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Ontario Mathematics Gazette

OAME – ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION

AOEM – ASSOCIATION ONTARIENNE POUR L'ENSEIGNEMENT DES MATHÉMATIQUES

Vol. 57 #1
Sept 2018

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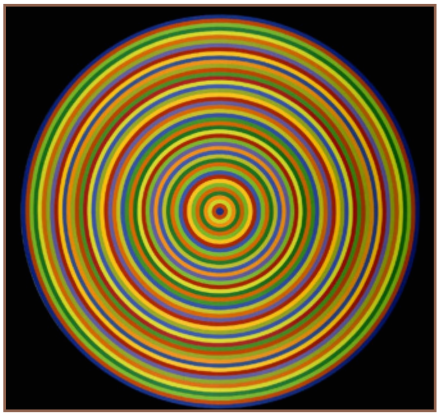

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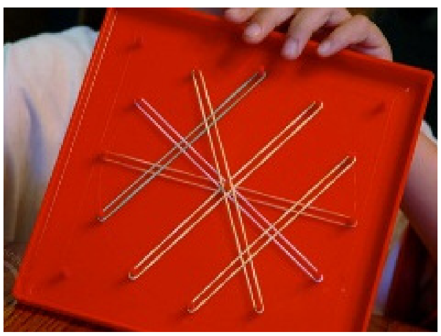

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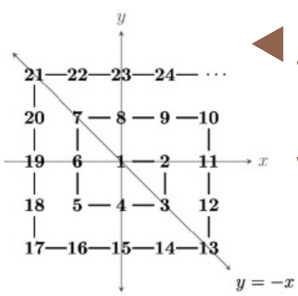

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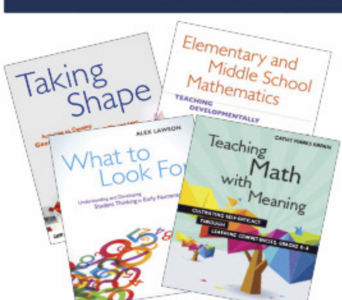

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Submission of Articles

The *Ontario Mathematics Gazette* (OMG) is looking for news items, articles, and good ideas that are useful to mathematics teachers and mathematics teacher education. We are seeking submissions, preferably from mathematics teachers K–12 and other mathematics education professionals, that describe innovative and creative approaches to mathematics teaching.

Please keep in mind the following criteria when making submissions to the *Gazette*:

- The ideas/activities must be of interest to the readership.
- The ideas/activities must be fresh and innovative.
- The mathematics content must be appropriate for the readership.
- The mathematics content must be accurate.
- The article must be well written and easily understood.
- The article and its ideas must be free of sexual, ethnic, racial, or other bias.
- The article must not have been previously published, nor should it be out for review by other publications.
- The article must be original.

Articles are to be word-processed, MS Word is preferred, and prepared according to the *Publication Manual of the American Psychological Association*, Sixth Edition (2009). However, please use single-line spacing (not double) and only one space after each period. Articles should not exceed five numbered pages of text, and figures, images, and photographs should be placed in the text close to where they belong, with captions. The photographer's permission is required, and for photos of students under the age of 18, the written permission of a parent or guardian is required.

Please submit your article in one blind file (i.e., identity of author is not evident), and include author names, contact information including email and mailing addresses, photos, biographies, and all content removed for blinding in a second file. Please email these two files to Tim Sibbald at **gazette@oame.on.ca**.

Upon review, you will be notified whether your article has been accepted for publication (as is, or pending minor or major revisions) or rejected. The Editor reserves the right to edit manuscripts prior to publication. Once an article is published, it becomes the property of OAME.

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▲ About the *Ontario Mathematics Gazette*

The *Ontario Mathematics Gazette*—ISSN 0030-3011—is indexed in the Canadian Education Index and is published four times per year. Its Canadian Publication Mail Product Sales Agreement Number is 40051074.

Submission of Advertisements

Advertisements for publication in the *Ontario Mathematics Gazette* should be sent to **Robert Sherk** at the above address. Courier is recommended to avoid possible delays. Deadlines for advertisements are January 23 for the March issue, April 1 for the June issue, July 1 for the September issue, and October 1 for the December issue.

Full-page advertisements are to be on 8.5" by 11" paper with a minimum of 0.5" margins and single sided. Each advertisement should be print ready, and colour advertisements should have no bleeds.

Advertising Rates

Advertising rates are available by telephoning, emailing, or writing to the Advertising Manager.

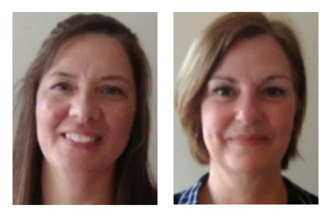
▲ Editor's Report

TIMOTHY SIBBALD, OCT, PhD
EMAIL: gazette@oame.on.ca

Tim Sibbald is the current Gazette *editor and a Past President of OAME. He is an associate professor in the Schulich School of Education, Nipissing University, with a focus on mathematics education in both pre-service and graduate programs.*

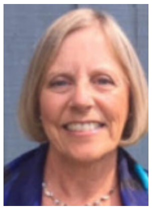
Welcome to the September *Gazette*! In this editorial, I want to present a challenge I perceive with the landscape of professional dialogue in mathematics education. It is a complex topic, and you will find this editorial has twists and turns, but it will also introduce you to some contests that have prizes!

The role of *Gazette* editor has been likened to the hub of a wheel. Items come in from many sources, the editor interacts with *Gazette* content, and then steers it to the next step of the process. This requires two essential elements. The first is a strong team, which we have with our *tour de force* associate editors, Anne Yeager and Jacqueline Foster, along with our proofreader, Gitta Berg, and long-standing graphic artist and layout expert, Penny Clemens. The other ingredient is the actual content, where we encourage first-time writers, while maintaining a high content standard. We are, in essence, a low-threshold/high-yield publication. We support writers with less experience, but maintain a review process for articles that does not reveal to reviewers the identity of the author, nor are authors aware of who the reviewers are. This is referred to as the "double blind" procedure, which is the gold standard to avoid bias within publishing.



Penny Clemens (left), our graphic designer since 1990, and Jacqueline Foster (right), associate editor.

What is important about the strength of this process is that it is systematic thinking and produces an outcome that has a high degree of clarity and consistency. This holds a special place in the modern world, but is also one of the major challenges within a technological society. Teachers have had to develop the student skill of critical appraisal of Internet materials because the Internet as a whole is a conduit that makes no guarantee about the quality, validity, or veracity of what it delivers.



Anne Yeager (Associate Editor)

"What is necessary for some students for learning is good for all students." This expression originated from Anne, during her extensive experience as a math and special education teacher of 30 years for the Upper Grand District School Board. Her commitment and proven results of this mantra led to a secondment with the Ministry of Education as an Education Officer. While there, she focused on curriculum and resource development for Grades 7–12. Anne was the inaugural winner of the OAME Award, "Leadership in Mathematics Education." Now retired, Anne enjoys travel, tennis, cycling, skiing, and gardening. She stays connected with the math community as a long-term Board Member of OAME and a long-standing Associate Editor of the OAME *Gazette*.

There is no doubt that the Internet has benefits, particularly in that it has made content more accessible. However, the need to be circumspect is more important than ever before. As an example of high-quality content, there is a wonderful paper-folding math problem that can be found at **www.youtube.com/watch?v=SOgn6J12NWE**. First there is the math problem itself that requires working with radicals and has two equivalent solutions that appear quite different—the more elegant solution is the one you want. There are subtleties to the video, such as the way the paper is flipped and numbers appear that were not there beforehand. It is clever, but more important is the added educational value that teaches about "hash functions." The video does not provide the answer, but it does provide a means to verify whether you have the solution. This is good pedagogy that also models a data-security method, which is where more details of hash functions can be found.

As good as the Internet is for such resources, the downside is the ease with which one can defeat the best intentions that oblige the audience to apply effort to obtain the solution. One can find the solution to the paper-folding problem quite readily, but that is counter-productive. This diminishes the acceptance of the Internet when a problem has a definitive solution.

Learning objects that involve an educational process are an alternative that addresses this problem. We are fortunate in Ontario to have Ministry of Education-developed learning objects with **www.mathclips.ca**. There are others, such as the **National Library of Virtual Manipulatives**, but again, it is necessary to apply critical appraisal because individual learning objects have varying fits to the curriculum. Increasingly I find difficulties with implementing learning objects. For example, I explored the wave-rider game (**www.bbc.co.uk/bitesize/higher/games/wave_rider/**) that supports teaching trigonometric transformations. At that time, I played with it and had no issues. However, on revisiting it, I had messages that Flash and Java needed to be up-to-date. Moreover, I switched browsers because the illustrious "ghost in the machine" (Flash and Java were up-to-date) was most easily beaten by this approach.

For teachers, there is a plethora of resources. One that has recently crossed my desk is a collection of e-books by Bob Albrecht (see **http://i-a-e.org/downloads/cat_view/86-free-ebooks-by-bob-albrecht.html**). These present a challenge for teachers because of the need to appraise the resource at the same time as trying to learn from it. However, practitioners are uniquely positioned to give ideas a "trial by fire" and simply test ideas in the classroom. I would welcome a couple of reviews of such books. (A guide for writing book reviews is available.)

To address the changing environment, teachers can make use of social media. Educators should heed the Ontario College of Teachers **Professional Advisory** that has been updated over the years (most recently in September 2017). However, as teachers, the use of social media can extend into the planning process, and there are substantive behaviours around social media that are challenging. Often social media are not systematic, reviewed, or based on developed thinking. They are often temporary and reactionary in nature. They are a form of idea dissemination, but they seem shallow in the sense of a low ceiling for engagement, as well as a low ceiling for the degree of detail, not to mention that the social/dissemination aspect seems to take on a world of its own. How often do social media items lead you to concrete actions?

There are, however, benefits. Since @OAMEwrites began tweeting, we have had a couple of volunteers for reviewing articles. We have a person who has contributed an article as a direct result, and an expression of interest in the development of a new column. The @OAMEcounts feed is also an active site for sharing. It is also unfair to suggest that the brevity of social media did not exist in the past. The news has always been a source of brief snippets, and a simple example that came to my attention recently was a map of the locations of residential schools in Ontario (see Figure 1, which is derived from **www.cbc.ca/news2/interactives/beyond-94-residential-school-map/**). The nugget it drew attention to was that residential schools were not simply in northern Ontario. There is much more to learn, but this is an example of the sort of short piece that encouraged further learning and existed before social media reinvented the idea.

Figure 1: *Location of residential schools in Ontario*

A recent radio broadcast about Google classrooms (see **www.cbc.ca/radio/spark/episode-401-1.4694935/as-google-for-education-tools-enter-classrooms-across-canada-some-parents-are-asking-to-opt-out-1.4694939**) has also been a cause for thought. At what point is there too much technology in a classroom? There was a point to involving technology in schools when students needed to learn about the use of technology. However, it is worthwhile to ask if we are beyond that point and technology is being used because it is available. This is not about Google. As an alternate example, it is discouraging that there has been research promoting the use of many vertical surfaces in classrooms, when the primary cause for the reduction was technology that narrows the classroom focus to a single illuminated vertical panel. Examples of this are the digital projector and SMART Board®. The days when one gets a dozen students working simultaneously "at the board" has too often become one student at a time on a single SMART Board.

Before you think this is a curmudgeon rant, let me assure you that I hold teachers in high regard and appreciate that they are doing their professional best to navigate a strange new world. I am embedded in that technological world, and was the first generation to have calculators in high school (my older brothers had slide rulers), and have taken full advantage of technology. Yet it is concerning that there is little discussion about how to balance the use of technology with other modes of teaching. In the *Gazette*, we often get articles focused on the use of technology or other means, but it is relatively rare to have bridging articles.

An example of the challenge is consideration of how to teach symmetry. My personal favourite is with kinesthetic paper folding and holding it up to a light. The use of Mira™ boards is also quite common and effective. Mirrors can be used as well. I have been alarmed by technology that will automatically generate a line of symmetry, as this requires no conceptual consideration on the part of a student about the meaning of symmetry. A better approach is technology that allows one to place a line and drag it to the position where one thinks it belongs. However, this is not equivalent to paper folding if the technology uses lines with a particular orientation (e.g., vertical). In this case, for technology to meet the same conceptual playing field as the hands-on approach, it must present the line with no given information—the line must be at a random orientation.

I could go on, but there are other details to attend to. Consider, for example, the recent news that the International Math Union is leading a global project directed at having UNESCO proclaim March 14 the international day of mathematics. If the project is successful, "pi day" in 2020 will be the first.

(left to right, back) Rebecca Jean Paul, Kimberly Simpson, Anne Holness, Nicole Bell, Mike Lieff, Marian Small, Chris Atkinson, Suhana Kadoura, Melissa Black, (front) Lara Eager, Monique Sack, Chelsea Cleveland, Anne Fitton, Kelly Littlemore. Missing: Jill Lazarus

The OAME 2019 conference is expected to be open for proposals as you read this *Gazette*. Consider submitting a proposal, but also consider rendering your proposal as an article for the *Gazette* and go for double billing. Based on my own experience, I can say that publishing in the *Gazette* is a very effective way of ensuring you have the conceptual idea to a point that is beneficial for focusing on developing a presentation. For those who have presented at OAME 2018, an article is a very effective consolidation.

Contests

Before speaking about what is in this issue, I want to introduce some contests that will run through to March 15, 2019. Some have prizes, and we hope you will encourage your students to engage!

Contest Details

Student contests will be done with two divisions: K–8 and 9–12. Prizes will be 1st – $100, 2nd – $75, and 3rd – $50 in each division for each contest (with the exception of the last contest). Entries must be submitted by a teacher, who does not have to be a member of the OAME—i.e., share the contest! In cases with younger students, on the advice of the teacher, alternative prizes with an equivalent value, will be considered (i.e., if the teacher advises a gift card is more appropriate than the monetary award, we will consider the possibility).

Permission to publish entries with an acknowledgement is required. Judging will be done in two stages with volunteers (if you wish to volunteer, email **gazette@oame.on.ca**) narrowing the field, and then either the board of directors or a membership vote in March. Entries can be submitted at any time to **gazette@oame.on.ca**.

General criteria:

- All representations (photo, model, written explanations) make it thoroughly clear what mathematical concept or pattern is being highlighted.
- Photos are, where feasible, taken in a manner that highlights characteristics of the concept or pattern (e.g., a picture showing a "circle" should be taken in a way that the circle appears as a circle and not an ellipse).
- Explanations will use grade-level math terminology and show a consistent use of the terminology through the explanation.
- Clearly outlined multiple interpretations that highlight different interpretations of the underlying concept will be taken into consideration.
- The spirit of the contest is that entries that appear in the *Gazette* will be instructive to a wide audience.
- Judges do not have to accept any entry that is not considered to have sufficient merit. (Teachers may include a note of explanation if they feel the judges may not appreciate the merit of an entry.)

Best Mathematical Photo by a Student

The emphasis is on mathematical concepts, so the photo must have an explanation by the student. This will explain, with proper mathematical terminology, what mathematical content the student sees in the picture. Note that a clean photo is required, but the entry may be augmented to highlight shapes (e.g., a photo of a rainbow with circular arcs added in GeoGebra should be accompanied by a copy of the rainbow without the construction). Please include details of the grade level of the student because, within each division of the contest, we will consider grade level.

Best Geometric Construction

For the K–8 division, a pattern with an explanation is required.

For the 9–12 division, a "visual proof" with an explanation is required.

Professional Teacher Challenge

This contest, which also includes prizes, requires a 3D model, video, augmented reality, or virtual reality demonstration of a mathematical concept. There are no divisions for this particular contest, and retired teachers are welcome to participate. An explanation must be included.

Presidential Prestige Contest

Finally, a contest that has no prize money, only prestige, and the ideal response will come in the form of a letter to the editor that identifies the Past Presidents in the photo and perhaps includes a printable story involving some of them or another Past President. (They will be asked for permission to print the story.) Note that you can always go to the OAME website and consult "Ye Old Archives."

In this issue…

The Executive Directors speak about changes that are coming in the Annual General Meeting (that you can attend). The new President, David Petro, speaks about engagement in the OAME with an interesting retrospective. Kerri Evershed reports on the Ontario Math Olympics for 2018. Details of award winners are also provided.

Angelica Mendaglio speaks about activities at the Fields Institute for Mathematical Sciences and the Margaret Sinclair Memorial Award. Ann Arden, Michael Tang, and Jimmy Pai write about their experience at the Canadian Mathematics Education Study Group annual meeting. Jacqueline Hill reviews the NCTM publication *Catalyzing Change in High School Mathematics: Initiating Critical Conversations,* while David Petro writes about his experience attending the NCTM Annual Conference.

In *Hey, It's Elementary*, Lynda Colgan draws attention to the achievements of former graduates and the use of infographics. *In the Middle* has Carly Ziniuk sharing her interest in the connections between art and math. *Technology Corner*, by Mary Bourassa, explains what is available in the way of teacher resources from Desmos. The Provincial Digital Team presents tools for demonstrating visual patterning and connections to expressions. *What's the Problem?*, written by Shawn Godin, resolves a question about obtuse integer triangles. *Assessment Abby* provides many resources for clarifying assessment and evaluation.

A new feature that arose as a result of @OAMEwrites is an interview that Ann Arden conducted with Peter Taylor. I am also delighted to have Ron Lancaster return to the *Gazette* with a new column. Ron's new column draws out the impetus of mathematical questions through things he has photographed.

Also in this edition are two book reviews. Kelli Gates reviews an online book, *Children's Mathematical Learning,* and Andrew Skelton reviews *Burn Math Class*.

Lastly Jeff Irvine provides a double header of articles. The first looks at the familiar handshake problem, but considers it through the Pólya Heuristic. The second considers how we motivate students. ▲

▲ OAME Awards

Award for Leadership in Mathematics Education

This award recognizes an educator who has demonstrated leadership by contributing in a significant way to the development of mathematics teachers and enhancement of mathematics education in Ontario through innovation and/or research-related activities.

The criterion for this award seems to have been written with this year's winner in mind. It is rare to find a teacher who has not heard of **Dr. Marian Small**, and very unlikely you would come across a teacher who has not used something in his or her own teaching inspired by Marian's work. It is difficult to capture the impact that Marian has had on the teaching of mathematics in this province and beyond. Her passion to see and spread a message of having math taught in a way that values every student's ability is evident in her work. She has run countless workshops in every province and authored over 20 books since 2012. She has recently become a proud grandparent. Marian has proven herself to be steady, humble, and always smiling.

▲ President's Message

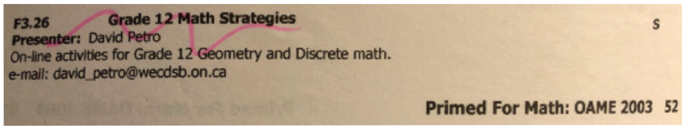
DAVID PETRO
EMAIL: David.Petro@oame.on.ca

David Petro, the current President of OAME, is the math, science, and IT consultant at the Windsor Essex Catholic District School Board. He is a large proponent of exploiting technology for the educational benefit of students.

Let's go back a few years. Sometime in 2002, my department head invited me to a meeting of something called SWOAME (the South Western Ontario Association for Math Education), which turned out to be the local chapter of something called OAME. It was in a restaurant, and only a few people were in attendance. We talked about math, had some friendly discussion, and by the end of the evening, I was the new chapter representative. Somewhere between then and now, a lot has happened, and I find myself a completely different teacher and the President of the OAME. Not that you have to get involved at the provincial level to do so, but that first step turned out to be pivotal in shaping the evolution of my teaching philosophy. At the time, I really didn't even know about OAME. I had been teaching math and science for almost ten years and wasn't aware that math teachers had a professional organization. I'd like to think that now, a higher proportion of math teachers and administrators know not only about OAME, but know about its worth.

As the incoming President, I'm supposed to have some sort of vision. Part of my vision is to continue the quest to make OAME a "household name." I would like pre-service teachers to know where they can go for support. I want current teachers to know that they can find quality professional development at our Annual Conference (we topped 2000 participants this year—the highest ever) or at our Leadership Conference (with Dan Meyer, Cathy Fosnot, and Graham Fletcher this November), but also in their local chapters (we have 15 to choose from across the province). I want administrators to know they can count on us to help their teachers reach their students, and I want our current government to know they can rely on us to help guide them to smart decisions about math education.

When I started going to provincial meetings as the chapter rep, it wasn't long before I started to meet people from all over the province, and it wasn't long until I was invited to speak. In fact, at one of my first meetings, Jaqueline Hill (who would eventually be President in 2007 and is now the NCTM representative) cornered me and suggested I submit a proposal for OAME 2003. Me? What would I talk about? Was there something I had to say that maybe no one else would? As it turns out, even 15 years ago, I was keenly interested in technology and online learning. Though you can see by my sparse description, I was definitely a newbie when it came to presenting at OAME.


F3.26 Grade 12 Math Strategies S
Presenter: David Petro
On-line activities for Grade 12 Geometry and Discrete math.
e-mail: david_petro@wecdsb.on.ca
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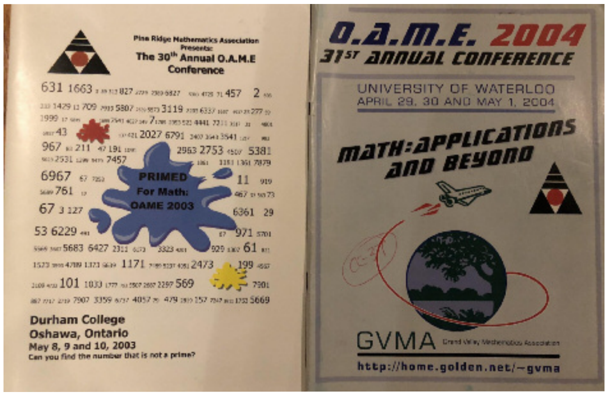
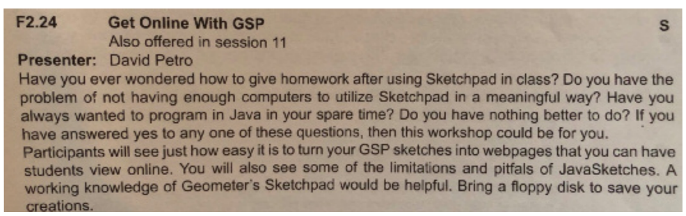



F2.24 Get Online With GSP S
Also offered in session 11
Presenter: David Petro
Have you ever wondered how to give homework after using Sketchpad in class? Do you have the problem of not having enough computers to utilize Sketchpad in a meaningful way? Have you always wanted to program in Java in your spare time? Do you have nothing better to do? If you have answered yes to any one of these questions, then this workshop could be for you. Participants will see just how easy it is to turn your GSP sketches into webpages that you can have students view online. You will also see some of the limitations and pitfalls of JavaSketches. A working knowledge of Geometer's Sketchpad would be helpful. Bring a floppy disk to save your creations.

