MBF 3C Unit 2 – Trigonometry – Outline

Day	Lesson Title	Specific Expectations
1	Review Trigonometry – Solving for Sides	Review Gr. 10
2	Review Trigonometry – Solving for Angles	Review Gr. 10
3	Trigonometry in the Real World	C2.1
4	Sine Law	C2.2
5	Cosine Law	C2.3
6	Choosing between Sine and Cosine Law	C2.3
7	Real World Problems	C2.4
8	More Real World Problems	C2.4
9	Review Day	
10	Test Day	
TOTAL DAYS:		10

C2.1 – solve problems, including those that arise from real-world applications (e.g., surveying, navigation), by determining the measures of the sides and angles of right triangles using the primary trigonometric ratios;

C2.2 – verify, through investigation using technology (e.g., dynamic geometry software, spreadsheet), the sine law and the cosine law (e.g., compare, using dynamic geometry software, the ratios a/sin A , b/sin B , and c in triangle ABC while dragging one c/sin C of the vertices);

C2.3 – describe conditions that guide when it is appropriate to use the sine law or the cosine law, and use these laws to calculate sides and angles in acute triangles;

C2.4 – solve problems that arise from real-world applications involving metric and imperial measurements and that require the use of the sine law or the cosine law in acute triangles.

Unit 2 Day 1: Trigonometry – Finding side length

MBF 3C

Description

This lesson reviews Trigonometry Material from the Grade 10 course – specifically solving sides of triangles using the three trigonometric ratios.

Materials

BLM2.1.1
Scientific calculator

Assessment Opportunities

Minds On…

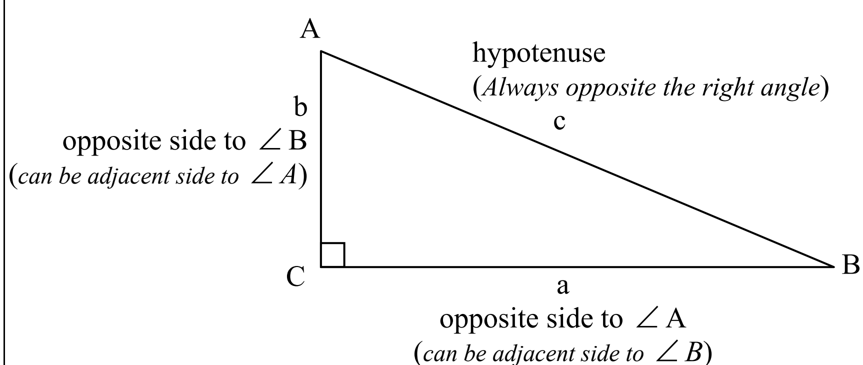
Whole Class → Discussion

Write the mnemonic SOHCAHTOA on the board and see what the students can recall from last year's material.
Use this to re-introduce the three primary trigonometric ratios; Sine, Cosine and Tangent

Sine → Opposite over Hypotenuse → SOH
Cosine → Adjacent over Hypotenuse → CAH
Tangent → Opposite over Adjacent → TOA

Action!

Use the following diagram to aid in identifying a right triangle.



These are the primary trigonometric ratios when we look at ∠A:

$$\sin \angle A = \frac{\textit{opposite side to } \angle A}{\textit{hypotenuse}} = \frac{\textit{length of side } a}{\textit{length of side } c}$$

$$\cos \angle A = \frac{\textit{adjacent side to } \angle A}{\textit{hypotenuse}} = \frac{\textit{length of side } b}{\textit{length of side } c}$$

$$\tan \angle A = \frac{\textit{opposite side to } \angle A}{\textit{adjacent side to } \angle A} = \frac{\textit{length of side } a}{\textit{length of side } b}$$

Perform a Think Aloud on the first example to find the missing side.

