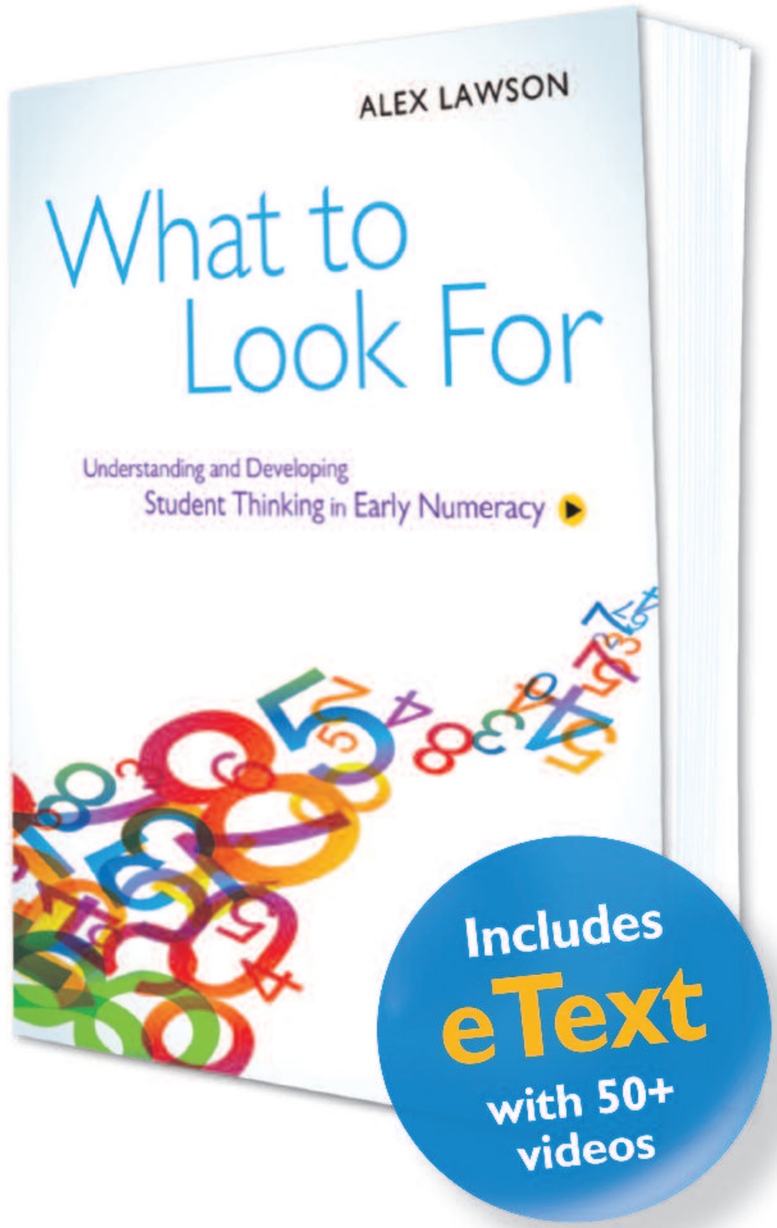

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Submission of Articles

The *Ontario Mathematics Gazette* (*OMG*) is looking for news items, articles, and good ideas that are useful to mathematics teachers and mathematics teacher education. We are seeking submissions, preferably from mathematics teachers K–12 and other mathematics education professionals, that describe innovative and creative approaches to mathematics teaching.

Please keep in mind the following criteria when making submissions to the *OMG*:

- The ideas/activities must be of interest to the readership.
- The ideas/activities must be fresh and innovative.
- The mathematics content must be appropriate for the readership.
- The mathematics content must be accurate.
- The article must be well written and easily understood.
- The article and its ideas must be free of sexual, ethnic, racial, or other bias.
- The article must not have been previously published, nor should it be out for review by other publications.
- The article must be original.

Articles must be word-processed in MS Word, double-spaced with wide margins, not exceeding 10 numbered pages of text, and prepared according to the *Publication Manual of the American Psychological Association, Sixth Edition*. Figures and diagrams should be drawn by computer, if possible, or drawn in black ink in camera-ready form. Embedded images must also be submitted separately in jpeg or tif format. Proof of the photographer's permission is required, and for **photos of students** under the age of 18, the written permission of a **parent or guardian is required**.

You must submit **one complete copy** of your article, embedded with any tables, figures, and captioned photographs or graphics, to the Editor, Dan Jarvis, along with **separate files for each of the text, graphics, and/or photographs**. Please email all files to Dan Jarvis at **dan.jarvis@oame.on.ca**.

Your name should not appear anywhere in your article, including websites, so that your article can be sent out for blind review. Your name, full mailing address, and email address must be included on a separate sheet. Upon review, you will be notified as to whether your article has been accepted for publication (as is, or pending minor or major revisions) or rejected.

The Editor reserves the right to edit manuscripts prior to publication. Once an article is published, it becomes the property of OAME.

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Advertisements for publication in the *Ontario Mathematics Gazette* should be sent to **Robert Sherk** at the above address. Courier is recommended to avoid possible delays. Deadlines for advertisements are January 23 for the March issue, April 1 for the June issue, July 1 for the September issue, and October 1 for the December issue.

Full-page advertisements are to be on 8.5" by 11" paper with a minimum of 0.5" margins and single sided. Each advertisement should be print ready, and colour advertisements should have no bleeds.

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▲ Editor's Message

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Dr. Daniel Jarvis is Professor of Graduate and Mathematics Education in the Schulich School of Education at Nipissing University, North Bay, Ontario. His research interests include instructional technology, integrated curricula, and mathematics of the workplace.

As the calendar year winds down, we are often drawn to reflect on the various personal, local, and global events that have occurred over the past 12 lunar cycles. Like many of you, I'm continually amazed with the speed at which new technologies are developed and adopted into our 21st century lives. These rapid transitions, as exciting as they may be, can often lead to somewhat embarrassing moments, like trying to finger-swipe your laptop screen, thinking it's a tablet; responding to individuals in public spaces, only to find they were actually communicating virtually through a concealed hands-free device; or avoiding near-death collisions with solid objects (or people) as you read/respond to emails en route to your next meeting (not legal while driving; less obviously non-advisable while perambulating).

As the focus on STEM (Science, Technology, Engineering, Mathematics) subjects continues to enjoy various degrees of increased interest at all levels of education, and in light of the recent re-boot of the *Star Trek* prequel films and the parallel ramp-up to the *Star Wars* sequel trilogy (*The Force Awakens*, no doubt, in a theatre near you), it behooves us to reflect for a moment on the technologies (usually involving mathematical insight and calculations) from these two ridiculously popular sci-fi franchises that have become (or have approached) actual realities in our post-modern world. Devoted "Trekkers", for example, celebrate the realization of transparent aluminum armour, communicators (think cellphone), hypospray, tractor beams, phasers, universal translators, telepresence (not to be confused with teleportation), and tricoders. S*tar Wars* fans (who regrettably lack a comparable moniker, it seems) similarly recognize the cinematic origins of holograms, intelligent droids, force fields, Luke Skywalker's macro-binoculars and his mechanical forearm prosthesis.

To what degree has new technology actually permeated our daily lives, and those of our students? Let's just focus on how we obtain information. Not sure about the names of the previous 22 Canadian Prime Ministers? Wondering when the Blue Jays last won the World Series? Dying to know the most commonly suffered human phobias (for those interested, such a list would include: arachnophobia [spiders], ophidiophobia [snakes], acrophobia [heights], agoraphobia [open/crowded spaces], cynophobia [dogs], astraphobia [thunder/lightning], claustrophobia [small spaces], mysophobia [germs], aerophobia [flying], carcinophobia [cancer], thanatophobia [death], glossophobia [public speaking])? (Top 100 Phobias List: **www.fearof.net**) Just "Google it," right?! In fact, don't bother typing anything at all, just speak these and other questions into a SmartPhone and Siri (on the iPhone, for example) will search it out on your behalf.

The advent of Google (Search, Maps, Earth, Translate, YouTube, etc.), Wikipedia, MOOCs, Learning Management Systems, social networking, increasingly inexpensive computers and mobile devices, and open-source software has led to remarkable changes in education—including both opportunities and ongoing challenges. How do (or can/should) these tools affect the learning, teaching, and assessment of mathematics? As usual, technology and these broader related questions will form part of the content in the pages that follow.

The December 2015 issue (V54N2) of the *Gazette* includes three articles, nine regular columns, the elementary school-focused *Abacus* insert, along with several other special features.

In their article entitled *Ontario Mathematics Olympiad (OMO): An OAME Tradition*, authors Paul Alves and Judy Mendaglio (OAME Past-President and President Elect, respectively) reflect on the history of, and local OAME chapter approaches to, the annual provincial OMO competition. Tim Sibbald offers an extensive exploration of the Morley Triangle, using GeoGebra software as a vehicle for algebraic, geometric, and numeric insights into the intriguing properties of this unique trigon. *Learning Elapsed Time Through After-School Activities*, co-authored by Miwa Takeuchi and Robin Coyle, presents the reader with an effective strategy for using number lines, along with analysis of authentic lived experiences of students, to reinforce the often perplexing measurement concepts related to time.

Regular columns include the following highlights: OAME President, Tim Sibbald (President's Message) discusses positive feedback, service opportunities, and a divisibility quirk; Assessment Abby (eponymous) focuses on group-work assessment and parental/guardian involvement; Greg Clarke, Agnes Grafton, Ross Isenegger, and Markus Wolski (Provincial Digital Learning Resources) highlight their interactive Number Line by Mathies, which allows students to represent numbers and to build deeper understandings of various operations; Mary Bourassa (Technology Corner) explores the Activity Builder feature in Desmos, which allows teachers to create investigations and visually track individualized student learning that may occur at different rates; and Carly Ziniuk (*In the Middle*) presents a math assignment focusing on StarBucks' new 916 mL cup size, the "Trenta" (reminds me of the 3L Coke bottles we witnessed in Ecuador last Spring—mucho litros, amigos!).

Todd Romiens (OAME/NCTM Report) explains the privileges and benefits of NCTM affiliation; Stewart Craven (Fields Institute Report) reports on the September 26, 2015 meeting of the Math Education Forum; Shawn Godin (What's the Problem?) heads out in search of buried pirate treasure; Ann Kajander (MB4T) examines teaching strategies and tools relating to the multiplication of integers; and Lynda Colgan (Hey, It's Elementary) showcases her new resource toolkit, *Inspiring Your Child to Learn and Love Mathematics*, through which she encourages parents/guardians to act as teaching partners in their child(ren)'s mathematical learning. In the *Abacus* insert, co-editors Mary Lou Kestell and Kathy Kubota-Zarivnij continue their Volume 54 focus on relational thinking, here looking at the operation of subtraction.

Volume 54 Issue 2 also includes several special features: (i) a letter to the editor by Tom Griffiths in response to the recent book review of Hoshino's *Math Olympian* (2015); (ii) an overview of the upcoming OAME Annual Conference "Leap into Math" 2016 by co-chairs Victoria and Jim Baumgart; (iii) an obituary regarding the late Frances Schatz—former *Abacus* editor, NCTM Rep, and *Gazette* contributor—written by friend Jack Weiner; (iv) two Ontario mathematics education researcher profile highlights; and (v) further excerpts from Lerman's *Encyclopedia of Mathematics Education* (2014) regarding a variety of contemporary math education issues.

So, as each of you look back at 2015, with its combination of educational successes and challenges, and ahead to your next journey (2016) around the nearest star, may the math force be with you, as you boldly go (or "go boldly," if you suffer from *splitinfinitivobia*!) where you've never gone (pedagogically-speaking) before.

References

Lerman, S. (Ed.). (2014). *Encyclopedia of mathematics education.* London, UK: Springer. ▲

▲ President's Message

TIM SIBBALD
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Have you ever considered that one of the toughest things a teacher learns to do is to provide a full repertoire of positive feedback? We learn that you "catch more flies with honey," but making positive feedback effective without seeming awkward or forced takes time to develop. Providing encouraging remarks that simultaneously help students improve is an art. Furthermore, it is an essential part of all evaluation because we need students to be motivated, at least to some extent, by the process. Nowhere is this public face of teaching clearer than quality report card comments. They take time because they are encapsulating so many interactions the teacher has had with the student.

Even when teachers have mastered the art of delivering positive feedback, they often have difficulties doing the same with their peers. It is easier to be collegial than evaluative when talking to another teacher. This is surely why teachers change the subject to last night's game or what one is doing on the weekend, rather than engage in discussions leading to professional growth. This is one of the primary reasons teachers so rarely observe each other's classes. It is not that we can't make time in the day—we do that for everything else. Rather, it is, in my opinion, concern regarding the feedback that will be received. We know the positive feedback is the thin veil of a deeper assessment that may miss the nuanced details of the context.

There is one form of positive feedback that circumvents the concern. It is through a nomination for one of the OAME's various awards. They are a realistic and credible way to acknowledge a colleague, which provides positive feedback without being evaluative. Some teachers would argue that they don't want to nominate anyone because the person may lose, but, as a person who has lost a couple of award nominations in my career, I can personally vouch for the powerful message that is received, simply by being nominated, which in itself is encouraging and motivating. Please consider acknowledging a colleague this year!

On a similar, but distinctly different, note, you have an opportunity to experience democracy by running for an elected position in the OAME. This could be within your chapter, the board of directors, or the executive. The democratic process also requires that we, the membership, take the time to vote! We collectively have an exercise in mathematics-education democracy. At the very least, I would ask you to participate by voting; at the very most, consider bringing your skill set to the electorate to learn about democracy in a different way.

Within chapters, voting occurs at different times; however, it is worthwhile to use the election period as a point for talking about roles in chapters. If we want to encourage new members, we need to actively engage in mentoring, and what better time to do that than during the winter months. In our chapters, we have room for more helpers—"Many hands makes for lighter work." As teachers, we know that having students come out for an event is what makes the event, yet we often make the same excuses as our students when it comes to our own engagement in chapter meetings.

One position that is not elected, but transitions now and again, is the role of *Gazette* editor. We have been fortunate to have had strong editorial leadership. The role is one that comes with timelines and production flow, but is clearly an opportunity for professional development that is second to none. Currently we are looking for an individual who is interested in mentorship through the Spring, with an eye to become the editor in the Fall. The role entails several people who contribute to the final product. The editor does not do the whole job, but is a pivotal contributor to the process. Anyone who would like more details is welcome to contact me.

As with all good math-education columns, I would be remiss if there wasn't a math moment in this column! Consider that the holidays are coming, with 15/12/25 being a key focus for many. If this is written 151225 (or any other order of the three component values), a curious number arises because each digit appears exactly twice. Is this number divisible by three? Recall that a number is divisible by three only if the sum of its digits is also divisible by three (and the sum of the digits in the sum, and so on…). So, in the case of a number where each digit appears twice, there must be an even sum of digits. This guarantees that the number is not divisible by three. In fact, 151225 = 5 x 5 x 23 x 263. Consider asking your students how many times a date with dual digits will happen next year.

On behalf of the OAME, I would like to take this opportunity to deliver best wishes for the holiday season. In our profession, it is a busy time, though there are often moments to pause and reflect on what we do. Most math teachers were inspired by a combined love of teaching and intrigue about mathematics. I hope you find time to tender the flames of mathematical intrigue as we approach "2 x 2 x 2 x 2 x 2 x 63," or should I say $2^5(2^6 - 1)$? Best wishes for a mathematically intriguing new year! ▲

▲ In the Middle: Starbucks Math – Part One

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Carly Ziniuk teaches Grade 9 Mathematics, Grade 12 Data Management, and AP Statistics at the Bishop Strachan School in Toronto, Ontario, Canada. She is very involved in using real-life data to engage her students in solving problems.

This is the first piece of a two-part activity looking at the math of Starbucks. This piece concentrates on the size of the cups, motivated by the larger size of the newest cup. The second part, which you will see in the upcoming issue, looks at the nutritional data that you can find on the Starbucks website and compares the Canadian and American nutritional information. Both of these activities centre on finding data in the real world and connecting it to the real problems our students might ask themselves.

Ideally, for this activity, students bring in a reusable coffee mug. It works really well to have access to a graduated cylinder. With a trip to your local Starbucks, you might be able to obtain some paper cups and sleeves. Although in Canadian locations the Trenta is available for iced drinks only, to avoid wasting so much plastic, I use paper cups instead.

If you don't want to do the entire experimental measurement components, there are a series of questions that you can complete with the rates and ratios given. I have also included some photos my students took with the cups taken apart. These photos could also be shown instead of the actual measuring.

Have you seen Starbucks' new cup size, the Trenta? Although it can be tricky to find this extra large cup size (it has been in some Canadian cities since 2014), you might be curious about the math behind the iconic coffee chain.

Take some time to examine the Trenta—all the opportunities for great questions might leave you with a caffeine-induced buzz!

Starbucks super-sizes with new 31-oz. "Trenta"

The Globe and Mail, January 17, 2011

Short, tall, grande, venti. Just when you thought you'd mastered the Starbucks lingo, the coffee company has announced it will launch yet another size, the "Trenta"—its biggest cup yet.

The 31-ounce size—basically the equivalent of about five industry- standard coffee mugs—will be made available only for iced coffee, iced tea, and iced tea lemonade drinks in the U.S., Reuters reports. Over the next few weeks, Starbucks will roll out the new size and it will be available in all of its U.S. coffee shops by May 3.

The Trenta-sized drinks will cost 50 cents more than similar drinks in the seven-ounce smaller venti size. Reuters adds that sweetened beverages in the new size will have fewer than 230 calories, while unsweetened Trenta drinks will contain fewer than 90 calories.

1. According to the article, it takes five standard mugs of coffee to get to 31 oz.
   a. How many "industry-sized" coffee mugs fill your cup?
   b. Although it might not be the most common Starbucks order, imagine your mug of coffee with one cream and one teaspoon of sugar. To maintain these proportions, how many creams and how many teaspoons of sugar would your mug need?
   c. How many creams and how many sugars does the Trenta need for these proportions?
2. It is interesting to note that the Trenta is a strange mathematical departure for Starbucks.

Starbucks Name	Imperial Volume	Metric Volume
Short	8 oz.	
Tall	12 oz.	
Grande	16 oz.	
Venti	24 oz.	
Trenta	31 oz.	

   a. Using the table, determine what the strange change is.
   b. Since the 2011 *Globe and Mail* article, the Trenta is now available in some Canadian locations. Unfortunately, the measures for volume are given in imperial measures. One oz. is approximately 29.5 mL. Complete the table to determine whether

the Trenta is a full litre.

c. Why do you think the Trenta didn't follow suit mathematically?

3. Using our cups and mathematical modelling,

a. How tall will the Trenta be?

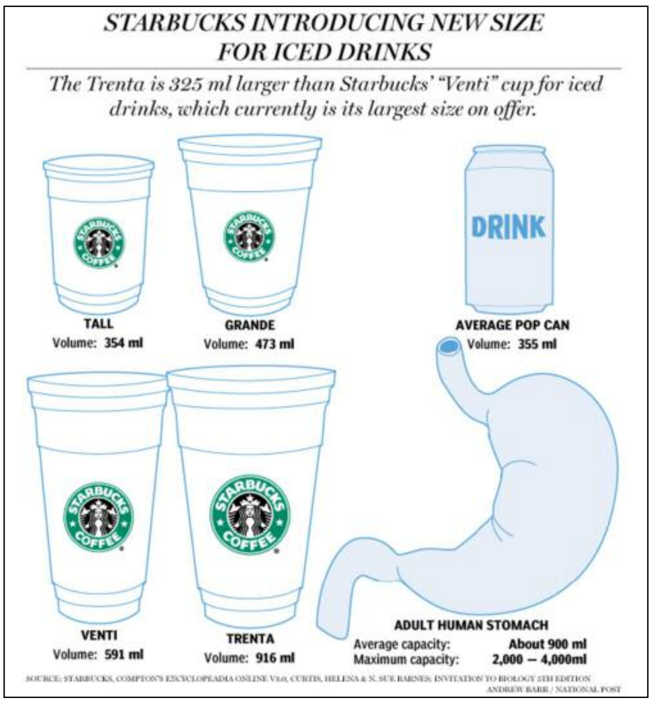
b. How wide will the mouth of the cup be?

c. What will its circumference be? Can you fit your two hands fully around the cup at the top?

d. How much paper will be used to make the cup?

e. What fraction of each cup is the paper sleeve?

4. Shortly after the Trenta was announced, *The National Post* put out this compelling graphic about a potential problem with the Trenta. Interpret this graphic in your own words.



5. The article maintains that most drinks will only cost 50 cents more. Now, again, although the Trenta is not available in all locations, and since pricing for Starbucks varies by location, we can use Toronto pricing to consider whether this 50-cent increase follows the other drinks, or whether you are truly getting a good deal.

Note: Short-drinks pricing is not available on the menu, but is available on request. Use your math as well, to determine a fair cost for these items.

Size	Iced Coffee	Iced Americano	Iced Misto	Iced Latte	Iced Caramel Macchiato	Java Chip Frappuccino
Short						
Tall	2.35	2.40	2.45	3.20	3.70	3.95
Grande	2.70	2.70	2.60	3.95	4.45	4.55
Venti	3.30	3.30	2.85	4.20	4.70	5.10
Trenta						

6. You may know about the Fair Trade movement, where the workers who produce and harvest the products, in this case coffee, ensure prices that are fair, safe labour, and environmental sustainably practices. As a result, the coffee costs more to pay for this care. Although pricing varies significantly, a reasonable approximation is that conventional coffee (unwashed, unroasted beans) costs approximately 70 cents/pound for every $1/pound for Fair Trade USA Certified.

a. How much would a Trenta of your favourite beverage above cost if you decided to make it Fair Trade?

7. NPR's look at the issues around uses and abuses of caffeine reported in 2006 that six ounces of standard coffee contained about 100 mg of caffeine, which was sufficient for most people to get enough of a physiological effect (a caffeine "buzz").

