

Grade 11 U/C Summative Assessment Unit

Activity: Investigation - Spindles and Spaces

Day: 7

Purpose of Activity: Assessment

Overall Expectations Addressed:

OCV.01

· demonstrate facility in manipulating polynomials, rational expressions, and exponential expressions;

OCV.03

· communicate mathematical reasoning with precision and clarity throughout the course.

Activity Description:

Students investigate calculations needed to determine equal spaces between the spindles of a railing. An algebraic model is required that describes the amount of space based on the number of spindles.

Management Suggestions:

Students will require graphing calculators/graphing software..

Popsicle Sticks can be used to model a railing and give students a visual representation of a railing.

Introduce the problem to the whole class.

In groups, have students brainstorm ideas of how to proceed.

Students work independently to complete task.

Assessment: See rubric attached.

Guidelines for Solution: See attached solution

SPINDLES AND SPACES

Scott is a homebuilder. He builds railings in which he places spindles. Spindles are vertical posts that are equally spaced beneath a horizontal bar. Scott would like a mathematical model to help him determine the amount of space to put between each spindle.

The railing must meet the following criteria:

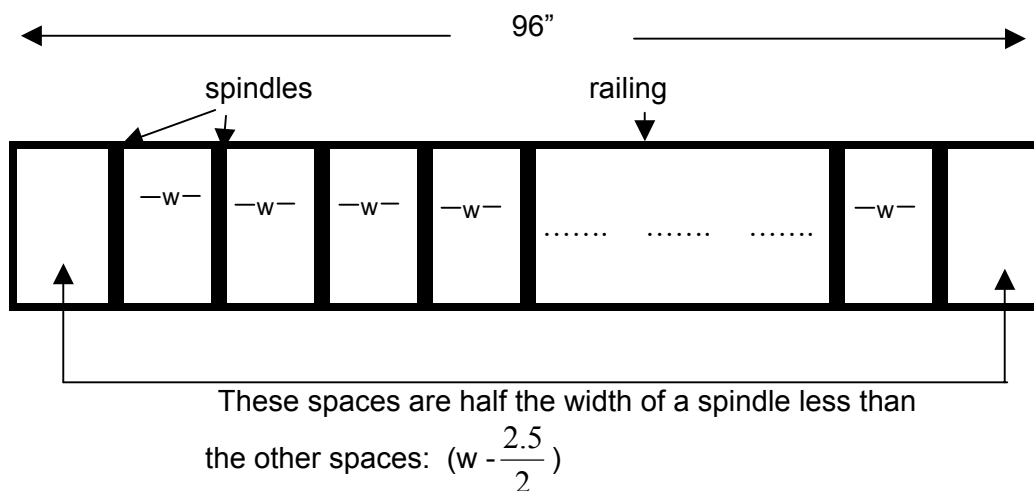
- The spaces between each spindle must be equal except for the ones at either end. These spaces are smaller. They are half the width of a spindle less than the other spaces.
- The number of spindles needs to be minimized since spindles are costly.

Help Scott determine what width of space to use between the spindles.

Create a mathematical model to determine the width of space between each spindle in terms of the number of spindles. Use an 8' (96") railing that has $2\frac{1}{2}$ " wide spindles. Explain your thinking.

For safety reasons, the maximum width of a space is 4".

Make a fully justified recommendation to Scott.



Generalize your model to determine the width of the spaces for total railing of length L and spindle width s .

SPINDLES AND SPACES – SOLUTION

For the 96" rail:

Let n represent the number of spindles and w represent the width between each spindle.

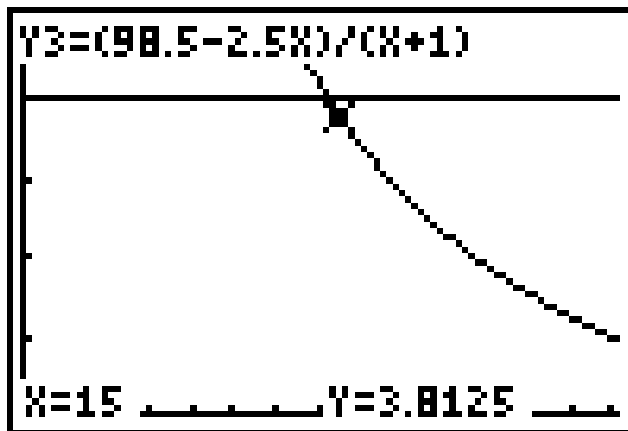
$$96 = n(2.5) + (n-1)w + 2(w - \frac{2.5}{2})$$

$$96 = 2.5n + nw - w + 2w - 2.5$$

$$98.5 = w(n+1) + 2.5n$$

$$\frac{98.5 - 2.5n}{n+1} = w$$

Graphing this relationship where width is represented by y and number of spindles is represented by x yields the following graph and table of values:



X	Y3	Y4
11	5.9167	4
12	5.2692	4
13	4.7143	4
14	4.2333	4
15	3.8125	4
16	3.4412	4
17	3.1111	4

X=11

In order to keep the width between the spindles less than 4", the least number of 2.5"-spindles that can be used is 15 with a spacing of 3.8125" between them.