But 50 people still came to hear what I had to say. I learned from all the mistakes I made and got a little more refined for presenting at OAME 2004. I was still talking about online learning, but now specifically with Geometer's Sketchpad™ (GSP). I'm not sure if I was ahead of my time or whether that's just how good GSP was and still is (even now, I'm still making online **websketches** with GSP). Since then, I have only missed presenting at one OAME conference and have continued to talk about how online tools can benefit both students and teachers.

So it makes sense that another part of my vision as President is to somehow exploit the online environment to enhance mathematics in Ontario, whether that be in classrooms with students learning via online tools, or outside of the classroom, helping teachers personalize their own professional development, I think that OAME can be there to help figure out the best way to do that. It's 2018, and we are literally one-sixth the way into the 21st century, yet people still talk about "21st-century learning" as if it's something new.

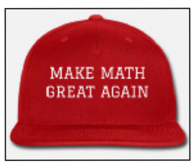
That thought led to the theme of this year's OAME Leadership Conference: Learning in the 21st Century. Our keynote speakers were chosen because they are currently exploiting online tools for the benefit of math teachers, leaders, and students. Dan Meyer started by giving away high-quality resources that were engaging for students and that caused them (and their teachers) to think. He eventually built a community of thousands of teachers who constantly talk about math in a deep way. Cathy Fosnot is well known for her work drawing out student thinking and numeracy skills, but now has developed tools to help deliver professional development to teachers remotely. Graham Fletcher (formerly from Ontario) has embraced the online community that organically grew out of Twitter, called the Math Twitter Blog-o-Sphere (#MTBoS), and used it to shape his own teaching. It is also a vehicle for all the great resources he develops. There is a lot of talk about too much Internet being a bad thing, but it is a marvelous tool that we have, for free, that can help us become better teachers.

Don't get me wrong. Despite this talk of the online environment being so great, I can't forget to speak of the personal connections I have made through the OAME in the last 15 years. For example, our current Webmaster and technology guru, Greg Clarke, and I started attending the OAME Board of Directors meetings at the same time. Both of us were pretty green back then, and both of us still talk about things we want to get done. (Maybe we'll get to that podcast yet, Greg.) I could fill an entire *Gazette* with the connections I have made over the years, but let me focus on just one weekend in 2004. NWOAME was having a mini-conference, and Bill Otto (current Vice-President) had invited me to speak. I had only recently met Bill, but we always had great conversations about teaching senior math courses. Now 14 years later, we can still have those conversations, but more importantly, in my current role, I know I can rely on him as a savant about the OAME constitution.

While there, in 2004, I presented alongside others that I was meeting for the first time. There was Anne and Rod Yeager. They have been involved with OAME and the Ontario Ministry of Education for many years since then.

(Anne is currently a Director of OAME and an associate editor of the *Gazette*.) But more than anything, over the years, they opened my eyes to the fact that teaching Applied students were not a "punishment," but a gift. Also presenting that weekend in 2004 were Fred Ferneyhough and Mary Bourassa. I quickly learned that he was the guy to go to if I needed any TI-84 knowledge, and now he is part of the dynamic duo (along with Lynda Ferneyhough), who make up our Executive Directors (and literally run the administrative side of OAME). Mary, of course, writes the Technology Corner column in the *Gazette*, but shares many resources online. You can see her day-by-day reports about her Grades 9 and 10 classes on her blog, **Making Math Meaningful**, as well as the work she did starting the site, **wodb.ca**, based on Christopher Danielson's book *Which One Doesn't Belong?*. As I look back now, I find it shocking that all those years ago, I was presenting in Thunder Bay with teachers who would have so much impact (and continue to have impact) on teaching and learning in Ontario.

So that's where I begin, hoping to make more people know what OAME is and what it can do for them, while exploiting online tools for teaching and learning. I hope you will stay with me for the journey, can learn something along the way, and make those connections with teachers from all over the province to "Make Math Great Again" (I realize it has always been great, but as the new President, I couldn't resist). ▲



▲ Executive Directors' Report

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Lynda and Fred Ferneyhough have been the Executive Directors of OAME since 2010. They taught in the Peel District School Board for over 30 years and had a three-year stint in the United Arab Emirates. Both of them served OAME as Chapter Representatives for CHAMP, Directors, and Vice-Presidents. During their career, they were Department Heads, and authors for McGraw-Hill Ryerson. They are both certified as instructors for Texas Instruments (TI), and Fred continues to coach for TI in the United States.

A year ago, Bill Otto wrote an article on the OAME Constitution, By-Laws, and Terms of Reference. Next month, the membership will be asked to consider three changes to our Constitution at the Annual General Meeting. That meeting will take place on Saturday, October 13 in Brampton. Any member is welcome to attend the meeting, either in person or online. Details will be sent out to members in September.

As was mentioned last year, our official documents are split into three portions: the Constitution (which defines the organization), the By-Laws (which provide the structure for the organization), and the Terms of Reference (which outline how the organization implements the structure). Any changes to the Constitution must be sent out to members at least four weeks prior to the Annual General Meeting and must be approved by a two-thirds majority of members present at the meeting. The Board of Directors can make changes to the By-Laws and Terms of Reference at any of their meetings.

This year, three changes to the Constitution are being proposed. The first change is a matter that has been discussed several times in the past. Currently, the term for the President is one year, and it is being proposed that the President's term be extended to two years. The Strategic Planning and Review Committee was tasked with proposing changes, and the motion to affect this change will be sent out to all members. The reason for this change is fairly simple. Several Presidents have indicated that, by the time they completed their year in the role of President, they had attained an understanding of how to chair the organization and implement their vision, except they were at the end of their term. Almost every President has indicated a desire for a second year to continue the work. If the membership approves this change, a person who is elected as President will serve one year as President-Elect, two years as the President, and a fourth year as the Past-President.

If this change is approved, there are several additional changes that will need to be made to the By-Laws and Terms of Reference, including a transition plan to show the composition of the Executive over the next few years, and modifications to the planning of the Leadership Conference and the Elections Committee. When the Annual General Meeting concludes, the Board of Directors will be presented with motions for consideration that will address these issues.

Our organization and structure allow for individuals to propose motions to change our Constitution, and I took advantage of that concept to request the next two changes. Currently, we have several members who have been on the board in varying capacities for many years. As the Executive Directors, we value that corporate memory, and we'd like to ensure that it will always be present. To that end, you will find a motion to create a new board position entitled "Emeritus Executive member." We constantly need the energy and ideals that younger members bring to us, but we also find that there is a wisdom in older board members that needs to be retained, cherished, and respected. The Emeritus member of the executive will address that desire, while allowing younger members to take on elected positions. The Emeritus member will be appointed and only have voting privileges if there is a tie on a matter before the Executive.

The final proposed change is to expand the Board of Directors by two members, from specific portions of our community. When we elect board members each year, we accept nominations for one person from K to Grade 6 and one from Grade 7 and above. We have been fortunate in the past to have individuals from post-secondary teaching positions join the board, but more often than not, candidates are found from the pool of Grade 7 to Grade 12 teachers. The final change to the Constitution calls for one new board position to be created for teachers from our colleges and one from our universities. It's easy to say that we value the input and opinions of our post-secondary members, but this motion allows the organization to formalize that concept by actively seeking two new members from these sources.

The revised Constitution will be posted after the board meeting. Feel free to read through it and ask questions that arise. ▲

▲ Interview with Peter Taylor (Mathematician, Queen's University)

ANN ARDEN
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Ann Arden is a math teacher in the Ottawa-Carleton District School Board and is currently an instructional coach. She has also taught in the Faculty of Education at the University of Ottawa, and her son's teacher this year was a former student! Ann is very interested in assessment to improve learning, especially through the use of conversational and observational evidence. Ann is on Twitter as @annarden.

Ann Arden interviewed Peter Taylor, a mathematician and professor at Queen's University, on June 15, 2018 in Ottawa. The interview took place after spending two days working with Peter, several teachers, and over 100 students in four math classes. Ann was interested in interviewing Peter because he has a vast experience, has been deeply involved in mathematics education as a mathematician, and has a unique voice in mathematics education, particularly in his narrative approach to writing problems. Peter has long had a passion for developing curriculum and resources based on rich problems, as demonstrated, for example, by textbooks he has written in a narrative style, and the problems he posed to students in the Math and Poetry course he taught for many years with Maggie Berg at Queen's. Ann had the good fortune to take the Math and Poetry course in her undergraduate studies, and to work with Peter on several projects in recent years. The students Peter has worked with this year in Toronto and Ottawa were deeply engaged by his problems and his stories, and enjoyed working on the challenging problems he presented.

Thank you for taking the time for this chat today.

I've heard you define mathematics as the "abstract study and creation of structure." Can you explain what you mean by this and why mathematics is useful to learn?

I think that what we all need to do is understand the structure of the systems within which we work and play. These structures are getting more and more sophisticated, and technology has a lot to do with that. We make decisions all the time, and it's hard to get that right unless we can see the big picture. So that kind of thinking and understanding is what our students need to do in every subject. Math is a particularly significant subject in that regard because it works with structures *abstractly*. That gives it a potentially wide applicability to different kinds of systems in business, in culture, in technology, studying the universe, biological systems, and so forth. So, an early study of mathematics gives you a handle on analyzing structure.

Furthermore, it turns out that mathematical structures typically have a remarkable, even unexpected, beauty. That's a good reason for studying it, because students respond to beauty and elegance.

Can you explain a bit about the research work you do as a mathematical biologist?

I construct mathematical models of the evolution of behaviour. Over the past 50 or 60 years, mathematics has become an indispensable tool in studying the process of evolution. So if you want to understand why an individual might respond in a certain manner, you need to understand the overall structure in which that happens. For example, when you get the flu, why does your temperature go up? To answer that, you have to understand what the host and the pathogen are each trying to accomplish in the battle they are waging. Certainly they have different objectives—so there's conflict. It turns out that the temperature increase is not directly caused by the pathogen, but is a response by the host to make life more difficult for the pathogen. Unless you understand who's causing what and for what reason, you might treat the disease in the wrong way. Mathematical models are what we use to analyze these conflicts.

I'm going to move on now to some of the math curriculum work you have done. You have been doing this for a long time. I remember seeing some of your earlier work—the fire, air, earth, and water books you presented at OAME. I'm going to ask you a few questions about that work.

In a recent paper (Taylor, 2018), you wrote: "Our worry about student knowledge and performance has led us to construct a curriculum that is narrow in scope and technical in character. There is little room for wonder and beauty in such a design."

Can you say a bit more about what you mean?

That paper was part of a special issue of the journal *Education Sciences*, edited by Jo Boaler, who was a keynote speaker at OAME 2018. I give a link to that issue in the references. It's open access. Another paper in the issue (Raymond, 2018) provides an interesting history of that narrowing that seems to have started in the middle of the 20th century. Ironically enough, it likely came out of the success that mathematics had in solving the problems faced by science and engineering, with the enormous possibilities that technology presented to us. That is still the case today through the emphasis on gaming and artificial intelligence. Anyway, the subject rapidly became so important and critical that technical proficiency jumped to the front of the line. Then you had the math wars and the fallout from that. That whole theme is what Lockhart's lament was all about; it painted a picture of what would happen if *music* had instead become the indispensable subject that would propel us toward technological growth.

Anyway, in this narrowing, what was lost was the sophistication and structure of the subject that it was felt high school kids were not ready for. In English class, for example (also a fundamental subject in high school), you do find that sophistication, in the range of texts that the students study and the nature of their discussion, but that's absent in mathematics.

There are articles in the newspapers right now, where employers say it doesn't really matter how much math a student knows. They want to know things about creativity, ability to communicate, ability to learn and do new things. It seems to me that education professionals have ignored that for a long time. It's in the front 50 pages of the curriculum document, but somehow it gets ignored.

I'm interested that you mention English. In your paper, you say that, "English also has a strong generative component, typically lacking in school mathematics. Students in English understand that they are expected to write creatively; but in math class, they will claim that it is unfair to be asked to solve a problem they have not seen before." Where does that difference between English and Math come from?

That's a good question. The English–Math analogy is an analogy. As with any analogy, it can inform as well as mislead. In English, you can read an article without understanding too much about the background issues, and without a full understanding, you can make some progress in terms of coming up with new ideas. In math, it's harder to do anything without a certain level of context and technical mastery. The language [of math] is special and you see this in the University itself. When I go to a colloquium on biology, political science, or English, I can understand a lot of what's happening. If any of those people came to a math colloquium, they are unlikely to understand the first thing about what's going on. And it's not just about the definitions of the words—what's an abelian group?—it's that to make sense of any discussion of abelian groups, you need to understand a whole collection of preliminary results.

In many ways, that has held mathematics educators back for a long time. To get around that, you need to choose your problems carefully. I try to work with elementary problems, or more precisely, to find elementary problems that can lead students toward sophisticated forms of mathematical thinking. For example, contour diagrams (that we have just been working with) can generate a lot of mathematical thinking, but they have an elementary nature.

You see a distinction there between knowing mathematics and thinking mathematically?

Absolutely. There isn't really much advanced mathematics used in the workplace. If you stop people on the street and ask how much advanced mathematics they use, you'll find hardly any. It's only a few specialists who use much mathematics in their work. But what almost everyone needs to know is how to think mathematically.

So what does that mean—"thinking mathematically"?

You know, it's probably ultimately about being able to go up to the structural level and use that to see how to take the problem apart and understand how the pieces relate. I think that's the best definition I can come up with.

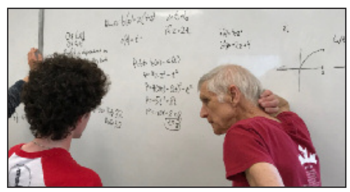
*You have spent a lot of time working on creating good, rich, accessible math problems. I was fortunate to be one of many students who took your Math and Poetry course (taught along with Dr. Maggie Berg) at Queen's, and was part of a large class with varied backgrounds who engaged with the trains problem (see **http://mast.queensu.ca/~math9-12/trains.html**, and see Shawn Godin's column in issue 56(3) in the* Gazette *archives), the ant problem (see the references for a link), and many others. In all your experience writing mathematics curriculum resources, what have you learned about the qualities of a good problem for high school students?*

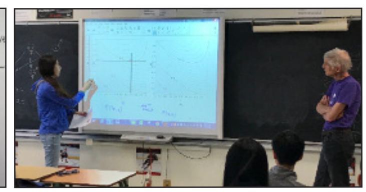
A sense of mystery, wonder, and surprise are all great assets. It's important to have some immediate access, something "hands on" that can be played with right away. That's a "low-floor" requirement, but then I want a "high ceiling" as well, possible pathways to a rich structure.

*In your most recent project (**math9-12.ca**), you've had a chance to work with students and teachers at several Ontario schools with these rich problems you have created. I know you have really enjoyed these experiences, and have come away feeling very optimistic about the way high school students can handle rich mathematics. Can you explain a bit about your experiences?*

Well, it's always seemed to go well. I've enjoyed it, and my sense is that most of the students and teachers have enjoyed it too. My feeling is that the experience has been significant for the students' mathematical learning. But lately, it's mostly just one day at a time. A visit to a school recently was the first time I've had two days in a row with several classes. Many years ago, I had the chance to spend a whole semester with a class—I did that twice in fact. But even with that, how was I to measure the success of the intervention? I actually believe that the problems I have been working with are better activities for both the students and the teachers than what is typically found in the high school classroom. But how am I to find out if that's really the case?—certainly not with the normal scientific protocol of having an "experimental" class and a "control" class, and pre- and post-testing each one and comparing scores. My idealistic view has been that teachers will see how wonderful my problems are and will sign up in droves to use them in their classrooms [said with a smile]. Seriously, I think there is a lot of organization and support that needs to be done before anything like that could happen. Lots to do!

Indeed there is. Thank you very much for your time and this conversation.





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Lockhart, P. (2002). *A mathematician's lament* (unknown edition). New York, NY: Bellevue Literary Press. A shorter version of Lockhart's Lament that might be of interest to readers can be found at **www.maa.org/external_archive/devlin/LockhartsLament.pdf**.

Raymond, K. (2018). M is not just for STEM: How myths about the purposes of mathematics education have narrowed mathematics curricula in the United States. *Educational Sciences, 8*(2), 47. doi:10.3390/educsci8020047. Retrieved from **www.mdpi.com/2227-7102/8/2/47**

Taylor, P. (2018). Teach the mathematics of mathematicians. *Educational Sciences, 8*(2), 56. doi:10.3390/educsci8020056. Retrieved from **www.mdpi.com/2227-7102/8/2/56/htm**

The special journal issue mentioned in the conversation can be found at **www.mdpi.com/journal/education/special_issues/Dispelling_Myths_about_Mathematics**.

A set of problems from Peter Taylor's *In Process* book that includes the ant problem can be found at **www.mdpi.com/journal/education/special_issues/Dispelling_Myths_about_Mathematics**. ▲

A New Lens on a Familiar Problem—The Handshake Problem

JEFF IRVINE
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Jeff Irvine has been a secondary math teacher in Waterloo and Peel District School Boards, a secondary math department head, and a secondary vice-principal. Jeff has taught at three faculties of education and at Sheridan College. For several years, he was an Education Officer in the Curriculum and Assessment Policy Branch of the Ontario Ministry of Education, where his portfolio was Grades 7 to 12 mathematics for the Province of Ontario. Jeff is co-author or contributing author for 11 high school mathematics textbooks. He is currently an instructor of mathematics education at Brock University, where he is pursuing a PhD.

At a party, everyone shakes hands with everyone else. If there are 12 people at the party, how many handshakes occur? Generalize for *n* people at the party.

This problem is a great example of what Marian Small calls an *open-routed* problem. There is only one correct answer, but there are numerous ways to get that answer. Along the way, there are many opportunities to do mathematics, including representing, selecting tools and strategies, reasoning and proving, reflecting, communicating, and of course, problem solving. To many of us, the handshake problem is not new. The richness of the problem has two valuable parts. One is its simplicity and connection to real life. This problem provides an example of a *low-floor*, *high-ceiling* problem that has multiple entry points. The multiplicity of solution methods results in successful outcomes for the majority of students, since almost everyone can construct and explain a solution to the problem.

George Pólya's problem-solving heuristic is provided in the front matter of every mathematics curriculum document in Ontario: understand the problem, make a plan, do the plan, reflect/look back. Here are some of the ways that Pólya's heuristic can be applied to the handshake problem.

Understand the Problem

- *Role play and fishbowl:* Have a subset of students in the class act out the problem. The rest of the class observes. Very quickly this role play will lead to three important ideas:

1) The need to organize the information. One way to do this is to introduce new people to the party, one at a time, and count the new handshakes.
2) That it takes two people to make a handshake. So Julie shaking Sarah's hand is the same handshake as Sarah shaking Julie's hand, and counts as only one handshake (i.e., in mathematical terminology, handshaking is *symmetric*).
3) Finally, that it is similar, but simpler. It is easier to start with fewer than 12 people and gradually increase the number.

- *Model the handshakes, using a geoboard:* This approach (see Figure 1) illustrates all the important ideas of the role-play route.
- *Straws, spaghetti, or string:* Connect the "people," using a manipulative. Some students really like to take a hands-on approach.
- *Make a diagram:* This is the most traditional route, but because it is pictorial, it actually requires a bit more abstract thinking.

Figure 1: *Model with a geoboard*

Table 1: Handshakes as a table of values

Number of people	Number of handshakes
1	0
2	1
3	3
4	6
5	10
6	15

Whatever method is chosen will typically lead to a table of values (see Table 1) or equivalent representation, such as a list, ordered pairs, or graph.

Make a Plan/Do the Plan

Whatever representation is chosen, once students get a table of values, they usually compute finite differences to identify that the relation is quadratic. There are then a lot of ways to proceed.

- *General term from patterning:* Students in Grade 8 or Grade 9 may gravitate to this patterning approach. I call this the "parachute" approach, in contrast to the "domino" approach of recursion. The general term only involves the number of the term, not what value came before. So students look at what would need to be done to the term number n to get the corresponding value t_n.
- *Quadratic regression, using technology:* $y = ax^2 + bx + c$. Students, especially in Grade 10, may jump directly to this method.
- *Recursion relation:* This relies on seeing the values in the table by "how many handshakes are added when the next person arrives at the party," which leads to $t_n = t_{n-1} + (n - 1)$, $t_1 = 0$, . This approach is popular in Grade 11.
- *Use finite differences and linear systems:* This is an interesting approach, often seen in Grade 10 or Grade 11, and connects to students' work with systems of equations. Since finite differences show that this is a second-degree relation, it has the form $t_n = an^2 + bn + c$. Using the first three terms in the table of values gives a system of three equations in three unknowns that can be solved for a, b, c.

$$t_1 = a + b + c = 0$$
$$t_2 = 4a + 2b + c = 1$$
$$t_3 = 9a + 3b + c = 3$$

- *Permutations and combinations:* For Grade 12 Data Management students, this is a chance to decide whether order matters:

$$P(n,r) = \frac{n!}{(n-r)!}$$
$$C(n,r) = \frac{n!}{(n-r)!r!}$$

The specific handshake problem is $C(12,2)$ and the general problem is $C(n,2)$.