One of the reasons Starbucks coffee is so popular with its patrons is the strength of the coffee. Strength in this case does not mean strength of taste or flavour, but practically, the amount of caffeine. NPR reported that a Grande-sized Starbucks coffee had 400 mg of caffeine.

a. How many milligrams of caffeine would you get from your mug filled with standard coffee?

b. How many milligrams of caffeine would you get from your mug filled with Starbucks coffee?

c. How many milligrams of caffeine would you expect to get from a Trenta-sized cup of Starbucks coffee?

d. How many "buzzes" could you get from that Trenta-sized cup?

8. Interestingly, the other iconic Canadian coffee chain, Tim Hortons, plays a little with their coffee sizes south of the border. According to the 2014 Rrroll Up The Rim To Win Rules:

*NOTE: The cup size names are the same in Canada and the U.S., but the ounces differ. In Canada, the cup sizes are 10 oz. (medium), 14 oz. (large), and

20 oz. (extra large). The cup sizes in the U.S. are 14 oz. (medium), 20 oz. (large), and 24 oz. (extra large).

a. What do you notice about the mathematical differences between Starbucks' and Tim Hortons' cup-size increments?

b. Although the Tim Hortons website only provides the information about Canadian franchises, the caffeine amount in the Tim Hortons coffees are also quite different, with a Small 80 mg caffeine, Medium 100 mg, Large 140 mg, and Extra Large 200 mg. Using the sizes in oz. and the caffeine in mg as two variables, create two linear equations and compare the two companies.

c. What do you notice about the size-to-caffeine relationships between the two companies?

Starbucks Math – Part One – Answers

1. According to the article, it takes five standard mugs of coffee to get to 31 oz.

   a. How many "industry-sized" coffee mugs fill your cup?

   3 1/5 is the size of a standard cup, so 6.2 oz. Many travel mugs can be as much as 3 of these (so approximately 20 oz.).

   b. Although it might not be the most common Starbucks order, imagine your mug of coffee with one cream and one teaspoon of sugar. To maintain these proportions, how many creams and how many teaspoons of sugar would your mug need?

   If you had a travel mug, you would need three creams and three sugars.

   c. How many creams and how many sugars does the Trenta need for these proportions?

   If you had a Trenta, you would need five creams and five sugars.

Starbucks Name	Imperial Volume	Metric Volume
Short	8 oz.	236 mL
Tall	12 oz.	354 mL
Grande	16 oz.	472 mL
Venti	24 oz.	590 mL
Trenta	31 oz.	914.5 mL

2. It is interesting to note that the Trenta is a strange mathematical departure for Starbucks.

   a. Using the table, determine what the strange change is.

   The other sizes are all divisible by four, but the Trenta is just 1 oz. short of 32 (which would be).

   b. Since the 2011 *Globe and Mail* article, the Trenta is now available in some Canadian locations. Unfortunately, the measures for volume are given in imperial measures. One oz. is approximately 29.5 mL. Complete the table to determine whether the Trenta is a full litre.

   No luck, it is over 85 mL short!

   c. Why do you think the Trenta didn't follow suit mathematically?

   I have no clue! Seriously, why?

3. Using our cups to look for a pattern,

   a. How tall will the Trenta be?

   When we looked at the paper cups, the heights were 9.3 cm, 11.6 cm, 15.5 cm, and 17.1 cm for the first four cups. If you make a line using technology, the line falls around 25 cm for the height.

   b. How wide will the mouth of the cup be?

   The top two cups have the same diameter because they want to use the same lids. This makes it hard to use math to make a decision. If they use the same lids, then it will be the 8.6 cm diameter value again.

   c. What will its circumference be? Can you fit your two hands fully around the cup at the top?

   The circumference for the top two cups is 27 cm. See notes above.

   d. How much paper will be used to make the cup?

   ***Teacher's note:** This could be harder than it looks, since it is probably not a truncated cone, but we can estimate it is… or you can take the cups apart to use the net on 1 cm graph paper to check. This is a lot of fun to do the math on this!*

   e. What fraction of each cup is the paper sleeve?

   Our work shows that for the short cup, it is approximately 1/3, but for the Venti, it is a little more than 1/6 (since the cups are almost 2x as high).

4. Shortly after the Trenta was announced, *The National Post* put out this compelling graphic about a potential problem with the Trenta. Interpret this graphic in your own words.

This looks like, on average, your body can only handle one Trenta! A tall drink is approximately the same size as a pop can, and three of them are approximately the size of your stomach.

5. The article maintains that most drinks will only cost 50 cents more. Now, again, although the Trenta is not available in all locations, and since pricing for Starbucks varies by location, we can use Toronto pricing to consider whether this 50-cent increase follows the other drinks, or whether you are truly getting a good deal.

   Note: Short-drinks pricing is not available on the menu, but is available on request. Use your math as well, to determine a fair cost for these items.

   I used a linear model for all of these costs and extrapolated to the two endpoints with the equations, but even just using the graphs works well enough.

Size	Iced Coffee	Iced Americano	Iced Misto	Iced Latte	Iced Caramel Macchiato	Java Chip Frappuccino
Short						
Tall	2.35	2.40	2.45	3.20	3.70	3.95
Grande	2.70	2.70	2.60	3.95	4.45	4.55
Venti	3.30	3.30	2.85	4.20	4.70	5.10
Trenta	4.56	4.50	3.83	5.66	6.16	6.69

Every one of these items is a good deal if it is just 50 cents more!

6. You may know about the Fair Trade movement, where the workers who produce and harvest the products, in this case, coffee, ensure prices that are fair, safe labour, and environmental sustainably practices. As a result, the coffee costs more to pay for this care. Although pricing varies significantly, a reasonable approximation is that conventional coffee (unwashed, unroasted beans) costs approximately 70 cents/pound for every $1/pound for Fair Trade USA Certified.

   a. How much would a Trenta of your favourite beverage above cost if you decided to make it Fair Trade?

   70 cents becomes 1 dollar. So every 1 cent of conventional coffee costs 1.43 cents if it is Fair Trade.

Trenta	4.56	4.50	3.83	5.66	6.16	6.69
Fair Trade Trenta	6.51	6.43	5.47	8.09	8.8	9.56

7. NPR's look at the issues around uses and abuses of caffeine reported in 2006 that six ounces of standard coffee contained about 100 mg of caffeine, which was sufficient for most people to get enough of a physiological effect (a caffeine "buzz").

   One of the reasons Starbucks coffee is so popular with its patrons is the strength of the coffee. Strength in this case does not mean strength of taste or flavour, but practically, the amount of caffeine. NPR reported that a Grande-sized Starbucks coffee had 400 mg of caffeine.

   a. How many milligrams of caffeine would you get from your mug filled with standard coffee?

   *If your travel mug had 20 oz., then (20/6)*100 or approximately 333 mg of caffeine.*

   b. How many milligrams of caffeine would you get from your mug filled with Starbucks coffee?

   *If a Grande cup has 16 oz. for 400 mg, then your travel mug would be (400/16)*20 approximately 500 mg of caffeine.*

   c. How many milligrams of caffeine would you expect to get from a Trenta-sized cup of Starbucks coffee?

   *If a Grande cup has 16 oz. for 400 mg, then the Trenta would be (400/16)*31 approximately 775 mg of caffeine.*

   d. How many "buzzes" could you get from that Trenta-sized cup?

   This is the equivalent of almost 8 cups of standard coffee!!!!!

8. Interestingly, the other iconic Canadian coffee chain, Tim Hortons, plays a little with their coffee sizes south of the border. According to the 2014 Rrroll Up The Rim To Win Rules:

   ***NOTE:** The cup size names are the same in Canada and the U.S., but the ounces differ. In Canada, the cup sizes are 10 oz. (medium), 14 oz. (large), and 20 oz. (extra large). The cup sizes in the U.S. are 14 oz. (medium), 20 oz. (large), and 24 oz. (extra large).

   a. What do you notice about the mathematical differences between Starbucks' and Tim Hortons' cup-size increments?

   You might notice that their extra large is the same as the Starbucks Venti; however, the others are smaller in Canada, but larger in the U.S.

   b. Although the Tim Hortons website only provides the information about Canadian franchises, the caffeine amount in the Tim Hortons coffees are also quite different, with a Small 80 mg caffeine, Medium 100 mg, Large 140 mg, and Extra Large 200 mg. Using the sizes in oz. and the caffeine in mg as two variables, create two linear equations and compare the two companies.

   *TimHortonsCaffeine = TimHortonsVolume*10 (Note the difference in units)*

   StarbucksCaffeine = StarbucksVolume(25)*

   c. What do you notice about the size-to-caffeine relationships between the two companies?

   Starbucks caffeine is 2.5 times the amount of caffeine as Tim Hortons! ▲

▲ Call for Manuscripts

The *Ontario Mathematics Gazette* is inviting manuscripts relating to all grade levels, including post-secondary education.

Instructions for submission of manuscripts are found on page 1 of any *OMG* issue.

Contact the Editor for further details.

▲ Learning Elapsed Time Through After-school Activities

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Robin Coyle has been teaching for 12 years in the TDSB, the last 6 of which have been at George Webster ES, an inner city model school. She has a Math Specialist degree and is passionate about proving a program where math makes sense to all students.

Abstract: A teacher in an urban school designed culturally and linguistically relevant lessons on measurement by utilizing students' timelines on evening activities. The lessons opened up multi-layered learning. Students learned about measurement, language, and also about their classmates. The lessons also facilitated English Language Learners' participation in mathematics lessons.

Teaching in Linguistically Diverse Classrooms

Many schools in North America are becoming linguistically diverse. In Canada, 17.5 percent of the total population (5.8 million people) reported speaking at least two languages at home (Statistics Canada, 2011). In the United States, 21 percent of households reported using languages other than English (United States Census Bureau, 2013). With a vision of providing high-quality mathematics instruction to all the learners, the National Council of Teachers of Mathematics (2014) states: "Our vision of access and equity requires being responsive to students' backgrounds, experiences, and knowledge when designing, implementing, and assessing the effectiveness of a mathematics program" (p. 60). *Culturally relevant pedagogy* is a pedagogical approach, where cultural practices of students' community and home are meaningfully embedded in mathematics learning (e.g., Leonard & Guha, 2002; Torres-Velasquez & Lobo, 2004). Under Ontario's Equity and Inclusive Education Strategy (2009), culturally relevant pedagogy has been embraced in Ontario to set high academic expectations for all learners (Litearcy and Numeracy Secretariat, 2013).

The lessons we introduce here stem from a teacher's efforts to make mathematics curriculum culturally relevant in one of the most linguistically diverse schools in an urban city of Canada. The lessons were also guided by the idea of *identity texts*. The term *identity texts* is defined as "the products of students' creative work or performances carried out within the pedagogical space orchestrated by the classroom teacher. (…) The identity texts then hold a mirror up to students in which their identities are reflected back in a positive light" (Cummins & Early, 2011, p. 3). This idea of identity texts has been used to enrich classroom practices, especially for students whose identities tend to be historically undervalued.

Like many classrooms in urban cities in North America, the classroom we introduce here is linguistically and ethnically diverse. In this Grade 5 classroom, the majority of students were English Language Learners (ELLs), either the children of immigrant families, whose home language was not English, or the children who grew up in non-English-speaking countries. The students' home languages included Bengali, Cantonese, Farsi, French, Mandarin, Romanian, Russian, Spanish, Tamil, Urdu, and Vietnamese. This diversity of students' language background presented a unique challenge to teachers because bilingual teaching could not be directly applied.

Making Measurement Lessons Culturally and Linguistically Relevant

As a teacher of an urban classroom, I (Robin) have been trying out various teaching strategies to maximize students' backgrounds and their languages. Some of the effective teaching strategies are to pair up students who share the same language, and to create a sharing time for various computation strategies (for example, Bengali students introduced their unique use of body parts for counting). In order to foster a sense of community in the classroom, I use community circles as a routine activity, where students and the teacher share stories about themselves and come to know each other better. Mathematics talk in each unit is typically organized in the following forms: 1) checking students' prior knowledge and understanding, 2) sharing learning goals, 3) checking students' strategies during problem solving, and 4) sharing various strategies and consolidating students' understanding. Daily mathematics talk serves as formative assessment, where the teacher offers linguistic support when ELLs are struggling to express their ideas. Models and diagrams are effective tools for ELLs to communicate their thinking processes.

In reaching ELLs in mathematics classrooms, one of the challenges that many teachers can face is a lack of teaching materials that are carefully designed for learning both the language and mathematics simultaneously. For example, available textbooks are often not relevant to many ELLs' lives outside the school, and their linguistic density can be an obstacle for mathematical learning. In order to make mathematics learning relevant and meaningful to students, collegiality at the school is essential. For this project, an ESL teacher at the school initially shared the idea of identity texts.

The curriculum strand chosen was measurement, on the topic of calculating elapsed time. In the province of Ontario, Canada, the topic of elapsed time is addressed in Grade 4 and Grade 5 (Ontario Ministry of Education, 2005). By the end of Grade 4, students are expected to solve word problems that involve time intervals. By the end of Grade 5, students are expected to learn how to estimate and determine elapsed time, given the duration of events expressed in minutes, hours, days, weeks, months, or years. In the curriculum, using various tools (including a timeline) and strategies are expected.

Typical word problems used in the unit of measurement attempt to connect to students' everyday lives, but often are generic, as seen in the following problem.

> If you wake up at 7:30 a.m., and it takes you 10 minutes to eat your breakfast, 5 minutes to brush your teeth, 25 minutes to wash and get dressed, 5 minutes to get your backpack ready, and 20 minutes to get to school, will you be at school by 9:00 a.m.? (Ontario Ministry of Education, 2005, p. 69)

Although this kind of problem may have relevance to students' lives, it does not offer students the necessity to measure elapsed time, using a relevant tool (e.g., analog and digital clock, stopwatch), and also to estimate elapsed time. In addition, especially in a classroom with students from diverse backgrounds, there is no single "normal" everyday activity that students engage in outside the school.

The topic of elapsed time can be mathematically challenging for students because they have to coordinate hierarchical units such as hours and minutes (Kamii & Russell, 2012). When identifying the duration between 8:30 a.m. and 11:00 a.m., children have to understand that hours are at a higher hierarchical level and minutes are at a lower hierarchical level. In addition, students can experience difficulties because hours and minutes do not follow a base-ten system.

After determining prior knowledge of reading time and calculating elapsed time, the students engaged in generic time problems involving the relationship between units of time and solving elapsed time in a variety of ways. If students were struggling with understanding a hierarchical relationship between hours and minutes, I (Robin) revisited the concept by using a clock model with five-minute intervals and one-minute intervals. The culminating lesson began about halfway through the unit. As a first step, the students brought a worksheet home and recorded their Wednesday evening activities in a table format, as a preparation for the upcoming lesson on elapsed time. They were asked to record the time that they arrived home and the start time and end time of each activity that they engaged in until bedtime (by using the units of hours and minutes). How to use analog and digital clocks and stopwatches were introduced in prior lessons. In order to record elapsed time of each activity, students chose to use tools that were available at home. Once students completed a table, they created a timeline to represent their evening activities. The idea of an interval and an open number line were introduced and modelled in previous lessons. Diverse representations of time intervals (e.g., a number line with intervals and an open number line) and supporting students to measure elapsed time of each evening activity were one of the most effective pedagogical tools. In the class, students identified how long each of their evening activities lasted and how much time passed from the time they arrived home and to the time they went to bed.

Students' Learning through the Lessons

Students' timelines showed not only their typical Wednesday evenings, but also portraits of their identities. Everyone spent the evening differently. The timeline depicted how and when they ate dinner, what kinds of hobbies they had, and how religion was significant to some students' daily lives. Once students completed their timelines, with the help of an ESL teacher in the school, a book titled *A Collection of Timelines and Reflections on When We Get Home* was created. The book is a collection of students' photos, their timelines, and their reflections. In the book, every student is smiling and proudly holding up their completed timeline with the calculated elapsed time. Students' reports on their evening activities uniquely represented a part of who they are. This book became community identity texts, in which students projected their identities, and through which teachers and other students could learn more about who those students are.

One of the successful aspects of the lessons was to highlight various strategies to calculate elapsed time. Many students were successful in showing their understanding of hierarchical relationships between hours and minutes. One common strategy was to write down all the time intervals between the starting time and the ending time of each activity (e.g., homework, eating dinner, video game) and adding the elapsed times of each activity, as seen in Figure 1: Fariha's Timeline (all the students' names are pseudonyms). Another strategy was to use different sizes of intervals (or "jumps," as students called them) to visualize the hierarchical relationship between hours and minutes. After calculating hours (bigger jumps) passed from the start time to the end time, minutes (smaller jumps) were calculated (as seen in Figure 2: Jessica's Timeline). Some students rounded up minutes to the unit of 15 minutes (e.g., 12 minutes were rounded up to 15 minutes), in order to estimate and determine elapsed time.

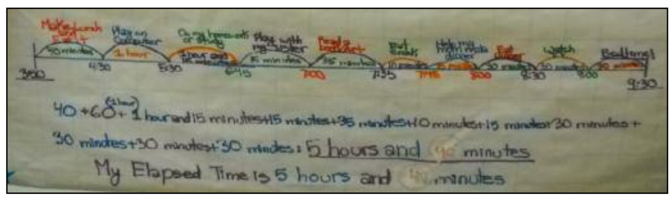

Figure 1: *Fariha's Timeline*

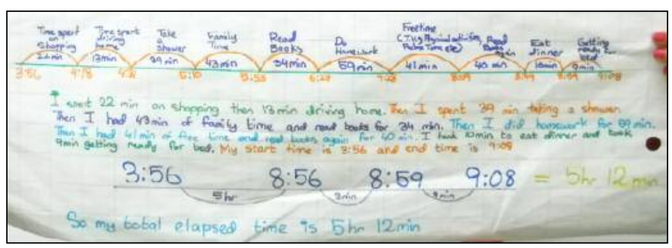

Figure 2: *Jessica's Timeline*

All students, including newly arrived ELLs, participated in this activity. For example, Hassan was one of the newly arrived ELLs. With the help of a timeline, he grappled with the terms and concepts of elapsed time and wrote: "elapsed time: 6 hours 30 minutes – it took me 6 hours 30 minutes to come home and to go to bed." The sentence he wrote is not extensive, but showed his emerging understanding of key grammatical rules, including tense, use of the to-infinitive (to come home, to go to bed) with a formative subject, "it." These grammatical rules were acquired along with his emerging understanding of elapsed time. In the book that we created, we included all of the students' work. Other students' work gave ELLs an opportunity to see a variety of ways to express elapsed time, which helped expand their linguistic repertoire (e.g., “I spent 39 minutes taking a shower,” “I did my homework for 59 minutes,” as seen in Jessica’s timeline).

In their reflections, students expressed how helpful this lesson was for them. For example, Afra wrote, “Making this timeline helped me with adding time and I used to have trouble adding time, thanks to the timeline I don’t have trouble adding time now!” For some students, a metaphor of “jumps” in time intervals helped them understand the difference in units of elapsed time. Eila wrote, “I have learned that the jumps in a timeline depend on how much time has passed from one time to another.”

Students also described the importance of estimation. Faced with computational complexities of adding time, students can face difficulties in grasping a realistic sense of elapsed time. To make a complex computation easier, some students started with estimating or computing rounded-up numbers. Ipsa said, “I learned to round it to the nearest tens to make it easier for me.” Considering students were initially facing a challenge in understanding and computing elapsed time, students’ works and their reflections demonstrated their developed understanding on this topic.

Many students reflected how this activity helped them understand their classmates better, including their similarities and differences. Students compared each other’s timelines and listed similarities and differences, using Venn diagrams. Diversity in after-school activities was a big realization for students and the teacher. As represented in Malikah’s timeline (Figure 3), she devotes a significant amount of time for prayer. It is also noteworthy that her timeline showed that she spent substantial hours for academic tutoring outside the regular classes (she categorized it under “doing homework”). Looking at timelines of other students, I (Robin) wrote, “The timelines help us in the classroom community because we can know each other better and we will have better understandings of each other.” As a teacher, I also appreciated this activity for understanding my students better, as stated in my reflection: “I learned so much about my children, including recent immigrant students. I had no idea what time they were having a dinner and what their evenings were like—so it was a big ‘ah-hah’ for me.”

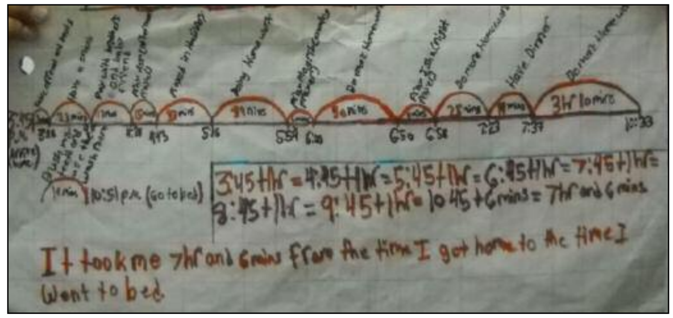

Figure 3: *Malikah’s Timeline*

Conclusion

What we highlighted is how a measurement lesson can be enriched as a tool to promote students’ mathematical understanding and language development, while honouring students’ lives outside the school and their identities. Such pedagogical approaches address multiple dimensions of learning simultaneously (Figure 4). In the lessons we introduced, students developed a deeper understanding of elapsed time, various strategies to determine elapsed time, along with a better understanding of their classroom community. Students’ identities—the sense of who they are—were inscribed in the book created, based on the students’ timelines. Language-learning opportunities were embedded in these measurement lessons. Students came to understand and use the term *elapsed time* and also the patterns of sentence structures used to describe elapsed time. Students described their evening activities in writing, read others’ descriptions, and wrote reflections about what they learned through sharing each other’s timeline. The seemingly simple question, “What did you do at home yesterday evening?” can open up multi-layered mathematics lessons.

