

Unit 7: Acute Triangle Trigonometry (5 days + 1 jazz day + 1 summative evaluation day)

BIG Ideas:

Students will:

- Solve acute triangles using the primary trigonometric ratios, sine law, and cosine law
- Solve real-world application problems requiring the use of the primary trigonometric ratios, sine law, and cosine law including 2-D problems involving 2 right triangles

DAY	Lesson Title & Description	2P	2D	Expectations		Teaching/Assessment Notes and Curriculum Sample Problems
1	Remember SOHCAHTOA? <ul style="list-style-type: none"> • Solve right angled triangle problems using SOHCAHTOA • Solve questions involving 2 right triangles (NO 3-D triangles) 	R	R	TF1.01 ✓	solve problems, including those that arise from real-world applications (e.g., surveying, navigation), by determining the measures of the sides and angles of right triangles using the primary trigonometric ratios;	
		N	N	TF1.02	solve problems involving two right triangles in <u>two dimensions</u>	
2*	Investigating Sine law <ul style="list-style-type: none"> • Investigate Sine Law using GSP • Solve problems involving Sine Law <p><i>Lesson Included</i></p>	N	R	TF1.03 ✓	verify, through investigation using technology (e.g., dynamic geometry software, spreadsheet), the sine law and the cosine law (e.g., compare, using dynamic geometry software, the ratios $\frac{a}{\sin A}$, $\frac{b}{\sin B}$, and $\frac{c}{\sin C}$, and in triangle ABC while dragging one of the vertices);	 (with GSP)
3*	Investigating Cosine Law <ul style="list-style-type: none"> • Investigate Cosine Law using GSP • Solve problems involving Cosine law • Discuss when to use Sine Law vs. Cosine Law vs. SOHCAHTOA <p><i>Lesson Included</i></p>	N	R	TF1.03 ✓	describe conditions that guide when it is appropriate to use the sine law or the cosine law, and use these laws to calculate sides and angles in acute triangles;	 (with GSP)
		N	N	TF1.04 ✓		

4,5	Solving Problems Involving Sine and Cosine Law <ul style="list-style-type: none"> • Tie up loose ends from days 2 & 3. • Discuss when to use Sine Law vs. Cosine Law vs. SOHCAHTOA • Do applications using sine and cosine law • Solve more questions involving 2 right triangles (NO 3-D triangles) • Emphasize choosing appropriate tools for the question. <p>Activity Included</p>	N	N	TF1.04 ✓	describe conditions that guide when it is appropriate to use the sine law or the cosine law, and use these laws to calculate sides and angles in acute triangles;	
		N	N	TF1.05 ☑	solve problems that require <u>the use of the sine law or the cosine law in acute triangles</u> , including problems arising from real-world applications (e.g., surveying; navigation; building construction).	
		N	N	TF1.02	solve problems involving two right triangles in <u>two dimensions</u>	
6	Review Day (Jazz Day)					
7	Summative Unit Evaluation					

NOTE: * Depending on Technology access, you may wish to do both investigations (Day 2 and 3) on one day and then applications on the next day.