Note the differences in the results if we consider we're looking from ∠B instead:

$\sin B = \frac{b}{c}$

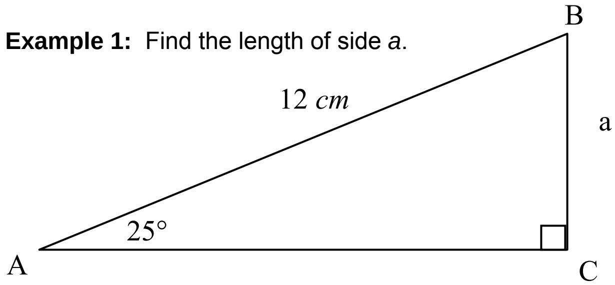
$\cos A = \frac{a}{c}$

$\tan A = \frac{b}{a}$

Word Wall:
Sine
Cosine
Tangent

.

Example 1: Find the length of side *a*.



Script for Think Aloud: We want to find side a. First, I want to examine the triangle to determine what information is given to us.
We have $\angle$A and the hypotenuse. Since, a is the side opposite to $\angle$A, we will need to use the sine trigonometric ratio which is:

$$\sin A = \frac{\text{opposite side to } \angle A}{\text{hypotenuse}}$$
$$= \frac{\text{length of side } a}{\text{length of side } c}$$

Next, I want to put in the information that we know into our equation. We'll replace A with 25° and c with 12. So we get,

$$\sin 25° = \frac{a}{12}$$

I am going to use the calculator to find the decimal value of the ratio for $\sin 25°$. I want to make sure the calculator is in the proper mode. I want it to be in decimal mode. Okay now the calculator shows that the ratio is worth 0.42261826174 and this we can replace sin 25° in the equation, giving us

$$0.422618 = \frac{a}{12}$$

To solve for a, I want to multiplying both sides by 12.

$$12 \times 0.422618 = \frac{a}{12} \times 12$$

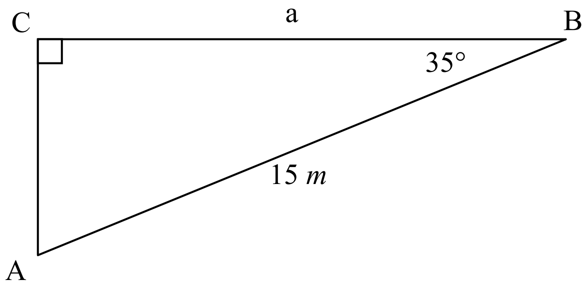
and the result becomes,

5.071416 = a

$\therefore$ the length of side *a* is approximately 5.1 cm long.

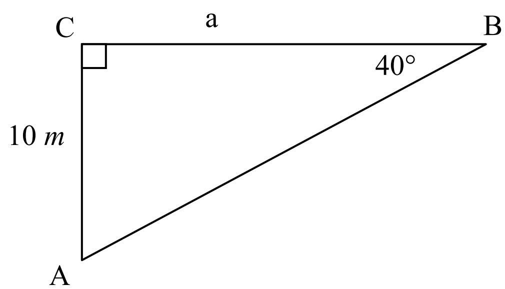
Ask students to do example 2 with a partner. Emphasize with the class to make sure they select the appropriate trigonometric ratio.

Example 2: Find the length of side a.



Take up example 2 with the class.
Give example 3 and discuss with the class possible strategies to solve for a.
One possible strategy is to use $\angle$ A (50^0 sum of the angles in a triangle)
Another possible strategy is as shown below.
Ask student to individually try example 3 on their own. Have students share their solutions.

Example 3: Find the length of side a.



$$\tan B = \frac{\textit{opposite side to } \angle B}{\textit{adjacent side to } \angle B}$$

$$\tan 40° = \frac{\textit{length of side b}}{\textit{length of side a}}$$

$$0.8390996 = \frac{10}{a}$$

$$a \times 0.8390996 = \frac{10}{a} \times a$$

0.8390996 a = 10

To solve for a: divide both sides by 0.8390996.

$$\frac{0.8390996a}{0.8390996} = \frac{10}{0.8390996}$$

a = 11.9175363687

∴ the length of side *a* is approximately 11.9 m long.

$$\cos B = \frac{\textit{adj side to } \angle B}{\textit{hyp}}$$

$$\cos B = \frac{a}{c}$$

$$\cos 35° = \frac{a}{15}$$

$$0.819152 = \frac{a}{15}$$

12.28728 = a

∴ the length of side a is approximately 12.3 m long.