In general,

$$w = \frac{(L + s) - sn}{(n + 1)}$$

where L is the total length of the railing, s is the width of a spindle, n is

the number of spindles and w is the width between the spindles.

Spindles and Spaces

Achievement Rubric

Category	Level 1	Level 2	Level 3	Level 4
Application	<ul style="list-style-type: none"> Algebraic models evident but with errors 	<ul style="list-style-type: none"> Algebraic models approximate data with minor errors 	<ul style="list-style-type: none"> Creates accurate algebraic model 	<ul style="list-style-type: none"> Creates algebraic models that take all facets of the problem into account
Thinking, Inquiry, and Problem Solving	<ul style="list-style-type: none"> Selects a few appropriate tools and uses these to make a limited attempt to solve the problem 	<ul style="list-style-type: none"> Selects some appropriate tools and uses these to solve some of the problem 	<ul style="list-style-type: none"> Chooses appropriate tools and uses these to solve most of the problem 	<ul style="list-style-type: none"> Chooses appropriate tools and incorporates these in execution of plan to solve problem and verify solution
	<ul style="list-style-type: none"> Reasoning is evident but lacks consistency 	<ul style="list-style-type: none"> Reasoning is evident and logical in parts of the problem 	<ul style="list-style-type: none"> Reasoning is logical and consistent within context 	<ul style="list-style-type: none"> Reasoning is logical and consistent and is able to make generalization
Communication	<ul style="list-style-type: none"> Explanations and justifications lack clarity Either mathematical or narrative form is present, but not both 	<ul style="list-style-type: none"> Explanations and justifications are understandable but lack clarity Both mathematical and narrative forms are present, but the forms are not integrated 	<ul style="list-style-type: none"> Explanations and justifications are clear Mathematical and narrative forms are present and integrated 	<ul style="list-style-type: none"> Explanations and justifications are clear and thorough A variety of mathematical forms and narrative are present, integrated and well chosen

Rubric for Assessing the Collection of the 3 Performance Tasks:

Financial Decision Making, SAD, and Spindles and Spaces

Category	Level 1	Level 2	Level 3	Level 4
Application (Financial Planning) Application (SAD)	<ul style="list-style-type: none"> Attempted solution applies few concepts of mortgages, annuities and compound interest Trigonometric model selected is only partially appropriate to the data 	<ul style="list-style-type: none"> Solution applies some concepts of mortgages, annuities and compound interest Trigonometric model selected is generally appropriate to the data 	<ul style="list-style-type: none"> Solution applies sufficient concepts of mortgages, annuities and compound interest Trigonometric Model selected is appropriate and relates to the data 	<ul style="list-style-type: none"> Detailed solution thoroughly applies concepts of mortgages, annuities and compound interest Trigonometric model selected is appropriate and efficient and relates to the data
Application (Spindles and Spaces)	<ul style="list-style-type: none"> Algebraic models evident but with errors 	<ul style="list-style-type: none"> Algebraic models approximate data with minor errors 	<ul style="list-style-type: none"> Creates accurate algebraic model 	<ul style="list-style-type: none"> Creates algebraic models that take all facets of the problem into account
Application (All tasks)	<ul style="list-style-type: none"> Selected models used with some inaccuracies 	<ul style="list-style-type: none"> Selected models used but may contain minor errors 	<ul style="list-style-type: none"> Selected models are used correctly 	<ul style="list-style-type: none"> Selected models are used correctly and are verified or supported
Thinking, Inquiry, and Problem Solving	<ul style="list-style-type: none"> Models and plans were used that address few aspects of the problems Reasoning is evident but inconsistent 	<ul style="list-style-type: none"> Models/plans were used that address some aspects of the problems Reasoning is evident and logical in parts of the problems 	<ul style="list-style-type: none"> Appropriate plans were used Reasoning is logical and consistent within contexts 	<ul style="list-style-type: none"> Appropriate plans were used that integrate all aspects of the problems/contexts Reasoning is logical and consistent and relates to broader contexts
Communication	<ul style="list-style-type: none"> Few mathematical forms are used Explanations and justifications lack clarity Either mathematical or narrative form is present, but not both 	<ul style="list-style-type: none"> Some mathematical forms are used correctly Explanations and justifications are understandable but lack clarity Both mathematical and narrative forms are present, but the forms are not integrated 	<ul style="list-style-type: none"> Most mathematical forms are used correctly Explanations and justifications are clear Mathematical and narrative forms are present and integrated 	<ul style="list-style-type: none"> Consistently uses mathematical forms correctly Explanations and justifications are clear and thorough A variety of mathematical forms and narrative are present, integrated and well chosen