Unit 7 : Day 2 : What do we do now?		Grade 11 U/C
Minds On: 15	Description/Learning Goals <ul style="list-style-type: none"> • Students will verify the sine law using dynamic geometry software • Students will apply the sine law to various triangles to determine unknown sides and angles 	Materials <ul style="list-style-type: none"> • BLM 7.2.1 • BLM 7.2.2 • Computers with GSP 4.0
Action: 40		
Consolidate:20		
Total=75 min		
Assessment Opportunities		
Minds On...	Groups of 2 → Think Pair Share Students will attempt to determine the length of a side and the measure of an angle of a non-right triangle using the tools they already have (SOH CAH TOA, Pythagorean theorem, angle sum of a triangle etc). Students will complete BLM 7.2.1. Once they have a solution, they will share their answer with their partner. Learning Skills(Teamwork/Initiative): Students work in groups of 2 or 3 to complete BLM 7.2.1.	
	Whole Class → Discussion Discuss the various methods used to solve the problem. Have some of the groups present their solutions to the class. Point out that most students used a multi-step approach. Wouldn't it be nice if we could find a way to solve a question like this in only one step?	
Action!	Groups of 2 → Investigation on GSP Students will investigate the properties of sine law. Students will complete BLM 7.2.2 and record their observations on the handout. Mathematical Process: Reasoning and Proving (Students will reason inductively by considering specific cases and identifying patterns.)	
Consolidate Debrief	Whole Class → Discussion Review of the properties discovered on GSP. Students will share their findings from the investigation. Whole Class → Lesson As a group, solve two problems using the sine law to solve for a side and solve two problems using sine law to solve for an angle.	
<i>Application Skill Drill</i>	Home Activity or Further Classroom Consolidation Select textbook questions that ask students to use Sine Law to solve for a side or an angle. You should also include some questions that involve triangles in contextual situations. NOTE: All triangles must be acute. For some sample questions, you may want to investigate the following website: http://www.gov.pe.ca/educ/docs/curriculum/521Aunit3.pdf	

7.2.2 Looking for a Shortcut – GSP Activity

Creating the Diagram

1. Open Geometer's Sketchpad™ (GSP).
2. Use the **straightedge tool** to construct an acute triangle.
3. Use the **text tool** to name the vertices **A**, **B**, and **C**.
4. Use the **text tool** to name the sides of the triangle **a**, **b**, and **c**. The side “a” must be opposite the angle “A”, the side “b” must be opposite the angle “B”, and the side “c” must be opposite the angle “C”.

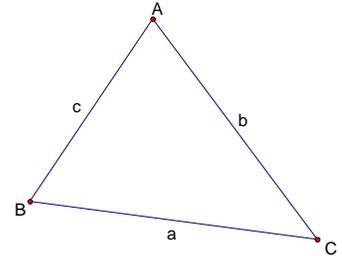


Figure 1

Your diagram should be similar to Figure 1. If it is not, please ask for assistance.

Instructions

1. Select the points **C**, **A** and **B** (in that order). Keeping these selected, from the **Measurement** menu, select **Angle**. The measurement for $\angle CAB$ should appear on your screen.
2. Repeat step 1 for points **A B C**, and **B C A**.
3. Select the side **a**. From the **Measurement** menu, select **Length**. The measurement for the length of side **a** should appear on your screen.
4. Repeat step 3 for sides **b** and **c**.

$m\angle CAB = 72^\circ$
 $m\angle ABC = 64^\circ$
 $m\angle BCA = 44^\circ$
 $a = 6.02 \text{ cm}$
 $b = 5.68 \text{ cm}$
 $c = 4.42 \text{ cm}$

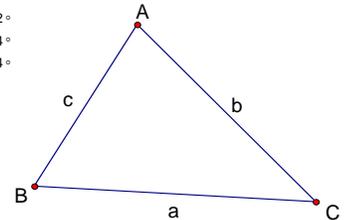


Figure 2

Note: Your diagram should be similar to Figure 2; however, your measurements do not have to match.

5. From the **Measurement** menu, select **Calculate**. A calculator will appear on the screen. Select **Functions**, and select **sine**. Now select $m\angle CAB$ on your sketch. It should appear on your calculator screen. Click on the bracket symbol “)” to close the sine function. Select **OK**.
6. Repeat step 5 for side **b** and $m\angle ABC$ and side **c** and $m\angle BCA$.

Part A

7. From the **Measurement** menu, select **Calculate**. Now set up the ratio $\frac{a}{\sin(\angle CAB)}$ by first clicking on the side **a**, and then click on the “÷” key, and finally selecting $\sin(m\angle CAB)$ on the sketch. Select **OK**. The ratio should appear on your sketch.
8. Repeat step 7 for side **b** and $m\angle ABC$ and side **c** and $m\angle BCA$.
9. Highlight the three ratios and from the **Graph** menu, select **Tabulate**.
10. Drag the corners of your triangle to create a different triangle, and double click on the table to record these new measurements. Record these numbers in the table at the top of the next page. Repeat until you have data for 4 different triangles. Answer the questions below the table.