Alternate solution:

$$\tan A = \frac{\textit{opp side to } \angle A}{\textit{adj side to } \angle A}$$

$$\tan A = \frac{a}{b}$$

$$\tan 50° = \frac{a}{10}$$

$$1.191754 = \frac{a}{10}$$

11.91754 = a

∴ the length of side a is approximately 11.9 m long

Concept Practice *Skill Drill*	**Home Activity or Further Classroom Consolidation** Students complete BLM2.1.1.		

MBF3C Name:
BLM2.1.1 Date:

Diagrams are not drawn to scale.

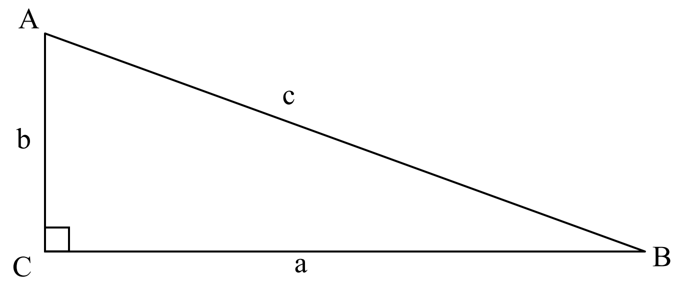
1. Based on the following diagram use the values given to find the missing/indicated side:

(a) $\angle A = 55°$, c = 25 *m* → find *a*

(b) $\angle A = 65°$, c = 32 *cm* → find *b*

(c) $\angle B = 15°$, c = 42 *m* → find *b*

(d) $\angle B = 35°$, c = 55 *cm* → find *a*



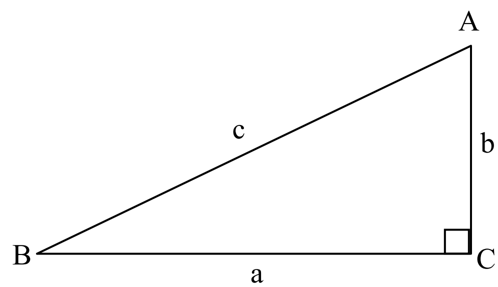
2. Based on the following diagram use the values given to find the missing/indicated side:

(a) $\angle A = 75°$, b = 52 *m* → find *a*

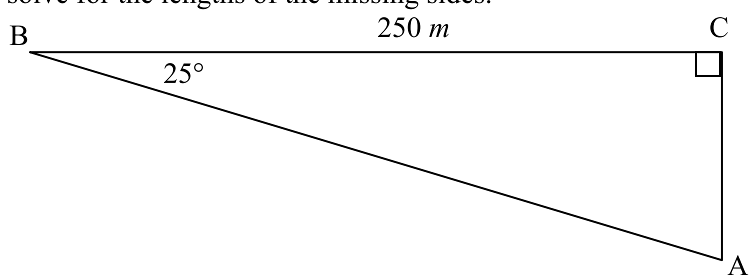
(b) $\angle A = 64°$, a = 23 *cm* → find *b*

(c) $\angle B = 18°$, a = 24 *m* → find *b*

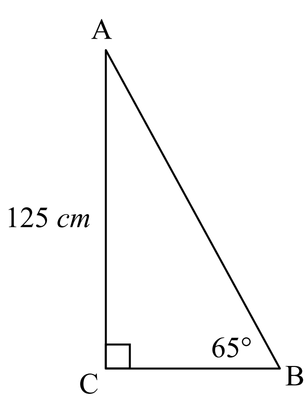
(d) $\angle B = 31°$, b = 58 *cm* → find *a*



3. Given the following diagram solve for the lengths of the missing sides.



4. Given the following diagram solve for the lengths of the missing sides.



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BLM2.1.1 **Solutions:**

(Note: answers should be within a decimal place depending on accuracy of numbers used.)