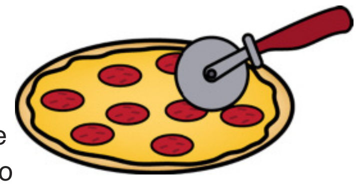
Reflect/Look Back/Extend

In the TIPS4RM materials, there are more detailed examples of how the handshake problem could be addressed at various grade levels. The basic handshake problem provides a starting point to extend the problem to other similar situations. Some of these extensions are given below, sorted into *near* extensions and *far* extensions. Near extensions include:

- *Fire towers:* In northern Ontario, there are 21 fire towers scattered across the region. Every tower must be able to communicate directly with every other tower. How many messages must be possible?
- *Points on a plane:* On a plane, there are 15 different points, no three collinear. How many different line segments can be constructed?
- *Chords of a circle:* There are 16 distinct points on the circumference of a circle. How many different chords can be constructed using these points as endpoints?
- *Diagonals:* How many diagonals are there in an icosagon?

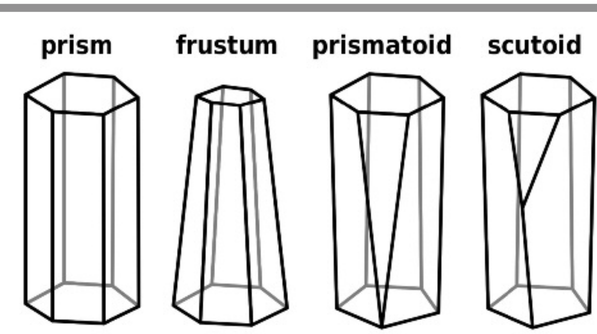
Far extensions use the methods from the handshake problem to tackle other situations.

- *Pizza cuts:* What is the maximum number of pieces of pizza resulting from 6 cuts? Cuts must be straight, but do not need to pass through the centre. Find a formula for the maximum number of pieces for n cuts. Materials: Pizza-sized circle, spaghetti for the cuts. [Each new cut divides all existing pieces into two.]

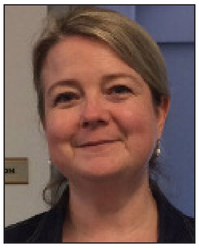


Factors: How many factors does the number 360 have? [In the prime factorization, 5 x 2^3 x 3^2, each factor is either included or not included. So 5 is either in or out, 3 can be included 0, 1, or 2 times, 3 can be included 0, 1, 2, or 3 times].

The handshake problem is an example of an open-routed problem that can be used across grades, and provides a rich ground for students to explore, predict, and justify. There are many other possibilities for student-generated problem posing and problem solving. The *low-floor*, *high-ceiling* criteria give us a way to identify those problems that are most suitable for use. Best results are obtained when student groups are allowed choice in solution methods, and then shared using gallery walks, stay and stray, or other sharing strategies. The emphasis on not just solving the problem, but with the students' ability to explain their thinking, justify their conclusions, and express their results in comprehensible ways, highlights the current thrust in mathematics education in Ontario. ▲



▲ BOOK REVIEW: Helping Educators Understand the Why and How of the Mathematics We Teach Our Students



KELLI GATES
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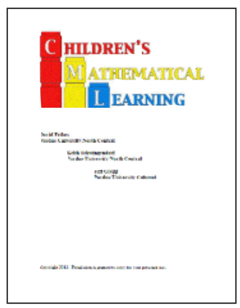
Kelli Gates is the Elementary Math Consultant for Trillium Lakelands District School Board, supporting all elementary schools K–8, and supplemental support in Grade 9 Applied, as well as an additional qualification instructor of Primary/Junior Mathematics at Lakehead University. Kelli is a member of the MAC^2 chapter and was co-chair of the 2016 Annual e-Conference. Teaching students mathematics is a privilege, and it is an honour to be part of their math learning journey.

Feikes, D., Schwingendorf, K., & Gregg, J. (2018). *Children's mathematical learning*. Retrieved from **www.cmlproject.com**



Children's Mathematical Learning (CML) is a resource to support educators, specifically pre-service teachers as identified in the beginning of the document. It is intended to help the reader understand how children develop conceptual understanding of math ideas. Although the resource has sample problems in each chapter, it is to be used as an educator learning resource. This makes it a valuable companion for educators, as it provides support in understanding the why and how of a student's math learning journey. When we understand the why, it gives the purpose of seeking more for us and our students.

There are three highlights from this resource that need to be noted. One is the discussion questions at the end of each section in each chapter. These can support learning and discussion during professional learning sessions of math content. They can be the start of going deeper in different areas based on student learning need. A second feature are practice questions at the end of each section in each chapter. Again, these questions would support educators in understanding the math by doing the math directly related to the learning in the resource, and building a deeper understanding of the content in order to be responsive and targeted in instruction. Finally, there are videos to support the learning in the resource. Videos of students participating with math support educators in understanding their engagement and behaviours. This supports triangulation of assessment and demonstrating how observations and conversations clarify where a student is in his or her learning.

There are two areas where I feel math educators in Ontario need to be mindful when using this resource. First is to remember that this is based on the United States Common Core. Although content is similar, we need to be aware of concepts that may be introduced earlier in the common core than in the Ontario curriculum. An example of this occurs in the Integers chapter, where they discuss multiplication of integers. Although the explanation and content is valuable, we need to remember that this area of learning is introduced in our Grade 8 expectations. This resource is tagged as a K–6 support resource, and multiplication of integers is not a part of this grade continuum in Ontario. A second area to be mindful of is the terminology. The front matter in this resource does note that it is to be used as a supplement; however, some of the terminology differs from that which is found in resources we use in Ontario classrooms. An example of this appears in the Whole Numbers chapter when speaking about division problem types. A quotative division question is referenced as a measurement or repeated operation model. Being aware of this will ensure the words we are using with colleagues and with students are common for expectations found in our Ontario curriculum.

There are a few components I wish this resource had delved into more deeply, and I will highlight two. First, it does a good job of telling educators how students learn concepts, but it does not tell you where to go to learn more, or what to do if a student is stuck. I think of it as a "So what?," "Now what?" scenario. The resource supports the "so what?," but would support educators further if it went deeper with the "now what?" A second component is going deeper into supporting educators with developing fact fluency. CML notes that learning facts is not a matter of simple memorization (chapter 3), which is important for math educators to understand. I believe educators hear this and want to support students, but sometimes need ideas and strategies that do not impose speed and competition. If this resource had gone further in these two areas, it would be a more well-rounded tool for math educators.

Overall this resource is a good document to have in schools to support professional learning. It is cost effective and supports content learning of all areas of our elementary math curriculum. Paired with rich, research-based resources that support students in their math learning, this resource, complete with support videos, will help educators in preparing our elementary mathematicians for their math learning journeys. ▲

▲ Mathematical Snapshots: The Art of Noticing, Wondering, and Questioning

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Ron Lancaster is an Associate Professor at the University of Toronto, where he teaches mathematics courses for pre-service middle and high school teachers. He has over 20 years of experience teaching Grades 7–12 mathematics. Ron's professional activities include consultations and conference presentations in North America, Asia, England, the Middle East, Africa, India, and Europe. Ron is an author for the NCTM (The Mathematical Lens) and member of the Advisory Board for the Museum of Mathematics in New York. He is the recipient of the 2015 Margaret Sinclair Memorial Award Recognizing Innovation and Excellence in Mathematics Education awarded by the Fields Institute.

With each issue of the Ontario Mathematics *Gazette*, Mathematical Snapshots provides teachers with a close-up view of the world around us through a mathematical lens. The column will consist of mathematical questions related to photos and videos. While the real-world connections are important, the powerful feature of the column is the focus on modelling three habits of mind that are of great value to students: noticing, wondering, and asking questions.

Notice: *Transitive verb (i.e., requires an object to be meaningful); To pay attention to or become aware of.*

Wonder: *Intransitive verb (i.e., an action that does not have to have an object); To be affected or filled with wonder; marvel.*

Question: *Noun; 1. An interrogative sentence calling for an answer: an inquiry. 2. A subject or inquiry or debate; a matter to be decided; problem.*

(*Funk and Wagnalls Standard College Dictionary* (Canadian edition), 1982)

These habits of mind encourage all of us to slow down, to observe, and to be curious.

Solutions to the problems below will be provided in the next issue of the *Gazette*. Finally, teachers and their students are encouraged to submit photographs and videos along with their own noticings, wonderings, and questions.

Photograph 1: *Noticing and Wondering about Graphic Art*

Notice the clever combination of a hammer and roots from a tree. Wonder what the connection is between the roots and the business. Wonder why this name was chosen for the business.

Figure 1: *Geometry of the photograph*

Notice the rectangles ABCD and EFGH (Figure 1). Wonder about the ratio of their areas. Wonder why squares were not used instead of rectangles. A square would be more appropriate and catchy. Wonder why they did not incorporate a square root sign into the design (maybe hide it in the hammer or the roots). Notice the phone number. Wonder if the 3-digit number or the 4-digit number is a perfect square.

Mathematical questions for photograph 1:

(a) Estimate which of the following is closest to the area of rectangle EFGH expressed as a percentage of rectangle ABCD:

0%, 10%, 20%, 30%, 40%, 50%, 60%, 70%, 80%, 90%, 100%

(b) Redo the design of the sign so that ABCD and EFGH are squares. Provide a written description with dimensions of your design, including the rectangles APQD and RBCS, so that another person could draw your sign by reading the description.

(c) Suppose that in the design that uses squares, the area of square EFGH is 20% of the area of square ABCD. If the length of the side of square ABCD is 1 m, determine the size of square EFGH.

(d) Are the numbers 359 and 1178 perfect squares? How do you know? Give an example of a phone number (ignore the area code) with a 3-digit number then a 4-digit number, for which both the 3-digit number and the 4-digit number are perfect squares. How many possible phone numbers are there for which both the 3-digit number and the 4-digit number are perfect squares?

Photograph 2a: *Noticing and wondering about a storefront*

Photograph 2b: *The address*

Notice the fractional address (see Photograph 2b). Notice that the 1/2 is raised to resemble an exponent. Wonder if this is a fractional address or if Square Root Restorations and Carpentry made the sign and were having fun by disguising the real street number (3). Notice the prices for bike rentals (Table 1). Wonder if these prices include the HST (Harmonized Sales Tax, 13%). Wonder if you would rent a bike for 2 hours or for half a day.

Table 1: Prices for bike rentals	
Full Day	$30.00
Half Day (up to 3 hours)	$20.00
Hourly	$12.00
Overnight	$40.00
Weekend	$60.00
Helmet Rental	$4.00

Mathematical questions for photographs 2a and 2b:

(a) Search online for information about fractional addresses. Where are they typically used? Why are they used?

(b) What is the total cost of renting a bicycle for a day if the HST
  (i) is included?
  (ii) is not included?

(c) Suppose you only have 2 hours to go out for a bicycle ride. Compare the cost of renting a bicycle for 2 hours with renting one for half a day. How much money would you save by taking the half-day option? Express the savings as a percentage.

Photograph 3: *Noticing and Wondering*

Notice the pattern in the stacked cups from the top row to the bottom row (1, 2, 3, 4). Wonder how many cups there are all together. Wonder how much it would cost to buy this many cups of gelato. Wonder how tall the pile is. Wonder how many cups there would be if another row were added. Wonder if there is a formula for the total number of cups if there are n rows. Wonder how many flavours are available.

Mathematical questions for photograph 3:

(a) Determine the total number of cups for each case listed in Table 2. In looking at the totals, what patterns do you see?

Table 2: Counting stacked cups	
Number of rows	Total number of cups
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

(b) Find a mathematical model for the total number of cups.

(c) What information would you need in order to determine the height of the pile of cups shown in photograph 3? Would you be able to determine the height, knowing only the volume of a cup?

Photograph 4: *Noticing and wondering about stacked barrels*

Notice the pattern formed by stacking barrels from the top row to the bottom row (2, 4, 6). Wonder how many barrels there are all together. Wonder how many barrels there would be if another row were added. Wonder if there is a formula for the total number of barrels if there are *n* rows. Wonder if there is a connection between the patterns and formulas for the cups and barrels.

Mathematical questions for photograph 4:

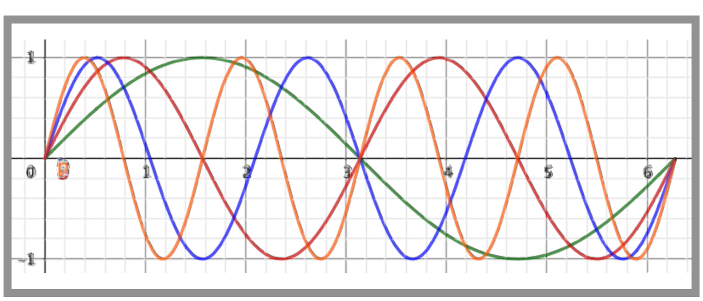
(a) Determine the total number of barrels for each case listed in Table 3. In looking at the totals, what patterns do you see?

Table 3: Counting barrels

Number of rows	Total number of barrels
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

(b) Find a mathematical model for the total number of barrels.

(c) Make a connection between the formulas for the total number of cups (Question 3b) and the total number of barrels (Question 4b). ▲



▲ NCTM Report: BOOK REVIEW: A look at "Catalyzing Change"

JACQUELINE HILL
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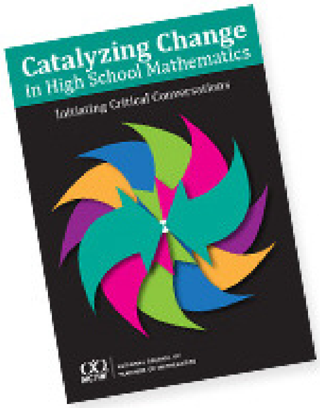
Jacqueline Hill is a Grades 9–12 teacher of mathematics, as well as an online instructor for York University. She is a Past President of OAME and OMCA, as well as the recipient of the award for Exceptional Teaching in Secondary Mathematics. She also wrote the "Director's Dialogue" for the Gazette *for a number of years.*

National Council of Teachers of Mathematics. (2018). *Catalyzing change in high school mathematics: Initiating critical conversations.* Reston, VA: Author. ISBN: 9781680540147

Mathematics learning and teaching has been around since the dawn of time. This book brings forth the conversation around the need for change in high school mathematics. The Pythagorean theorem may have been studied since before 600 BC, but the people and students studying it now have a different historical context. The mathematical ideas are still needed; however, they are needed for an ever-changing world.

> The goal of *Catalyzing Change* is to create the synergies necessary to ensure that students' high school mathematics education provides them with the enduring understanding and skills, motivation, reasoning abilities, and mathematical enjoyment that each and every student deserves and must have to become a fully engaged member of democratic society, capable of achieving his or her full personal and professional aspirations (NCTM, 2018, p. 94).

The Task Force and Writing Team is extensive for this project. The two collaborating writers—Matt Larson (Lincoln Public Schools and University of Nebraska-Lincoln) and Robert Q. Berry III (University of Virginia)—are both NCTM members, with Matt Larson being the outgoing President of NCTM. Both have authored other works. Matt, with other writers and independently, has written on the topics of professional learning communities, needing coherence and focus in high school mathematics, and one district's journey to promote access and equity, while Robert Q. Berry III has authored works on informing teachers about identities and agency, as well as professionalism and enhancing



classroom practice. The major reviewers on the team are eight in total, and the reviewers in general were composed of 61 people (with others wishing to remain anonymous).

The book is a fabulous read! All the way through the book, I felt the team was putting students and the ever-changing world's needs first. The book was broken up into the following areas: the purpose of school mathematics, creating equitable structures, implementing equitable instruction, essential concepts in high school mathematics, organizing high school mathematics, next steps, and conclusions. With over 125 current cutting-edge references, the book was exceptionally well researched. The book has a "More 4U Content" section on the NCTM website, where reflecting questions are posted and discussion along with relevant resources are provided.

Two main ideas came to forefront, the first being that every student has the ability to do well and thrive in mathematics, and the second being the summary of "Essential Concepts" in Number, Algebra and Function, Statistics and Probability, and Geometry and Measurement.

This is the best book that I have ever read on catalyzing change! It takes the reader from what needs to happen, to essential ways to make it happen, and finally, to the next steps (actions for teachers, schools, districts, policy makers, post-secondary educators, and NCTM).

A key recommendation that made me stop and breathe and then reread it was the notion that high schools should focus on the essential concepts in a common shared pathway for two to three years. The notion is that this approach ensures the highest-quality education for all students (NCTM, 2018, p. 7). This is a notion that we are presently working with in the province of Ontario. As a seasoned teacher, it is easy to reflect on a time when we tried to destream Grade 9 students in the early 1990's. This concept was not well received, lacked the shared pathway for a period of years that *Catalyzing Changes* says is necessary, and eventually fell into disfavour.

In summary, this book, from beginning to end, takes the reader on a journey to help students. Some of the conversations may be difficult to have, but they are necessary to ensure students are best able to fulfill their potential in life. ▲

▲ OAME/NCTM Report – The 2018 NCTM Annual Meeting and Exposition

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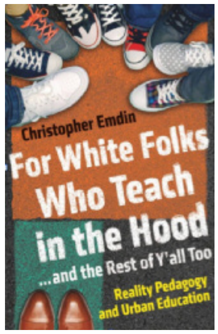
David Petro, the current President of OAME, is the math, science, and IT consultant at the Windsor Essex Catholic District School Board. He is a large proponent of exploiting technology for the educational benefit of students.

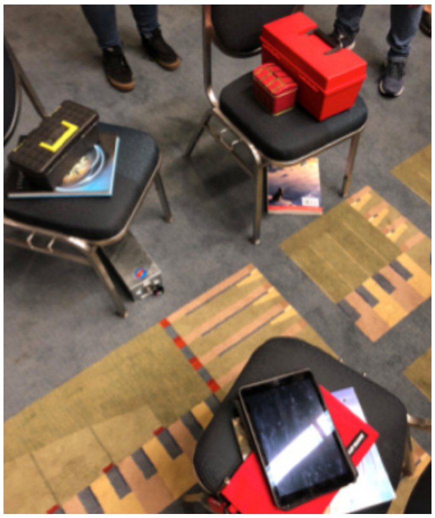
After years of being an NCTM member, but only ever attending our OAME annual conference, I had the pleasure of attending the 2018 NCTM Annual Conference this year. What a spectacle (in a good way)! NCTM has its annual conference in a different place in the continental United States each year. This year, it was held in Washington, DC and there were over 9000 participants. For comparison, this year, the OAME Annual Conference had in the neighbourhood of 2000 participants and was the largest in the history of the OAME. So I can't imagine the amount of planning that went into the NCTM annual conference.

The event took place from Wednesday to Saturday. However, with 662 sessions and over 900 presenters, you had to start to make some decisions and pre-planning before arriving, or you could be wasting your time wandering the convention centre (which spanned three blocks), wondering where to go. Whether it be using their online app or perusing the 242-page program, you had to find some way to get the most out of your conference.

Because I only have our annual conference to compare to, I am going to describe my experience through that lens. At our conference, you pick your sessions ahead of time, and if a session you want to go to is full, you have plenty of time to find a replacement and choose it instead. At NCTM, however, there is no pre-registration for sessions. That means, if you get to a session and you see a lineup out the door like this one from Graham Fletcher, then you better have at least one or two back-up plans because by the time you get across to the other building, then you might find that is full too. Also, if you are presenting, you might find people leaving early so they can be sure to get into their next session. This leads to some strategic planning. For example, if there is a session you are interested in that may be very popular, then look at the sessions in the rooms close to it during the previous time slot (if not the same room).



The opening keynote was on Wednesday night and Dr. Chris Emdin was the speaker (**http://chrisemdin.com/**). I had not heard of him and really had no expectations, but he was amazing. His connections of context to students from the inner cities was inspiring. For more information, see his site for videos, items relating to #HipHopEd, as well as his book *For White Folks Who Teach in the the Hood... and the Rest of Y'all Too: Reality Pedagogy and Urban Education* (2016).



After the keynote, I attended the NCTM Affiliates meeting. NCTM Affiliates are independent organizations that align themselves with the NCTM. The OAME is one of the Canadian affiliates. Many of the affiliates are like OAME in that they are math organizations in individual states. In our case, we were grouped with the Affiliates-at-Large, which are other organizations not affiliated with states. A couple of groups were Women in Mathematics Education (WME) (**www.wme-usa.org/**) and The Council For Technology In Mathematics Education (**www.ciese.org/~ihor/CLIME/**). We discussed ideas to help move NCTM forward, including how to stay relevant when there are so many free high-quality materials online. We then acquired an intimate view of the future of NCTM at the Delegate meeting and heard suggestions from the affiliates on how to improve NCTM's future.

I saw some amazing sessions, whether it be how to create an escape room in your math class, to Robert Kaplinsky's "Problems Worth Solving" (**https://robertkaplinsky.com/**), to Annie Fetter's "Sense Making" (**https://annie.mathematicalthinking.org/**). Probably the best session, in my opinion, was Sarah VanDeWerf's "Engaging Students in Seeing Structure." This turned out to be a freight train of resources to help students think deeper in engaging ways (**https://saravanderwerf.com/**). Even if you weren't able to attend, you can still see some of the main events at their main page (**www.nctm.org/annual/**). I suggest the "Ignite" and "ShadowCon" as some great starting points.

It was also good to see that Ontario was well represented. Ron Lancaster is a long-time staple of NCTM, and this year was no different. Other OAME members that represented us well were Cal Armstrong, Fred Ferneyhough, Kyle Pearce, and Sunil Singh.

There was also more to see than just the sessions. One could spend hours in the exhibit area, where smaller vendor-based sessions were constantly ongoing. I was continually seeing people with whom I had only interacted online. You can go to #MTBoS game night and see how much fun it is to have a massive Rock, Paper, Scissors tournament, or go to Desmos Trivia Night and see Dan Meyer hosting in a way that would make Alex Trebek jealous. Of course, math is everywhere, including in the massive amount of art found in the convention centre.