7.2.2 Looking for a Shortcut (continued)

Triangle	$\frac{a}{\sin \angle CAB}$	$\frac{b}{\sin \angle ABC}$	$\frac{c}{\sin \angle BCA}$
1			
2			
3			
4			

What do you notice about all the ratios for a given triangle? _____

Write a mathematical expression that summarizes the relationship between these ratios for a given triangle.

Part B

11. Now you are going to calculate three new ratios. From the **Measurement** menu, select **Calculate**.

Now set up the ratio $\frac{\sin(\angle CAB)}{a}$ by first clicking on $\sin(m\angle CAB)$, and then click on the “÷” key, and finally selecting the length of side **a** on the sketchpad. Select **OK**. The ratio should appear on your sketch.

12. Repeat the step 7 for **m∠ABC** and side **b** and **m∠BCA** and side **c**.

13. Enter these numbers in the first line of the second chart below.

14. Highlight the three ratios and from the **Graph** menu, select **Tabulate**.

15. Drag the corners of your triangle to create a different triangle, and double click on the table to record these new measurements. Record these new numbers in the table below. Repeat until you have data for 4 different triangles. Answer the questions below the table.

Triangle	$\frac{\sin(\angle CAB)}{a}$	$\frac{\sin(\angle ABC)}{b}$	$\frac{\sin(\angle BCA)}{c}$
1			
2			
3			
4			

What do you notice about all the ratios for a given triangle? _____

Write a mathematical expression that summarizes the relationship between these ratios for a given triangle.

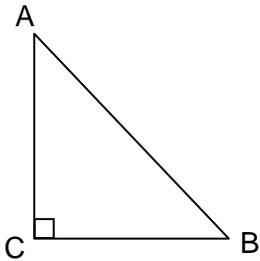
Unit 7 : Day 3 : Cos I said so		Grade 11 U/C
Minds On: 15	Description/Learning Goals <ul style="list-style-type: none"> • Students will verify the cosine law using dynamic geometry software • Students will apply the cosine law to various triangles to determine unknown sides and angles 	Materials <ul style="list-style-type: none"> • BLM 7.3.1 • BLM 7.3.2 • Computers with GSP 4.0
Action: 40		
Consolidate:20		
Total=75 min		
Assessment Opportunities		
Minds On...	Pairs → Think Pair Share Students will attempt to determine the length of a side and the measure of an angle for a non-right triangle using the tools they already have (SOH CAH TOA, Pythagorean theorem, angle sum of a triangle, Sine Law etc). Students will complete BLM 7.3.1. Once they have a solution, they will share their answer with their partner. Learning Skills(Teamwork/Initiative): Students work in groups of 2 or 3 to complete BLM 7.3.1.	
	Whole Class → Discussion Discuss the various methods used to solve the problem. Have some of the groups present their solutions to the class. Point out that most students used a multi-step approach. Wouldn't it be nice if we could find a way to solve a question like this in only one step?	
Action!	Pairs → Investigation on GSP Students will investigate the cosine law. Students will complete BLM 7.3.2 and record their observations on the handout. Mathematical Process: Reasoning and Proving (Students will reason inductively by considering specific cases and identifying patterns.)	
Consolidate Debrief	Whole Class → Discussion Review of the properties discovered on GSP. Students will share their findings from the investigation. Whole Class → Lesson As a group, solve two problems using the cosine law to solve for a side and solve two problems using cosine law to solve for an angle.	
<i>Application Skill Drill</i>	Home Activity or Further Classroom Consolidation Select textbook questions that ask students to use Cosine Law to solve for a side or an angle. You should also include some questions that involve triangles in contextual situation. NOTE: All triangles must be acute. For some sample questions, you may want to investigate the following website: http://www.gov.pe.ca/educ/docs/curriculum/521Aunit3.pdf	

7.3.2 Cos I said so! – GSP Activity

First, let's revisit the Pythagorean Theorem. In the space below, state the Pythagorean Theorem in your own words. Include a labelled diagram.