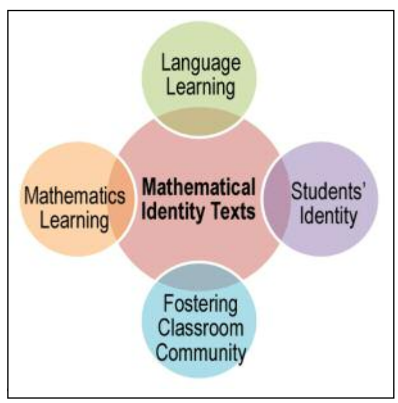

Figure 4: *Multidimensional Learning through Creating Mathematical Identity Texts*

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Acknowledgement:

We would like to acknowledge Jennifer Fannin for her pedagogical suggestion to generate identity texts. ▲

▲ Letter to the Editor

Dear Editor

This is with reference to Tim Sibbald's book report on *The Mathematics Olympian*.

I also found this book an excellent read, as did my wife, who is not a math teacher. We plan to make the book available as a Christmas gift to all our senior students in the Math Challenge at Western program.

I was disappointed at the concerns expressed about mathematics contests. I have been involved in mathematics contests in all their aspects for over 55 years, and still am.

We use the contests as the main teaching aid in our program, which incidentally involved 370 students and 10 volunteer teachers last year. We find that the contests provide a much more interesting set of material for the students than would a textbook and curriculum. Many of our students, especially elementary, describe maths at school as boring. This is a complaint we do not hear with the material we use. The questions in the contests are designed by some of the best teachers in the country—they are varied; they are from simple to extremely challenging; and they are a route to digressions into learning and teaching some wonderful mathematics. I believe that much more mathematics would be learned in the classroom if contests accompanied the use of textbooks.

I believe that the concerns expressed by Mr. Sibbald were more about the competition aspect than the mathematics. From my experience, the competition aspect is a very minor part of the scenario, and affects an incredibly small proportion of those involved. For most of the students, any competition is with themselves. Can they improve on their last performance, or can they get a perfect paper?

The only students who are really competing seriously with each other are those with the goal of representing Canada in the International Olympics, and this is a very small select group. I have worked with many of these students, and they do not exhibit the aggressive and negative aspects of competition as often seen in sports and politics. The students involved are most likely to appreciate another's better solution than their own, as opposed to reacting negatively to another student's better response.

Overall, I find that students learn mathematics well and in a pleasant way through doing math contests. I also find that they learn how to learn. The competition aspect is not significant.

Tom Griffiths
Past President OAME

▲ Hey, It's Elementary: Parents as Mathematics Teaching Partners: A Toolkit for Ontario Families

LYNDA COLGAN
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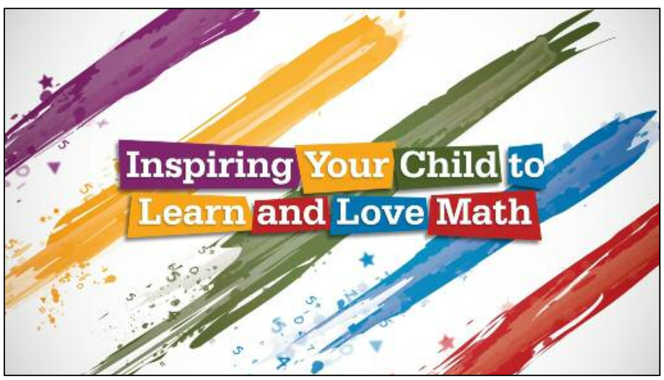
Lynda Colgan's career has included roles as a classroom teacher, a university professor, and newspaper columnist. Her contributions to mathematics and its teaching have been recognized through awards such as the Marshall McLuhan Foundation Distinguished Teacher Award. Lynda always exhibits a passion for mathematics and views her professional mission as dispelling the myth that math is the bad guy.



Inspired by my (favourite) high school teacher, mentor, colleague, and friend—the legendary Peter Saarimaki—I have committed much of my career as a mathematics educator to the inclusion of parents (including guardians and family members) as valued and valuable members of a child's teaching team. Initially, I followed closely in Peter's footsteps, taking my lead from him as I embarked on my role as district-wide, K–Grade 12 Math Coordinator for the Board of Education for the City of Scarborough; thus, I included parents in every initiative. With Shirley Fairfield, I became an early pioneer in bringing *Family Math* to Scarborough schools and families, and a one-person distribution centre for Peter's popular *Family Math Calendars*, purchasing them for every school, school council, community centre, after-school program, and math club in the city. I conducted mathematics workshops for parents in local libraries, encouraging them to use TVO programs such as *Mathica's Mathshop* at home to support informal learning. I coordinated family-oriented Saturday afternoon Scarborough *Math Trail Adventures* (based on the models created by Dr. Eric Mueller at Brock University) and recruited parents to be facilitators for regional and provincial Math Olympics events.

Since coming to Queen's, my passion for and commitment to parent engagement has never waned. For nearly two decades, I have sponsored summer *Family Math* courses for teachers and school council members; generated a successful *Family Math* calendar and a set of *Family Math* placemats; coordinated a city-wide math party (Kingston Counts!); facilitated numerous *Family Math* nights and Parent Math courses in local schools; presented at many *Parent Engagement* workshops; written hundreds of math-focused newspaper columns; and brought math to the wider community through my children's book (*Mathemagic: Number Tricks*) and family-focused TVO television series (*The Prime Radicals* and *mathXplosion*).

Over the past three years, I have been very fortunate in that the "reach" of my efforts to engage parents has been greatly enlarged. As a regular featured speaker at *Parent Forums* held across Ontario by the Outreach Team at the Educational, Quality, and Accountability Office (EQAO), I have had the opportunity to work closely with parents, school councils, and district-wide Parent Involvement Committees, from Windsor to Cornwall, and Picton to Fort Frances.

With each presentation came more invitations to work with and support parents in communities across the province. The call for high-quality, accessible resources for parents about math was loud and clear, and fell upon important ears: Penny Maidens, Education Officer in the Parent Engagement Branch at the Ontario Ministry of Education (MOE), and Frank Kelly, Executive Director of the Council of Ontario Directors of Education (CODE).

CODE (an advisory and consultative organization composed of the CEOs of each of the 72 District School Boards in Ontario—Public, Catholic, and French Language) and the MOE have had a successful track record in terms of Parent Tool Kits. Three Tool Kits (*Planning Parent Engagement, What Parents Can Do to Help Their Teens Succeed* and *What Parents Can Do to Help Their Children Develop Healthy Relationships*) and two Guide books for parents have been developed to help families as they guide and support their children/teens throughout school and life.

Happily for the mathematics education community and school councils across the province, a new Parent Tool Kit has recently been added to the exemplary collection: *Inspiring Your Child to Learn and Love Math*. This comprehensive suite of print and video resources was developed by myself; my Research Assistant, Kimberly Betts; the production/audio-visual team from GAPC: Ken Stewart, Hoda Elatawi, Dean Tardioli, Jamie deRooy, Andrew Huggett, and Cheryl Simard; print/web developer John James; and research assistant/writer/editor *par excellence*, Kate Keating. It can be found in electronic form at **www.ontariodirectors.ca/parent_engagement-math/en/index.htm**. Hard copies of the complete toolkit were slated for distribution to schools and school councils across the province in November 2015.

The Kindergarten, Primary Division (Grades 1 to 3), Junior Division (Grade 4 to 6), and Intermediate Division (Grades 7/8) have identical structures. Each contains a Video, Resource Guide, Fact Sheet, Additional Resource Guide, and Implementation Guide.

The short videos show parents and children learning and doing mathematics together in the playground or at the kitchen table, formally and informally. The activities and strategies were designed to be easily replicated by families across the province.

The comprehensive resource guide describes the general characteristics of children in Kindergarten or one of the specific grade bands and presents a thumbnail of the math that children of that age will be doing in class. Examples of children's work, screenshots from interactive whiteboards, and photographs of word walls are embedded so that parents can see authentic *examples that elicit or answer the question, What are you teaching my child?*, and provide a window into mathematics through the eyes of their children. While the resource is explanatory and rich in detail, it is written in jargon-free language that is at a reading level that will be accessible across demographics.

Double-sided, one-page overviews, or fact sheets, are designed to be photocopied and distributed at school council meetings, parent–teacher interviews, in school newsletters, or included in take-home materials at *Family Math* programs, and provide at-a-glance summaries of the resource and an abbreviated suite of suggested resources.

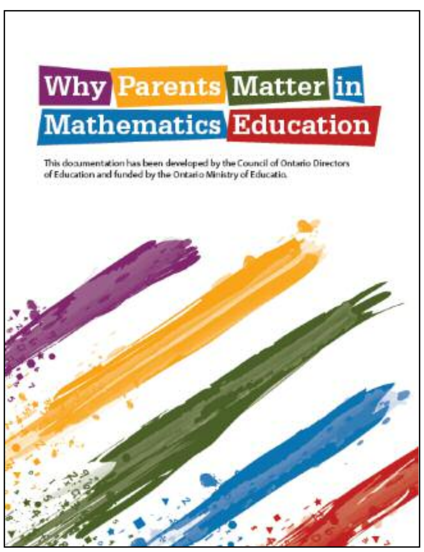
The *Additional Resource* component includes many suggestions about books, television programs, games, websites, and apps that provide meaningful and engaging complements and supplements to formal math programs.

One of the major goals of the I*nspiring Your Child to Learn and Love Math* toolkit is to empower parents to support their children's mathematics growth and achievement. The toolkit celebrates all parents as key stakeholders—acknowledging that they know their children better than anyone and are important members of the mathematics teaching team. Research tells us that 90 to 95 percent of all human behaviours are learned through modelling; thus, the *Implementation Guide* component of the toolkit is designed to enable parents to become positive math role models and mentors by providing the tools, resources, and support that they need to lead workshops for other parents in their communities. Modelled learning is strongly influenced by the quality of the relationship that exists between the model and the learner—the higher the regard, the more likely a person is to follow an example, which makes parents such influential models for other parents. When parents see other parents being positive about math, hear them talking about ways to support math at home, and watch them easily do things they think they themselves cannot do, they are more likely to pick up strategies and transfer them to their own homes. As Edgar Guest, the "people's poet" of the last century, so succinctly said in his poem *I'd rather see a sermon*:

I'd rather see a sermon
than hear one any day;
I'd rather one should walk with me
than merely tell the way.
The eye's a better pupil
and more willing than the ear,

Fine counsel is confusing,
but example's always clear;
For to see good put in action
is what everybody needs.
I soon can learn to do it
if you'll let me see it done;
I can watch your hands in action,
but your tongue too fast may run.
And the lecture you deliver
may be very wise and true,
But I'd rather get my lessons
by observing what you do;
For I might misunderstand you
and the high advice you give,
But there's no misunderstanding
how you act and how you live.

We created the *Parent Tool Kit, Inspiring Your Child to Learn and Love Math* specifically for parents of children in the elementary grades in Ontario (Junior Kindergarten to Grade 8). Our goal was to provide parents with the most significant research-based information to help them be the best, most knowledgeable, and most confident supporters for their child's mathematics education. In addition, we hope that educators might also want to share the toolkit with other educators in their school and with friends and family members in their communities who are struggling to find the information they need to help their children navigate the K–8 mathematics program. However parents and educators choose to use it, we hope that this toolkit will help strengthen your knowledge and understanding, which in turn will reinforce children's success in mathematics… with your help and support.



The following excerpt is taken from the introduction to the resource booklet that contains all five modules:

PARENTS AS PARTNERS IN MATH EDUCATION

"You are their idol. They look up to you. They don't do a good job listening to you, but they do a fantastic job imitating you." Karl Subban on being a role model to your children.

Parents and teachers are the most influential players in children's lives—their attitudes and behaviours have enormous impact on what children believe and achieve.

Did you know that of all the waking hours children have from the time they are born until they graduate from secondary school, *they spend only about 15% of that learning time at school* and the rest somewhere else, primarily at home with their parents?

There is no doubt that there are many demands on the 85% of time that parents and children are together, enjoying family time, playing, and engaging in all of the responsibilities of daily living: including homework.

In a recent study, over 90% of parents surveyed agreed that parental involvement is important, but 80% of those parents said they needed more information about how to help their children at home. This document is a response to that request.

The purpose of this resource is to reinforce what you are already doing at home and to extend those efforts by learning how to maximize the benefit of those all-important learning opportunities that take place at home.

Our goal is to help you to support your child's math learning by providing strategies and practical suggestions to build and reinforce positive mathitudes, reduce math anxiety, and enhance achievement by using everyday objects and routines.

This book will provide information that complements the videos. The package will introduce you to what is happening in math class and why math teaching has changed. You will also learn why your active involvement is key to your child's success and how to make a positive difference in your child's mathematics education.

▲

OAME/AOEM AWARDS FOR EXCELLENCE IN MATHEMATICS EDUCATION

The purpose of the Ontario Association for Mathematics Education's Awards is to formally recognize individuals and educators who demonstrate an outstanding contribution to mathematics education in Ontario. The successful candidate of each award enhances the learning environment, providing opportunity for students to do, see, hear, and touch mathematics in a profound and meaningful way. If you know someone who qualifies for this honour, please nominate her/him/them for one of the following awards.

OAME AWARDS

OAME/AOEM Life Membership Award recognizes an educator who has contributed in a significant way to OAME/AOEM; demonstrates outstanding leadership in mathematics education; and has accumulated ten or more years of membership in OAME/AOEM.

Award for Outstanding Contribution to OAME/AOEM and Mathematics in Ontario recognizes an individual who has made a significant contribution to OAME/AOEM, but who is not necessarily an educator or a member of OAME/AOEM.

Award for Exceptional and Creative Teaching in Elementary Mathematics recognizes an exceptional and creative elementary teacher who demonstrates excellence in mathematics education and contributes to the overall development of students.

Award for Exceptional and Creative Teaching in Secondary Mathematics recognizes an exceptional and creative secondary teacher who demonstrates excellence in mathematics education and contributes to the overall development of students.

Award for Leadership in Mathematics Education recognizes an educator who has demonstrated leadership by contributing in a significant way to the development of mathematics teachers and enhancing mathematics education in Ontario.

Secondary School Department Award for Exceptional and Collaborative Mathematics Teaching recognizes a secondary mathematics department which fosters collegiality, teamwork, and excellent classroom teaching; contributes to the overall development of students; and demonstrates leadership in the mathematics education community.

Elementary School Staff Award for Exceptional and Collaborative Mathematics Teaching recognizes an elementary school (a school accommodating any or all of Grades JK–8), whose staff fosters collaborative and excellent mathematics teaching; contributes to the overall development of student learning of mathematics; and demonstrates leadership in the mathematics community.

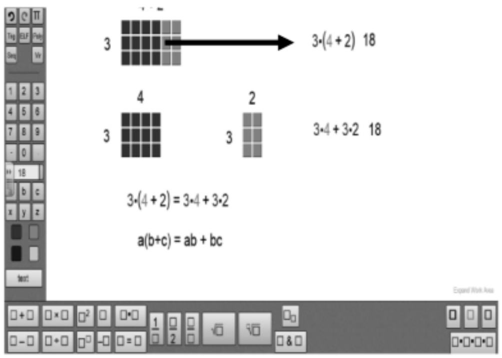
OAME/AOEM Awards Nomination Process

1. Complete the Award Nomination Cover Page appropriate to the specific award, found at **www.oame.on.ca**, or contact the Executive Directors at **EDs@oame.on.ca** for an electronic version. You may include a one-page letter of recommendation.
2. Provide up to three letters of support, of no more than two pages each. These supporting documents may be gathered from a variety of sources to reveal a full picture of the nominee. The support documents will provide precise information about the nominee's contributions as they relate primarily to the specific Award.
3. Include the nominee's Curriculum Vitae of up to five pages. (C.V. not required for Secondary School Department Award and Elementary School Staff award)
4. Send the completed cover page and supporting documents by January 15, 2016 to: OAME Executive Directors, 4751 Mack Avenue, Town of Plympton-Wyoming, ON N0N 1J6, or email: **EDs@oame.on.ca**

DEADLINE: Nominations are due by January 15, 2016 to the Executive Directors (contact info above).

The awards will be presented at the Annual OAME Conference in May 2016.

Unlocking Mathematics for **All** Students



The distributive Rule: 3×(4+2) = 3×4 + 3×2

The need for Foundation (*a teaching tool that complements other teaching methods*) was conceived during preparation for a weekly hour tutoring a gifted Grade 5 student. Getting him to learn a simple programming language and model the abstract concepts of Mathematics seemed a good idea.

However, there wasn't any language that avoided the confusing and tedious process of coding – a task at which computers are much better than humans.

Some months later, a search for a method of using the computer to do the coding uncovered the expression $\square^{\square}$ in WORD. The two boxes could be considered entry points to a function (*each now called a "doorway variable"*) rather than symbols of equal value.

Mathematical expressions could be built by dragging sub-functions into doorways of other functions. This would be step by step function composition. With this approach, neither computer programming nor coding would be required from teacher or student. With the aid of a systems engineer, work began on the development of a prototype.

Its calculation facility included functions needed in high school (*such as trigonometric, polynomial, factorial, exponential and logarithmic functions*).

A serendipitous discovery was the fact that Foundation is a common base for both Arithmetic and Algebra. In Foundation there is no such thing as separate Arithmetic and Algebra: Arithmetic is Foundation with numbers and no letter variables; Algebra is Foundation with at least one letter variable. There is no barrier to overcome in order to get to Algebra from Arithmetic. This offers a way of tackling Algebra and abstraction from Grade 12 down to Grade 1.

It has been recognized in the literature on the teaching of Algebra that it would be pedagogically beneficial to teach Mathematics through functions instead of operators, which are an abstract concept.

Abstraction also results from the writing of Mathematics symbols on paper. This takes successive abstract statements, e.g.

$$x + 4 = 11$$
$$x = 11 - 4$$
$$x = 7$$

This is like a series of snapshots. The numbers, variables, expressions and equations in these snapshots are abstract objects. Each new snapshot is created from the last snapshot through abstract rules.

By contrast, the Foundation prototype with functions was found to generate concrete, moving processes, more like a video and more like the real world.

Significant success was demonstrated in an 18-month Grades 1 & 2 pilot project (*described in "Unlocking Mathematics for All Students" at www.FoundationNotation.com*). The Foundation Prototype offered a fresh and concrete way of introducing mathematical concepts that could make a considerable difference for all young students before they expand their capability in abstraction.

Building virtual manipulatives on a screen was a powerful introduction to building physical manipulatives, the abstraction students faced, and the paperwork they did.

In the pilot project, Foundation was motivating, engaging and easy to understand and learn. It worked with existing lesson plans and helped prepare students for Mathematical exercises on paper. As a model of pure Mathematics, Foundation was used for constant Inquiry Based Learning.

The focus on notation could help students perceive Mathematics as a developing science. They could also be helped to understand what mathematical expressions are by watching them be both composed and calculated one step at a time.

Problem solving has involved four steps:

1) Define the problem well
2) Identify the solution in words
3) Change the words to notation
4) Calculate the answer.

After 40 years of good research, the methodology for teaching the first two steps is well understood and well taught. Research has also established that calculating an answer by hand in the fourth step does not teach Mathematics.

However, if the computer does the calculation, the number of steps is still four, with the last two being:

3) Change the words to notation
4) Code the notation

Coding also does not teach Mathematics. Addressing this issue was the purpose for which Foundation was created. By turning Mathematics notation into a computer language, it can remove the last step. Thus Foundation is a complement to current teaching, not in conflict with it.

It could be argued that the difficulty of the third step has been undervalued. It expects students to move almost daily from the concrete concepts of the Real World to the abstract concepts of Mathematics notation.

Foundation builds a bridge between them that consists of physical manipulatives, virtual manipulatives and functions. This bridge is no longer necessary once students develop abstraction capabilities. This gives meaning to notation for all students and provides them a way of reaching back from Mathematics notation to the Real World.

Students who have not used Foundation in their early years can learn it quickly. If the use of Foundation in higher grades brings understanding to all students in the same way that it did in the Grade 1 & 2 pilot project, all high school learners could be motivated to achieve their maximum capability in Mathematics.

The features of the prototype have been redesigned, expanded and reprogrammed; on-line self-teaching has been added; and it has been turned into an app ("*Foundation Notation" for $10 in Macintosh and Windows 8 versions*).

▲ In Memoriam:

Frances Schatz

May 11, 1946 – May 25, 2015

BY JACK WEINER

It is with sadness that we are writing this obituary. However, we compose it with a sense of celebration of a wonderful life well lived.

Frances Schatz (née Brown) was born in Paris, Ontario. She attended the University of Waterloo, graduating in 1969 with a Bachelor of Mathematics. She taught at Oshawa Central Collegiate on a letter of permission for one year, but was unable to attend Teachers' College at that time. In 1970, she married Frank Schatz and they settled in Kitchener. They met on a blind date while she was still a student at U of W. She graduated from OISE with her Teacher's Certificate in 1988. For 45 years, she was the go-to math tutor for the Kitchener Waterloo region and beyond, as hundreds of successful students would happily attest. Frances and Frank have two children, Anita and Bill.

Anita stated: "She always wanted to make math real for her students. For example, to help with fractions, she would bake apple pie. Of course, the students were rewarded with a slice for correct solutions. Such was her talent for teaching, that her students always ended the lesson full. Her personal library was populated with books such as *Calculus of Friendship*, from which she took more examples to make math come alive."

Jack stated: "It was not exactly rare for me, teaching first-year calculus at the University of Guelph, for a student to introduce himself or herself to me this way: 'I was taught by Mrs. Schatz and she said to say hello.' Invariably, those Frances Schatz students were well prepared and did superbly in my course."

Her exceptional work as a tutor won her a prestigious University of Waterloo Descartes Teacher Recognition Award in 1989.