However, I do have one complaint. There was too much to see and do! You can't help but have a case of FOMO (Fear Of Missing Out), not to mention being in an American history buff's ideal location. I'm not that history buff, but I did manage to get a rainy run in and at least see a few sites. They could put this conference in any city and it would be just as good. There is so much to do that is related to math, you don't even notice your location. Speaking of which, if you can find your way to next year's event, I strongly recommend it. It's in San Diego, and I am going to try to get there myself. I hope to see you there. ▲

▲ Fields Institute MathEd Forum Report

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Angelica Mendaglio is an instructional designer at Vretta Inc. in Toronto, Ontario, where she helps to create interactive digital mathematics lessons and activities for middle school students.

As we start a new school year, so too, the Fields Institute begins a new season of discussion, lectures, and research on math education at all levels. The Fields Institute for Research in Mathematical Sciences is a centre for mathematics research more broadly. The building itself, located in downtown Toronto, is beautiful, with mathematics-inspired sculptures and plenty of chalkboard space to support visitors wanting to collaborate.

The promotion of mathematics education has also always been an important part of the Institute's work. The Fields is home to the Centre for Mathematics Education (CME), which is dedicated to research in mathematics education. The CME oversees the *Fields Mathematics Education Journal*, which is an international peer-reviewed online journal that can be accessed for free online at **fieldsmathed.springeropen.com**. The CME also organizes Math Circles at the Fields, which is an opportunity for middle school and high school students to work on challenging problems to stretch their math knowledge or train for an upcoming contest. The Circles meet on Saturday afternoons, and are open for students in the Toronto area. The CME administers the Math Knowledge Network, which you can learn more about at **www.mkn-rcm.ca**.

Fields MathEd Forum

One of the key activities organized as part of the CME is the Fields MathEd Forum. The Forum is an open monthly meeting, where participants learn about and discuss current research and teacher practices in mathematics education at all levels. The Forum convenes on the last Saturday of every month at the Fields Institute. Teachers from K–12 and beyond, teacher candidates, math education researchers, and anyone interested in the challenges associated with teaching and learning math can find something interesting at the Forum.

Each meeting typically has a central theme, with presenters invited to speak about their research or classroom experiences. The themes and speakers are selected by the Forum's steering committee, a dedicated team of teachers, researchers, and other members of the math education community. One of this year's themes will be computational thinking, which relates to expressing a problem in a way that a computer would be able to follow. This includes algorithms, and has interesting connections to computer programming. Another theme will focus on semiotics, cognitive science, and the teaching and learning of STEM.

The Forum is also planning to host a special day on the importance of statistics and probability in memory of Burke Brown, a fixture at the Forum. Burke championed the importance of statistics and probability for all people, though it is often overlooked in math curricula. He worked on ways of making these ideas accessible to students outside the classroom, including the Taming of Chance Story Competition in 2015, which challenged students and adults to answer the question, "What would our world be like if the normal curve had never been discovered?" The winning submissions of this competition can be found at **tamingofchance.vretta.com**.

One highlight of the Forum is the annual Research Day, which occurs on the last Saturday in January. During this meeting, teachers, doctoral candidates, and other researchers present their current research to the Forum, including a collection of poster presentations to peruse during lunchtime. This is always a very interesting meeting, especially when the research has been conducted by classroom teachers. Anyone who is interested in presenting at this year's Research Day is encouraged to contact one of the members of the Forum steering committee.

The year is shaping up to be a very engaging one at the Forum, with great topics and speakers. If you are interested in the Forum, you can sign up to receive agendas for meetings as they approach. One of the most rewarding aspects of attending the MathEd Forum is to hear and participate in the discussions that occur after each presentation and during lunch. However, if you are not able to travel to Toronto to join in person, you can watch the presentations remotely through the Fields website.

Margaret Sinclair Memorial Award

Every year, the Fields Institute and the CME recognize an outstanding educator with the Margaret Sinclair Award. This award is given to someone who demonstrates innovation and excellence in math education at the elementary, secondary, college, or university level across Canada, and the winner is invited to give a talk at the Fields Institute the following year. The winner in 2018 was Dr. Peter Liljedahl of Simon Fraser University, who is perhaps most well known for his thinking classrooms. Thinking classrooms incorporate vertical non-permanent surfaces to encourage student collaboration and low-stakes exploration, as well as make student thinking easily visible for the teacher. Peter Liljedahl was also the keynote speaker at the OAME 2017 Leadership Conference, and one of the keynote speakers at the OAME 2018 Annual Conference. He is an engaging speaker, and his talk at the Fields is sure to reflect the spirit of the award. ▲

▲ Provincial Digital Learning Resources – What's New? Algebraic Reasoning – The Power of Visual Representations




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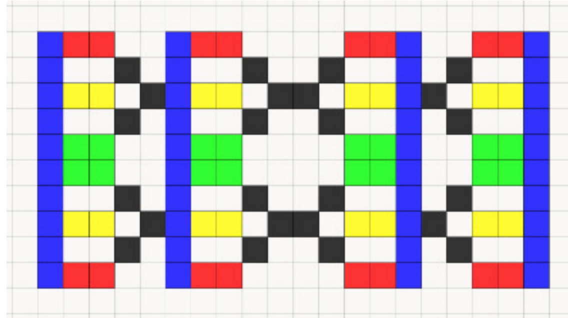
Greg (Simcoe Muskoka Catholic District School Board), Agnes (Brant Haldimand Norfolk Catholic District School Board), Markus, and Ross (Near North District School Board) are all currently working as Project Leads assigned to develop the digital resources found at mathies.ca. Greg retired as of June, so this is his last regular article with the team. Best wishes to Greg, and know you will be missed.

One of the reasons that mathies learning tools are so powerful is that they provide a means to create visual representations that can then be manipulated to solve problems and share mathematical thinking.

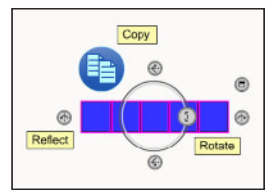
At the beginning of professional development sessions on mathies learning tools, we like participants to have a hands-on introduction to one of the tools. The recent mathies tools share a lot of common functionality, and an introduction to one makes participants more confident using the others. For the Colour Tiles tool, they might be asked to construct the pattern below.



Challenge: Can you create the pattern in fewer than 25 steps?

We challenge participants to use features of the tools that allow the pattern to be built using fewer steps. For example, rather than dragging 10 blue tiles to the left edge one at a time, the multiplier button ×10 can be used to bring 10 tiles out all at once.

As participants work to create the pattern in as few steps as possible, they are encouraged to discover various features of the tool, including:



- setting the multiplier to 1, 2, 5, or 10 to drag groups of tiles to the workspace
- selecting a group of tiles
- rotating a group of tiles
- copying or reflecting a group of tiles
- seeing how to Undo/Redo steps through the actions with the tool (Participants can use this feature to count the number of steps used and see if it is less than 25.)

Participants are then asked to:

a) write an expression for the number of blue tiles
b) write an expression for the total number of tiles

Participants are encouraged to write expressions that connect to how they actually built the pattern. Expressions such as these emerge from the discussion:

Number of blue tiles is:

a) 4 x 10 (4 groups of 10)
b) 4 x 5 x 2 (4 groups of 5 doubled)
c) 5 x 2 x 4 (5, two times, quadrupled)

Total number of tiles is:

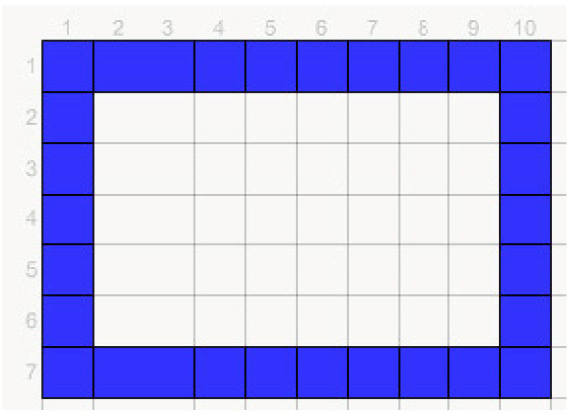
a) (5 + 3 x 2 + 3) x 2 x 2 x 2
b) (10 + 6 x 2 + 6) x 2 x 2

What does each expression reveal about how the pattern was built?

This activity not only serves as an effective introduction to the use of a mathies tool, but can lead to some interesting discussions about equivalent expressions. The visual pattern corresponding to these expressions makes them come to life in a concrete way. There is a video on the support page for Colour Tiles (see link below) that reveals one way to create this pattern.

Border Problem

In this adaptation of the border problem, the objective is to determine the number of blue tiles in this 7 by 10 rectangle, without counting each individual square.

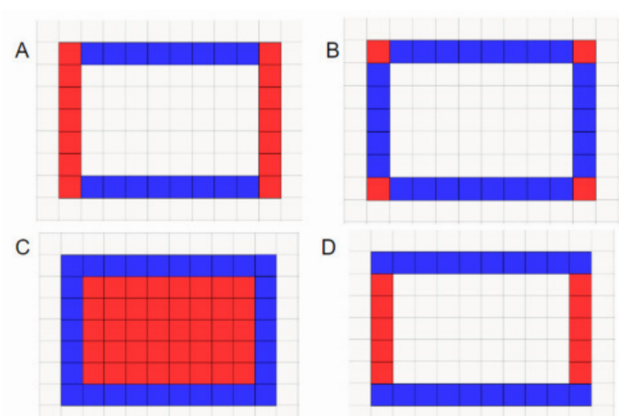


First, use the mathies Colour Tiles tool to build the rectangle.

Next use the Colour Palette  to recolour the tiles to illustrate your thinking as you determine the total number of border tiles.

How can you use the various features of the tool to build this rectangle efficiently?

Does the approach to building it influence the way you determine the total number of blue tiles?



Here are some possible ways to colour the tiles.

Which image best helps you to see:

a) 2 x 10 + 2 x 5

b) 2 x 7 + 2 x 8

c) 2 x (10 + 5)

d) 4 + 2 x 5 + 2 x 8

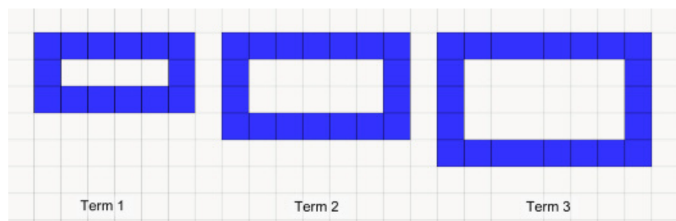
e) 10 x 7 – 8 x 5

If you created the rectangle by setting the multiplier to allow you to drag out groups of 10 tiles for the top and bottom, and groups of 5 tiles to form each side, then you might have coloured the tiles like those shown in image D and come up with 2 x 10 + 2 x 5 as the corresponding expression. Alternatively, you might have seen two "L" shapes, each made up of one blue horizontal strip and one red vertical strip, and hence come up with 2 x (10 + 5).

Each of the listed expressions corresponds to one or more of the coloured images.

Which ones might also be connected to how the rectangle was created using the tool?

Once again, there is power in seeing each of these expressions come to life visually. It facilitates making connections between the visual representation of the border and the arithmetic expressions for the number of blue tiles, which exemplifies moving from early patterning to algebraic reasoning. Mathematical processes, including making connections, representing, reasoning, and proving, are engaged by this task.

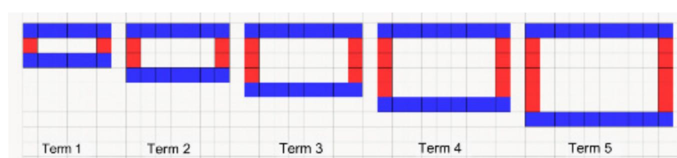


This numerical example is a good bridge to exploring more generalized algebraic expressions.

What if the 7 by 10 rectangle (shown in blue in C above) were one term in the following sequence of rectangles?

Which term would it be? Some discussion might be necessary to establish that the original rectangle is the fifth term of this sequence.

One approach is to build several terms of this sequence and colour-code them as before. Here is one possibility using the same thinking as described for image D above.



Write an expression for the number of border squares for the *n*th figure.

One of the numeric expressions that describes the fifth term of this sequence is 2 x 10 + 2 x 5. Thinking about a general term for this sequence can be addressed by considering which values in the expression remain constant and which vary? How does the coloured diagram help you to see that? In each diagram, there are two blue horizontal strips and two red vertical strips. This helps us to see that the factors of 2 are constant and leads us to generalize the expression as 2 blue strips + 2 red strips. Now, how long is each strip? How is the number of tiles in each strip related to the term number?

The number of tiles in each red vertical strip is the same as the term number.

The number of tiles in each blue horizontal strip is five more than the term number.

So, the total number of border tiles, for term n, is $2n + 2(n + 5)$.

Colouring the sequence in alternate ways, as indicated above, might result in one of the following algebraic expressions:

a) $2(n + 2) + 2(n + 3)$

b) $2n + 2(n + 3) + 4$

c) $(n + 2)(n + 5) - n(n + 3)$

d) $2(2n + 5)$

e) $2m + 2n$

Let the math discourse begin! Are these expressions equivalent? How do you know?

Do you need to use two variables, or is one sufficient? Why might it be better to use only one variable?

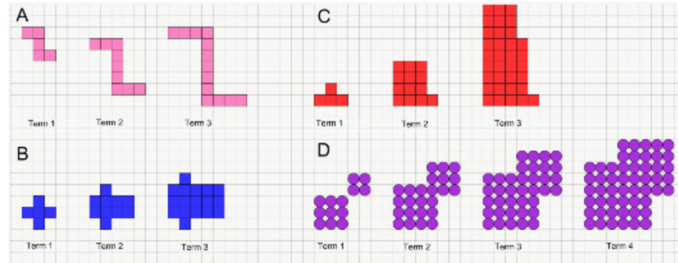
Now students have a reason to want to learn some of the basic algebraic principles, like the distributive property and collecting like terms, all with strong visual representations to support their learning.

Other Visual Patterns

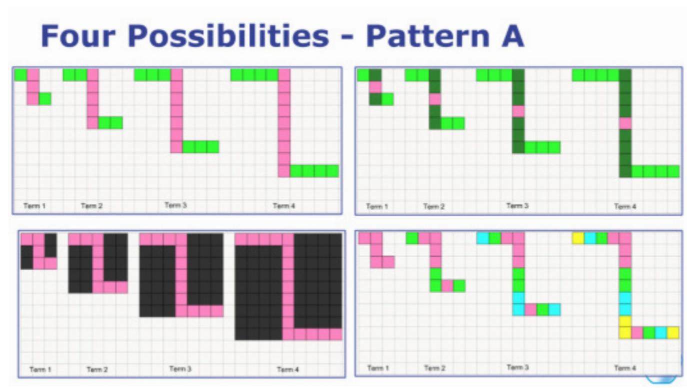
One good source of visual patterns is **www.visualpatterns.org**, started by Fawn Nguyen, one of the featured speakers at the recent OAME Annual Conference. There are over 250 patterns currently, and there is a mechanism to submit your own to the site.

Students or teachers can be given a collection of patterns and be asked to choose one (or more) for which they generate a pattern rule. They might be asked to:

1. represent the first few terms, using the Colour Tiles Tool, looking for symmetries or structures that would save them from adding tiles one by one
2. colour the tiles to make the structure explicit
3. use the annotation feature to add explanations and the pattern rule



In some of our sessions, to save time and to show many possibilities, we asked participants to match expressions to a colouring.



Which possibility is the "best" to help you see:

a) $2n + (2n + 1)$

b) $4n + 1$

c) $(2n + 1)2 - 2(n[2n])$

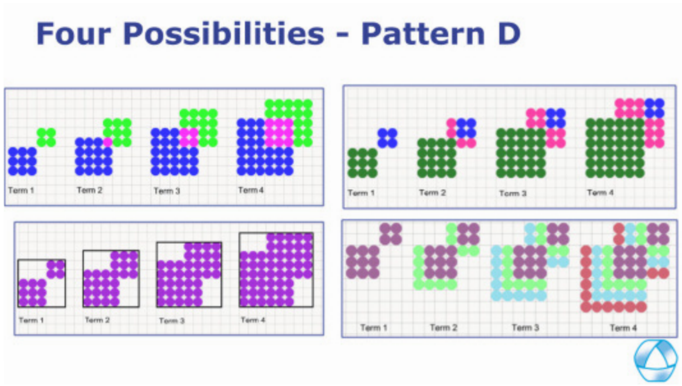
d) $5 + (n - 1)*4$

e) $2(2n) + 1$

Each of these pattern rules and colourings exploits a structural feature of the visual representation; for example, a rotational symmetry (by 180 degrees) is possible at the top right. Exploiting that rotational symmetry is possible in Colour Tiles, since it has the ability to quickly copy the tiles at the top left, rotate the copy, and move it to the bottom right.

As students practise this, they will begin to see some strategies that are mathematically useful, like using negative space (bottom left) and first differences (bottom right – where the amount added at each term is coloured differently).

As students investigate Pattern D, they might notice that its relationship will not be linear, since the number added to each term, the finite difference, changes (see bottom right below).



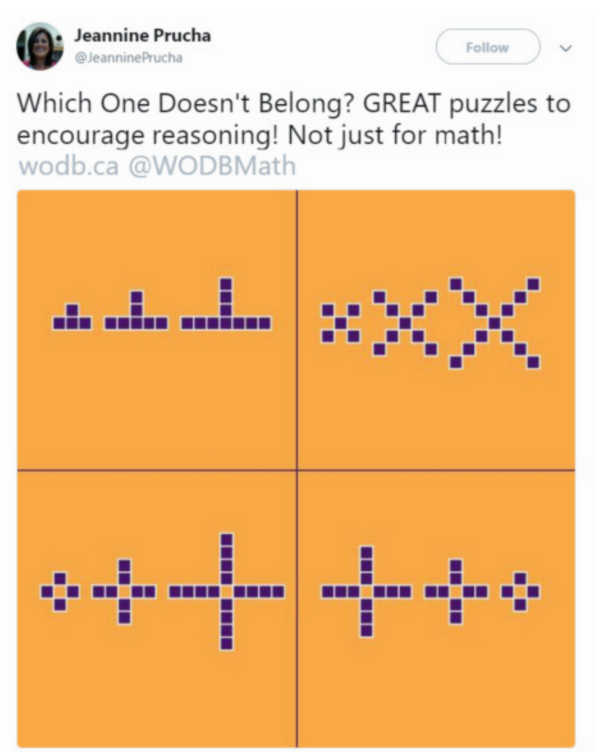
However, the use of negative space is still a very useful strategy here (bottom left). Each term is made up of a square with two 2 by 3 rectangles removed. The expression,

$$(n + 4)^2 – 12$$

matches this representation and is an expression for the pattern rule. The pattern rule is a quadratic function, and is expressed as a completion of a square in a completely visual way, without formal algebraic manipulation.

Any of these observations are best made after students (or teachers) have had ample time to construct, colour, relate quantities to the term numbers, and test various pattern rules themselves.

Another website with visual representations is **wodb.ca** (Which One Doesn't Belong?), created by Ontario's own Mary Bourassa and grown by the math community. Typically, four representations are presented, and the reader is challenged to find reasons why each one doesn't belong. This builds students' communication and creativity.



In elementary school, students can use WODB activities as part of their number talks; in fact, "Numbers" is one whole section of the site, together with "Shapes," "Graphs," and "Incomplete Sets."

It has been fun to see teachers in professional development sessions start to think about algebra in a richer way as they use visual representations. You can access mathies webinars given as part of the Renewed Mathematics Strategy at **rms.thelearningexchange.ca** and follow the links to On-Demand Professional Learning. Perhaps this article or the on-demand sessions might provide the basis for sessions that you have with some colleagues.

Feedback and Future Requests

Please feel free to send us your feedback about any mathies tool, using the Feedback Form button inside the Information Dialog, accessed from the button. Visit the support wiki page for more examples and detailed descriptions of the functionality of the tool. (With the imminent closure of wikispaces, the support pages will be migrated to **support.mathies.ca**).

You can also send your comments to **WhatsNew@oame.on.ca**. You can share your experiences on Twitter, using the hashtag #Onmathies, and follow or message us at @ONmathies. There is an increasing set of interesting posts of student and teacher work on Twitter.

To be among the first to find out about the latest digital tool developments, sign up for our email list at **mathclips.ca/WhatsNewEmailList.html**. ▲

Math decor at Humber College (OAME 2018)

▲ BOOK REVIEW: BURN MATH CLASS? OR MAYBE JUST A LIGHT CHAR?

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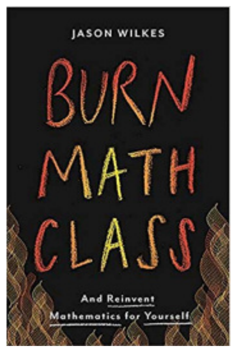
Wilkes, J. (2016). *Burn math class: And reinvent mathematics for yourself.* New York, NY: Basic Books.



Upon receiving this book and flipping through the pages, my first instinct was admittedly to dislike it! Despite a life of being encouraged not to judge a book by its cover, I did. Having read and digested the entire book and better understanding its motivation, however, my thoughts have certainly evolved.

I had originally approached this book as a reflection on mathematics learning written by an educator driven mad by a perceived focus on rigidity, unmotivated rules, and memorization that resulted in his desire to "burn" all of mathematics. If approaching this book from this direction, a reader might be forgiven for being annoyed by a number of the suggestions made by the author, such as replacing mathematical terms such as "polynomials," "reciprocal," and "Taylor series" with more colloquial "plus-times machine," "handstand," and "nostalgia device." From this perspective, the book will feel gimmicky in its push to be informal.

Partway through the book, however, I came across the following passage that changed my opinion of the book entirely.