What condition(s) must be met before this theorem can be used to calculate the length of a side of a triangle?

In triangle ABC below, $\angle C$ is a right angle. What happens to the length of side c as the measure of $\angle C$ decreases?



Now, explore - gather some data!

1. Open The Geometer's Sketchpad™ .
2. Use the Segment tool to construct an acute triangle.
3. Use the Label tool to name the vertices A, B, and C.
4. Also use the labelling tool to name the corresponding (opposite) sides a , b , and c .
5. Measure $\angle ACB$ (often called simply $\angle C$).
6. Measure the lengths of sides a , b , and c .
7. Select point C. Drag point C until $\angle ACB$ is 90° .

7.3.2 Cos I said so! (continued)

8. What is the Pythagorean relationship for this triangle? Write the equation that you would use to calculate the length of the hypotenuse.

Model - Set up the Pythagorean relationship on the *Sketchpad*

9. Next, you will use *Sketchpad* to calculate the value of $a^2 + b^2$.
- From the **Measure** menu, choose **Calculate**.
 - When the calculator appears, click on length of side a , select \wedge , 2, and $+$ on the calculator.
 - Then click on the length of side b and select \wedge and 2 on the calculator.
 - Click on OK.
 - *Sketchpad* should show the sum of the squares of the two shorter sides of the triangle.
 - Move this calculation to a clear spot on your sketch.
10. Next you will have *Sketchpad* calculate the value of c^2 .
- From the **Measure** menu, choose **Calculate**.
 - Click on the length of side c , select \wedge and 2 on the calculator and click on OK.
 - *Sketchpad* should now show the value of c^2 .
 - Move this value to a spot underneath the calculation for $a^2 + b^2$.
11. Is $a^2 + b^2 = c^2$? If not, are they close? Why might they be off by a little bit?
12. What happens if you move vertex C further away from line segment AB?
13. Does the Pythagorean theorem property still apply to this triangle? Why not?
14. Now we need to calculate the expression $a^2 + b^2 - c^2$ (to determine how much the Pythagorean theorem is off by).
- From the **Measure** menu choose **Calculate** and fill in the operations. (Refer to steps 9 and 10 for help.)
 - Place this expression by itself in a clear spot on your sketch.

7.3.2 Cos I said so! (continued)

Manipulate/transform the geometric model and record changes to the numeric model

15. Move point C to five other locations and record the data in the table below. Make sure one of your triangles has $\angle ACB = 90^\circ$, and make sure $\angle ACB$ is never more than 90° .

Observations:

Triangle	a	b	c	$\angle ACB$ (or $\angle C$)	$a^2 + b^2$	c^2	Missing Part ($a^2 + b^2 - c^2$)
1							
2							
3							
4							
5							

16. Construct the expression $2ab \cos(m\angle ABC)$. Here's how:
- From the **Measure** menu, select **Calculate** and input $2 * \text{length}(a) * \text{length}(b) * \cos (m \angle ACB)$ and then choose **okay**. Note: **cos** is found in the **functions** menu.
 - Place this near your expression for $a^2 + b^2 - c^2$ from above.

Infer/conclude

17. What do you notice about this new value and the value of $a^2 + b^2 - c^2$? Are they the same? Do you think it was just a lucky guess?

19. Based on your observations, does the Pythagorean relationship apply to acute triangles?

20. What modification would be needed to make the Pythagorean theorem work for acute triangles? (What expression do we need to include on the right side of the equation $c^2 = a^2 + b^2$ so that it works for acute triangles?)

$$c^2 = a^2 + b^2 \underline{\hspace{2cm}}$$

7.4.1 Whose Law is it Anyway? (Teacher Notes)

This is intended as a Consolidation Activity.