1. (a) a = 20.5*m* (b) b = 13.5*cm* (c) b = 10.9*m* (d) a = 45.1*cm*
2. (a) a = 194.1*m* (b) b = 11.2*cm* (c) b = 7.8*m* (d) a = 96.5*cm*
3. b = 116.6*m* c = 275.8*m*
4. a = 58.3*cm* c = 137.9*cm*

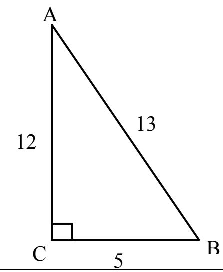
Unit 2 Day 2: Trigonometry – Finding Angle Measure		MBF 3C
	Description This lesson reviews Trigonometry Material from the Grade 10 course – specifically solving angles in triangles using the three trigonometric ratios.	Materials Scientific Calculator BLM2.2.1

Assessment Opportunities

Minds On…

Whole Class → Discussion

Draw a right angled triangle on the board (*as shown*).
Pose the question to the students: Given the sides how would you find the measure of the missing angles?



Students should be able to relate the trigonometric ratios learned yesterday to begin to make the connection to finding angles.

Have a discussion that then leads into the lesson shown below.

Action!

Whole Class → Guided Instruction

Ask students to reflect on the primary trigonometric ratios for $\angle$ A and to connect it to the above triangle :

$$\sin A = \frac{\textit{opposite side to } \angle A}{\textit{hypotenuse}} = \frac{\textit{length of side a}}{\textit{length of side c}}$$

$$\cos A = \frac{\textit{adjacent side to } \angle A}{\textit{hypotenuse}} = \frac{\textit{length of side b}}{\textit{length of side c}}$$

$$\tan A = \frac{\textit{opposite side to } \angle A}{\textit{adjacent side to } \angle A} = \frac{\textit{length of side a}}{\textit{length of side b}}$$

Specifically for the above triangle:

$\sin A = \frac{5}{13}$ $\cos A = \frac{12}{13}$ $\tan A = \frac{5}{12}$

$\sin A = 0.3846154$ $\cos A = 0.9230769$ $\tan A = 0.4166667$

Show students how to use the calculator to solve for the angle in any of the above cases by accessing the inverse of each of the trigonometric functions.

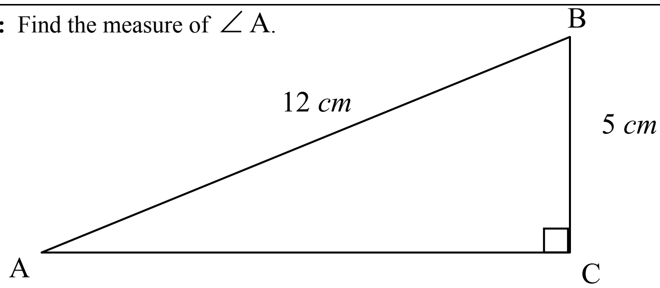
$\angle A = 22.619864948$ $\angle A = 22.619864948$ $\angle A = 22.619864948$

It really didn't matter which trigonometric ratio we chose to use in order to find the correct angle. Usually angles are rounded to the nearest degree. Therefore $\angle$ A is approximately 23°.

Do example 1 with the students.

NOTE to students:: Every calculator is different. Some require you to enter the value first and then do the 2^{nd} button and then the Sin button, others require you to hit the 2^{nd} button, then sin before entering the value. Test to see which order your calculator uses.

Example 1: Find the measure of $\angle$ A.



First we need to examine the triangle to determine what we should do:
We have the side opposite $\angle$ A and the hypotenuse and we need to find the measure of $\angle$ A. This would be easiest using the sine trigonometric ratio.

$$\therefore \sin \angle A = \frac{\text{opposite side to } \angle A}{\text{hypotenuse}}$$

$$\sin \angle A = \frac{\text{length of side } a}{\text{length of side } c}$$

$$\sin \angle A = \frac{5}{12}$$

$$\sin \angle A = 0.41666666667$$

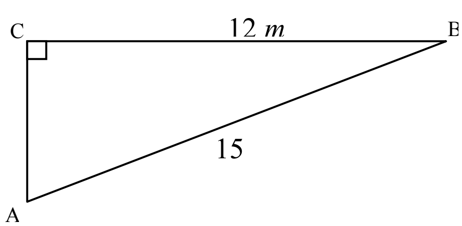
Use your calculator to find $\angle$ A $(\sin^{-1})$

$$\angle A = 24.62431835$$

$\therefore \angle$ A is approximately 25°.