Even in its final form, the book will inevitably contain numerous instances of the following sins: typos, hyperbole, poorly worded sentences, repeating myself, contradicting myself, sounding too arrogant, sounding too insecure, saying "I'll never do X!" and then promptly doing X, saying "I'll never do X!" and then later doing X (but not promptly)… unintentionally alienating or offending innocent readers, experimenting with the medium in ways that some will find distracting, too many prefaces, too many digressions, too many dialogues, too few dialogues, too much meta-commentary, the use of arcane Greek and Latin words, despite having made fun of them (and the people who use them) for being more pretentious than is necessary (p. 16).

I suddenly realized that the book is actually a reflection on mathematics learning, written by a learner of mathematics. The author wrote the book to share his personal journey through learning mathematics and to share the explorations and ideas he found valuable, inspiring, and motivational. From this perspective, the book is a rare and honest view into the mind of a passionate learner, and certainly inspired me to reflect honestly and critically on my pedagogical practice.

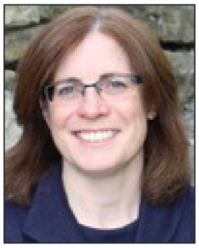
The overall concept of the book is to throw away everything you know about mathematics education and approach the entire subject anew from a strictly exploratory methodology. The author starts with the basics of addition and multiplication and moves quickly through a wide range of advanced topics, but always from a strictly exploratory and inquiry-based setting, and always with a focus on how the mathematics is created. Since the book is such a personal exposition, I strongly believe that every reader will have different likes and dislikes within the book. So, in a book review such as this, I think it is perhaps valuable to share my take-aways from such a book.

I disliked the attempt throughout the book to replace formal language with informal, but equally regimented, language. In a critique that learning mathematics is akin to learning a new language, the equation "(Number)*(food)^number" is presented, which is, arguably, equally inaccessible as a new language. I was sometimes frustrated by the author's claim that certain ideas were not necessary, suggesting in one sentence that the math "doesn't require division" and then encouraging the reader to "cancel the t's." There are also some distracting dialogues between the author and an omnipresent being called "MATHEMATICS."

I loved the terrific discussion on the different meanings of the symbol "=" and how we shouldn't take for granted that learners are fluent in these differences! I enjoyed the focus on exploratory learning, and there are some great examples that helped motivate me to explore inquiry activities in a variety of other settings. I liked that the author's motivation is "to create students who know how to think," rather than "to create students who know things about math," and he discusses many positive ways to realize this goal.

I believe, through this book, the author managed to accomplish his overall goals. I got a valuable and informative insight into the mind of a learner and came away with ideas and inspirations that I can apply in my own classroom. I may not have been convinced to "burn" mathematics, but perhaps it should be singed a little! ▲

▲ Technology Corner: Desmos Activity Bundles



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Mary Bourassa teaches mathematics at West Carleton Secondary School in Ottawa. She is a strong advocate for the appropriate use of technology in the classroom. She has presented workshops internationally, authored mathematics resources, is a Past VP of OAME, and a Past President of COMA. An award-winning teacher, Mary continually strives to learn new and better ways of helping students learn and love mathematics.

In addition to their free graphing calculator, Desmos is a powerhouse of rich activities. Teachers access these activities through the **teacher.desmos.com** site, where they can search for a particular topic or check out the Activity Pick of the Week. All of the activities on this site have either been created by the folks at Desmos or have been edited by them to ensure sound pedagogy and mathematical correctness. These activities allow students to work at their own pace, while giving the teacher feedback of their progress. There are teacher tips included in the activities, which can be seen by working through the student preview. The teacher tips provide sample answers along with guidance about where to use teacher pacing, which allows you to restrict which screens the students can see, where to pause the activity for consolidation, when to use overlay mode, and other helpful hints to consider when running the activity. Along with the stand-alone activities, Desmos has created a number of bundles of activities, which thoughtfully work through each topic.

The "Linear Bundle" consists of seven activities that build up students' knowledge of slope and the *y*-intercept. It begins with "Polygraph: Lines." Polygraph activities are similar to the "Guess Who?" game. Students are paired up and assigned roles: one picker and one guesser. The picker chooses one of the 16 graphs presented. The guesser must ask questions to determine which graph was chosen. Each question can only be answered with "Yes," "No," or "I don't know." The feedback given allows the guesser to eliminate one or more graphs before asking a new question. This develops the need for common language—in this case, "steepness," "*x*-intercepts," and "*y*-intercepts." Figure 1 shows the 16 graphs that students will see in Polygraph: Lines.

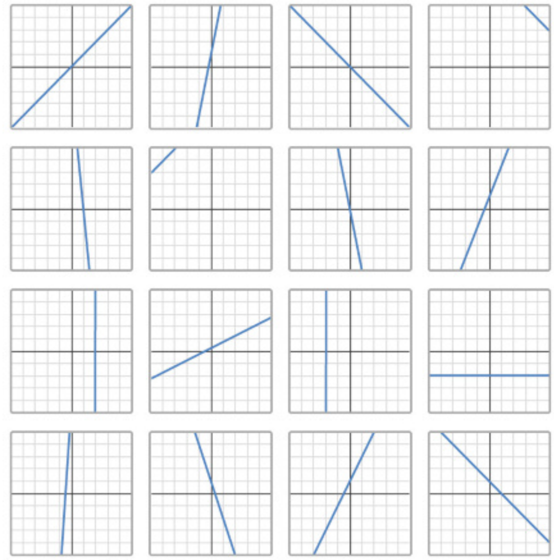

Figure 1: *Polygraph: Lines*

"Polygraph: Lines, Part 2" introduces more formal vocabulary, and the teacher tips included provide a framework for all teachers to make this a successful lesson. A sample screen is shown in Figure 2.

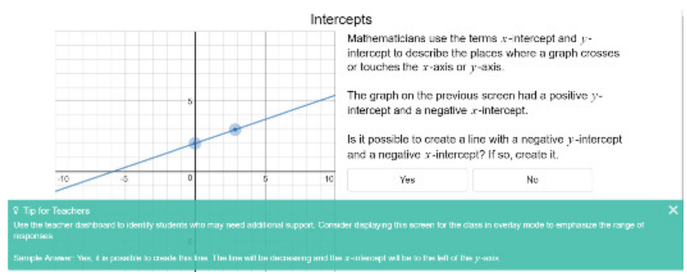

Figure 2: *Polygraph: Lines, Part 2*

Now that slope has been defined, students can put this knowledge to use in "Put the Point on the Line," the third activity in this bundle. Here, students are given two points and must estimate where to place a third point so that all three are collinear (i.e., pass through the same line). There is no grid to begin with so that students can see the greater precision afforded by one being provided. They then use the changes in both *x*-values and *y*-values between two points to determine where the third point should be placed, as shown in Figure 3.

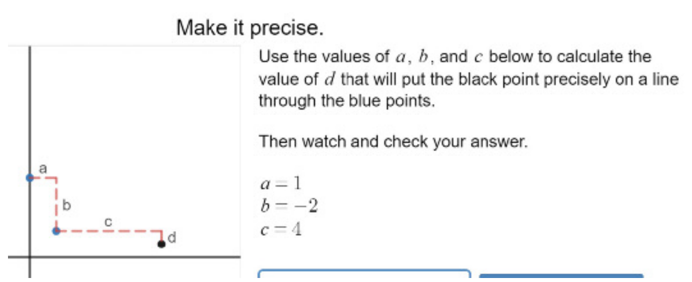

Figure 3: *Put the Point on the Line*

Students get feedback through the graphic display of their point and the line, once they enter a value for *d*, as shown in Figure 4.

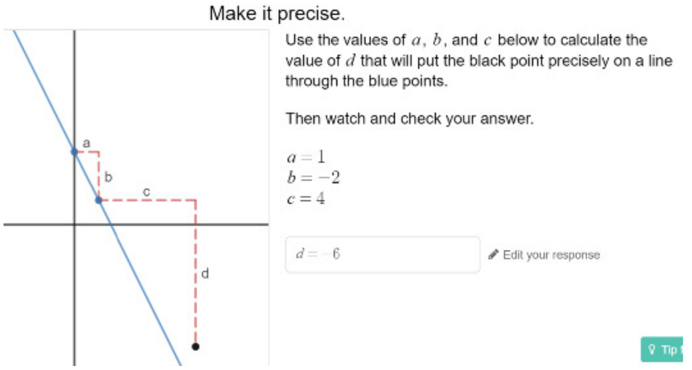

Figure 4: *Put the Point on the Line – Feedback*

Students can also edit their response, and watch to see that the line now goes through all three points, as shown in Figure 5.

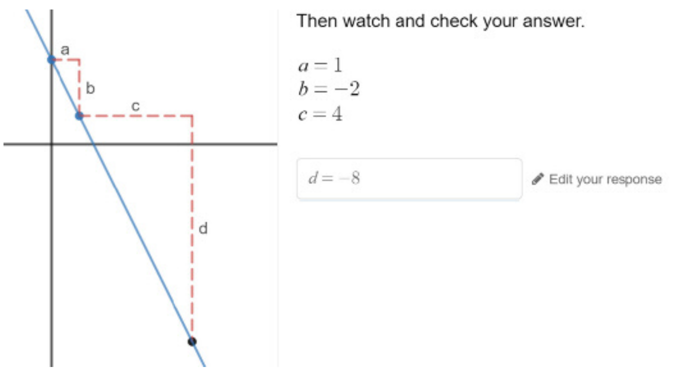

Figure 5: *Put the Point on the Line – after editing one's response*

The fourth activity in this bundle is actually two activities: “Match My Line” and “Land the Plane.” The former really provides a lot of practice determining the equations of lines. They begin with a *y*-intercept of 0 and grow in complexity as students progress through the activity. A sample screen is shown in Figure 6. When the student clicks the “Check My Work” button, each of their equations is graphed, giving immediate feedback on their work.

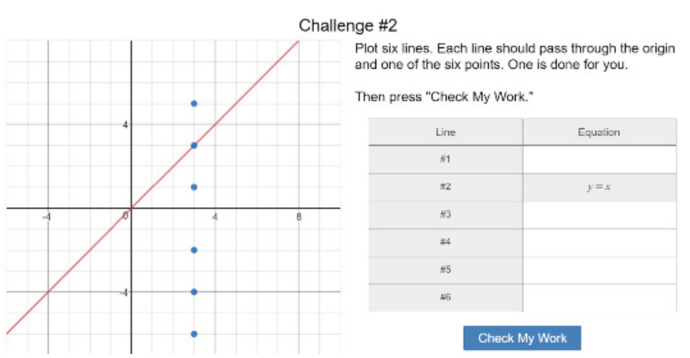

Figure 6: *Match My Line*

“Land the Plane” is a much quicker activity, where students need to enter the equation representing the path that a plane must follow to safely land on a runway, as shown in Figure 7.

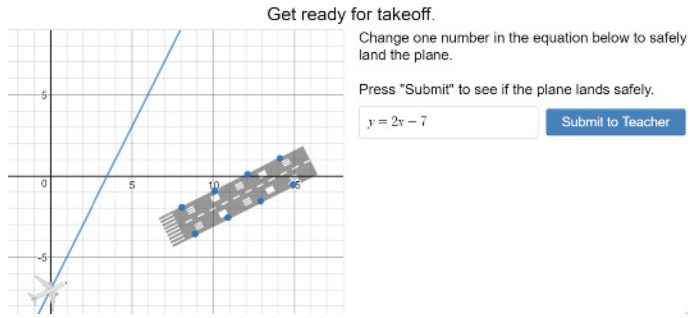

Figure 7: *Land the Plane*

This activity finishes with a creator challenge, where students design their own challenge. Each of these is placed in the class gallery so that students can then try to complete each other's challenges. This type of extension provides meaningful work for students who complete the activity quickly, without negatively impacting those students who take a little longer to work through the main activity screens.

The fifth activity is a card sort, where students need to group a set of nine cards (tables, graphs, and equations) based on whatever criteria they choose.

Next in the Linear Bundle is “Marbleslides: Linear.” I don't know anyone who doesn't love Marbleslides! Students must edit or write their own equation(s) so that the marbles touch all the stars, at which point, the word “Success” appears on the screen. A sample screen is shown in Figure 8.

Marbleslides activities are fun and reinforce students' understanding of linear relations. Students also quickly become proficient at restricting the domain of their line. Built into the activity are screens that help students to demonstrate their understanding of the material, using a predict-then-verify model. The many challenges that follow give all students the opportunity to test their skills and provide enough extension to keep all students interested, regardless of the speed with which they work through each screen.

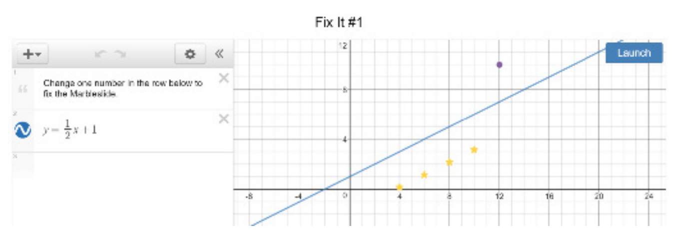

Figure 8: *Marbleslides: Lines*

The last activity in this bundle shows students how linear models can be used. Students will create a linear model for the relationship between the price and the number of pieces in a LEGO™ set. They must also interpret the meaning of the slope and justify the y-intercept used. A sample screen in shown in Figure 9.

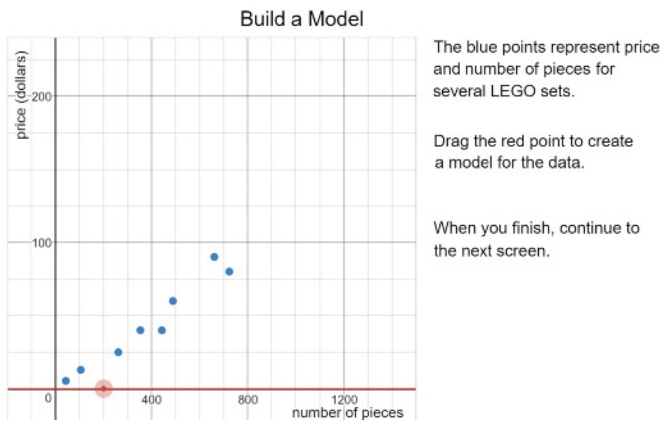

Figure 9: *LEGO™ Prices*

Throughout this bundle of activities, students become comfortable and then proficient at working with linear equations. Other bundles focus on quadratics, exponentials, functions, transformations, modelling, and more. These and many more await you at teacher.desmos.com. ▲

▲ OAME Awards

Life Membership Award

The Life Membership Award is given to recognize a person or persons who have contributed in a significant way to OAME/AOEM. It is an acknowledgement that they have demonstrated outstanding leadership in mathematics education, and have accumulated ten or more years of membership in OAME/AOEM. This description applies to the recipients of this year's award. **Dave** and **Sue Hessey** served as executive directors of OAME/AOEM from 2001–2010. Their tireless work—whether it was Sue's almost maniacal ability to reply to a request within seconds at all times of the day, or Dave's steady hand and calm demeanour when there seemed like too many plates were spinning—have made OAME/AOEM the vital organization that it is today. The award is given, on behalf of all members past and present, as a way to thank Dave and Sue for their contributions.

▲ Hey, It's Elementary: An Important Lesson from Mitch & Greg and Brittany & Sarah: No one will ever complain that you've made things too simple to understand.

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Dr. Lynda Colgan is a Professor at the Faculty of Education, Queen's University. In addition to her teaching responsibilities in the BEd and Graduate programs, she is involved in research and knowledge-mobilization projects with the Council of Ontario Directors of Education, TVO/TFO, the Ontario Ministry of Education, and the Mathematics Knowledge Network.

Every May for the last eight years, on the Saturday before Mother's Day, I have hosted *Science Rendezvous Kingston*. The goal of *Science Rendezvous* is to celebrate all things Science, Technology, Engineering, and Math (STEM) related: inventions, people, discoveries, and careers. By bringing STEM to the people of southeastern Ontario, we hope to inspire more children, adolescents, and young adults to see themselves in the role of scientist, innovator, inventor, and researcher. From robots and medical simulators, to stealth computer displays (made possible by laser light magic!) and heart ultrasounds (performed on site by cardiologists from Kingston General Hospital), *Science Rendezvous Kingston* has grown to become the largest pop-up science discovery centre in Canada. It was attended by over 4400 people in 2017 and 2018, and made possible by some 400 volunteers (faculty, students, community scientists, museum curators). It is a day full of fun and informal STEM learning. You can check out our program and download our take-home book (filled with great kitchen sink and backyard STEM experiments) at **educ.queensu.ca/coc/science-rendezvous**.

While there were many "Wow!" moments throughout the day on May 12, 2018, including our green *Chemistry Magic Show*, a microscopic scavenger hunt by *Art the Science*, and an opportunity to climb Leonardo da Vinci's Self-Supporting Bridge, the huge highlight this year was a live performance by Mitchell Moffitt and Gregory Brown, aka *AsapSCIENCE* (**www.asapscience.com/**).

Mitchell and Gregory (Figure 1) are graduates of the University of Guelph. Mitch was always captivated by the world around him and understanding the science in our daily lives. After earning a BSc from the University of Guelph, he became fascinated by YouTube's potential to teach and reach people around the world. Gregory attended the University of Guelph for Biological Science and minored in Visual Art. Passionate and curious about teaching, he then went on to earn his BEd from the University of Toronto. The two teamed up and *AsapSCIENCE* was born with the intent to educate, entertain, and inspire people to have a similar love of science. Their success is evident because within two years, *AsapSCIENCE* had earned over 2.6 million subscribers and nearly 100 million views! Mitch and Greg apply their skills as educators, visual and musical artists, and scientists to research, create, and evolve their online educational channel to the positive advancements in science, education, and technology. Their work has been enthusiastically endorsed by Bill Nye and Neil de Grasse Tyson, two of the most famed science communicators of all time.

Figure 1: *Mitch and Greg*

Figure 2: *Mitch and Greg with book winner*

Their live show at The Rogers K-ROCK Centre, introduced by Kingston's own official Town Crier, Chris Whyman, was attended by almost 1000 people, singing and dancing along to the guys' energetic rendition of *The Periodic Table Song* and posing excellent questions for the duo to consider as future episodes. Twenty-five lucky (and *very* excited) children won a copy of their book, *AsapSCIENCE: Answers to the World's Weirdest Questions, Most Persistent Rumors, and Unexplained Phenomena*, as well as an opportunity to meet Mitch and Greg at a VIP meet and greet (Figure 2).

Watching from backstage, I was struck by their seemingly effortless presentation: invitational, inspirational, educational, and accessible to learners across a wide age span. It was clear that not only were the duo mesmerizing the crowd, they were *teaching* the crowd.

It was at that "Aha!" moment that reminded me of a 1986 paper by David Berliner: *In Search of the Expert Pedagogue*. The paper is worth reading again for many reasons. It speaks to professional growth in teaching, the various types of knowledge required by an educator, and, perhaps most importantly, the need to celebrate and acknowledge the daily complexity of the roles carried out by classroom teachers. After his extensive qualitative study, conducted with great care and respect for his participants, Berliner concluded by saying:

> We will continue our pursuit of the expert pedagogue. If we ever feel really secure that we have found a few of these elusive beasts, we will study them in great depth and share those findings with those who also await their capture. Like the search for the Yeti and for Bigfoot, we expect to have a good many false sightings and a good deal of fun along the way (p. 13).

As I heard Mitch and Greg speak, moved along to their music, and attended to their unadorned, line-drawn illustrations of Bigfoot, I was reminded more of the lovable (well, at least after his sore tooth was fixed!) Abominable from the *Rudolph the Red-Nosed Reindeer* animation, and just as certain that here stood the elusive beasts described by Berliner: *expert pedagogues*.

Not long after being part of the very large "classroom" taught by Mitch and Greg, I had an opportunity to attend an interesting presentation at *Innovate Kingston*, an annual showcase for student entrepreneurs vying for funding and support. At that event, I learned about Brittany Foerster and Sarah Nersesian, two Queen's graduates who were pitching their project, aimed at making science more accessible: *Designs that cell*.

As scientific illustrators, through their venture, Sarah and Brittany (Figure 3) create cartoon-style, but scientifically accurate, graphics for students and researchers. The accessible illustrations aim to be an alternative for imposing, jargon-heavy scientific research. "It's not that anyone doesn't have the same level of intelligence. It's just if you're not in that field, you don't speak the language," Sarah said about the complex research that scientists churn out. Recognizing that we live in a world where science isn't always communicated to the general public, the team set out to address the problem through creativity and their unique spin on the art of communication.

Figure 3: *Sarah and Brittany*

As I reflected on these two recent encounters, I was taken back to a panel presentation by educational researchers and education officers about mathematics teaching and learning at a major symposium that took place last February in Toronto. It did not take long for me to come to the realization that I had no idea what was going on: the expert panelists were spouting an inaccessible language of jargon and acronyms, talking to and over one another, and using terms that I am certain must have baffled other audience members too (although most were educational professionals in some capacity).

Interestingly, there was an artist creating a Graphic Recording (i.e., a live drawing of the meeting in the style of the one shown in Figure 4) on a large piece of paper, attempting to synthesize the essential elements of the event into a combination of words and images. The goal of a graphic recording is to pull out the most important points and distill key messages into their most potent form. The artist, though trying her best, was stymied by the fact that almost all of the words being thrown about represented complex concepts with multiple nuances, meanings, and layers. As a result, the end product, though aesthetically interesting, colourful, "fun," and accurate in terms of the language of the day, was not informative. It did not communicate the exchange of thoughts, information, ideas, and messages between the panelists themselves and among the panelists and the audience. Why? The artist was not an expert in mathematics education: the absence of deep reflective knowledge of the pedagogical, curricular, philosophical, and belief foundations led to a *picture*, but not one that conveyed clear, unambiguous information, i.e., one worth 1000 words.