Preparation:

1. Make one copy of BLM 7.4.4 on cardstock (or thick paper of some sort) for each student and cut the cards out. If you have time, you may wish to laminate these cards for future use.
2. Make one copy of BLM 7.4.2 for each student
3. Make overheads of BLM 7.4.3 and use large post-its to cover each triangle so that they can be 'revealed' during the activity. (You may wish to enlarge the triangles if you have a poor overhead projector)
4. BLM 7.4.3 (student version) can be copied for each student to complete as homework after the activities.

Setting the scene:

Your classroom is now a Court Room and each of the triangles is a defendant. The students are the jurors who will be making their verdict using the cards.

1. Hand out the cards to each student as well as BLM 7.4.2
2. Show one triangle on the overhead projector; instruct students to record the given information on the chart on BLM 7.4.2.
3. Students will then choose a card to declare their verdict for the defending triangle. Once everyone has chosen ask the students to simultaneously raise their cards. To ensure that each triangle gets a fair trial, students should show their verdict without looking at other student's.
4. Ask individual students to explain their verdict to the class and allow others to change/correct their verdict.
5. Once a unanimous (and correct) verdict is made have students complete the last cell of the chart. (Make sure students only write down the part of the formula required to solve for the unknown.)
6. Repeat the above process for all triangles.
7. They can now solve for the unknowns in each triangle as home practice.

7.4.2 Whose Law Is It Anyway?

- Write the Sine Law and the Cosine Law for $\triangle ABC$.
- Decide whether the unknown in each defending triangle can be determined using the Sine Law, Cosine Law, SOHCAHTOA, Sum of Interior Angles, Pythagorean Theorem or Insufficient Evidence. Complete the chart to discover the general conditions that guide the use of each.

Name of \triangle	Find	Given	Given	Given	State the law required and the formula, using the correct letters.
$\triangle ABC$	b				
$\triangle PQR$	p				
$\triangle DEF$	$\angle F$				
$\triangle TUV$	u				
$\triangle XYZ$	y				
$\triangle GHJ$	h				
$\triangle NSW$	$\angle N$				
$\triangle KLM$	$\angle K$				

- Describe the conditions that guide when it is appropriate to use the Sine Law or the Cosine Law.