Frances made significant contributions to mathematics education in Ontario. She was the editor of The Abacus, the elementary mathematics education journal of the Ontario Association for Mathematics Education (OAME), from 1996 to 2004. During many of those years, she experienced a dearth of submissions. Rising to the challenge, she would choose a theme or two and create a well-designed, easily implementable, eminently student- (and teacher-)friendly 12 pages on her own. From 1998 to 2004, Frances was the National Council of Teachers of Mathematics (NCTM) representative on the board of OAME. She, with husband Frank, attended all board meetings, offering valuable comments during council discussion, and at breaks and lunch (and OAME conferences), setting up an NCTM exhibitor's booth to share the resources of that North America-wide organization. She contributed numerous articles to OAME's other journal, the *Ontario Mathematics Gazette*.

Frances Schatz: loved and respected by her family, her students, her colleagues. Frances Schatz: missed by all. Rest in peace, Frances. ▲

▲ Mb4T (Mathematics by and for Teachers): Multiplying Integers

ANN KAJANDER
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Ann Kajander is an experienced classroom teacher currently teaching mathematics and mathematics education at Lakehead University. Her research interests relate to teachers' enhanced learning of mathematical concepts. She and her classroom-teacher colleague, Tom Boland, have written and published a new book for teachers called Mathematical Models for Teaching: Reasoning without Memorization.

As mentioned in the September column, the development of integers historically was not in response to real-world contexts, but rather as a way of dealing with expressions such as 10 + 2 × ____ = 4. Thus, constructing classroom "real world" examples becomes more of a challenge. For example, we might attempt to construct a problem related to this calculation such as the following:

> I have $10. Then I carry out two transactions of the same amount. Now I have $4. What might the two transactions have been?

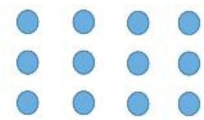
One way to answer the word problem might be to say, "So I spent $3 two times." On the other hand, we might want students to respond, "Each transaction was (-$3)." It is important to understand that, in this context, either response is correct: the "2 × (-3)" in the number sentence aligns with "two instances of *spending* $3." Algebraically, we see that (-3) is the correct answer to the equation 10 + 2 × ____ = 4, but in words, the answer can be described in various ways. However, it seems quite awkward to talk about -$3. At a certain point in mathematics, we may need to be honest about the use of integers. They help us solve mathematics equations, but are sometimes hard to apply directly to the real world, when a descriptor—such as "spending" $3 or "below" sea level—may be clearer. Nevertheless, conceptual understanding of the operations can still be constructed using manipulatives, and this remains important in learning. The following discussion illustrates the mathematical ideas behind using coloured counters to model the multiplication of integers concretely.

Multiplying by a Positive Value

To model multiplication problems in which the first number is positive, it can be helpful to use the repeated addition interpretation of multiplication, in which case the model is relatively straightforward and familiar.

Example:

Consider 3 × (-4). Using repeated addition, we can think of this as three groups of negative four. This can be modelled by constructing 3 groups of 4 negative counters



(shaded counters are used here to represent negative counters), which illustrates that 3 × (-4) = (-12).

Using examples similar to the previous, it is fairly easy for students to establish that a positive number of groups of (either) a positive or a negative number is analogous to repeatedly adding that value.

We can also use number lines to model products that begin with a positive value. To use a number line to model 3 × 4, we can start at zero and show 3 jumps of 4. We can use a similar model for 3 × (-4), but instead, the jumps are 4 units to the *left* (toward the negative) each time.

A Negative Times a Positive

If we knew that the commutative property held for integers, we could simplify an expression such as (-3) × 4 by simply calculating 4 × (-3) , similar to the method as above. However, students may not have evidence of this generalization at this stage.

To model multiplication as repeated addition, start at zero and then add groups. But to model the multiplication of (-3) × 2 using this idea, we would need to add *negative* 3 groups of 2. An easier conception is to think of the expression as 0 - (-3) × 2; then we can think of starting at zero and then *removing* 3 groups of 2. In a sense, the (-3) models repeated subtraction (rather than repeated addition).

Going back to an idea introduced in the previous column, recall that numbers can be represented in many different ways, using zero pairs. So we could start with zero—actually a number of zero pairs—and then think of modelling (-3) × 2 by *removing* the 3 groups. We begin with a representation of zero… one such representation being a sea of zero pairs.

Starting with the above model of zero with a lot of zero pairs, we can think of effecting the negative 3 groups by subtracting or *removing* 3 groups of 2 positive counters.

After the removal, the model then shows 6 unpaired negative counters—a net value of -6. This illustrates that (-3) × 2 = (-6). Note that had we used the commutative property and *changed* the question from (-3) × 2 to 2 × (-3), and answered it as already established earlier in the section, we would have achieved the same result. Hence we conclude that the commutative property seems plausible with integers.

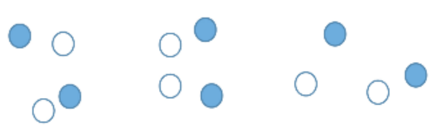
Negative Times a Negative

Surprisingly, the counters or chips model just explored applies to the famous *negative times a negative* situation, with no further effort or adjustments!

Example:

Consider (-3) × (-2) and think *remove 3 groups of 2 negative counters*.

Let's begin with a representation of zero as before:



Now, we can use the same reasoning as before to interpret "-3 times" as *remove* 3 groups, but this time, we are removing the negative (coloured) counters. Effectively, this is also analogous to repeated subtraction. When we remove 3 groups of (-2) from the model of zero shown, we are left with 6 more positive counters than negative ones.

So the model then shows +6. We conclude that (-3) × (-2) = 6. Students should construct and solve a few similar examples themselves, in order to see the generalization.

As students become more and more familiar with these examples, they will soon learn to predict exactly how many zero pairs are needed in a model. For example, in the problem just completed, we might have started with 12 zero pairs, but really we only needed 6 of them to solve the problem.

There are also other approaches that are helpful for some students when thinking about integer products, which are illustrated in our recent mathematics book for teachers (Kajander & Boland, 2014). It is suggested that a choice of methods be available in learning, as sometimes one particular method will click with a student when another has not. For example, a negative in front of a value is sometimes used to signify a change in direction, which can be informally modelled by students as they think about walking on a number line. We can think of showing a trip of (-5) in two different ways: either as walking backwards 5 (from zero), or, reversing direction (from facing positive) and then walking 5. (Try it!) Then we can interpret (-5) × (-3) as *turn around, and make 5 trips of backwards 3*. (Try it! Where do you end up?)

This directional interpretation aligns with the idea of vectors in mathematics. For example, we can think about (-1) as a unit vector going in a negative direction. Then to interpret "– (-1)", we see that the original (-1) vector is *reversed* in direction by the extra negative sign in front—so the final result ends up going in the opposite direction—back to positive! (What would happen if we keep multiplying by (-1), i.e., keep adding negative signs?)

Eventually students may generalize that the product of two like-signed integers is always positive, and that the product of two unlike-signed integers is always negative. Relying on rules alone, without engaging students in activities that allow them opportunities to develop understanding, is dangerous—and can result in misinterpretations, such as thinking that – 3 – 5 is 8 ("because there are two negatives"). After modelling enough examples, students will soon begin to construct and internalize the general procedures for integer products on their own. The difference is that if they forget, they have something to fall back on—reasoning!

Reference

Kajander, A., & Boland, T. (2014). *Mathematical models for teaching: Reasoning without memorization.* Toronto, ON: Canadian Scholars' Press. ▲

A Provincial Digital Learning Resources – What's New? Number Line by Mathies (Version 1.0 Released Fall 2015)

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Greg is from the Simcoe Muskoka Catholic District School Board and Ross is from the Near North District School Board. Both are currently on assignment with the Ontario Ministry of Education as Provincial Math Leads, working on the CLIPS project, in conjunction with their colleague, Agnes Grafton, from the Brant Haldimand Norfolk Catholic District School Board. Markus Wolski is a teacher with the Bluewater District School Board and has been working on the CLIPS project for the past two years.

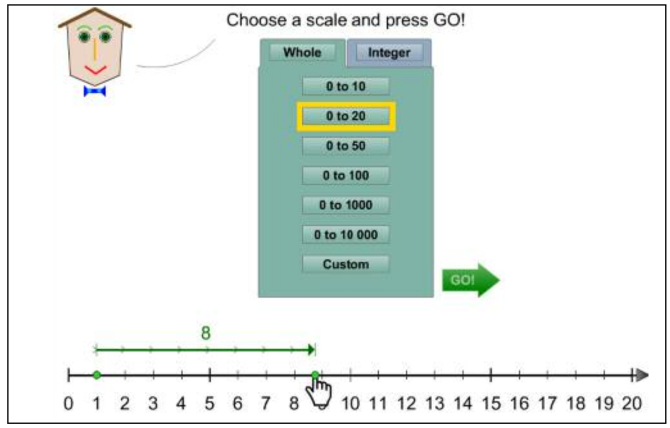
Visit **mathies.ca** to access the latest digital learning tool – *Number Line by mathies*.

This powerful mathematical tool can be used to represent numbers and build an understanding of various operations. The current version of this tool has been designed to model addition and subtraction of both whole numbers and integers. This tool can also be used to represent, order, and compare numbers. Currently, only a desktop version is available.

Encyclopedia of Mathematics Education (S. Lerman, 2014) Excerpts

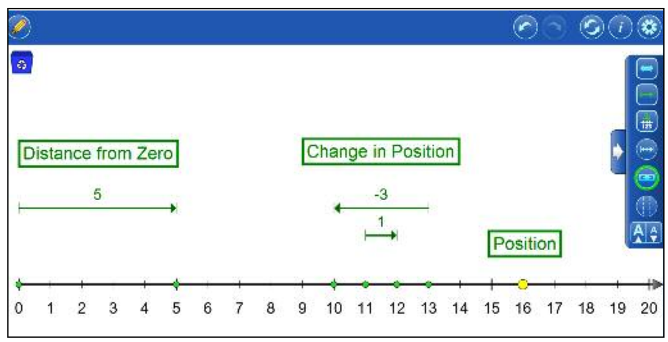
"The word algorithm probably comes from a transliterated version of the name al-Khwarizmi (c. 825 CE), the Arabic mathematician who described how to solve equations in his publication al-jabr w'al-muqabala.
(continued on page 27)

Getting Started



From the opening screen, choose a scale to begin exploring the number line and press GO!. To access Integer scales, click the Integer button. Future development plans include incorporating fraction and decimal scales.

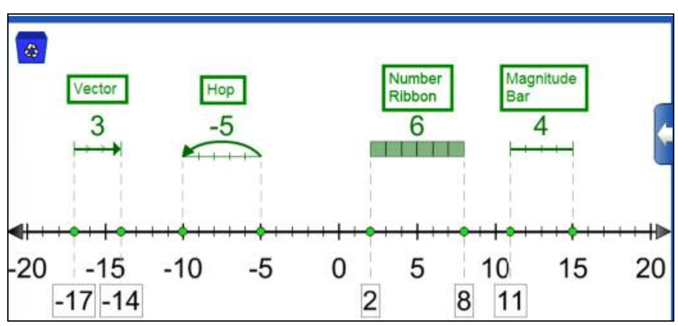
Representing Numbers



Numbers can be thought of as:

- *distances from zero*. Click on zero and drag along the number line.
- *changes in position*. Click at any starting point and drag along the number line.
- *positions* on the number line. Click the number line to create a point.

Currently, the tool has a choice of four different representations.



Vectors or hops can be used to represent directed values. Number ribbons or magnitude bars can be used to think about just the number's magnitude.

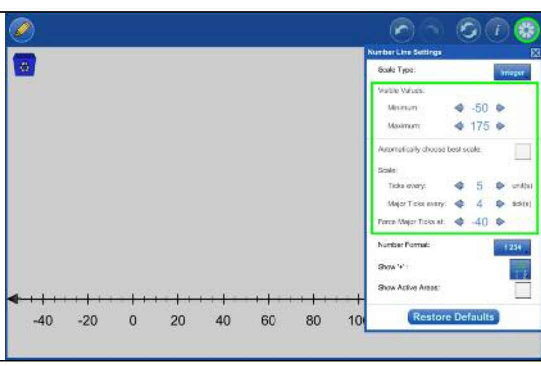
To adjust an object in the workspace, grab one of its endpoints (shaded blue and pink above) with your mouse and drag the endpoint to shorten or lengthen it. You can move the entire object by dragging and dropping it from the middle of its body (shaded green above). A single click on an object reveals a recycle bin that can be used to delete just that object.

Adjusting the Settings

The panel at the right contains a number of widgets that can be used to customize the workspace or the objects displayed in it.

- Change Number Line Orientation (from horizontal number line to vertical number line—and back)
- Choose Representation Object (vector, hop, number ribbon, magnitude bar)
- Show/Hide Labels (determine whether labels appear on objects and/or the number line)
- Show/Hide Unit Ticks on Objects
- Allow/Disallow Object Linking
- Show/Hide Drop Dashes
- Increase/Decrease Label Size

To access even more options that let you customize the tool, click on the settings button. From here, you can choose an exact range, modify the tick placement, and much more. Don't be afraid to try out settings to see what they do. You can always click the undo button if you don't like the effect.



In the example above, the scale has been customized to allow for multiples of 5 and 20 to be explored.

Zooming and Panning

You can change the visible values on the number line, using methods similar to Google Maps.

Zoom in on a position by:

- double-clicking on some white space above the number line, or
- using the mouse wheel

Zoom out by:

- double-clicking on some white space above the number line, while holding down the SHIFT or CTRL key, or
- using the mouse wheel in the opposite direction

Zoom to fit by:

- clicking a glowing black arrow on either end of the number line to show all the objects outside the current scale, or
- clicking a non-glowing black arrow, while holding SHIFT, to shorten the range of values displayed to the necessary ones

To pan:

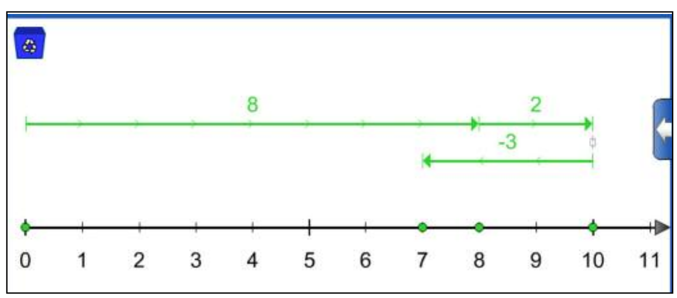
- click below the number line and drag either left or right

The Tool in Action

Objects can be linked together to model a story.

Whole-Number Example

Sandy's father gave her 8 cookies. Her mother gave her two more, then she gave three to her brother.

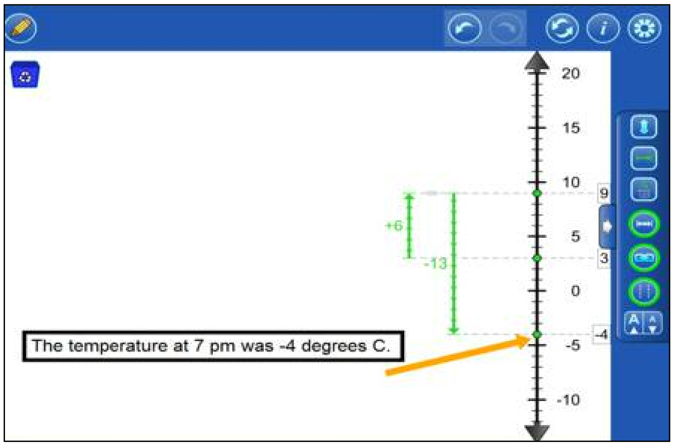


Integer Example

The temperature at 7 a.m. was +3°C.

It rose 6°C by noon and then dropped 13°C by 7 p.m.

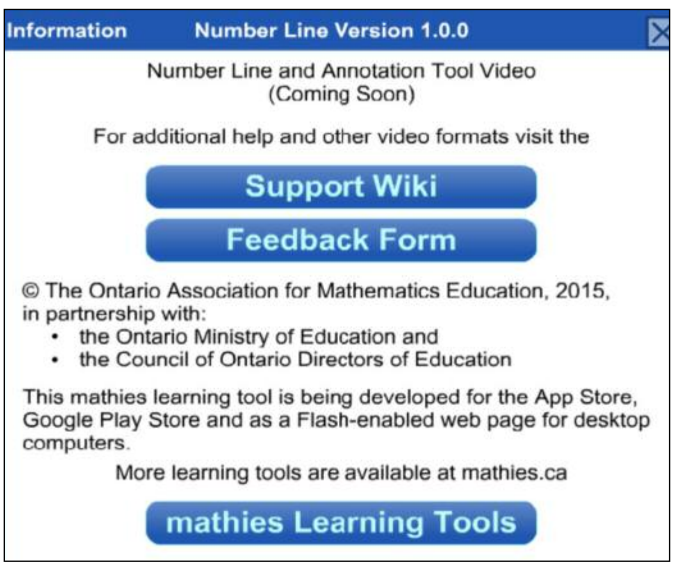
What was the temperature at 7 p.m.?



Note: The above screenshot uses the built-in annotation tool that provides a vehicle for students to communicate their thinking.

Feedback and Future Requests

This digital tool and others found at mathies.ca are developed to meet the needs of Ontario students and teachers. Although the initial release of this tool is designed for desktop use, it can be accessed on mobile devices, using the Puffin Academy app. Try out the Number Line tool, then send us your feedback, using the Feedback Form button inside the Information Dialog.



We would love to hear how you and your students used the tool. How did it support student learning? What mathematics did the tool reveal? What did you like? What additional features would you like to see? You can also send your comments to **WhatsNew@oame.on.ca**. To be among the first to find out about latest digital tool developments, sign up for our email list at **http://mathclips.ca/WhatsNewEmailList.html**. ▲

▲ Ontario Mathematics Olympiad (OMO): An OAME Tradition

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Paul is the department head of math at Fletcher's Meadow Secondary School and past president of OAME. Paul is an advocate for the appropriate use of technology and the use of a variety of methods and tools for assessment and You can follow him at the Twitter handle: @paul_math.

JUDY MENDAGLIO
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Judy Mendaglio has worked as a high school and college math teacher and is currently a sessional instructor in math education at Western University. Judy believes that all students will experience the joy of mathematics if they see interesting contexts and problems.

There are many highlights in the OAME calendar each year—the Leadership Conference, the provincial annual conference (don't forget OAME 2016 "Leap into Math" from May 5–7 in Barrie), and the many local events sponsored by OAME chapters. Many of these events are geared to promote one of the two parts of OAME's mission—promoting excellence in mathematics educators. The second part of OAME's mission is promoting excellence for students. As an organization, we do this by advocating for a curriculum that will prepare students to meet the mathematical challenges in their lives, by fostering familiarity with the methods of mathematics and an admiration for its principles, and by giving recognition and respect to their own mathematical achievements. One of the primary vehicles that allows the organization to promote excellence for students is the Ontario Mathematics Olympiad (OMO).

OMO is a fun and challenging mathematics competition for students in Grades 7 and 8. The competition is made up of individual and team events. The teams are required to solve problems and communicate their understanding in various applied situations. It is a great way for the students to feel a sense of competition, to collaborate toward a common goal, and to meet students from schools in other parts of the province—all this while experiencing the joy of mathematics!

The provincial playdowns usually take place in early June and are sponsored by one of the 15 OAME chapters. The first provincial OMO took place in 1995 and was hosted by COMA at Lisgar Collegiate in Ottawa. It is very fitting that this year being OMO's 20th anniversary, it was once again hosted by COMA on June 5–6 at Carleton University. Next year's OMO will be hosted by SAME and will again be taking place in June. Set **www.oame.on.ca** as one of your "favourites" for more information as it becomes available.

The teams who get to travel to the provincial playdowns are selected in various ways by the chapters. The dates of the regional playdowns vary from chapter to chapter. If you are interested in getting involved either as a volunteer or to participate with your students, please visit the Chapter Directory at the OAME website for the contact information of your chapter representative.

At a recent OAME board meeting, chapter representatives shared some of the different approaches they have adopted to run these regional OMO playdowns. Although the manner in which the local playdowns occur varies in scope and methodology, some common themes emerged from the sharing. What is consistent across all chapters is their reliance on volunteers to run the events. Sometimes all volunteers are active members of the chapter, and some chapters seek board of education support, while others receive support from faculties of education and teacher candidates. Also, all of the playdowns provide a creative space for students to collaborate and compete in the hopes of moving on to the provincials. What everyone—teachers, parents/guardians/family members, and volunteers—come away with is the pleasure of watching young students working collaboratively on challenging math activities. A common refrain from those watching the events is "Wow! Math never looked like that when I did it." Lastly, the students have an amazing time participating and interacting with their peers! with their peers!

OMO continues to be one of the signature events for OAME. Its success has grown because of the OAME chapters' insistence on providing an opportunity for students to experience solving mathematical problems in an atmosphere that fosters collaboration and communication. However, this success hinges on the support of the organization's members and each chapter's volunteers. As you start this new school year, why not consider either volunteering within your chapter to help with the event or participating with your students. Check OAME's website for more information on your chapter's regional playdowns and the provincial Ontario Math Olympics! ▲

▲ Leap into Math at OAME 2016

The 2016 OAME annual conference will be held at Georgian College in Barrie, Ontario from May 5–7, 2016. Hosted by MAC2, the conference will feature amazing speakers and spectacular special events and programs, all held in a state-of-the-art facility.

Leap outside the box in your mathematical experiences with our extraordinary keynote speakers. Robert Lang, our Thursday keynote speaker, is recognized as one of the foremost origami artists in the world, as well as a pioneer in computational origami and the development of formal design algorithms for folding. Lang will also be the guest speaker at the OAME Banquet, where attendees will have an opportunity to try their hand at origami. Author and award-winning educator Steven Strogatz, our Friday keynote, shows how synchrony occurs spontaneously in nature, from atoms to solar systems—almost as if the universe had an overwhelming desire for order. Saturday delegates are invited to explore the mathematical content and educational value found in the Simpson's and Futurama during an interactive talk with Professor of Mathematics and pop culture enthusiast, Sarah Greenwald.