Figure 4: *Creating a graphic recording*

It is very hard work to communicate complex academic ideas clearly, accessibly, and accurately… but it is possible, as Dr. Angela Pyle from the University of Toronto recently demonstrated by winning a prestigious award at the 2018 annual conference of The American Education Research Association from the Early Education & Child Development Special Interest Group. The award was to recognize her innovative and highly visual infographic (Figure 5) about the findings of her study of the continuum of play in full-day Kindergarten classrooms. Determined to share the results of her research as widely as possible, Angela sought ways to debunk the wildly misunderstood, but commonly held, definitions and "rules" for *play-based learning* by publishing in venues beyond traditional academic and professional journals. Her infographic, which concretizes poet David Kresh's observation that *the understanding of atomic physics is child's play compared with the understanding of child's play*, has been well received by multiple audiences and is setting a new standard for the flow of information from researcher to those interested in or impacted by the topic.

There is much to be learned from Mitch and Greg, Brittany and Sarah, the unknown Graphic Recorder, and Angela. They understand that in order to improve understanding about anything, we must figure out how to talk and write clearly about *it*. The expert communicators are never guilty of "dumbing down" the content that is at the heart of their lessons—instead, they democratize knowledge, opening it up by making complex concepts simple, but not simplistic.

I once heard it said that "no one will ever complain that you've made things too simple to understand." There is empirical evidence to support this. Princeton researchers ran three experiments. In the first experiment, they asked students to read and rate admissions essays. The first was a low-complexity essay; the second was a moderate-complexity essay; and the third, a high-complexity essay. The study showed that the simple essays were given higher ratings than the moderately complex ones, and the moderately complex ones were given higher ratings than the highly complex ones. In other words, the more complicated vocabulary negatively influenced raters' assessment of the text. In a related series of five experiments, Oppenheimer (2006) found that people tended to rate the intelligence of authors who wrote essays in simpler language as higher than those who authored more complex works.

Given that Ontario's curriculum documents and implementations for math are intended for multiple audiences, including parents and the broader community, perhaps it is time for accessible communication about what we as mathematics educators are teaching in today's classrooms. Mathematicians strive for *beauty* through *elegant visual proofs*, i.e., ones that are simple and brief. Given all the research data that supports contemporary approaches to teaching and learning mathematics, its educational leaders must strive to generate their own version of elegant visual proofs to debunk persistent myths such as "discovery math." Simple and plain just like Mitch and Greg, Sarah and Brittany, and Angela, we need to pitch something (*teaching math*) more complicated than physics (according to Berliner) in a way that is not only engaging, but also faithful to the evidence.

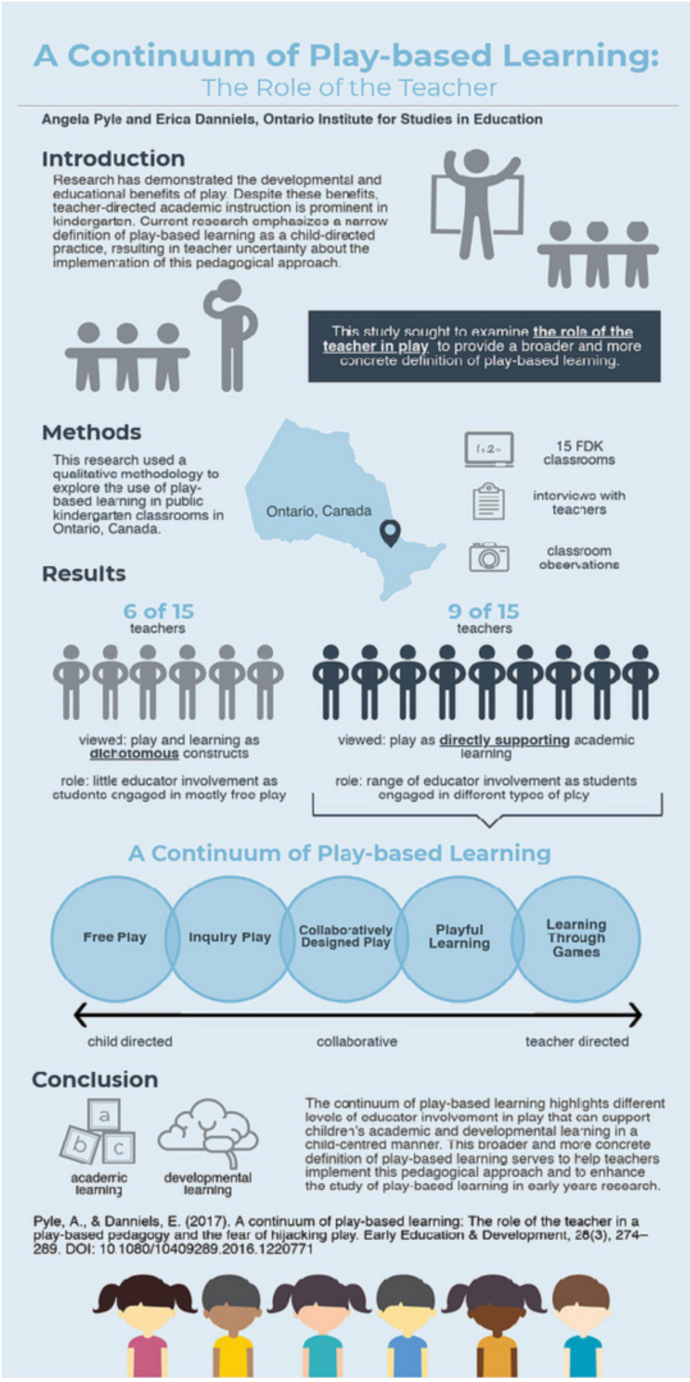

Figure 5: *Research findings as an award-winning infographic*

If we, as a community, cannot do that, then the words often (mis)attributed to Einstein may be true: we do not really understand because we cannot explain it to our grandmothers.

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▲ Mb4T (Mathematics by and for Teachers): Examining the Use of Manipulatives and Models for Understanding

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The use of manipulatives and models in mathematics classrooms is widely accepted as an important support for students who are grappling with mathematical concepts in elementary education. The National Council of Teachers of Mathematics (NCTM, 2000) standards, for one, advocates for the use of manipulatives and models throughout Kindergarten to Grade 12. In a recent publication, the NCTM (2010) notes that "hands-on and virtual manipulatives are powerful tools for representing mathematical concepts, thereby adding to a toolbox of representations from which students can communicate their mathematical understandings" (p. 21). The Ontario curriculum also encourages the use of manipulatives: "Students should be encouraged to select and use concrete learning tools to make models of mathematical ideas" (Ontario Ministry of Education [OME], 2005, p. 15). A critical component of the use of models is the inclusion of the associated mathematical reasoning; the models alone are insufficient, so the purpose of this article is not to advocate for the use of manipulatives, but to look critically at how models and manipulatives are used in the classroom.

I have been teaching for 18 years, and as I reflect on my evolution as a mathematics teacher, I recognize many misunderstandings I had about how students learn mathematics and my role for guiding them on their mathematical journeys. One mistake I made in my mathematics classroom is the focus of this column:

something I have termed “procedures with a picture,” and it is something I have observed unintentionally being replicated in many other classrooms to this day. “Procedures with a picture” is tantamount to teaching the students a procedure by directly instructing students on *how* to use the manipulatives and models. In other words, the models and manipulatives are being used without the development of the related reasoning. And importantly, it is this mathematical reasoning that is the actual point of the models being used in the first place. Although a model provides a “picture” for students to see, without delving into the reasoning, it is not an effective way for students to deeply understand the mathematical concepts. In this column, I want to focus on an alternative way to use manipulatives where teachers set up a context that allows students to explore the mathematical concepts. The manipulatives are used to construct the models themselves, thereby developing understanding of the reasoning process (not just developing an algorithm of “steps” for using the materials). This alternative allows students to make connections between the mathematics and how they see it, by using the manipulatives. In this column, I give three examples in different content areas, using the method of directly instructing students, and then provide an improved way to use manipulatives and models with reasoning to construct the understandings. In the discussion, I summarize why I have highlighted the difference between the two approaches.

Example 1: Addition with Regrouping

For this example, the problem 36 + 19 is used to illustrate and contrast two different ways that manipulatives can be used to model this operation. The first way provides an example I personally used as direct instruction for teaching addition with base-10 blocks, and the second gives an example that allows students to construct understanding of the mathematics through a focus on reasoning.

Procedure with a Picture

This is one “procedure with a picture” that I used faithfully for many years in my own classroom. To teach addition and subtraction with regrouping, I used the “magic carpet” method with base-10 blocks. I learned this method at a workshop, and I thought it was an effective way to teach students how to use base-10 blocks for addition and subtraction. It also strongly aligned with ideas that I had learned during my experiences in my education program. I felt like I had found “the” method for teaching addition and subtraction in elementary school. In order to use this method, students need the “magic carpets” (which are cards that look like flying carpets and have single digits 0–9 on them), base-10 blocks, and the template.

When working with students, I first pulled out rods and cubes and explained that when you have ten cubes, they are the same as one ten rod. Then I showed the magic carpets. The magic carpets consisted of the numbers 0–9. An explanation was then given that numbers bigger than 9 could not fit on a carpet, and the blocks would have to be “regrouped” to the next column so that they could sit on the carpet for the answer when adding. As a note, the carpets are used differently in subtraction, but for simplicity, I am only focusing on addition in this column.

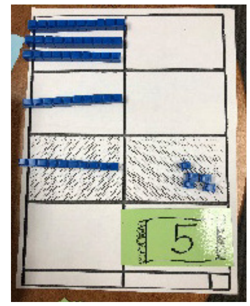

Figure 1: *Completed set up for 36 + 19*

To use the process, I gave students a template, a baggie with the magic carpets, and rods and cubes from the base-10 block kit. I demonstrated how to use the template, carpets, and blocks. In the first set of boxes, students would model 36 as three rods and six cubes; in the second set row of boxes on the template, the 19 as one rod and nine cubes (see Figure 1 for completed template). When completing the addition, students pull all the base-10 blocks together into the hatched area as an area to work out the sum. The hatched area of the template cues students to pause, think, and make sure everything is ready to determine the final sum. Since there are 6 + 9 cubes, the result is 15, but 10 of the cubes “fly” to the tens column and become a rod. The carpet of “5” could then be placed in the answer (see Figure 2). Since there is no carpet with the number 15, this cues students that something needs to be done with the 10 additional cubes in order to find the solution. The rods show 3 + 1, but then there is the additional rod in the hatched area that combines to give 5 rods. The resulting sum is 55.

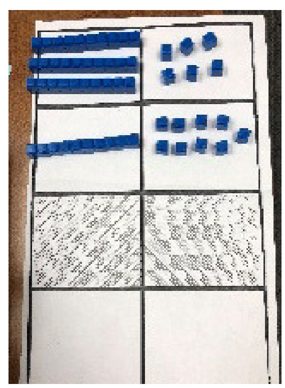
Figure 2: *Mat showing adding the ones*

Using the Manipulatives to Construct Understanding

Consider starting by providing a context, such as “Terry has 36 marbles, and Steve has 19. How many marbles are there in total?” Now students use the blocks to model the situation and make some observations themselves about what happened. Many methods for modelling could be used with the base-10 blocks that allows students to construct an understanding of the mathematics. One method is to use 36 ones and 19 ones and then to count the total cubes. This is going to become a painful process as the numbers get larger, but this can be a starting point to introduce using groups of 10 to help students count the total number in the answer. The use of rods to represent the tens can then naturally come from the discussions based on the experience with the models. For students using rods to build the question, they may combine the tens and ones to get 4 tens and 15 ones as the solution. This leads to answers like 40 and 15 or "415" being presented by students, but now there is a space to discuss what is happening in order to get a mathematically correct answer. This allows for students to reason through the model, and moving the extra ones to the tens column becomes something that comes out of the discussion and a problem the students are having as they are working with the mathematics. Now they are creating their own understandings of numbers instead of memorizing a formula that requires them to use the blocks in the way the teacher told them.

Example 2: Fraction Multiplication

An area model is a wonderful model to use when looking at multiplying fractions. The example of $\frac{2}{3} \times \frac{3}{4}$ will be used to illustrate the two different ways to teach the model for fraction multiplication, starting with the direct instruction and then a way to construct an understanding of multiplication that leads to an understanding of the model.

Procedure with a Picture

Similar to the previous example, this would involve a teacher instructing students on *how* to create the model for the procedure. The process could go something like this: In order to multiply two fractions, first draw a rectangle that is divided into quarters and shade three of them (see Figure 3). Second, in the other direction, divide the rectangle into thirds and shade two of them as shown by the hatch shading in Figure 4. The part that is both shaded and contains the hatch pattern gives the numerator, and the total number of little rectangles gives the denominator. In this case, the answer would be $\frac{6}{12}$. Correctly following this procedure would always lead to an accurate answer for any fraction multiplication question using the model.

Figure 3: *Image of* $\frac{3}{4}$

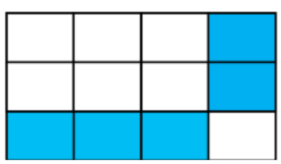

Figure 4: *Area model*

Using the Manipulatives to Construct Understanding

Instead of focusing on the model and the procedure, first start by providing manipulatives and talk about what multiplication means. If we interpret multiplication as "of," we can change the question to "What is $\frac{2}{3}$ of $\frac{3}{4}$?" Better still, we could provide a word problem that would lead students to make this conclusion for themselves: "Aaron's dad shovelled one-quarter of the driveway before lunch. After lunch, Aaron heads out and shovels two-thirds of the remaining snow before snow starts falling again. How much of the driveway was Aaron able to shovel before it began to snow?" Using fraction bars, a student could now illustrate the amount of snow still on the driveway ($\frac{3}{4}$) as shown in Figure 5. Now if Aaron shovels two-thirds of what is left, it becomes intuitive to find two of the three pieces left and note that Aaron shoveled $\frac{2}{4}$ of the total driveway. By looking at the fraction pieces as a set of *three* one-quarter pieces, it is easier to identify $\frac{2}{3}$ of the amount. Students can reason through the model by focusing on the meaning of fractions, as well as the context in order to solve the word problem. There actually is no reason to create an area model at all from the question, but instead, use reasoning and a model to determine the answer.

Since the area model does lead nicely into the standard procedure itself (see previous Mb4T column: Kajander, 2013), a teacher could set up the need for a new model. A problem like the following could help students create the area model for themselves: "Tara has $\frac{2}{5}$ of a cake left after her party. She decides to send $\frac{1}{3}$ of what is left home with a friend of hers. How much of the cake does Tara give to her friend?" A student could still model the $\frac{2}{5}$ with the given fraction manipulatives; however, the fraction pieces cannot be used to make the two pieces of fifths into three equal pieces. A teacher has now created a need for a new model, and paper can be helpful at this point. By having a sheet of paper cut into fifths, you can now fold or cut each strip into thirds in order

Figure 5: *Showing* $\frac{2}{3}$ *of* $\frac{3}{4}$ *with the rectangle*

Figure 6: $\frac{1}{3}$ *of* $\frac{2}{5}$ *with paper*

to determine the solution (see Figure 6). Some discussion will likely ensue around determining the size of each of the rectangles, since it is likely that students will at first only cut the $\frac{2}{5}$ into thirds. The entire sheet of paper (all the fifths) would need to be folded to help determine the size of each of the squares (as in Figure 6). These conversations serve the reasoning and mathematics that is being explored, instead of fitting the procedure being taught. An image like Figure 7 could now be explored, where $\frac{2}{5}$ is coloured in and then it is cut into thirds, with $\frac{1}{3}$ of what is coloured in being the answer of $\frac{2}{15}$ as shown by the hatched coloured portion.

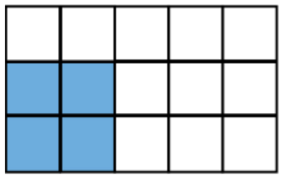

Figure 7: $\frac{1}{3}$ *of* $\frac{2}{5}$ *model*

Example 3: Algebra

Algebra is another topic where there is a tendency to rely on a "procedure with a picture" in an attempt to simplify the math for students, thereby removing the students reasoning through the solution. In this case, the pattern in Figure 8 will be used as an illustration to look at the difference between creating a procedure for using the model and a way to construct an understanding of algebra, using the students' ideas.

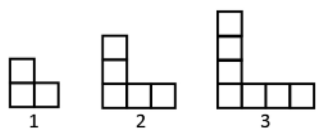

Figure 8: *Linear pattern example*

Procedure with a Picture

There are two ways this can be done as a procedure with a picture, either by first setting up the image in Figure 9 with different shading, or by asking students to always do this for themselves. The idea is to have students first figure out what stays the same and give this one colour (the shaded part of the image). The part that grows is known as the multiplier, so you determine the multiplier by how many sets of the picture number are represented by the remaining squares. For example, in picture 3 in Figure 9, there are six more squares, and this represents two groups of three (and it is the same for all previous pictures that you can multiply each picture by two to get the extra squares). Another way to look at it is to notice that each bar (represented by the hatch marks and the dots) is equal to the picture number, and there are two of them. The algebraic expression takes the form of $2n + 1$ (the amount multiplied by the picture number, the picture number represented as "n," plus the constant). This procedure works for any linear algebraic pattern, with some accommodation necessary if you are removing a constant amount (i.e., $2n - 1$).

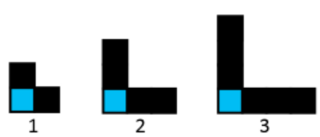

Figure 9: *Linear pattern procedure with a picture example*

Using the Manipulatives to Construct Understanding

To encourage reasoning about algebra, leave the pattern as it was in Figure 8, with all the squares the same colour. Now ask the question, "What do you notice?" Now there can be many answers, depending on the complexity of the pattern. In this simple example, students may say, "The 'L' always grows taller and wider," and "There is one square always in the corner and then both sides are the same." In the first quotation, the student is not seeing the square in the corner as a constant that does not change. Instead, the student sees two sides that are always growing. The expression could then be discussed as the vertical part of the "L" being one more than the picture number, and the horizontal bar being the same as the picture number, or $(n + 1) + n$. In the second statement, the student would see the same expression as the example in the "procedure with a picture" section. The two different ways lead to equivalent forms of the expression. By focusing on the reasoning of the pattern, all students access what they see in the pattern, and use their reasoning in order to create an algebraic expression. It is much easier to work with what a student sees in the pattern, rather than encouraging him or her to use the teacher's approach to patterns. Furthermore, allowing different forms of the expression leads naturally to discussions of simplifying expressions, showing how they are equal through the use of reasoning. Again, the reasoning and mathematical concepts are serving the model that students see, instead of just being given to them. (For a larger example, the article in a past issue of the *Ontario Mathematics Gazette* by Ann Kajander (2014) provides a detailed account of looking at patterning and algebra, using models and reasoning.)

Discussion

The examples shown advocate for allowing students to explore within a problem context, using manipulatives and reasoning. On the surface, the magic carpets seemed like a succinct way to "teach" regrouping to Grade 2 students. However, it was only slightly better than teaching them the procedure directly. What I had done was take an effective tool for students to discover concepts about addition and subtraction and turned it into a memorized procedure using the tool. What I did not understand at the time was why so many of my students were struggling: they were using manipulatives after all. The problem was not the manipulatives, but that the students could not remember all the "rules" that came with using them to get the correct answer. It is better to allow students to construct an understanding of addition by using the base-10 blocks through reasoning within a context. Students are then using the tool to support their understanding by reasoning through the mathematical concepts, rather than getting lost in all the "rules" for using the blocks.

It helps to look at manipulatives as a tool that can lead to foundational mathematical discoveries. As the NCTM (2000) notes, "students' understandings of mathematical ideas can be built throughout their school years if they actively engage in tasks and experiences designed to deepen and connect their knowledge" (p. 21). Looking at the line in the Ontario curriculum that speaks about the use of models, it becomes clear that "procedures with a picture" is not what is intended: "Students need to understand that making their own models is a powerful means of building understanding and explaining their thinking to others" (OME, 2005, p. 15). The difference between the two given examples in each content area supports the idea in the curriculum by allowing students to explore a carefully constructed context, using manipulatives. It may take longer for the procedure to be developed, but the mathematical understandings and reasoning becomes a part of the discussion instead of simply reiterating a prescribed procedure.

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▲ Report: Canadian Mathematics Education Study Group 2018 Annual Meeting

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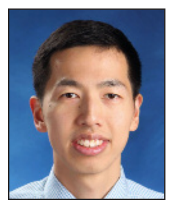
Ann Arden is a math teacher in the Ottawa-Carleton District School Board and is currently an instructional coach. She has also taught in the Faculty of Education at the University of Ottawa, and her son's teacher this year was a former student! Ann is very interested in assessment to improve learning, especially through the use of conversational and observational evidence. Ann is on Twitter as @annarden.



Michael Tang teaches mathematics and physics, as well as being a special education resource teacher in the Durham District School Board. Formerly, he was the math department head at Fraser Academy, an independent school in Vancouver for students with language-related learning disabilities such as dyslexia. He is passionate about helping students improve with problem solving, logical reasoning, and communication through mathematics engagement. Michael is on Twitter as @MichaelTang09.

Jimmy Pai is a mathematics teacher in the Ottawa-Carleton District School Board. He has presented locally and internationally on the topics of assessment and mathematics education. He strives to support students beyond what they believe they can do. He continues to crave discussions about how people learn to teach mathematics, as he sincerely believes that these discussions support personal journeys. He can be found on Twitter as @PaiMath.

This year, the Canadian Mathematics Education Study Group (CMESG) met in Squamish, British Columbia from June 1–5, 2018 for its 42nd annual meeting. A diverse group of 155 mathematicians, math educators, teachers, and graduate students discussed a variety of issues related to mathematics education in Canada. We were drawn to this meeting by the opportunity to work with others beyond our districts and province, and to gain national and international perspectives on mathematics education.

A unique feature of the CMESG meeting is the working-group format. All participants choose a group and spend 12 hours together over three days to gain an understanding of a current issue in math education. This makes the CMESG meeting a true conference—a place where conferring with each other and having conversations defines the majority of the programming.