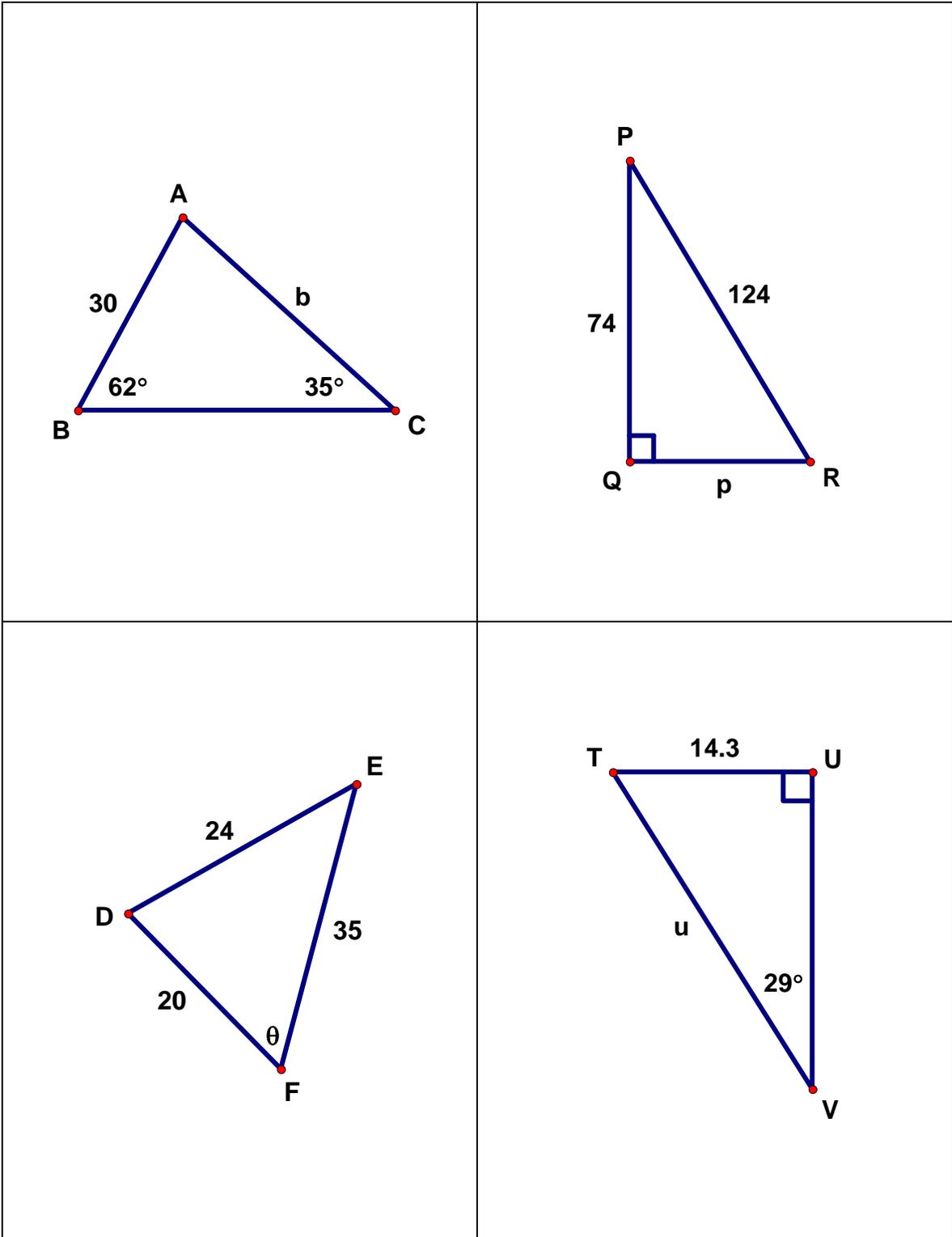
Sine Law is used to find
a **Side** when given –

an **Angle** when given –

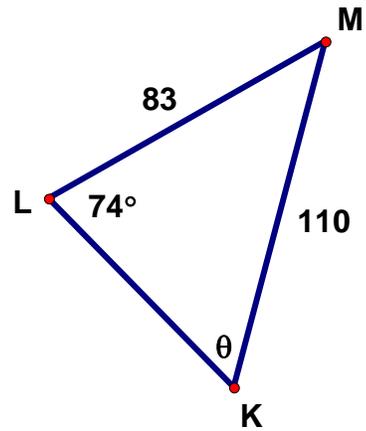
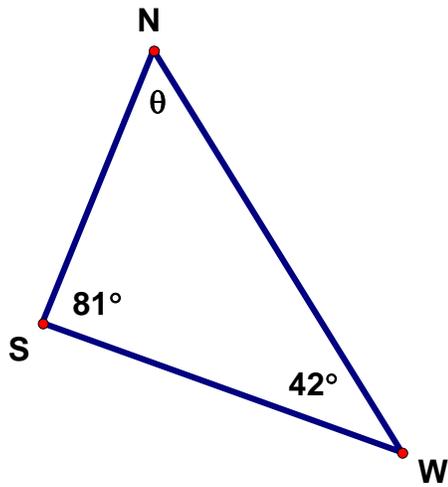
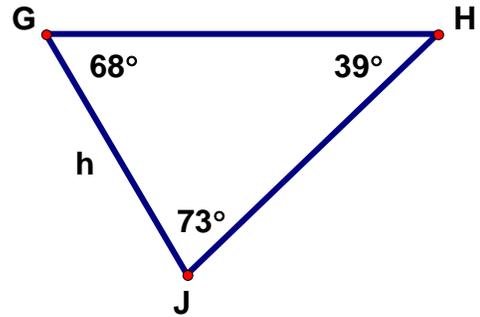
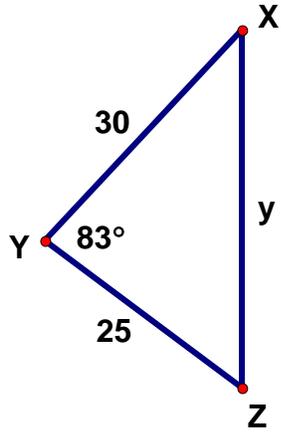
Cosine Law is used to find
a **Side** when given –