Robert Lang

Steven Strogatz

Sarah Greenwald

Our featured speakers are varied and plentiful at OAME 2016. They include: Marian Small, Cathy Bruce, Jill Gough, Joan Moss, Alex Lawson, Ron Lancaster, Nathalie Sinclair, Ruth Beatty, Chris Suurtamm, Lisa Lunney Borden, Connie Quadrini, Nicholas Jackiw, and

Don Fraser.

You should *Leap into Math* at OAME because there is something for everyone. OAME 2016 promises a return of the Ignite sessions, where invited speakers share their personal and professional passions, using 20 slides that advance automatically every 15 seconds, for a total of 5 minutes of enlightening entertainment per speaker. From the wine and cheese to the OAME Banquet, casino to shopping, rock-wall climbing to microbrewery tours, the planned evening entertainment opportunities are endless in Barrie and the surrounding area. OAME 2016 has new program events to offer as well. Delegates who decide to register on Wednesday night are invited to get to know the Georgian campus while doing math on the *Math Mystery Tour*. Delegates are also invited to participate in OAME 2016's *KenKen Tournament* on Friday afternoon. KenKen is a puzzle that combines arithmetic and logic. New to KenKen? Don't worry. You can find the basic rules of KenKen at **www.kenken puzzle.com/howto/solve**. Start practising today.

Interested in presenting at OAME 2016? Check the MCIS website to see if session proposals are still open, or contact **program2016@oame.on.ca** with your ideas.

Educators who are unable to make the trip to Barrie can *Leap into Math* virtually through our eConference. Watch for eConference details online and in our conference flyer.

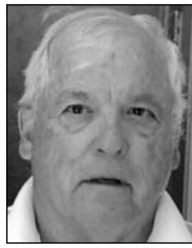
Be sure to follow us on Twitter (@oame2016) , and check out our website to stay up-to-date about conference happenings, including some exciting contest opportunities. Visit **www.oame2016.ca** for additional information about the conference and how you can *Leap into Math* at OAME 2016. ▲

(continued from page 23)

An algorithm comprises a step-by-step set of instructions in logical order that enable a specific task to be accomplished. Due to its nature it can be programmed into a computer, although some problems may not be computable or solvable by an algorithm. . . . We note that two algorithms to accomplish the same task may vary or be entirely different. For example, there are a number of different algorithms for sorting numbers into order, often much more efficiently than the bubble sort, such as the quicksort algorithm."

Thomas, M. O. J. (2014). Algorithms. In S. Lerman (Ed.), *Encyclopedia of mathematics education* (pp. 36–38). London, UK: Springer.

▲ OAME/NCTM Report



TODD ROMIENS
EMAIL: todd.romiens@oame.on.ca

Todd is a former instructor of K–12 teaching methodologies in the Faculty of Education at the University of Windsor. He is also a Past President and life member of OAME, as well as a previous editor of the Gazette.

This column will outline what it means for OAME to be an affiliate of NCTM—the conditions and the benefits for our organization.

As an affiliate, OAME falls under the overall vision statement of NCTM. Amended and approved in 2012, it states:

> NCTM is the global leader and foremost authority in mathematics education, ensuring that all students have access to the highest-quality mathematics teaching and learning. We envision a world where everyone is enthused about mathematics, sees the value and beauty of mathematics, and is empowered by the opportunities mathematics affords.

By its nature, NCTM has a more public voice and a wider scope, but OAME, as an affiliate, is expected to try and carry out this vision in Ontario. The purpose of our affiliation is to enhance both NCTM and OAME's capacity to build effective relationships in our community, namely through sharing information and ideas. Our organization is able to carry this out by attending NCTM conferences and ensuring OAME representation at Delegate Assembly meetings at annual NCTM conferences.

Because OAME is a provincial organization, it is classified as a "partner" affiliate and pays affiliation dues of approximately $350.00 annually. OAME's affiliation therefore entitles it to:

a) unlimited access to Online Affiliate Resources
b) participate as a delegate to the annual conference
c) sell NCTM documents at its Ontario conference

When someone from OAME joins NCTM, OAME receives $5.00 for a full individual membership or e-membership and a $3.00 rebate for renewals.

For OAME to maintain its affiliation status in good standing, a number of criteria must be met. These are mainly handled by Lynda and Fred:

a) Pay affiliation dues
b) Keep NCTM informed of changes in offices (including who the NCTM representative is for OAME)
c) Maintain membership in NCTM for both the president and NCTM representative
d) File an annual report, outlining membership and current activities
e) Maintain an approved constitution and bylaws
f) Promote membership in NCTM

NCTM also offers grant money to affiliates to support OAME's efforts to serve mathematics teachers in attaining NCTM's goals, to promote creative projects that raise public awareness about both organizations, and to increase membership. As much as $3000.00 may be granted if the project is deemed to have merit. OAME has never been the recipient of a grant, so if you have a worthwhile project, you are encouraged to submit an application. The deadline for applications is June 1, 2016. Other possible recognition depends on the number of NCTM members who belong to OAME.

The *Gazette* could also receive recognition if the following conditions are met: that the *Gazette* is published at least twice a year, in print and online; that OAME has been publishing the *Gazette* for at least two years; and that OAME supply the names of the publication's editor and the organization's president. Winners of the award for this recognition receive one free registration to the next annual NCTM conference, as well as recognition at the Affiliate's Delegate Assembly.

My role as your NCTM representative from OAME is to:

a) promote membership in NCTM
b) communicate information on NCTM activities to OAME members
c) be knowledgeable about NCTM services to affiliates
d) order and sell NCTM materials when required
e) attend the Delegate Assembly at the annual NCTM conference when required

I hope this provides OAME members with a clearer picture of our role in NCTM, and the advantages of being an affiliate organization. I encourage you to join.

Don't forget the annual NCTM conference in San Francisco on April 13–16, 2016. ▲

▲ Technology Corner: Desmos Activity Builder

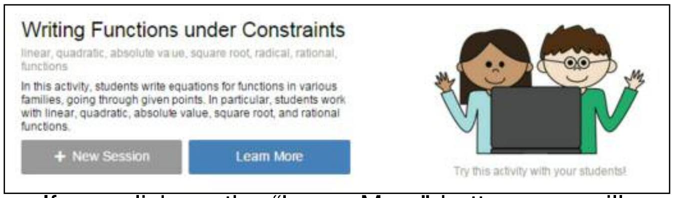
MARY BOURASSA
EMAIL: mary.bourassa@oame.on.ca
TWITTER HANDLE: @MaryBourassa

Mary teaches mathematics at West Carleton Secondary School in Ottawa. She is a strong advocate for the appropriate use of technology in the classroom. She has presented workshops internationally, authored mathematics resources, is a past VP of OAME and a Past President of COMA. An award-winning teacher, Mary continually strives to learn new and better ways of helping students learn and love mathematics.

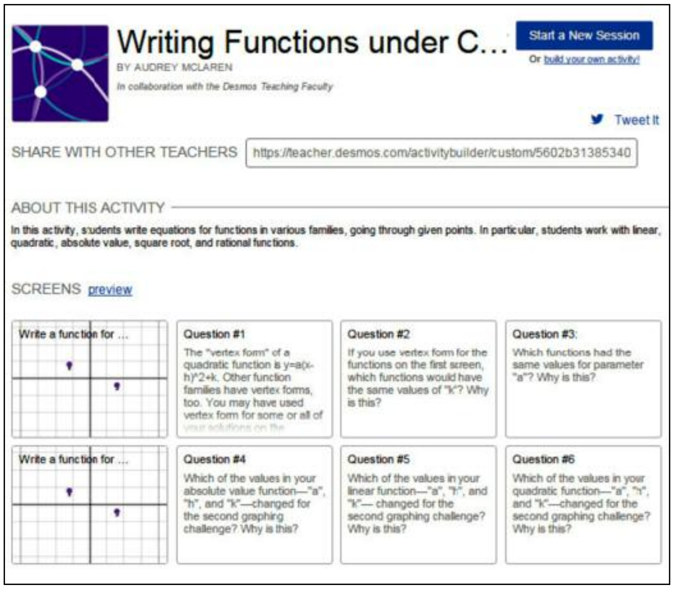
Over the summer, Desmos, makers of fantastic, free graphing software and amazing activities, released Activity Builder. This new tool allows teachers to build their own activities, essentially creating a series of graph screens and questions for students to answer along the way. Activity Builder provides teachers the framework to create an investigative lesson that students can move through at their own pace. Each student's answer to the questions is recorded for the teacher to see, as is the last graph from each screen.

If I have piqued your curiosity, head over to **teacher.desmos.com** and sign in (or create an account, if you don't already have a Desmos account). You can search using the box at the top left of the page. Here is what I got when I searched for "linear."

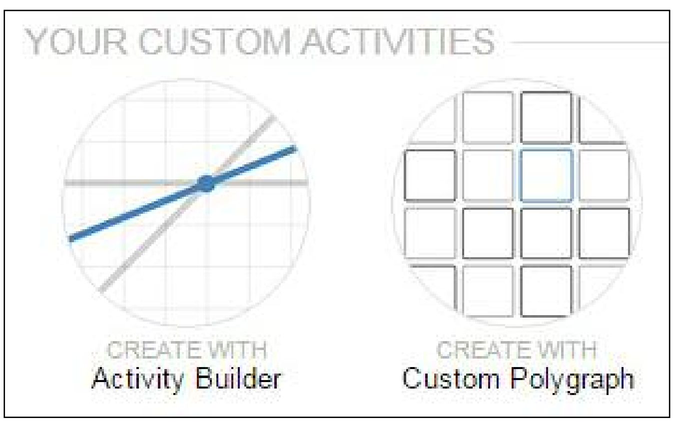
Some activities are from Desmos, but the majority were created by classroom teachers using Activity Builder or Custom Polygraph. If you click on an activity, you will be provided with more information and can start a new session. This means that you can run the activity with your students. To do this, have your students go to student.desmos.com and enter the code that Desmos created for you.



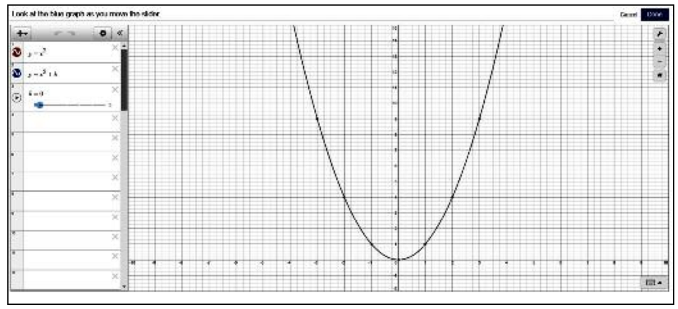
If you click on the "Learn More" button, you will see the layout of the entire activity, with thumbnails of each screen. From this page, you will be able to preview the activity without actually running it as a class. There is also a link on this page for you to share the activity with other teachers.



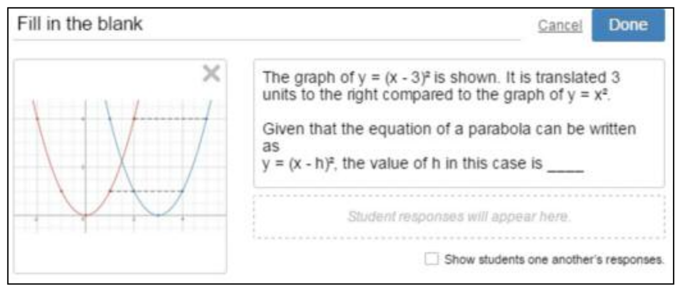
Interested in creating your own activity? Great! Start by going to **teacher.desmos.com**, scroll to the bottom of the page to the "Your Custom Activities" section, and select "Create with Activity Builder."



The next page will provide you with great instructions. You start by giving your activity a title, which can later be edited, then you build your screens. There are three types of screens: graph, question, and text. Text screens allow you to give instructions or explanations. Graph screens look just like the Desmos graphing calculator. You can have one or more graphs ready for students to work with, including graphs with sliders. Students can also enter their own equations on graph screens. The title area allows you to provide instructions for your students. Here is an example of a graph screen:

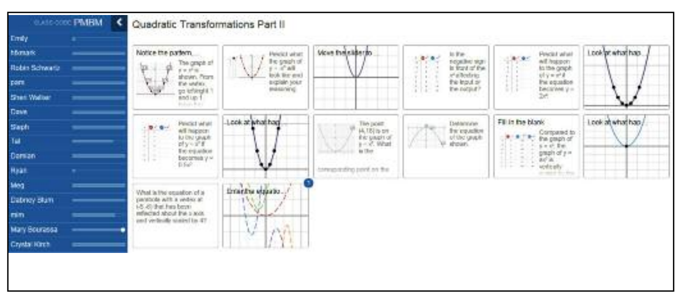


Question screens allow you to have an optional title, an optional image, and a question, with an answer box for students. You also have the choice of whether you want students to see each other's responses. Here is an example of a question screen:

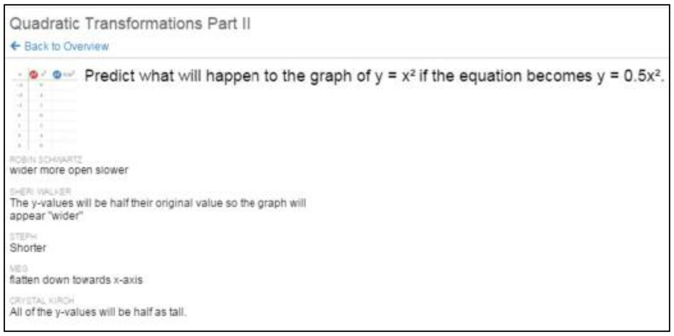


You can edit your screens and rearrange them as needed. You will be prompted to enter a description of your activity before you exit the editing process.

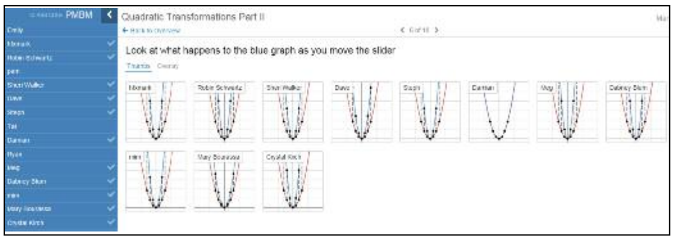
Once you run your activity with students, you can see an overview of the class. Each student has a progress bar, allowing a quick view of how the class is doing.

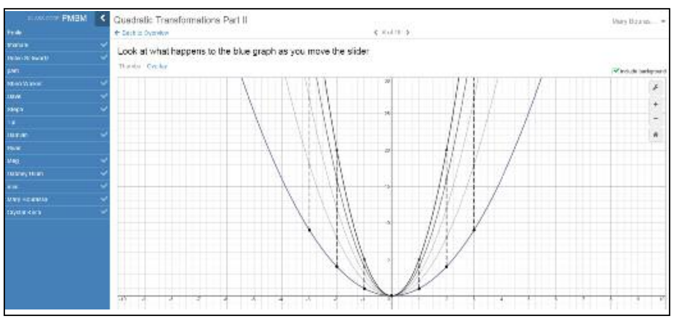


If you click on a question screen, you will see each student's answer:

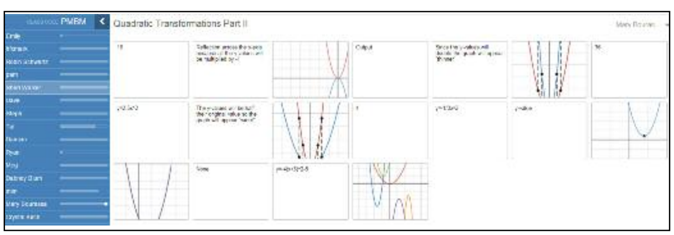


If you click on a graph screen, the left side will show you who has looked at that screen. You can also see a thumbnail of each student's screen or an overlay of all the results.





If you click on a student, you will see all of his or her screens for the entire activity.



Activity Builder is the perfect tool to address all of those curriculum expectations that begin with "Through investigation, using graphing technology…." Instead of trying to keep your entire class moving through an activity at the same rate, or making a handout for them to work through independently, Activity Building provides both differentiation and feedback to your students and to you. If many students are struggling with the content of a particular screen, you can stop the whole class and address the issue. Likewise, if they all understand the concepts being explored, you know that you can move on.

One more note about Desmos activities— you can find more on the Desmos Bank: **sites.google.com/site/desmosbank**. This is a searchable repository of activities that were created with the Desmos graphing calculator, Desmos Activity Builder, or Desmos Custom Polygraph. There are already activities that can be found using Ontario course codes (MPM2D, MHF4U, 1P, 2P, 2D), as well as more general categories. If you are looking for an activity for a particular topic, you can quickly find out if there is one ready for you to use. And be sure to submit the activities you create to the bank! ▲

▲ Mathematics Education Researcher Highlights

Math Education Researcher: Dr. Ruth Beatty, Lakehead University

Projects: Ruth has spent 10 years researching how children learn complex mathematical concepts, particularly in the domain of "early algebra." This work has been supported by a number of federal grants, including a three-year Canadian Graduate Scholarship (CGS) awarded by SSHRC, and has been published as *From Patterns to Algebra: Lessons for Exploring Linear Relationships* by Nelson Canada (co-authored with Dr. Cathy Bruce). Currently Ruth and Danielle Blair, provincial math lead, are working with members of Anishinaabe communities and educators from Ontario school boards to research the connections between Anishinaabe ways of knowing mathematics and the Western mathematics found in the Ontario curriculum. The goal of this research, funded by a SSHRC Insight Development Grant and by the Ontario Ministry of Education, is to collaboratively design culturally responsive mathematics instruction for First Nations students, and to learn from and incorporate Anishinaabe pedagogical perspectives in inclusive classroom settings.

▲ Math Exploration Using the Morley Triangle

TIM SIBBALD
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Tim is an Assistant Professor of mathematics education in the Schulich School of Education, Nipissing University. He has previously taught high school and been a mathematician in industry. He currently serves as the President of the OAME.

The Morley triangle is composed within a triangle by trisecting each angle and identifying the three points where neighbouring tri-sectors intersect (see Figure 1). The Morley triangle, ΔDEF, is an equilateral triangle. This discovery was made around 1900 and, unlike many high school geometry theorems, was unknown in antiquity. This article explores ways to bring this relatively modern geometry result to life in the classroom. The focus is primarily numerical because the triangle can be constructed and investigated using the trisection option in GeoGebra (software freely available at **www.geogebra.org**) and the spreadsheet facility that records values can be used for experimental investigation. The challenge that this paper addresses is how to relate the position, size, and orientation of the Morley triangle to the original triangle. The issue of proof is not addressed because it would be difficult for students to achieve in a constructive manner without significant guidance (several proofs can be found online at **www.cut-the-knot.com/triangle/Morley/**).

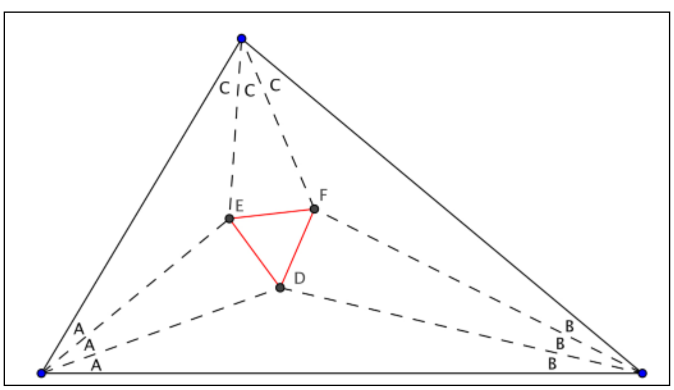


Figure 1. *The Morley triangle.*

Before reading further, consider that the orientation of this article is toward having students create questions about the Morley triangle that they can investigate numerically. Teachers will need to gauge their comfort level with doing this, but should consider asking open questions to promote student exploration. Consider asking your students to each make a triangle and have them measure the angles with protractors, then construct the trisectors and the Morley triangle. When I have done this, a sense of disbelief and awe have arisen. The idea can be adapted to creating a class set of triangles that follow a pattern (such as a common base and angles of 10, 20, 30,… degrees in one corner), where construction of Morley triangles allows for a gallery walk to see how the pattern among the original triangles becomes a pattern among the Morley triangles. Hypotheses may arise and discussion is likely with support from the teacher.

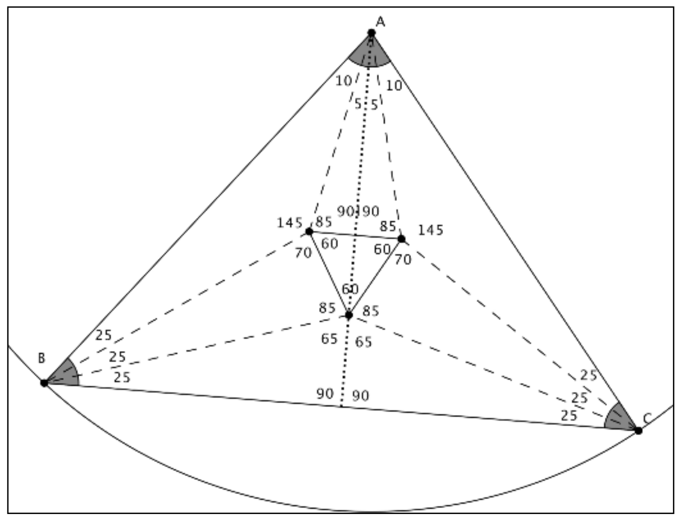


Figure 2. *Angle determination exercise.*

One case that can be addressed geometrically in Grades 9 and 10 is the Morley triangle constructed within an isosceles triangle. In Grade 9, students can work out the size of all the angles, given the size of one of the isosceles triangle angles. Consider the example in Figure 2 of an isosceles triangle with a unique angle of 30°, where students are to use the fact that the central triangle is the Morley equilateral triangle. In Grade 10 and above, trigonometry allows for considerably more detail. Consider the layout in Figure 3, where the dotted line AD is the line of symmetry. The angle *x* is the independent variable, and the challenge for trigonometry students is to determine *h* and *m* that define the location of the Morley triangle. Note that in the isosceles triangle case, the orientation of Morley's triangle is not considered because it shares the axis of symmetry, and GF is parallel to BC.