This year's working groups focused on:

- confronting colonialism in mathematics and mathematics education
- playing with mathematics/learning mathematics through play
- considering "mathematics curricula for the 21st century" at the secondary level
- robotics in mathematics education
- relation, ritual, and romance: rethinking interest in mathematics learning

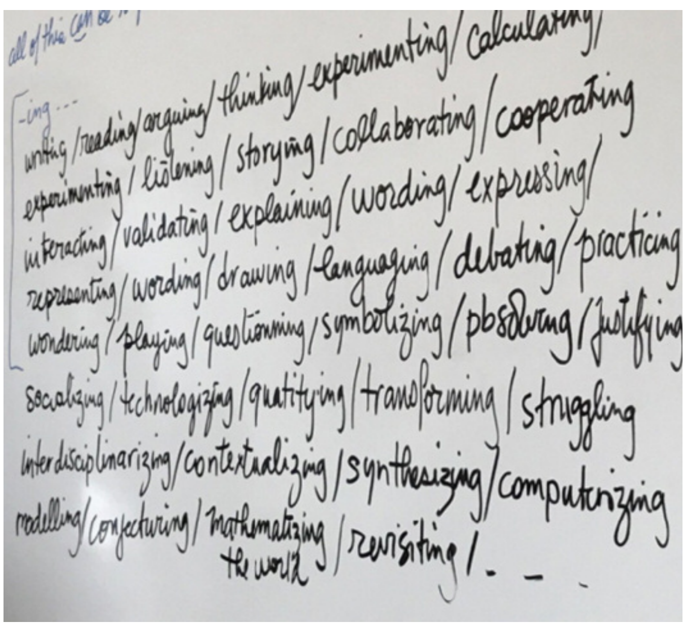


Figure 1: *A list of "-ing" words brainstormed by the 21st-century math curriculum working group*

Ann and Michael participated in the 21st-century math curriculum working group. Our group gravitated toward the idea that when developing a math curriculum, problem solving and thinking processes should be emphasized (Figure 1). We had the opportunity to take a deep dive into two math problems and consider what is important in teaching, learning, and assessment for current and future teachers and students. In addition, we discussed how teachers can be supported in using problems that effectively draw out mathematical processes.

Jimmy participated in a working group that focused on rethinking engagement of student interest. This was done by considering why and how concepts can be introduced through the structures and metaphors of ritual, romance, and relation. Participants worked with mental constructions of visual geometric and algebraic objects, videos of teachers working with students on patterns, and projective geometry. Ritual, for example, highlighted a sense of community that may be built in the classroom, and romance may be found in the mysteries offered by the concealing and revealing of information.

For more information about CMESG, teachers can find the proceedings, published annually, at **www.cmesg.org/past-proceedings**. These include summaries of the plenary talks, working groups, topic sessions, new PhD talks and poster, and interactive presentations from a gallery walk. Information about the group can be found at **www.cmesg.org**. Detailed program information, including the schedule, is usually posted in March for the June meeting. The conference rotates among Western, Eastern, and Central Canada. The next meeting, in June 2019, will take place at St. Francis Xavier University in Antigonish, Nova Scotia. ▲

▲ In the Middle: Math in the National Gallery of Canada?

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Carly Ziniuk teaches Grade 9 Mathematics, Grade 12 Data Management, and Advanced Placement Statistics at the Bishop Strachan School in Toronto, Ontario, Canada. She is very active in adopting real-life data to engage her students in solving problems.

Middle school visits to our nation's capital provide many opportunities to enhance History and Geography lessons. However, further opportunities await. A visit to the National Gallery can be a place to make connections between Canadian Art and Math. Even without the trip to Ottawa, your students can use the Gallery collection in your Math classes.

In the previous issue of the *Gazette*, I included fraction, area, and probability representations with the Guido Molinari painting *Vertical Horizontal Blancs*. Molinari was part of the Canadian group of artists known as Les Plasticiens. In the 1960s, Les Plasticiens artworks focused on geometric order and bright primary colours, and similar to the work of Piet Mondrian, sought to "objectify paintings, not paint objects." This phrase alone invites a mathematical discussion with teens. As Canada turned 100 years old, much of the well-received art of the period was of the form known as "geometric abstraction." The National Gallery of Canada has a large collection of Les Plasticiens, and also many other pieces inviting mathematical thinking.

Here are some additional places you might not be aware of that could engage your students and cover two curricular topics together! Many galleries include images from their collections online. These images can be projected on a SMART Board®, for example, and then you can draw your geometry lesson on top of a painting. These paintings can also be printed for the students to examine and dissect (in some cases, literally!); it is highly effective to ask a question of your class and have students consider multiple pieces in an artist's work or a set of paintings in a theme. A collection like the works of Escher, or metal box sculptures such as those by Rabinowitch and Delavalle, can motivate students to generalize a principle, such as tessellations or how areas change with a scaling factor, using specific examples.

In this column, you will find some paintings and sculptures that may motivate you to explore what the National Gallery site has to offer. The pieces are organized primarily by artist, but you may wish to take inspiration from the ideas and choose artists known by you or your students. Alternatively, you might consider what you can view personally with a combination math–art field trip.

M.C. Escher

Many middle school classrooms have captivating images on their walls by M.C. Escher. Did you know that M.C. Escher's oldest son, George, immigrated to Canada from the Netherlands? George Escher lives in Nova Scotia and donated many of his father's famous artworks to the National Gallery of Canada. The Gallery collection includes numerous letters between father and son, lithographs, diagrams, and writings about connections between his artwork and projective geometry.

Figure 1: *M.C. Escher (1957): Regular Division of a Plane II*

The National Gallery is proud of its collection of M.C. Escher works, and you can find teaching materials, including lesson plans relating Escher's art to the work of University of Toronto geometer, H.M.S. Coxeter. In particular, you can find the varied experimentation with tessellations in *Regular Division of the Plane*, a series of works from 1936–1957 (see Figure 1). Additional writing about the most recent Escher exhibits can be found at:

- Puzzles of Precision. M.C. Escher: The Mathemagician: **www.gallery.ca/magazine/your-collection/puzzles-of-precision-mc-escher-the-mathemagician**
- M.C. Escher – Mindscapes: **www.gallery.ca/escher/mindscapes/**

Claude Tousignant

A memorable painting in the National Gallery is the target-like piece *Chromatic Accelerator* (see Figure 2) by Claude Tousignant. Tousignant is a living Montreal artist, especially well known for his geometric abstraction work with stationary pieces simulating movement.

Chromatic Accelerator has over 40 brightly coloured rings, and the outside circle itself is over 2.4 m in diameter. Many viewers of the work see it as pulsing or rippling, even though it is a static two-dimensional image with no typical representations of the third dimension.

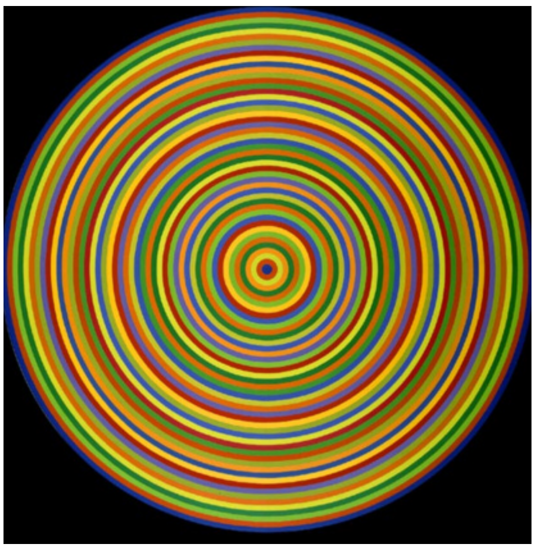


Figure 2: *Claude Tousignant (1967):* Chromatic Accelerator

Students need time to look at the work and to generate some comments on what the artist has done with the circles. Using the mathematical word "concentric" for this situation provides a bridge between the mathematical and artistic concept. Questions you and your students might consider with *Chromatic Accelerator* include the following:

1. It looks like the bands of colour in the work are equal distances apart. How does that affect the area? How does that affect the circumference?
2. What is the relationship between the circles as they expand outwards? Some suggestions to consider the relationship are:
    a. The circumference of the outer band is different from the circumference of the inner band.
    b. The area of the band can be determined by finding the area of the whole circle and subtracting the area of all the previous bands.

Students can break into groups. Each student can take a section of bands from the work so that each band was counted at least once and student data can be pooled (i.e., all brought together). Ideally, each student would have a copy of the work to do this.

3. Choose a set of five bands, with each band making one row on the following chart. What is the value of each characteristic?

Circle Number	Colour	Diameter	Inner Circumference	Outer Circumference

4. What do you notice about the relationship between the changing circumferences? How are they related?
5. Using the same set of five bands, can you determine the values for the following chart? Hint: Reduce your work by communicating with other students who examined circles near yours.

Circle Number	Colour	Diameter	Area of the Whole Circle	Area of Only That Band

6. What do you notice about the relationship between the changing areas? How are they related? Graph the circle number, the diameter, the circumference, and the area columns. What do you notice?

Jean-Marie Delavalle

The mathematically named *Black Volume* (see Figure 3) is an aluminum, acrylic-painted rectangular prism created by Jean-Marie Delavalle in 1990.

Figure 3: *Jean-Marie Delavalle (1990):* Black Volume

As a child, Delavalle emigrated from France to Canada in 1954. Recently, his related "Grandes Volumes" series was installed in the Christopher Cutts Gallery in Toronto, and you can see mages of these at **www.cuttsgallery.com/artists/jean-marie-delavalle/# /images**.

These questions consider both the original sculpture and a scale model, and then compare the surface area and the volume of the scale model to the original. The same examination can be done with many sculptures.

The National Gallery lists the dimensions of *Black Volume* as 182.7 cm x 61 cm x 23 cm. If you cannot see it in person, can the students physically present how big this would be? Could you store it in your classroom?

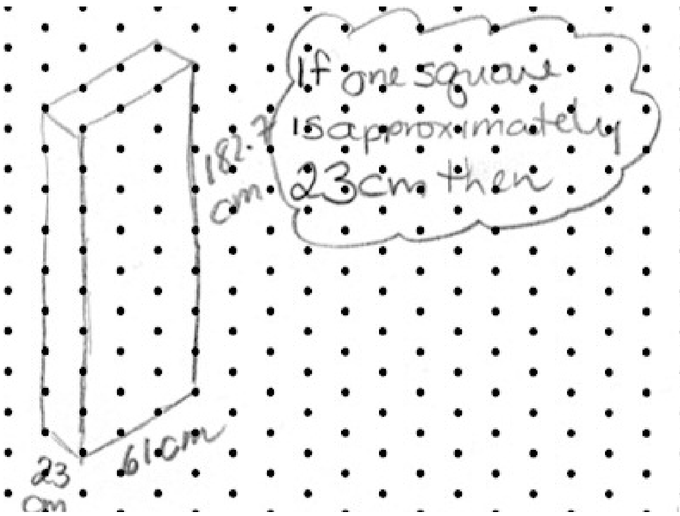


Figure 4: *Isometric Dot Paper for* Black Volume *with scale 1 unit : 23 cm*

1. Create a diagram using isometric dot paper (see Figure 4) or with 3-D imaging software (like SketchUp) in order to label the three dimensions carefully. Since you are unlikely to use a piece of paper almost 2 m tall, what would your scale need to be?
2. What is the volume of the work? What are the units used to measure the volume? What is the volume of your scaled piece? How are the volumes of the original and the scaled piece related?
3. What is the surface area of the piece? What are the units used to measure the surface area? What is the volume of your scaled piece? How are the volumes of the original and the scaled piece related?
4. Which face of *Black Volume* has the largest area? How much bigger is it than the other faces? Is it the same for the scale model? How do you know?
5. If you wanted to lay a long umbrella in the bottom of *Black Volume*, what is the longest it could be?
6. Estimate the angles formed between the umbrella and the sides of the work. Could you measure or calculate this using your scale model? Explain.
7. Draw the umbrella on a diagram of the circle for your scale model. What does your umbrella represent?
8. If you wanted to put the longest umbrella you could inside of *Black Volume*, how long could it be? (Why on Earth would you want an umbrella that long!—perhaps to protect the sculpture if it rains?)

Additional Pieces to Explore in the National Gallery Collection

Guido Molinari (1967): *Orange and Green Bi-Serial*

This piece, shown in Figure 5, invites fractions, patterning, and probability (much like the Molinari piece from the last issue).

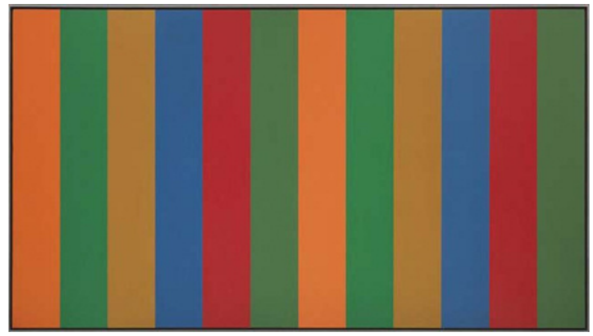


Figure 5: *Guido Molinari (1967):* Orange and Green Bi-Serial

John Elliott Woolford (1801–3): *Mosaic Floor in the Mosque of the Pharos*

This historical watercolour, in Figure 6, is based on the artist's travels to Egypt. You might consider discussions of symmetry, area, perimeter, characteristics of circles, and parallel lines.

Figure 6: *John Elliott Woolford (1801–3):* Mosaic Floor in Mosque of the Pharos

Kazuo Nakamura (1983): *Number Structure and Fractals*

Japanese-Canadian artist Kazuo Nakamura experienced internment as a teenager in British Columbia, and studied art at Central Technical School in Toronto. He founded the abstract group Painters Eleven, seeking to illustrate the "fundamental universal pattern in all art and nature." His grid paintings, like *Number Structure and Fractals*, in Figure 7, include Pascal and Sierpinski's triangles. Nakamura's public artwork can also be found in Toronto Pearson International Airport and Queen's Park.

Figure 7: *Kazuo Nakamura (1983):* Number Structure and Fractals

Clocks including Thoreau MacDonald (1929–30): *Tall Case Clock and Gathie Falk (1976): Picnic with Clock and Egg Cups*

These pieces are shown in Figures 8 and 9, respectively. There are many challenging questions with the changing angles and times on a clock. Did you ever consider using artworks with those? These sculptures are two appropriate examples.

Figure 8: *Thoreau MacDonald Clock (1929–30):* Tall Case Clock

Figure 9: *Gathie Falk (1976):* Picnic with Clock and Egg Cups

Royden Rabinowitch (1973): *Hollow Solid No. 8*

As with the Delavalle piece, this is a sculpture (see Figure 10), but with very different dimensions. Students can examine and contrast the two pieces in terms of their areas and volumes. The questions for *Black Volume* could be done for this work.

Figure 10: *Royden Rabinowitch (1973):* Hollow Solid No. 8

Additional Opportunities for Exploration

The Winnipeg Art Gallery, Glenbow Museum (in Calgary), and the aptly named Polygon Art Gallery (in Vancouver) have exhibited their math/geometrically themed collections previously. Unfortunately, their online curation is not as well organized or available publicly for teachers. The pieces chosen, however, were very similar to the ones presented here. The following locations explore math in art online extensively and have a vast set of teacher resources.

The Metropolitan Museum of Art (New York) – www.metmuseum.org

Check out their significant collection of art and history on Geometric Designs in Islamic Art, including lesson plans and curriculum planning, at:

- **www.metmuseum.org/learn/educators/lesson-plans/geometric-design-in-islamic-art**
- **www.metmuseum.org/learn/educators/curriculum-resources/islamic-art-and-geometric-design**

There is similar work done by the **Aga Khan Museum** (**www.agakhanmuseum.org**) in Toronto, but it is more elementary in its mathematical reach. The work at the Met is unparalleled. Pun intended.

The National Gallery of Art (Washington) – **www.nga.gov**

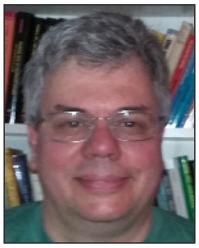
Their lesson plans for teachers are excellent and can be adapted to Canadian and international artworks. They also allow you to download high-resolution images. Resources for teachers can be found at:

- **www.nga.gov/education/teachers.html**
- **www.nga.gov/education/teachers/lessons-activities/6-8-lessons-activities.html**

Scroll down to the section on Art and Math—particularly check out the yummy look at *Thiebaud's Cakes*, which has a lesson plan including volume and surface area of cakes and the exploration of Fibonacci's sequence with the Calder mobiles.

Many local galleries have a docent or specially trained educator available to help you to identify pieces for your students to explore. The questions in this article may suggest what you are looking for mathematically. If time or location does not work for your class to visit in person, you can take an online field trip to extend and complement your lessons in geometry, measurement, probability, fractions, and number properties. Enjoy your artistic adventures! ▲

▲ What's the Problem? Playing with Pythagoras



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Welcome back, problem solvers. I hope you had a wonderful, restful summer, filled with time spent with family, friends, and some great math problems. Last time, I left you with the following problem:

> *A (non-degenerate) triangle has side lengths 10, 17, and* x*, where* x *is an integer. For which values of* x *is the triangle obtuse?*

This problem was inspired by question 23 from the 2017 Cayley Contest. The Cayley Contest is run by the Centre for Education for Mathematics and Computing (CEMC) at the University of Waterloo. Every year, the CEMC runs over a dozen contests in mathematics and computer science for students from elementary school through high school. They also have outreach programs, conduct school visits, provide resources for teachers and students, and much more. You can check out the CEMC's activities and resources on their website: **www.cemc.uwaterloo.ca**.

This problem begs to be explored with dynamic geometry. I created the sketch in Figure 1, using GeoGebra. In the sketch, segment AB has a fixed length of 17 units. The circle with centre A has a fixed radius of 10 units. In the top left corner, I have created a slider that will take on integer values from 1 to 30. I then created a second circle, with centre B and radius equal to the variable length, *n*, in the slider. The two circles intersect at point C, with AC being 10 units and BC being *n* units. From there, I measured the angles and we are set to explore!

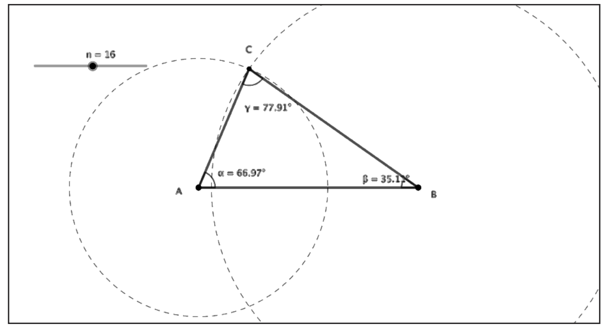


Figure 1: *An exploration with GeoGebra*

An interesting thing happens when we start playing with the slider; we see that not all values of *n* produce a triangle! If $n < 7 = 17 - 10$ or, $n > 27 = 17 + 10$, no triangle exists. In the case where $n = 7$ or $n = 27$, we get the limiting case, called a degenerate triangle, with angles of 0° *and* 180°. This leads to the triangle inequality which states that in a non-degenerate triangle, every side is shorter than the sum of the other two. The triangle inequality comes into play in the nice "12 sticks" problem discussed by Jeff Irvine in his article "A Focus on Student Understanding: Georg Pólya's Problem-Solving Heuristic" from the last issue of the *Gazette* (56(4), pp. 34–36).

As we play with our slider, we notice that when $7 < n$, $\angle ACB$ is obtuse. Similarly, when $19 < n < 27$, $\angle BAC$ is obtuse. So we have solved our problem. Yay!

The next question should be, "How could we have solved this problem without the aid of software?" The answer lies in one of the oldest and most well-known mathematical ideas, the Pythagorean theorem.

Recall that the Pythagorean theorem states that in a right-angle triangle, the square of the hypotenuse (the longest side, directly across from the right angle) is equal to the sum of the squares of the other two sides. This is usually presented in the form $c^2 = a^2 + b^2$, where *a* and *b* are the lengths of the legs (i.e., sides adjacent to the right angle), and c is the length of the hypotenuse.

There are many proofs to the Pythagorean theorem. The web page **www.cut-the-knot.org/pythagoras** provides 122 proofs, some with dynamic applets so you can explore the geometry involved. My go-to proof is #9 on the list. I usually give it to my Grade 9 students, and it can be shown by the diagrams in Figure 2.

Figure 2: *A proof without words of the Pythagorean theorem*

All you need to know is that the eight triangles are congruent right-angled triangles. All red line segments have length a, blue segments have length b, and green segments have length c. This proof is sometimes shown in the form of a puzzle, where you have a large square region and four small triangular pieces that fit in the region. If the triangles are placed around the perimeter, like the ones on the left in Figure 2, then you have a square (how do you know?) uncovered region in the centre. If the triangles are paired up and moved as in the diagram on the right, you have two square uncovered regions. Since the triangles cover the same area no matter where they are placed, the total area uncovered in the two diagrams must be equal, which leads to the Pythagorean theorem. The one on the right can also be used to show the algebraic identity $(a + b)^2 = a^2 + 2ab + b^2$, so you get more bang for your buck!

Now, suppose we have a right triangle with legs a, b and hypotenuse c, and we keep the lengths of a and b constant, while we allow the right angle to "shrink." What happens to the hypotenuse? It shrinks as well (and is no longer called a hypotenuse). What happens to the Pythagorean theorem? If we let C represent the "new" third side, we have, $< c$; hence, $C^2 < a^2 + b^2$. Similarly, if we allow the right angle to "grow," we have $C^2 > a^2 + b^2$.

Both of these ideas have also been known for a long time. They appear as propositions 12 and 13 from book 2 of *Euclid's Elements* (the Pythagorean theorem is proposition 47 from book 1). We can get a sense of how much larger the side opposite an obtuse angle will be by following the line of thought presented by Euclid.