an **Angle** when given –

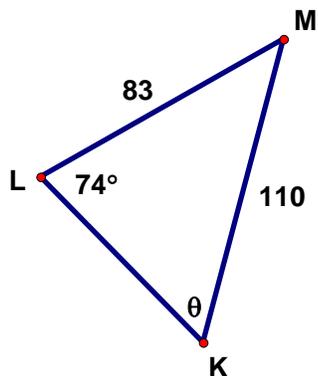
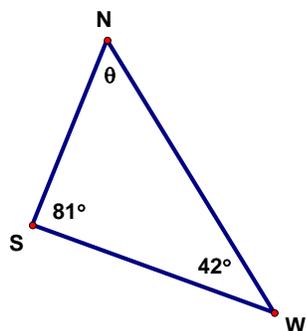
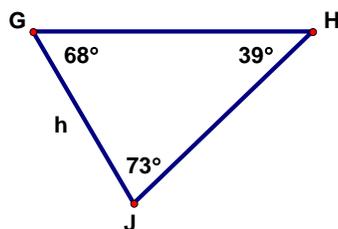
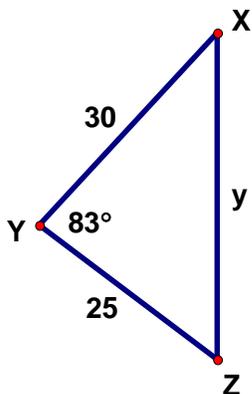
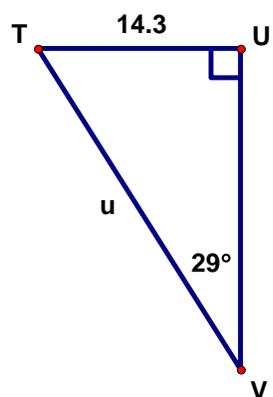
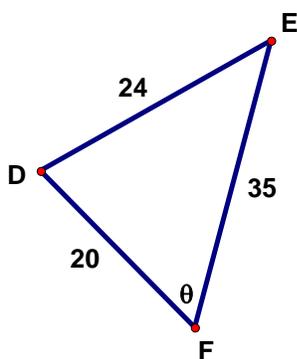
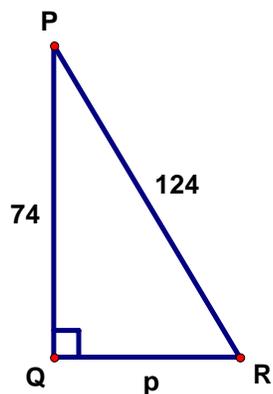
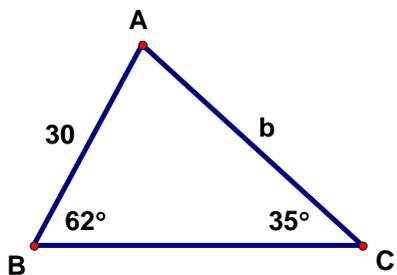
7.4.3 Whose Law is it Anyway? Defending Triangles



7.4.3 Whose Law is it Anyway? Defending Triangles (Continued)



7.4.3 Whose Law is it Anyway? Defending Triangles (Student Version)



7.4.4 Whose Law is it Anyway? - Voting Cards

**Sine
Law**

**Cosine
Law**

SOH CAH TOA

**sum of
interior Angles**

**Pythagorean
Theorem**

**Insufficient
Evidence
to Convict**