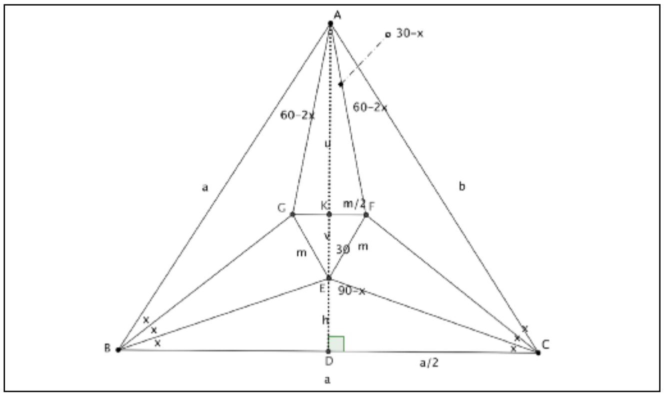


Figure 3. *The general isosceles Morley triangle.*

In the following examination, one could define $a = 1$ as one way to simplify the algebra for students. In Figure 3, triangle CDE gives the relationship defining h as:

$$h = \frac{a\tan(x)}{2}$$

Since the central triangle, EGF, is equilateral, $v = \frac{\sqrt{3}m}{2}$, and in triangle AKF, the relationship relating u to m is:

$$\tan(30 - x) = \frac{m}{2u}$$

$$u = \frac{m}{2\tan(30 - x)}$$

The relevance of this is that $u + v + h$ is the height of the outer triangle. It also allows substitution based on the three previous equations and leads to:

$$u + v + h = b^2 - \frac{a^2}{4} = \frac{m}{2\tan(30 - x)} + \frac{\sqrt{3}m}{2} + \frac{a\tan(x)}{2}$$

Using some algebraic equation solving, which will require teacher support, the relationship defining m in terms of a and x is:

$$m = \frac{\left(4b^2 - a^2 - 2a\tan(x)\right)\tan(30 - x)}{2 + 2\sqrt{3}\tan(30 - x)}$$

That is the extent of theoretical considerations for students in the high school environment. A little more will be stated at the end of the article for teachers who are interested in a more detailed look at the algebra behind this phenomenon. For the classroom, the focus becomes numerical in order to keep it accessible to students.

A Numerical Approach

When I first played with the Morley triangle, I was using Geometer's Sketchpad®, which had no facility for trisecting angles. This required explicitly calculating points corresponding to the trisections. It was feasible and I did manage it, but I never felt it was reasonable to suggest for other math teachers. That changed when I found that GeoGebra includes a tool for making angles with a specified size ("angle of a given size") and that the specification can be in terms of any other measured angle. In other words, trisecting angles is an option.

To make numerical investigation achievable, GeoGebra includes a spreadsheet. Any variable can be recorded to the spreadsheet (right click, "record to spreadsheet"). In the version of GeoGebra I am using (version 5.0.119.0), the recording of angles is in degrees only with the endless list option. Angles are in radians if a finite list is selected. Two additional details need to be recognized. Firstly, calculations in the spreadsheet should be done when recording of variables is turned off (red dots in column labels need to be clicked with the mouse). Second, if you wish to work in degrees, then calculations need to be done in GeoGebra, rather than copying the spreadsheet data to other software. If data is copied and pasted into another spreadsheet, it registers as text with the degree sign included; numerical calculation is no longer feasible. If you want your students to work with the values in other software, it requires recording a finite list and guiding your students to make a calculated column that converts the radians into degrees (i.e. , multiply by 180/π). Luckily, angles measured in degrees are fine for inspection and for manually using the values for graphing by hand or duplicating in a graphing calculator.

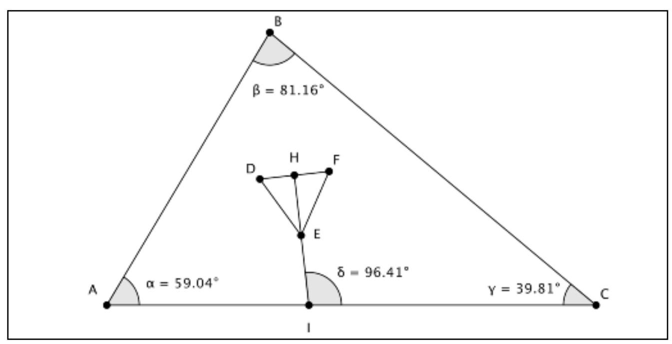


Figure 4. *Experimental set-up for measuring orientation.*

As a first numerical experiment, consider asking students to measure the orientation of the Morley triangle. Students can make a Morley triangle, or you can provide a template (see "concluding remarks" for accessing a template) to suit the amount of time you have for the activity. An experimental set-up for measuring the orientation is provided in Figure 4. Students should be guided to question how to measure the orientation, and encouraged to explore other approaches—Figure 4 is only one approach! Students might, for example, extend DE down to AC and measure the angle that is formed. Alternatively, they could measure DF relative to the horizontal by adding a line through D that is parallel to AC. The option shown uses a DE to construct a perpendicular bisector (HE) of the Morley triangle and extends it to the base of the original triangle (point I). The angle HIC then measures the orientation. Right clicking on each angle in GeoGebra allows recording to the spreadsheet. Forewarn students not to drag too fast; it is computationally intensive and GeoGebra does periodically run into difficulties—make sure students save their experimental set-up before any dragging that records to the spreadsheet. When I ran the experiment, I acquired the results shown in the screen shot in Figure 5. Note that the jumps in values in the spreadsheet correspond to computation being unable to keep up with the speed I dragged point B.

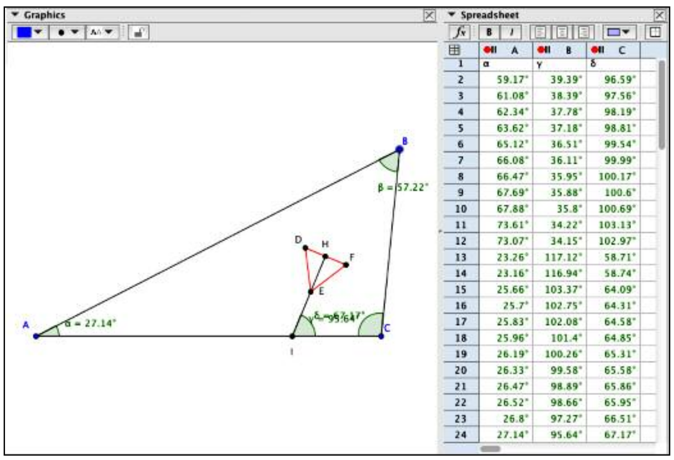


Figure 5. *Screenshot of orientation experiment.*

Following the data collection, students can turn the recording feature off by clicking the red "record" buttons at the top of each column in the spreadsheet. They can then create hypotheses by visually inspecting the columns and observing trends. This can lead them on a path of organized experimentation, where they constrain the point that is dragged. They may, for example, have the dragged point, B, affixed to a line that is parallel to AC. What would happen then? This helps tailor hypotheses and can be subsequently used for discussion in the classroom.

A second type of experiment is beneficial and suited to anyone wishing to have data that can easily be copied and pasted to other software. Have students design their own experiment involving only lengths. They could examine how the Morley perimeter varies with the triangle perimeter. Measuring areas would also work. To demonstrate what is feasible, this experiment will follow a hunch and see how the side length of the Morley triangle is related to the radius of the circumscribed circle for the triangle. The experimental set-up is shown in Figure 6. In the figure, the circumcentre is found as the intersection of the lines perpendicular to the midpoint of each side (point K). The circumcentre is the middle of a circle that passes through the three vertices (A, B, and C) of the triangle. The radius of the circumcircle is KC, which has been measured, while the side length of the Morley triangle is *m*.

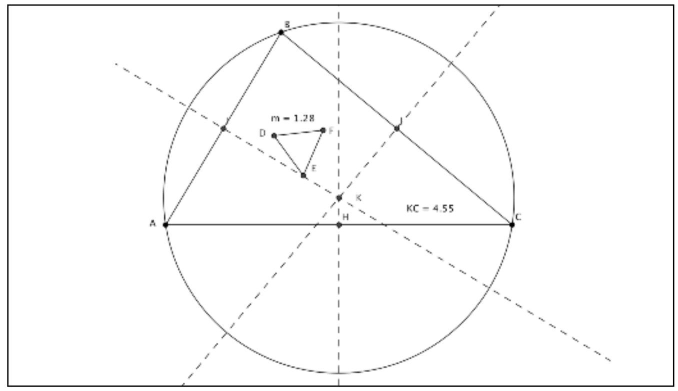


Figure 6. *Experiment to investigate lengths.*

Recording the data took multiple tries, and it might have been wise to reduce the amount of software running on my computer. However, when it was successful, I clicked on the red circles to turn off recording to the spreadsheet and that then calculated the ratio column on the spreadsheet. The ratio column is the ratio of the circumcentre radius to the Morley side length. I confess, I initially thought the value always seemed to be between three and four, and thought that I should see if I could defy that finding. The outcome of the experiment is shown in Figure 7.

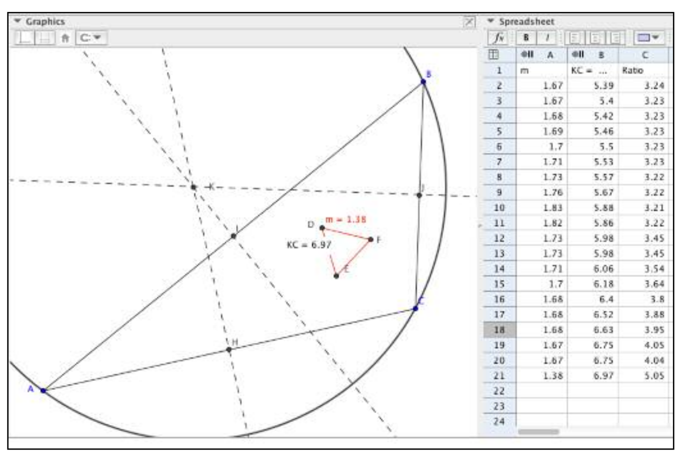


Figure 7. *Results of experiment concerning lengths.*

It was through some serious play that it became evident that as the largest angle in the triangle grew large, the circumcircle also grew large, but the Morley triangle shrank. Encourage your students to go the extra step in explaining: The Morley triangle is contained in the original triangle and is equilateral, so in some sense, it is confined by the minimum linear size of the triangle. In other words, the height of the obtuse triangle is becoming much smaller, and that reflects a constraint on the size of the Morley triangle. Perhaps that is not quite as clear as you would like? That is because there is a whole new experiment lurking that asks: How big is the largest equilateral triangle that fits inside a given triangle?

Why not elaborate on the algebra?

As mentioned earlier, this paper purposefully pursued a numerical approach. If you are wondering why, then consider a particular arrangement shown in Figure 8, where the triangle has been placed on a coordinate grid so that the three points are on the axes of the coordinate system. The three points are (-a, 0), (b, 0) , and (0, c). While this naming approach does not match the “standard” approach of having small letter sides opposite capital letter angles, it does aid clarity by having the coordinate a affiliated with the angle A. The coordinate arrangement, with the three points on the axes, is convenient for clarifying why theory can only be used to some extent; the triangle will be assumed to be acute in this case.

Within this set-up, the coordinates of point E, the Morley point closest to the x-axis, can be determined. In Figure 8, the tangent of angle A and tangent of angle B provide two equations in two unknowns. The two equations are:

$$\tan(A)=\frac{y}{a+x} \quad \tan(B)=\frac{y}{b-x}$$

When solved, the two coordinates are:

$$x=\frac{b\tan(B)-a\tan(A)}{\tan(B)+\tan(A)}$$

$$y=(a+x)\tan(A)=\frac{(a+b)\tan(A)\tan(B)}{\tan(A)+\tan(B)}$$

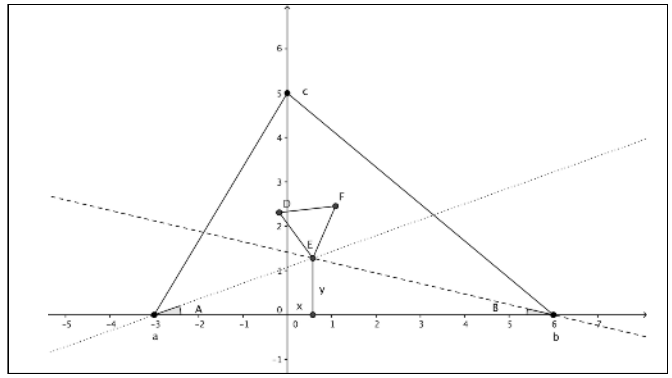


Figure 8. *An analytic geometry set-up.*

This resolves point *E* in terms of *a*, *b*, *A*, and *B*. To remove references to the angles requires resolution of tan(A) in terms of side lengths of the right triangle in the second quadrant. However, since *A* is one-third of the full angle, determining tan(A) requires solving the single-angle tangent in terms of the triple-angle tangent, which can then be replaced with a ratio $\left(\frac{c}{a}\right)$. Given that:

$$\tan(3A)=\frac{c}{a}=\frac{3\tan(A)-\tan^3(A)}{1-3\tan^2(A)}$$

What is important is to realize that this involves solving a cubic equation for tan(A). In general, that is beyond the theoretical scope of the high school curriculum, except using numerical means.

Similarly, tan(2A) can be expressed as a ratio of values based on coordinates in the Morley triangle. This provides quadratic algebraic connections between the coordinates, but does not provide any clear way to separate the variables in order to use high school approaches to quadratics.

There is quite a rich vein of mathematics that teachers could explore further, with an eye to including it in advanced functions. However, it is not as readily suited to teaching as the numerical approach. The remarks here are intended to make it clear that using methods that are not numerical will pose significant pedagogical challenges. I remain hopeful that others will develop these aspects.

Concluding Remarks

Thank you to the two reviewers who provided some good advice for this article. In particular, a template file and a file corresponding to Figure 6 are available at (**http://faculty.nipissingu.ca/timothys**) and also at the GeoGebra Institute of Canada website (**www.geogebra canada.org/home/resources**). ▲

▲ Mathematics Education Researcher Highlights

Math Education Researcher: Dr. Cathy Bruce, Trent University

Projects: Dr. Bruce is currently focused on two multi-year studies, one in the area of teaching and learning fractions, and the second in the area of mathematics for young children (M4YC). Her research in fractions is supported by the Ministry of Education in Ontario and has resulted in field-tested resources available to educators free of charge on the EduGAINS website. The research has involved developing a fractions learning pathway beginning with unit fractions and moving through to operations with fractions. The project has consisted of collaborating with educators in Collaborative Action Research cycles that focus on fractions teaching and learning to: a) develop precise, relevant, and meaningful tasks; b) field-test and revise tasks and instructional strategies and materials; and c) provide evidence for further developments and refinements of the fractions learning pathway (for related resources, see: **www.edugains.ca/newsite/DigitalPapers/fractions/fractions-Content.html**).

Her research on Math for Young Children (M4YC) began five years ago and continues with an emphasis on spatial reasoning and the role that spatial reasoning plays in mathematics learning. This project is funded by SSHRC in the form of three current research grants. Young children's mathematics learning is crucial for later success in school and for productive social contributions, as evidenced by research showing early math to be a key indicator of later achievement and grade retention. In practical terms, by engaging in Lesson Study with teachers of young children, Dr. Bruce has implemented a professional learning model that focuses on releasing the ceiling on what young children can and should do in Kindergarten to Grade 2 classrooms. Lesson study teams in the study explore the range of spatial reasoning skills and playful learning opportunities that support high-quality mathematics learning overall for young children (for related resources, see: **www.tmerc.ca**).

▲ Ask Assessment Abby: Group Marking

ASSESSMENT ABBY
EMAIL: assessmentabby@oame.on.ca

Ask Assessment Abby A³ is a regular column in the OAME *Gazette*, where teachers can share concerns and best practices about assessment, evaluation, and reporting of mathematics. Please send your questions to Ask Abby at **assessmentabby@oame.on.ca**.

Dear Assessment Abby,

My son came home with a group mark on his progress update. He got 80% on his individual part, but the group got 25% on the group aspect. The final mark that was assigned to my son was 65%.

Please advise.

From a Concerned Parent in the PRMA chapter.

Dear Concerned Parent:

I understand your surprise and would recommend that you speak to the teacher to clarify the assignment expectations, what was assessed, and how the grade was determined. Involving your son in this conversation may help him understand why this mark was achieved and how he can improve.

For teachers, the value of group work is articulated in the Ontario Ministry of Education's *Growing Success* document, and we should continue to support these activities. When it comes to evaluation, while we don't know specifically what happened in this case, to avoid such misunderstandings, I would recommend assigning individual, rather than shared, group marks. According to *Growing Success*, "Assignments for evaluation may involve group projects as long as each student's work within the group project is evaluated independently and assigned an individual mark, as opposed to a common group mark" (p. 39). I would also recommend including success criteria to clarify how individual marks are determined when students are involved in group projects. This may be new territory for mathematics teachers. I would suggest consulting with teachers of other subjects, perhaps Phys. Ed., Drama, or Music, where evaluating individual performance within a group environment may be more prevalent.

Keep math rich with students in mind,

Assessment Abby

If you have any questions or comments for Assessment Abby, email: **assessmentabby@oame.on.ca** ▲

▲ What's the Problem? Pirate Treasure Hunting

SHAWN GODIN
EMAIL: shawn.godin@ocdsb.ca

Shawn is the head of Mathematics, Business Studies, Law, and Computer Science at Cairine Wilson Secondary School in Orleans, Ontario. He is an advocate of mathematical problem solving. He currently helps create and mark math contests for the Centre for Education in Mathematics and Computing at the University of Waterloo. He currently works on math contests for the Canadian Mathematical Society and has worked as editor of their problem-solving journal, Crux Mathematicorum*. His Google Drive folder for supplementary material can be accessed at* **https://drive.google.com/folderview?id=0ByDIaUaj..8StpanhnUWo2bEV6ZE0&usp=sharing**.

Welcome back, problem solvers. Last time, I left you with the following problem:

A pirate leaves the following instructions: "The island where I buried my treasure contains a single palm tree. Find the tree. From the palm tree, walk directly to the falcon-shaped rock, counting your paces as you go. Turn a quarter-circle to the right, walk the same number of paces as you just counted, and plant a stick into the ground. Return to the palm tree. From the palm tree, walk directly to the owl-shaped rock, counting your paces as you go. Turn a quarter-circle to the left, walk the same number of paces as you just counted, and plant a stick into the ground. Connect the two sticks with a rope and dig beneath its midpoint to find the treasure." By the time the pirate's instructions are found, the palm tree has long since died, but the two rocks are still identifiable. Where should you look for the treasure?

This wonderful problem was sent to me by Ross Isenegger. It has appeared in presentations by Peter Taylor, who got it from George Gamow's book, *One, Two, Three, …, Infinity*. Ross pointed me to the article *Dynamic Visualization and Proof: A New Approach to a Classic Problem* by Daniel Scher from the September 2003 issue of NCTM's *Mathematics Teacher.*

Let's proceed as Scher did in his article by investigating the situation with dynamic geometry. In figure 1, points F, O, P, S_1, S_2, and T represent the falcon-shaped rock, the owl-shaped rock, the palm tree, the first stick, the second stick, and the treasure. It only takes a couple of moments playing with a dynamic geometry sketch to come up with the conjecture that the location of the treasure is independent of the location of the palm tree. Scher's article constructs a lovely geometric proof, in stages, inspired by an ongoing dynamic geometry investigation. I highly recommend looking up this article.

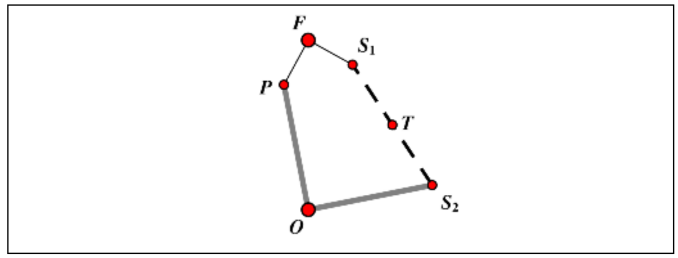


Figure 1

Delegating the geometric proof to Scher (or passing the buck), let's look at the problem from another point of view. Suppose we put these points in the Cartesian plane. We could, without loss of generality, fix F at (0, 1) and O at (0, -1). Grade 9 students could then explore the idea, using a Geoboard and working with slopes of perpendicular line segments. They will quickly see that the location of the treasure stays fixed. If they explore systematically, by keeping their location of the palm tree to the x-axis, as in figure 2, they can follow up computationally and then generalize algebraically.