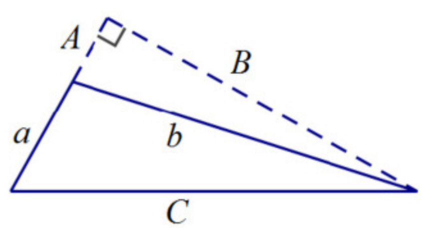


Figure 3: *Anticipating the Law of Cosines*

In Figure 3, we have an obtuse triangle with sides a, b, and C, where side C is opposite the obtuse angle. If we drop a perpendicular to side a, as shown in the diagram, and label the new segments A and B, then we have a right-angled triangle. We can now use the Pythagorean theorem to get $(a + A)^2 + B^2 = C^2$, which expands to give $a^2 + 2aA + A^2 + B^2 = C^2$.

Notice that we have a second smaller right triangle in Figure 3. Using the Pythagorean theorem a second time yields $A^2 + B^2 = b^2$.

Combining the last two results, we get $(a^2 + b^2) + 2aA = C^2$, which, if you use a little trigonometry, you can change into the Law of Cosines. Before the formal development of trigonometry, however, Euclid's statements were referring to areas of rectangles and squares produced using lengths associated to the original triangle. Figure 4 shows the "extra area" that would be needed to "complete" the Pythagorean theorem in this case. Can you "see" the terms from the algebraic expressions above in the diagram?

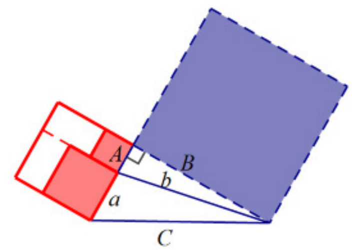


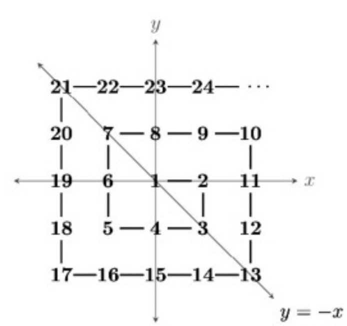
Figure 4: *The missing area*

Armed with our new knowledge, let's return to the original problem. In a triangle with sides 10, 17, and x, the longest side will be 17 or x, depending on the value of x. If 17 is the longest side and the triangle is obtuse, then we have $17^2 > 10^2 + x^2$, which simplifies to $x^2 < 189 < 196 = 14^2$, so $x < 14$. If we take into account the triangle inequality, we also have $x + 10 > 17$, so $x > 7$; thus, the triangle is obtuse and 17 is the longest side if $7 < x < 14$.

If we go through a similar argument where we let x be the longest side, we will get $19 < n < 27$, which I will leave for you to verify. There is still more to explore with the problem. You can investigate how changing the 10 and 17 alters the nature of the solutions. Have fun with your further investigations.

Now for your homework:

The rectangular spiral is constructed as follows. Starting at (0, 0), line segments of lengths 1, 1, 2, 2, 3, 3, 4, 4,… are drawn in a clockwise manner, as shown in the diagram. The integers from 1 to 1000 are placed, in increasing order, wherever the spiral passes through a point with integer coordinates (that is, 1 at (0, 0), 2 at (1, 0), 3 at (1, -1), and so on).



What is the sum of all of the positive integers from 1 to 1000, which are written at points on the line $y = -x$?

Until next time, happy problem solving! ▲

▲ Assessment Abby Addressing Stress

ASSESSMENT ABBY
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Ask Assessment Abby A³ is a regular column in the *OAME* Gazette, where teachers can share concerns and best practices about assessment, evaluation, and reporting of mathematics. Please send your questions to Ask Abby at **assessmentabby@oame.on.ca**.

Dear Assessment Abby,

I appreciate this column and the answers you have been providing since *Growing Success*. I wonder—who has been doing research on assessment and evaluation in mathematics, and would you provide some additional resources to continue my learning in this important area? Thank you.

In recent years, there has been an increase in mathematics education research that includes a focus on assessment and evaluation. Below is a list of resources that will help you get started. Much of the research is freely available online. You may be able to access some research through the Ontario College of Teachers library or by logging into a research database such as EBSCO Information Services or your university alumni library. Others may need to be purchased.

Keep math rich with students in mind,

Assessment Abby

Some Resources

Videos:

- EduGAINS – **www.edugains.ca/newsite/aer/index.html**
- The Learning Exchange – **thelearningexchange.ca/videos/assessment-in-a-knowledge-building-mathematical-classroom/**

Websites:

- Dr. Christine A. Suurtamm – **mathforum1314.files.wordpress.com/2014/02/dr-christine-suurtamm-research-paper-math-forum-2013.pdf**
- Math 4 the Nines – **www.math4thenines.ca/professional-learning.html#assessmentwiab**
- youcubed – **www.youcubed.org/resource/assessment-grading/**

Webcasts/Slide Shows:

- EduGAINS – **www.edugains.ca/newsite/math/webcasts.html**
- The Learning Exchange – **thelearningexchange.ca/wp-content/uploads/2015/10/Assessment-to-Promote-Mathematics-Learning.pdf**

Articles:

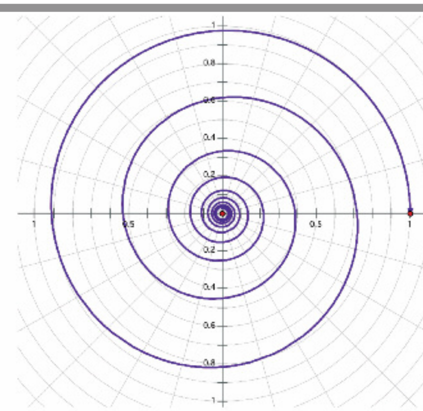
- National Council of Teachers of Mathematics (NCTM) – **www.nctm.org/News-and-Calendar/Messages-from-the-President/Archive/Cathy-Seeley/Assessing-to-Learn-and-Learning-to-Assess/**
- **www.nctm.org/Publications/Mathematics-Teaching-in-Middle-School/Blog/Promoting-Growth-Mindset-through-Assessment/**

e-Books:

- National Council of Teachers of Mathematics (NCTM) – **www.nctm.org/Store/Products/(eBook)-Annual-Perspectives-in-Math-Ed-2015—Assessment-to-Enhance-Learning-and-Teaching-(PDF-Downloads)/**
- Chapter 1 (free download) **www.nctm.org/Handlers/AttachmentHandler.ashx?attachmentID=EYMDZIYX%2FFE=**

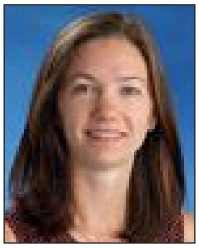
Other Resources

- Lim, L., & Colgan, L. (2004). Implementing multiple assessments in mathematics: An action research study of one teacher and his students. *Ontario Action Researcher, 7*(1.3), 1–6. Retrieved from **http://oar.nipissingu.ca/PDFS/V713.pdf**
- Suurtamm, C., & McDuff, A.R. (Eds.). (2015). *NCTM annual perspectives in mathematics education: Assessment to enhance teaching and learning.* National Council of Teachers of Mathematics. Reston, VA: NCTM.
- Suurtamm, C., Thompson, D.R., Kim, R.Y., Moreno, L.D., Sayac, N., et al. (2016). A*ssessment in mathematics education (ICME-13 Topical Surveys series).* Dordrecht, Netherlands: Springer. ▲



A relation or function depending on the coordinate system you choose.

▲ Ontario Mathematics Olympiad 2018



KERRI EVERSHED
EMAIL: kerri.evershed@peelsb.com

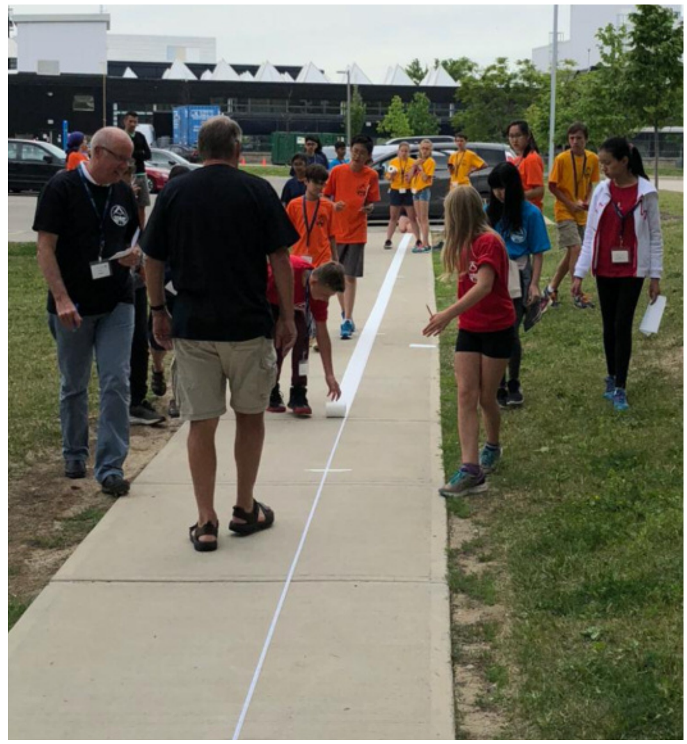
Kerri Evershed was the OMO Coordinator for CHAMP. She is an Instructional Coach with Peel District School Board, supporting schools in K–12 in all areas of education.

The OAME Ontario Mathematics Olympiad (OMO) is a fun, challenging mathematics competition for students in Grades 7 and 8. The students are required to solve questions based on the Grades 7 and 8 Ontario mathematics curricula. The CHAMP (Credit Humber Association for Mathematics Promotion) chapter hosted the 2018 OMO at Sheridan College, Oakville Campus, on June 8 and 9. Twenty-eight teams of four students, two from Grade 7 and two from Grade 8, from OAME chapters across Ontario, competed in a series of math challenges that used the theme of *Revolving around Mathematical Thinking*.

Figure 1: *Life-size Soma puzzle piece*

On Friday night, students and coaches had an exciting time team-building with other schools while building personal and life-size Soma puzzles (see Figure 1) and sculptures. This activity was facilitated by George Hart and Elisabeth Heathfield from Making Math Visible. Then, on Saturday, students competed in four different challenges: individuals, grade pairs, mixed pairs, and a team event.

In the pairs event, students used their Soma puzzle pieces to complete several challenges, including building animal structures and analyzing surface area and volume of the cubes.



The grade-pairs students solved exploding dots questions and various challenges using hex nuts.

Finally, the team event involved a full roll of toilet paper that was used for investigation and data collection about how it unrolls. The results were used to calculate where to place a partial roll so that it would stop just at a wall.

All mathletes worked collaboratively to demonstrate their understanding of the mathematical concepts, their ability to solve problems, and to communicate their knowledge. An amazing time was had by all—coaches, students, and event volunteers. We would like to congratulate all teams who participated and the top three placing teams:

- **First Place:** ISOMA Team 1
- **Second Place:** Derivative Dreamers from SAME
- **Third Place:** PRMA Team 1

First place: “ISOMA Team 1”

Second place: “Derivative Dreamers from SAME”

Third place: “PRMA Team 1”

Thank you to our sponsors: Spectrum, Sheridan College, and OAME.

A special thank you to all the team members who contributed to making OMO 2018 a success! ▲

▲ Motivating Students in Our Math Classrooms: An Example of Theory to Practice

JEFF IRVINE
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Jeff Irvine has been a secondary math teacher in Waterloo and Peel District School Boards, a secondary math department head, and a secondary vice-principal. Jeff has taught at three faculties of education and at Sheridan College. For several years, he was an Education Officer in the Curriculum and Assessment Policy Branch of the Ontario Ministry of Education, where his portfolio was Grades 7 to 12 mathematics for the Province of Ontario. Jeff is co-author or contributing author for 11 high school mathematics textbooks. He is currently an instructor of mathematics education at Brock University, where he is pursuing a PhD.

One of the nice things about education research is that it frequently reinforces what our professional teaching “gut” already told us. For example, we know that motivating and engaging our students is important to help them achieve their potential in math classes, and it’s no surprise that research says the same thing. There is abundant research available on this topic. For example, Jennifer Fredricks, Phyllis Blumenfeld, and Alison Paris (2004) published an extensive review of how engagement, one subdimension of motivation, impacted student achievement. Not surprisingly, students who were more engaged tended to have higher achievement. There is also a reciprocal relationship between motivation and achievement. Students who do well in math tend to like math more, and students who like math more tend to do better than those who do not. A 2014 EQAO study of 110 389 students found that students who did not meet the provincial standard in any of Grades 3, 6, and 9 had less positive attitudes about mathematics, had much less positive perceptions of their own ability in mathematics, liked math much less after Grade 3, and were less likely to connect new math concepts to previous knowledge.

But how exactly do we address this research on motivation in our classrooms? Bridging the gap between research and practice is not necessarily straightforward. I recently came across something that may help. It’s called the MUSIC model. Developed by Brett Jones, and based on theories in motivation, this model has been validated by research for both the elementary and secondary levels. It has also been research affirmed in a number of different countries, including the United States, China, and Iceland. It has nothing to do with music. The acronym MUSIC is just to help remember the five features of the model. I like it because it forces me to think about my classes from the students' points of view. The five elements of the model are:

- eMpowerment
- Usefulness
- Success
- Interest
- Caring

The elements are only in this order to facilitate the weak MUSIC acronym, not to indicate their relative importance. No one element is more important than another, but the more of the five elements that are addressed, the greater the impact on student motivation. As you read through this brief discussion of each element, think about the following question:

> When a new student enters your classroom, how long does it take for the student to decide that you are a "good" teacher, i.e., that you will meet his or her learning needs? [answer is at the end of the article]

eMpowerment: Research by Edward Deci and Richard Ryan, among others, showed that students are motivated when they perceive that they have some level of control over their environment and their lives. This does not occur much in school, where adults dictate most of what happens each day. So structuring your class to allow students some level of control some of the time yields great results in the area of motivation. This doesn't mean that students get to decide everything, or even most things. Providing some opportunities for students to control some aspect of their learning is not only possible, but also often easy to do. For example, we are doing more and more open-routed problems, where there is more than one valid way to get to the answer. [A while back, there was a teacher in my math department who insisted that students could only solve problems, using the techniques they had learned in his class. Any other solution was assigned a mark of zero. I asked him how he managed to have his students empty their heads of all other knowledge before entering his classroom.] Another good way to give students some control is through the use of choice (e.g., do 7 of 10 questions). There are lots of alternatives to provide students with choices, for example, menus, choice boards, tiered assignments, and others. One thing that really worked well for me in all my classes (Grades 9 through 12, Academic, Applied, College, Workplace) was cubing. Each student rolled a large six-sided die, and did as many homework questions as the die showed. For example, if the die came up 4, the student could choose to do any 4 questions for homework. The only caveat was that the student had to be able to demonstrate understanding of the math concept the next day. In my Applied Grade 9 class, students took this seriously. I often had students say, "I only had to do 2 questions last night, but I did 6, so I knew I understood the ideas."

Usefulness: We have been doing a lot better in this area recently, with problem-focused classes and real-world applications. My personal favourite is the reversing of traditional theory, followed by applications, to the applications-as-a-driver of theory. Usefulness to the student is not, "Trust me, you'll use this later." Usefulfulness to the student is: using it outside of school, using it in another class, someone they know using it (a relative, friend, etc.), or there are examples of someone using it in the real world. It can help if that someone in the real world is high profile to the student, like an athlete, astronaut, movie star, or celebrity. The last couple are tougher for math content, although shows like *The Big Bang Theory* help. Of course, not all math topics have real-world applications (factoring polynomials come to mind). It is better to not fake applications, but rather to be up front with students—that sometimes skills need to be developed, and this is one of those times.

Success: This is another area where teachers typically do a good job. Obviously, we want our students to succeed, and we automatically scaffold and structure our classes so that big tasks are broken into smaller subtasks on which students can be successful. This leads to bigger and bigger successes. Remember that our definition of success may not match the student's definition. In the first class of the semester, I used to ask students to estimate their desired final mark they wanted in my class. One student wrote down "40%." I asked him after class why his target was so low, and he responded that he wanted to do twice as well as he had the last time he took the course. This is an extreme example showing that, to a student, success is relative and personal. It is progress that we need to foster, not necessarily "meeting a teacher-imposed standard."

Interest: This is always a tough one. To paraphrase an old saying, "We can interest all of our students some of the time, and some of our students all of the time." However, interest is very personal. Traditionally, some math teachers try to relate content to sports, but some of our students don't like or care about sports. For example, some girls may not care about male sports, but are interested in female sports. The same goes for relating content to money. All students care about money to some degree, but it may not be interesting to them, just necessary. I've found a lot of success with allowing students to do Internet research on math topics, to discover something related to the math content that actually interests the student. I try to have them do this before we tackle the topic in class, so that they approach the content "pre-interested." For example, one assignment that I give is shown below:

Parabolas in Real Life

Part A:

- Do an Internet search for images of parabolas.
- Either print or bookmark 6 to 10 images to share with your group tomorrow.

Part B: In Your Group

- Select one image per person.
- Impose a grid and find an equation for your parabola.
- Find the coordinates of a point 2/3 of the height of the maximum or minimum of your parabola.

Caring: "Students don't care how much you know until they know how much you care." (modified by William Purkey from Theodore Roosevelt by replacing "people" with "students"). My friend, Sean Schat, does research in teacher care theory. He says that teachers need to be intentionally inviting so that students feel welcome in the classroom and that they are cared for by someone, hopefully the teacher. You can show caring in lots of different ways, but one way is to treat each student as you might want to be treated. In my class, simple things such as allowing students to go to the washroom without having to raise their hand and interrupt the class; or, not speaking when someone else (teacher or student) is speaking; and other similar classroom norms pay big dividends. If each student feels that your classroom is a safe and welcoming place where everyone is valued, he or she is more likely to come to class and participate. It's important to avoid value-laden word choices, like "trivial," "It's obvious," and my own personal favourite, "It is easy to see that…." I'm sure many of you can relate, since most of us heard these phrases repeatedly in university classes, where it was easy for the professor to see, but a total mystery to his or her students.

The MUSIC model gives us a way to bring research-affirmed ideas into our classes, without having to guess or to search for ways to implement them. By using MUSIC as a checklist for each lesson, we can address student motivation, without having to consider this goal as an add-on to all the other expectations we deal with.

Engaging and motivating students can be difficult. I had a large banner across the back of my classroom that said, "Mathematics Is Not a Spectator Sport." To truly learn, students need to be active participants in their own education—something that is definitely true in math class.

[Answer to the question about "good" teacher: Work by Peter DeWitt, and supported by additional research by Nalina Ambady across Kindergarten through university, says the student decides in the first *10 seconds* in the classroom. This has huge implications for us as teachers.]

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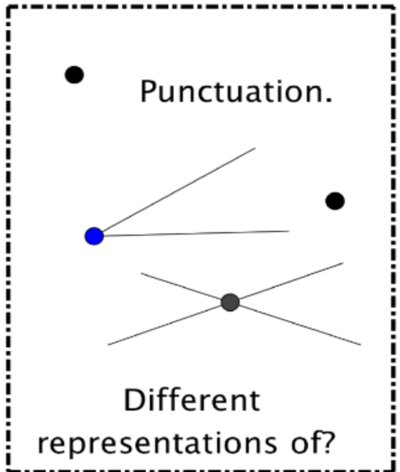
Jumble 2

Unscramble each word. Take the marked letters and solve the picture riddle.

O	T	D							
	☐								
D	I	R	E	O	P				
☐									
T	R	E	X	V	E				
			☐						
E	C	T	S	R	N	I	E	T	
☐									
A	N	T	E	I	D	R	C	O	O
						☐			



Solution: ☐☐☐☐☐

Stalactites hang from the ceiling

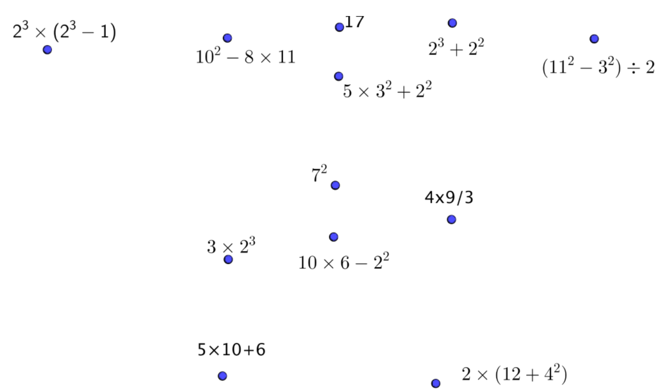
Instructions: This is a nomogram puzzle, where you have to determine which cells are filled in and which aren't. The values at the top of each column and beginning of each row show a series of numbers corresponding to the runs of consecutive boxes that are filled. For example, the second row says "3, 6," meaning somewhere in the row are three filled-in boxes, and after that are six filled in boxes.

There must be at least one empty box for each comma. Hence, there should be at least one empty box in between the three filled boxes and the six filled boxes.

	1	3, 1	8	10	1	7	8	4	2	6, 1	3, 2	1, 3
12												
3, 6												
3, 3, 2												
2, 3, 1												
2, 2, 1												
2, 2, 1												
2, 2												
2, 1												
1												
1, 1												
1, 2												
3												

Join the Dots

Connect dots with equal values. Decipher the "answer."



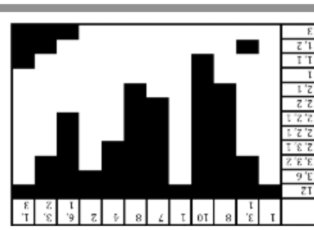
Solutions

Jumble 2

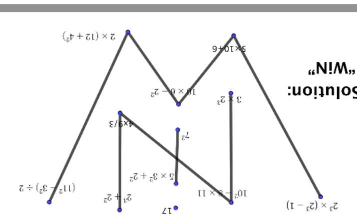
DIFFERENT REPRESENTATIONS OF?
1. DOT
3. PERIOD
2. VERTEX
2. INTERSECT
5. COORDINATE

Answer: POINT

Stalactites



Join the Dots



▲ 2018-2019 OAME Board

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