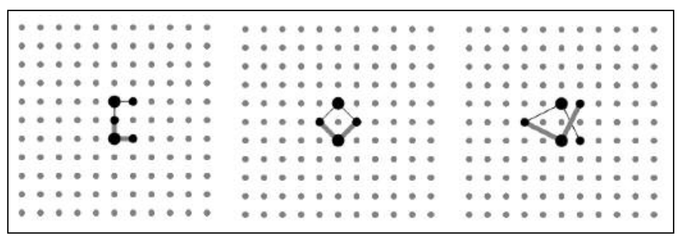


Figure 2

For example, let's consider the right-hand case from figure 2, where P is located at (-2, 0). Going from P to F, we rise 1 unit and run 2 units, so that $m_{PF} = \frac{1}{2}$. Thus, if $FS_1 \perp PF$, we must have $m_{FS_1} = -\frac{2}{1} = -2$. Since the lengths of these two segments must be equal, we must have the rise as -2 units and the run as 1 unit as we go from F to S_1; hence S_1 is located at (1, -1). Similarly, we find that S_2 is located at (1, 1). So the location of the treasure is the midpoint of S_1S_2,

$$\left(\frac{1+1}{2}, \frac{-1+1}{2}\right) = (1,0)$$

which is the same location that we get for every location of the point *P*.

Next we look at an algebraic generalization of a special case, when *P* is on the *x*-axis at the point (*a*, 0). Then, going from *P* to *F*, we rise 1 and run –*a*; thus, to go from *F* to S_1, we must run 1 and rise *a*. Putting this into algebraic notation, we see that S_1 is located at (1, 1 + *a*) and a similar argument puts S_2 at (1, –1 – *a*). Hence, it is easy to calculate that *T* is at (1, 0) and is independent of *a*.

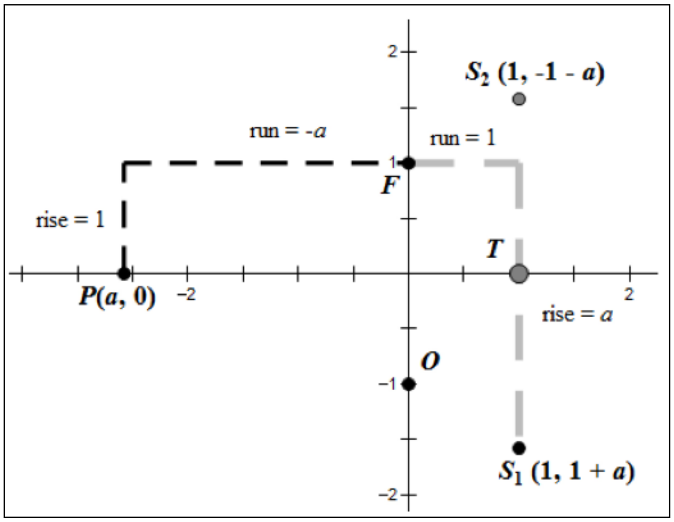


Figure 3

You can go through the same argument with *P* at any location (*a*, *b*) and show that *T*, indeed, is independent of *P*'s location. If you love your algebra, you can go one step further and show that if the rocks are located at $F(x_1, y_1)$ and $O(x_2, y_2)$, and the palm tree is at *P*(*a*, *b*), then the location of *T* depends only on the location of *F* and *O*, but I will leave that fun to you!

Finally, we will examine the solution from *One, Two, Three, …, Infinity*. In Gamow's treatment of the problem, he represents the locations as complex numbers. When we multiply a complex number by the imaginary unit, $= \sqrt{-1}$, and plot the results on the Argand(complex) plane, the point representing the product is the original point rotated counter-clockwise by 90˚. Thus, if we reformat the situation discussed in the last solution, we can represent the falcon-shaped rock and the owl-shaped rock with *i* and –*i*, respectively. If we let the complex numbers π *and* τ represent the locations of the palm tree and the treasure, respectively, then σ_1, the complex number representing the location of the first stick, can be calculated as

$$\sigma_1 = (\pi - i) \times i + i = i\pi + 1 + i$$

Note that subtracting *i* is equivalent to translating the palm tree (and the falcon-shaped rock) down 1 unit. Thus, the falcon-shaped rock is at the origin, so multiplying by *i* rotates the palm tree's location counter-clockwise about *F*, then adding *i* moves everything back so *F* is at its original position. Similarly, the location of the second stick can be represented by

$$\sigma_2 = (\pi - (-i)) \times (-i) + (-i) = -i\pi + 1 - 1$$

Thus, the location of the treasure is at

$$\tau = \frac{\sigma_1 + \sigma_2}{2} = 1$$

the same as in our first solution. Again, if we want to generalize, we can place *F* and *O* anywhere and use complex variables to represent them. The ensuing calculations will show that the location of the treasure does not depend on the location of the palm tree.

And now for your homework.

Pick any whole number between 20 and 100 inclusive. Add the digits together and subtract the sum from the original number. Finally, add the two digits from your last result and concentrate really hard.

I will reveal your answer in the next issue. Until then, happy problem solving. ▲

Encyclopedia of Mathematics Education (S. Lerman, 2014) Excerpts

"The impact of technological change in the workplace is another strong theme. For example, Hoyles and colleagues (2010) describe the emerging need to go beyond mere procedural competence with calculations, to interpret and communicate fluently in the language of mathematical inputs and outputs to technologies. The focus in these studies is on understanding the mathematical demands and affordances of workplaces in order to increase knowledge in and of the field among educators, employers, policy makers, and workers themselves and make evidence-based recommendations for good practice to equip learners to meet those demands."

Coben, D., & O'Donoghue, J. (2014). Adults learning mathematics. In S. Lerman (Ed.), *Encyclopedia of mathematics education* (pp. 15–23). London, UK: Springer.

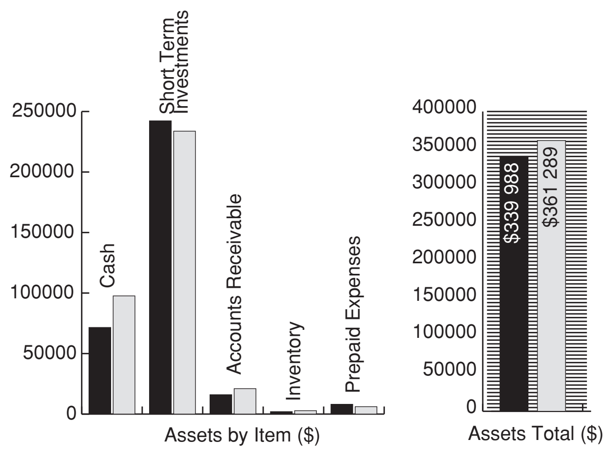
▲ OAME Financial Statement 2014-15

Notice to Readers

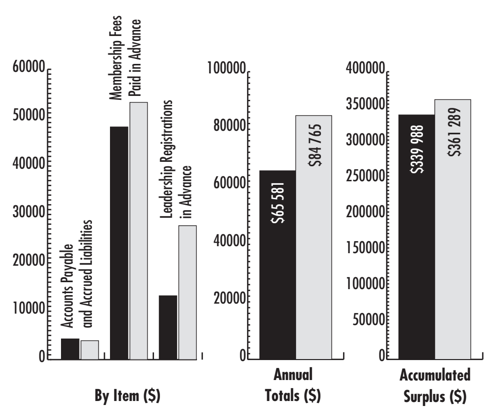
On the basis of information provided by management, I have compiled the statement of financial position of Ontario Association for Mathematics Education (Association Ontarienne pour L'Enseignement des Mathématiques) as at August 31, 2015, and the statement of operations, changes in net assets, and cash flow for the year then ended. We have not performed an audit or a review engagement in respect of these financial statements and, accordingly, we express no assurance thereon. Readers are cautioned that these financial statements may not be appropriate for their purposes.

London, Ontario, August 31, 2015, SKRYPNYK GROUP, Professional Corporation, Authorized to pratise public accounting by CPAofO

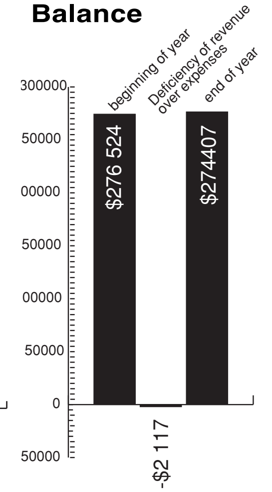
Current Assets



Liabilities and Fund Balance

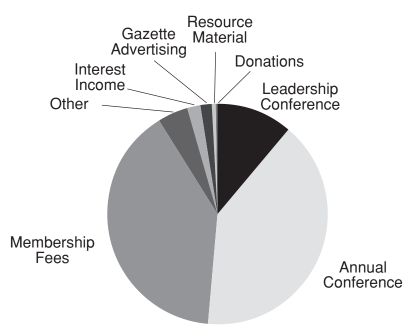


Operating Fund Balance

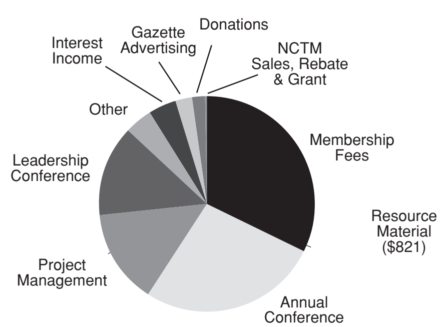


2015/2014

Revenue

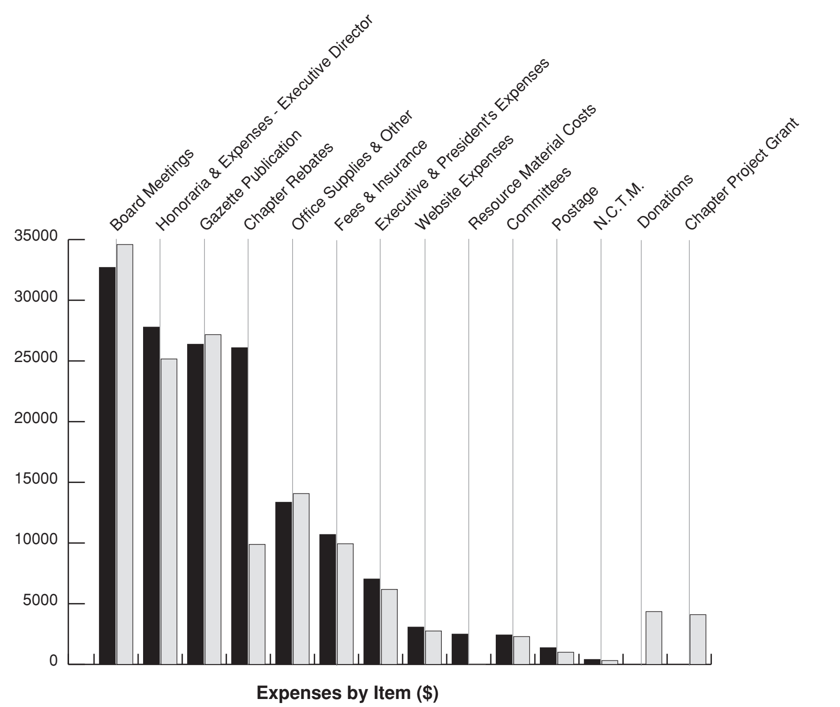


2015 ($261 062)



2014 ($214 762)

Expenditures



▲ Fields Institute MathEd Forum Report

STEWART CRAVEN
EMAIL: numeratecitizen@mac.com

Stewart Craven presently teaches courses for the Continuing Education Department and the Transition Years Program at York University. He is an active participant in the Fields Mathematics Education Forum.

Our first meeting of the academic year took place on September 26, 2015. As has been the case for the last couple of years, this meeting was well attended. I couldn't help but notice that the participants represented diverse areas of mathematics education and included mathematics professors, primary school teachers, masters and Ph.D. students, mathematics education professors, professional engineers, high school teachers, college professors, and mathematics education consultants. This is not an exhaustive list, but serves to demonstrate that the MathEd Forum audience varies widely and that all attendees are welcome.

This meeting highlighted talks by Amy Lin (IGNITE Your Passion) regarding several five-minute timed presentations that were given at the OAME annual conference. The purpose of the OAME presentations was to inspire mathematics teachers. Charles Anifowose reported on the winners of "The Taming of Chance" essay competition, while Ed Barbeau's presentation, "Problems That Get into the Underlying Structure," illustrated mathematics problems that help students think more deeply. Ed issued a challenge for teachers to try these problems in their classrooms. (You are encouraged to check out his website at **www.math.utoronto.ca/barbeau/home.html**.) Other presenters included Tony DeRose ("Pixar: The Math Behind the Movies"); Kristian Howald ("Mathematical Thinking in Modern Art Production"); and Joe Geraci ("Applied Math at the Cutting Edge of Research"). As you can see, the range of topics varied widely.

Once a year, the Fields Institute MathEd Forum is held at a university or college outside of Toronto. This year, the October meeting was held in Ottawa. The theme was "Enactivist Methodology in Mathematics Education Research," featuring Laurinda Brown (University of Bristol, UK), Jérôme Proulx (Université du Québec à Montréal), and Jean-François Maheux (Université du Québec à Montréal).

Throughout the year, the MathEd Forum Steering Committee seeks input regarding new themes and would appreciate your suggestions. Drop me an email at numeratecitizen@mac.com and I will pass your ideas along. Agendae and more information can be found at **www.fields.utoronto.ca/programs/mathed/forum/**. ▲

Encyclopedia of Mathematics Education (S. Lerman, 2014) Excerpts

"The weight of evidence from countries across the world indicates that ability grouping harms the achievement of students in low and middle groups and does not affect the achievement of high attaining students. Despite this evidence, ability grouping continues to be widespread in some countries—particularly English-speaking countries in the West, probably reflecting a common Western belief that students have a certain ability that is relatively unchangeable. Where countries recognize that high achievement is possible for all students . . . or hold equity as a central principle . . ., ability grouping is less prevalent and not used with young children. Deeply held cultural beliefs about learning and about what it means to be 'smart' are difficult to change, which may be the reason for the persistence of ability grouping in some countries, a practice that appears to benefit some students at the expense of others."

Boaler, J. (2014). Ability grouping in mathematics classrooms. In S. Lerman (Ed.), *Encyclopedia of mathematics education* (pp. 1–4). London, UK: Springer.

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Call for *Gazette* Editor

The *Ontario Mathematics Gazette* is taking applications for a two-year term as editor commencing with the September 2016 issue.

Job shadowing available now.

For more details, please contact:
Fred and Lynda Ferneyhough
519-899-4203
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NOMINATIONS FOR OAME/AOEM BOARD OF DIRECTORS 2016–2017

Each year, elections are held for a President-Elect, a Vice President (3-year term), and two Directors (up to a 3-year term) of OAME/AOEM. OAME/AOEM members running for Director will declare their intent to work on either Grades JK–6 or Grades 7–12 issues based on their personal experience, expertise, and interests.

Nominations for these positions are to be submitted by 4:00 p.m., January 31, 2016, to:
Paul Alves, Past President
paul.alves@oame.on.ca

Include a paragraph of the nominee's qualifications with the Nomination Form. A nominee must be a member of OAME. By March 2016, the Nomination Committee will publish a slate of candidates, including candidate bios, for consideration by the membership. Voting will be online and is open to all OAME/AOEM members. The election results will be released at the Annual General Meeting of the OAME/AOEM in May, 2016.

NOMINATION FORM

I nominate:____
for the position(s) of:

☐ President-Elect ☐ Vice-President ☐ Director

Nominee's Address: ______________________________________

____

E-mail: ____

Telephone: Home (_________)______________________________

Work (__________) _______________________________________

Nominator's Name : ______________________________________

Nominator's Address:______________________________________

____

E-mail: ____

Telephone: Home (_________)______________________________

Work (__________) _______________________________________

ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION

8 PAGE INSERT ELEMENTARY MATH

Abacus

Abacus

Volume 54 • Number 2 • DECEMBER 2015

EQUIVALENCE AND RELATIONAL THINKING - SUBTRACTION

TABLE OF CONTENTS

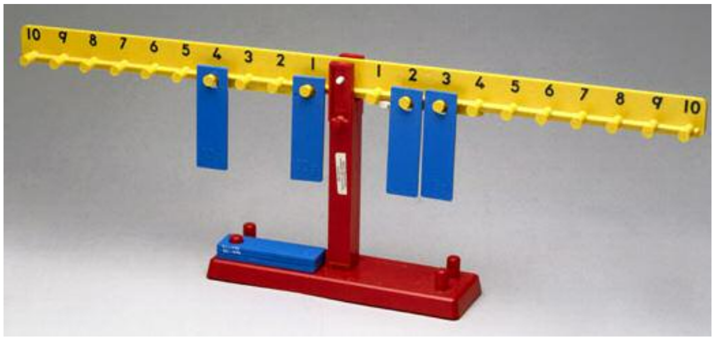
A Publication of the OAME/AOEM

ABACUS CO-EDITORS

MARY LOU KESTELL works on K to 12 mathematics professional learning at the provincial level. She has spent 40+ years working in all aspects of Ontario mathematics education and is a past-president of both OAME and OMCA. **maryloukestell@gmail.com**

KATHY KUBOTA-ZARIVNIJ currently focuses on K to 6 mathematics professional learning at the provincial level. She is a long-time OAME board member and OAME past president. In relation to her work in mathematics education, she uses complexity thinking to interpret mathematics teaching and learning for students and teachers. **kathkubo@rogers.com**

The balance metaphor is a widely used conceptual image to represent the concept, equivalence within the context addition expressions. This number line balance scale shows 4 + 1 = 2 + 3.



What would this number line balance scale look like to model:

- 4 + 1 - 2 = 2 + 3 - 2
- 4 + 1 - 2 < 2 + 3 - 1

Is it true that 8 – 3 = 8 – 4 + 1? How do you know that 17 + 21 – 21 ≠ 18 + 21 – 21? How might the number line balance be used to model these equations?

In this issue, the Research Summary and reasoning tasks (with multiple solutions) focus on equivalence and relational thinking, within addition and subtraction contexts. The reasoning tasks in this issue are designed for use with a three-part problem-solving lesson or a Mental Math mini lesson structure. Consider including in your mathematics program planning daily learning and practice for solving subtraction true/false number sentences and open number sentences.

RESEARCH SUMMARY – EQUIVALENCE AND RELATIONAL THINKING FOR SUBTRACTION

Carpenter, Franke and Levi (2003) challenged children's conceptions about equivalence and the equal sign within number operation contexts. They asked children about how the equal sign was used. Children are justifying their thinking, convincing others as they recognize and resolve conflicting assumptions and conclusions about equality, number operations and relationships. Exploring the relationship between terms on either side of an equal symbol enables students to develop an understanding of the relationship that exists between the terms. It is important that they discover this relationship through their own thinking. These discussions require children to develop ideas and understand properties of equality inherent in number equations. An example of the Subtraction Property of Equality is about maintaining equivalence when subtracting the same quantity from both sides of the equation

Developing Relational Thinking Using Subtraction Open Number Sentences

How could you compare the expressions on either side of the equal sign to determine the value of ? 45 – 7 = - 4

a) *Operational View of the Equal Sign*

When a student does not understand that the equal sign expresses a relationship between the expressions to its left and right, the student says that:

- is 38 because 45 - 7 = 38
- is 34 because 45 - 7 - 4 = 34

- Use base ten blocks to show that the quantities on the left and right sides of the equal sign must have the same value.
- Model how to determine the value of one side of the equation and then use that value to determine the unknown quantity on the other side of the equation.

b) *Comparative View of the Equal Sign*

When a student uses a comparative view of the equal sign, the student must determine the difference between the numbers on the left side of the equal sign, in order to determine the value of and tries to determine the value of by thinking comparatively:

- 45 – 7 = 38; 42 – 4 = 38; the quantities on the left and right sides of the equal sign must have the same value

- Look at the numbers on both sides of the equation and make some comparisons.
- How does 5 and 2 compare? If 5 is three more than 2, then how will the value of □ compare to 39

c) *Relational View of the Equal Sign*

When a student uses a relational view of the equal sign, the student compares the minuend in each expression relative to the subtrahend.

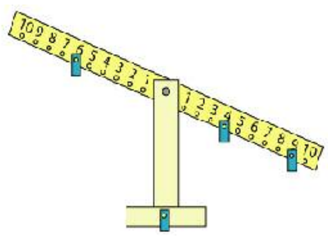
- 7 is three more than 4, so must be three less than 45

Further, Stephens (2006) investigated this subtraction equation, 90 – 59 = 99 - .

- the difference between the two numbers remains the same, if an increase of 9 in the first number (1st minuend) must be matched by an increase of 9 in the second number (2nd subtrahend)
- if the first number (1st minuend) increases to 99, then leaving the second number (1st subtrahend) as 59 changes the difference. So to keep the difference, the same the second number (2nd subtrahend) must increase by 9, giving 68
- if 90 increases to 99, then reducing the second number by nine to 50 gives an incorrect result: 90 – 59 is not equivalent to 99 – 50.

According to Stephens and Wang (2008), “a deep understanding of equivalence and compensation is at the heart of relational thinking. Students need to know the *direction* in which compensation has to be carried out in order to maintain equivalence. The direction of compensation that is appropriate for the operation of addition, for example, is inappropriate for subtraction. Some children reason incorrectly that a number sentence such as 87 – 48 is equivalent to 90 – 45. Failing to recognize that the relationship of subtraction is fundamentally different from addition, these children treat the direction of compensation for subtraction the same as for addition. Other children, however, explain, “in order for the difference to remain the same, the same number has to be added to each number in the expression”. For these children, 87 – 48 = 89 – 50.” (pp. 28-29). Furthermore, “relational thinking appears to be characterized by a capacity to see possibilities of variation between the numbers where the possibilities embody specific directions of change depending on the operations involved. Knowing that one number is greater or less than another number is no use unless one also knows the direction of change implied by these differences” (Stephens, 2006, p. 49).

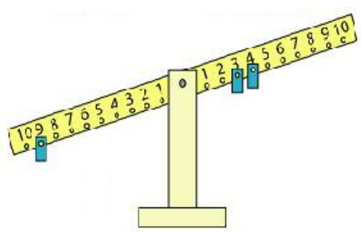
LINKS TO MANIPULATIVES: NUMBER LINE BALANCE SCALE



6 < 4 + 9

From which side of the equation does a quantity need to be subtracted to create equivalence?

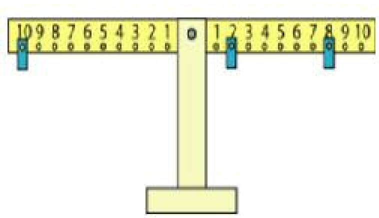
How much can be subtracted from the number line balance to create equivalence?



9 > 3 + 4

From which side of the equation does a quantity need to be subtracted to create equivalence

How much can be subtracted from the number line balance to create equivalence?



10 = 2 + 8

How much can be subtracted from each side of the number line balance to maintain equivalence?

LESSON LEARNING GOALS

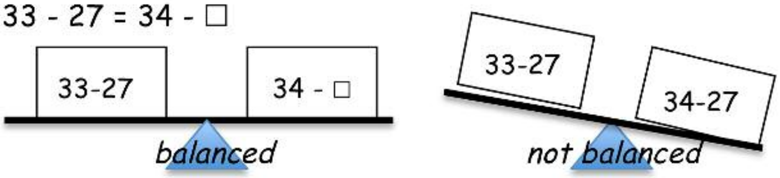
Identify the curriculum expectations (MOE, 2005) grade-before and grade-after the grade specific learning goal, in order to:
- see the development of mathematical knowledge, skills, and strategies across grade levels
- determine the mathematical focus of the problems chosen for Before (activation), During (solving problem), and After (Practice) parts of the lesson

BEFORE (Getting Started) – 5 to 10 minutes
- Activating students' mathematical knowledge and experience using a prompt or problem, related to the math in the lesson problem.
- Includes student responses to highlight ideas and strategies.

What is the Unknown Number?

What is □? Explain the method used.

33 - 27 = 34 - □



Solution 1
33-27 = 6
What number needs to be subtracted from 34 to give a difference of 6?
□ = 28

Solution 2
(1 more) 34 > 33
□ needs to be 1 more than 27
□ = 28

Solution 3
33-27 = 34–28
(33+1)-(27+1)
constant difference
□ = 28

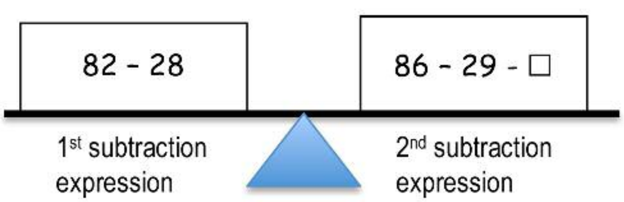
DURING (Working on It) – 15 to 20 minutes
- Understanding the problem – Teacher asks, "What information from the problem are we using to solve it? Explain." Teacher records in a list below the problem, the information that students identify.
- Students solve the problem on chart paper (landscape) with markers (visible for class discussion) in pairs or small groups.
- Teacher circulates to record different student solutions, in addition to the ones the teacher anticipated.

What is the Unknown Number?

What is □? Explain the method used.

82 – 28 = 86 – 29 - □

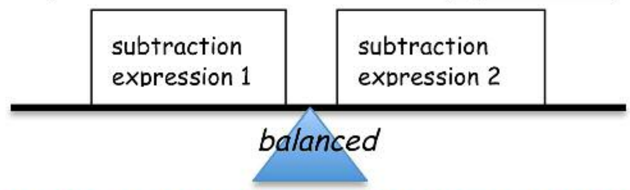
What information are we going to use to solve the problem?



□ *means the unknown number*

AFTER (Consolidation) – 20 to 25 minutes
- Teacher selects 2 or more solutions for class analysis and discussion in a sequence (1st, 2nd, 3rd, etc.) based on mathematical relationships between the solutions and the lesson learning goal.
- Students (authors) explain and discuss their solutions with their classmates; co-construction of success criteria towards the lesson learning goal
- During whole class discussion, solutions are organized (often re-organized) to show mathematical elaboration from one solution to the next and towards lesson learning goal.
- Teacher mathematically annotates (math terms, math symbols, labelled diagrams, concise explanations) on and around solutions to make mathematical ideas, strategies, and models of representation explicit.

*Visualize the quantity for each number expression so there is balance (**equivalence**)*



Same Difference *– the result of subtracting numbers in the first expression is the same quantity as the result of subtracting numbers in the 2nd expression*

Solution 1
Subtract numbers in the 1st expression to get difference
82–28 = 82–20–2–6
82–28 = 62 – 2 – 6
82–28 = 54 (difference)
Subtract 2 numbers in the 2nd expression, then subtract to get same difference as 1st expression
54 = 86–29– □
54 = 86–26-3 – □
(decompose 29 into 26 and 3)
54 = 60–3 – □ ; 57-54=3
54 = 57–□ □=3

Solution 2
Subtract numbers in the 1st expression to get difference
82–28 = (82+2) – (28+2)
84 – 30 = 54 (difference)
Get same difference; so subtract the difference in the 1st expression from the difference of 2nd expression
□ = 86-29 - 54
□ = (86+1)-(29+1)-54
(easier to subtract tens- +1 to each term -keep the difference constant)
□ = (87-30) – 54 = 3
57 – 54 = 3 so □=3

Operational
Difference from 1st Subtraction Expression Used for 2nd Expression

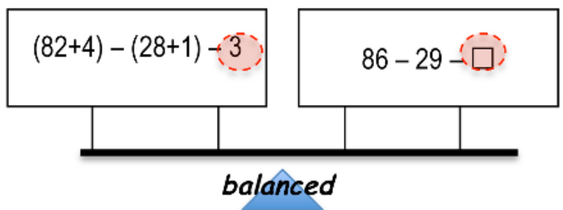
Operational
Subtract Same Difference from 2nd Subtraction Expression to get □

Note: Mathematical *annotations* include mathematical vocabulary, symbols, elaborations of mathematical details from solutions, labels describing the method/strategy and questions to further thinking. All *annotations* are records of students' mathematical discussion.

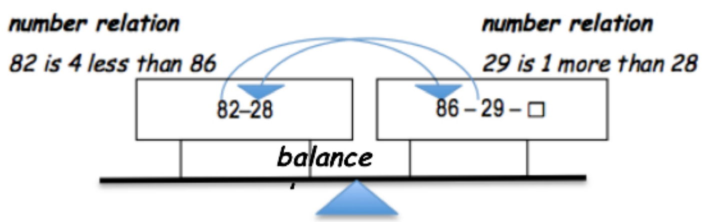
Grade 2	Grade 3	Grade 4
determine the missing number in equations involving addition and subtraction to 18	determine, the missing number in equations involving addition and subtraction of one- and two- digit numbers	determine the missing number in equations involving multiplication of one- and two-digit numbers

Equivalence or Equality
An equation indicating that two quantities (or expressions) are in balance; two expressions that are equivalent (e.g., 82 – 28 = 86 – 29 - □).

Compensating Method for Equivalence



Relational Method for Equivalence
Comparing pairs of numbers from expression 1 and 2.



There is a difference of 3 between LS and RS. □ = 3

Solution 3

82–28 = 86-29– □

Make pairs of numbers the same to know the unknown

82-28 = (82+4)-(29-1)- □

□ = 4-1

= 86–29 – 3

□=3

Solution 4

82–28 = 86-29– □

Comparing pairs of numbers from each expression
82 < 86 (4 more)
28 < 29 (1 more)

balance → 4 more (left side) minus 1 more (right side)

□=3

Compensating
Make Terms Same to See the Unknown

Relational
Compare Pairs of Terms Across 2 Expressions

AFTER (Highlights/Summary) – **5 minutes**

- Teacher and students revisit the student solutions for key ideas, strategies, and models of representation that are related to the lesson learning goal (anticipated success criteria)
- Teacher lists key ideas, strategies, and models of representation so the students can see how the mathematical details from their solutions relate explicitly to the lesson learning goal; summary of co-constructed success criteria

Highlights/Summary

Determine unknown number in subtraction equations using different methods:

- ***operational*** *– calculating the operations for both number expressions (e.g., 82-28 = 82-20-2-6 = 54)*
- ***compensating*** *– making pairs of numbers the same by adding or subtracting numbers (e.g., (82+4)-(28+1) = 86-29 -3*
- ***relational*** *– comparing pairs of numbers in expressions to know the number relations (e.g., 82<86 (LS 4 more); 28<29 (RS 1 more)*

AFTER (Practice) – **5 to 10 minutes**

- Teacher chooses 2 or 3 problems, similar to the lesson problem for student to solve in pairs as a scaffold and individually.
- Problems could vary by number (choice, size), problem contexts, or what is unknown or needs to be solved
- Students are asked to solve these problems using a strategy different from the one they used for the lesson problem.

What is the Unknown Number?

What is □? Explain the method used.

74 - 37 = 71 - 39 + □

Solution 1

74 - 37 = 71 - 39 + □
Relational Comparison
(3 more) 74 > 71
37 < 39 (2 more)
Balance each side - Make equivalent 2 expressions
3 more LS; 2 more RS
So □=5

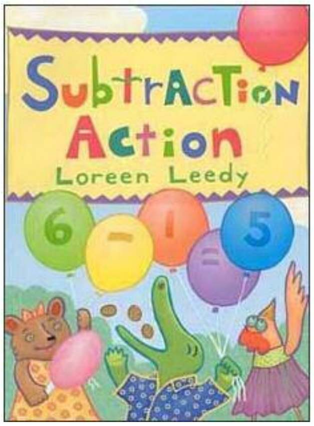
Solution 2

74 - 37 = 71 - 39 + □
Compensating
74-37
= (74-3)-(37+2)+5
= 71 – 39 + □

So □=5

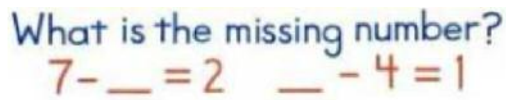
Note: A classroom board is longer proportionally than these 2 pages. Due to the space constraints on these pages, the mathematical *annotations* are recorded above the solutions with arrows, rather than on and around the solutions.

LINKS TO LITERATURE: SUBTRACTION ACTION



This non-fiction picture book, *Subtraction Action* by Loreen Leedy includes different representations of subtraction. Miss Prime and her students spend a day at the school fair and use subtraction in many ways. The class watches a puppet play in which cookies disappear from Little Red Riding Hood's basket. Another situation involves price slashing at two competing refreshment stands. Mathematical details include the definition of difference, writing a subtraction equation, regrouping, and three-digit equations. Tally and Otto use subtraction to sell snacks and Fay tries to break an obstacle-course record. A math problem is presented in every episode with the answers given on the back page.

Elaborate on the subtraction contexts by emphasizing equality of two subtraction expressions, with unknown subtrahend or minuends.

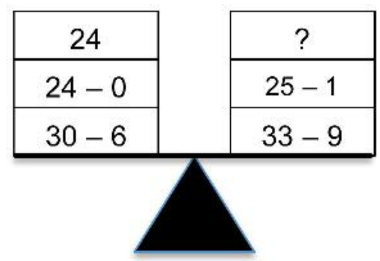


LET'S DO MATH – TRUE/FALSE NUMBER SENTENCES

Before (Getting Started)

What numbers could be subtracted to make a difference of 24?

- Record student responses on a drawn balance scale.
- Possible responses -> 24 – 0, 25 – 1, 26 – 2, and so on.
- Ask, "What subtraction expressions could be put on the other side to create a balance?"



During (Working On It)

Subtraction True/False Number Sentences:

True or False? 135 – 49 = 145 – 59

After (Consolidation)

Anticipating Student Responses:

Solution 1

135 – 49 = 145 – 59

L.S. = (135 + 10 – (49 + 10)
= (145) – (50 + 10 – 1)
= 145 – 59

True – compensation I took away more than I needed to and then added some back. I also used the common difference principle.

Solution 2

135 – 49
= 135 – 30 – 10 – 9
= 105 – 10 – 9
= 86

145 – 59
= 145 – 40 – 10 – 9
= 105 – 10 – 9
= 86

True – both expressions have a difference of 86.

Solution 3

Left side Right side

135 < 145 (+10)
49 < 59 (+10)

Differences between the minuend and subtrahend are the same.

135 – 49 = 145 – 59

True –constant difference. If I add a number to the minuend and the same number to the subtrahend, then I will have

the same difference as when I started.

Coordinating Discussion for Student Learning:

Why might solution 3 be chosen first for student discussion followed by solution 1, and 2?

- *Solution 3* – relational – constant difference – same number added to each of the terms in the subtraction question results in the same difference
- *Solution 2* – operational -> calculated each expression and compared the differences
- *Solution 1* – compensating -> I took away more than I needed and added some back. I also used common difference: when subtracting, if I add a number to the minuend and the same number to the subtrahend, then I get the same difference

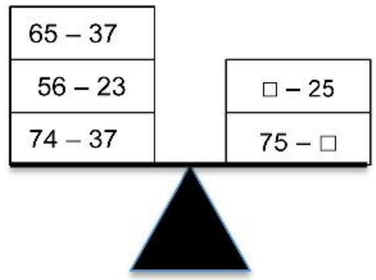
LET'S DO MATH – MENTAL MATH OPEN NUMBER SENTENCES

Before (Getting Started)

What is the value of ? Explain your method.

Record student responses on a drawn balance scale.

- Operational – subtract the terms in the 1st subtraction expression and use that difference to subtract the term from the 2nd expression to get value of
- Compensating – adjust value of terms in the 1st subtraction expression to be similar to terms in the 2nd subtraction expression
- Relational – Compare pairs of terms from each subtraction expression to determine how much greater one term is from another term



During (Working On It)

Subtraction Open Number Sentences

What's the value of ? 74 – 37 = 71 – 39 +

After (Consolidation)

Anticipating Student Responses:

Solution 1

74 – 37 = 71 – 39 + □

(74 – 3) – (37 + 2) + □= 71 – 39 + □

74 – 3 – 37 – 2 + □ = 71 – 39 + 5

Solution 2

74 > 71 Minuend 3 less
37 < 39 Subtrahend 2 more

So difference □ = 5

Solution 3

L.S.	R.S.
L.S. 74 – 37	R.S. 71 – 39 + □
= 74 – 30 – 4 – 3	= 71 – 40 + 1 + □
= 44 – 4 – 3	= 31 + 1 + □
= 37	= 32 + □ = 37
	□ = 5

Coordinating Discussion for Student Learning:

Why might solution 3 be chosen first for student discussion, followed by solution 1 and 2?

- *Solution 3* - operational – subtracted the terms in the 1st subtraction expression and subtracted the terms in the 2nd subtraction expression to get value of
- *Solution 1* - compensating – adjust value of terms in the 1st subtraction expression to be similar to terms in the 2nd subtraction expression
- *Solution 2* - relational – compare pairs of terms from each subtraction expression to determine how much greater side of the equation is from the other

NEXT STEPS FOR YOUR PROFESSIONAL LEARNING

Application to Your Classroom

- Consider how equivalence, reasoning tasks (e.g., True/False Equations, Open Number Sentences) develop students' understanding of number relationships and the use of different strategies for addition, subtraction and relational thinking.
- To suit your students' learning needs, select and design reasoning tasks that are relative to grade specific curriculum expectations in the Patterning and Algebra strand (equivalence) and the Number Sense and Numeration strand (addition and subtraction of 1 digit, 2 digit and 3 digit numbers; mental math addition and subtraction), being mindful of the transition from calculating and comparing the results of addition and subtraction expressions towards analyzing number relationships between addends, minuends and subtrahends when determining the equivalence of addition expressions.
- Incorporate daily practice of addition and subtraction strategies and relational thinking practices through daily mental math mini lessons, two times a day.
- Practise noticing the different methods and the breadth of the mathematics your students use to solve equivalence reasoning tasks, such as, carrying out the operations for each expression, comparing terms in the addition and subtraction expressions, adjusting the terms to make them similar in order to see the difference or unknown (compensating).
- Think ahead of the kinds of mathematical annotations you will record on and around the student solutions to make explicit addition and relational thinking strategies for solving equivalence reasoning tasks.

Suggested Readings

Some readings used in the Research Summary and in the development of the problems and solutions are listed below.

- Carpenter, T., Franke, M., & Levi, L. (2003). *Thinking mathematically: Integrating arithmetic & algebra in elementary school.* Portsmouth: Heinemann.
- Carpenter, T., Levi, L, Franke, M., and Zeringue,J. (2005). Algebra in elementary school: Developing relational thinking. *ZDM, 37*(1), pp. 53-59.
- Molina, M., Castro, E., and Mason, J. (2008). Elementary school students' approaches to solve true/false number sentences. *PNA 2*(2), pp. 75-86.
- Stephens, M. (2006). Describing and exploring the power of relational thinking. Conference Publication at **http://www.researchgate.net/publication/236661303**, pp. 479-486.
- Stephens, M. & Wang, X. (2008). Investigating some junctures in relational thinking: A study of Year 6 and Year 7 students from Australia and China. Journal of Mathematics Education, 1(1), 28-39.

CLOSING

Our next issue will focus on Equivalence and Relational Thinking within multiplication contexts. As well, the ways that teachers can use these mathematical ideas and pedagogical strategies within learning and teaching mathematics within a three-part problem solving lesson framework are outlined.

CALL FOR *ABACUS* SUBMISSIONS

Send your teaching strategy ideas, problems and student solutions to the *Abacus* Co-Editors via email

kathkubo@rogers.com or maryloukestell@gmail.com

	CONFERENCE DELEGATES (includes 1 year Digital OAME Membership)			RETIRED OAME MEMBERS	STUDENT DELEGATES
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Thursday only	205	230	240	160	100
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Saturday only	145	155	170	120	75
Thursday & Saturday	275	300	310	240	160
Friday & Saturday	295	320	325	240	160

REGISTRATION

PRE-REGISTRATION
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• Delegates may select the day(s) that they will attend the conference; sessions not available for booking

FULL REGISTRATION
OPENS FEBRUARY 1, 2016
• Early Bird Deadline – payment in full by February 29, 2016

REGISTRATION CLOSES
APRIL 1,2016

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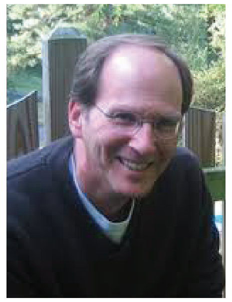
ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION

KEYNOTE SPEAKERS

DR. STEVEN STROGATZ



The Joy of X

Hailed as "a gifted and inspiring communicator" and "a first-rate storyteller and an even better teacher," Strogatz shows how synchrony occurs spontaneously in nature, from atoms to solar systems – almost as if the universe had an overwhelming desire for order. He answers the questions that we find ourselves asking over and over: Why do traffic jams seem to occur without any apparent cause? How do millions of neurons fire together to control our breathing? How can a movie theater full of strangers begin to applaud in rhythm? And are we really all separated from one another by just six degrees – even from Kevin Bacon?

DR. SARAH J. GREENWALD

Good News Everyone! Mathematical Morsels from The Simpsons + Futurama

Sarah J. Greenwald is Professor of Mathematics at Appalachian State University. Join us as Dr. Greenwald uncovers humorous mathematical references in popular television shows such as The Simpsons and shares her belief that popular culture can reveal, reflect, and even shape how society views mathematics.

DR. ROBERT LANG

From Flapping Birds to Space Telescopes: The Art and Science of Origami

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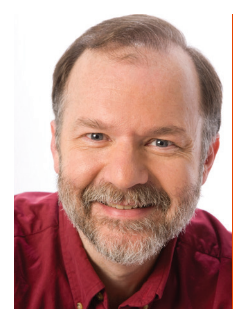

FEATURED SPEAKERS

- **RUTH BEATTY** *"Indigenous and Western Ways of Knowing Mathematics: Continuing to Build Connections"*
- **LISA LUNNEY BORDEN** *"Honouring Indigenous Knowledge in the Mathematics Classroom: A Learning Together Approach"*
- **CATHY BRUCE** *"Fractions Teaching and Learning: What's the big deal?"*
- **DON FRASER** *"Taking the Numb out of Numbers"*
- **JILL GOUGH** *"Deep Practice: Building Conceptual Understanding in Middle Grades"*
- **NICHOLAS JACKIW** *"Time's Arrow: Where School Geometry Comes From and Where It's Going"*
- **RON LANCASTER** *"I'm Thinking of a Number: Introducing Your Students to Micah Lexier, an Artist who uses Mathematics and Measurement to Visualize Information and Data"*
- **ALEX LAWSON** *"Why Just Knowing the 'Facts' Is Not Enough: The Mathematical Territory Between Direct Modeling and Numerical Proficiency at the Primary Level"*
- **JOAN MOSS** *"Expanding Horizons in K-3 Geometry and Spatial Thinking: New Approaches to Support all Learners"*
- **CONNIE QUADRINI** *"Supporting Students with Learning Disabilities in Mathematics"*
- **NATHALIE SINCLAIR** *"Moving Geometry in the K-4 Classroom: More than just vocabulary"*
- **MARIAN SMALL** *"Picture It!!"*
- **CHRIS SUURTAMM** *"Developing Algebraic Thinking"*

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Math Mystery Tour Wednesday May 4, 2016	**OAME BANQUET** Barrie Golf and Country Club Thursday May 5, 6:30 pm	Celebrating Math Educators and Past OAME presidents. In a hands on presentation, special guest speaker Dr. Robert Lang will guide the audience in practicing the art of origami.	Flying Monkey Brewery Tour Thursday May 5 Dinner at 5:00 pm • Tour at 7:00 pm
KenKen Tournament Friday May 6, 2016	Casino Rama, Orillia Friday May 6, 5:30	Rock Wall Climbing @ Gerogian College Friday May 6, Time TBD + Georgian College Fitness Centre Access	Wine and Cheese Thursday May 5, 4:00 PM – 5:30 PM Exhibitor's Area