Ask students to work with a partner to do example 2 and 3.

Example 2: Find the measure of $\angle$ B.



$$\cos B = \frac{\text{adj side to } \angle B}{\text{hyp}}$$

$$\cos B = \frac{a}{c}$$

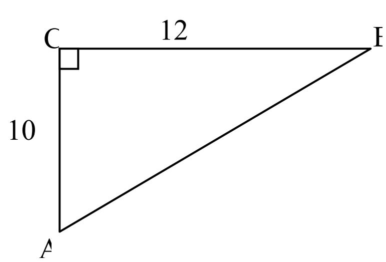
$$\cos B = \frac{12}{15}$$

$$\cos B = 0.8$$

$$\angle B = 36.86989765$$

$\therefore \angle$ B is approximately 37°.

Example 3: Find the measure of $\angle$ B.



$$\tan B = \frac{\text{opp side to } \angle B}{\text{adj side to } \angle B}$$

$$\tan B = \frac{b}{a}$$

$$\tan B = \frac{10}{12}$$

$$\tan B = 0.833333333$$

$$\angle B = 39.805571$$

$\therefore \angle$ B is approximately 40°.

			Some students might say using a different trig ratio and others might comment on the sum of the angles in a triangle.
Concept Practice Skill Drill	**Home Activity or Further Classroom Consolidation** Students complete BLM2.2.1		

MBF3C Name:
BLM2.2.1 Date:

Diagrams are not drawn to scale. **Round angle measures to the nearest degree. The side length answers should be rounded to one decimal place.**

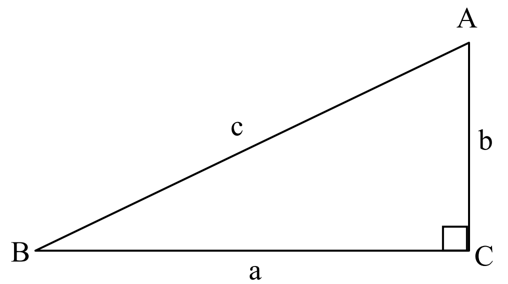
1. Based on the following diagram, use the values given to find the missing/indicated side:

(a) a = 58 *cm*, c = 124 *cm* $\rightarrow$ find $\angle$A

(b) b = 75 *m*, c = 215 *m* $\rightarrow$ find $\angle$A

(c) b = 64 *m*, c = 225 *m* $\rightarrow$ find $\angle$B

(d) a = 45 *cm*, c = 238 *cm* $\rightarrow$ find $\angle$B



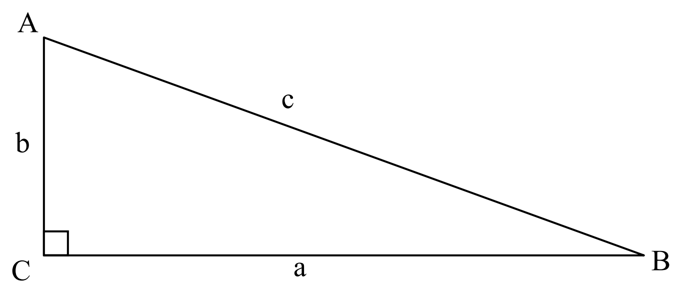
2. Based on the following diagram, use the values given to find the missing/indicated side:

(a) a = 55 *cm*, b = 137 *cm* $\rightarrow$ find $\angle$A

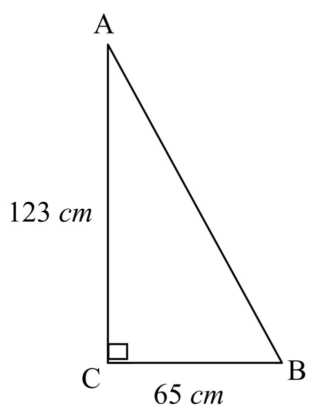
(b) a = 235 *m*, b = 68 *m* $\rightarrow$ find $\angle$A

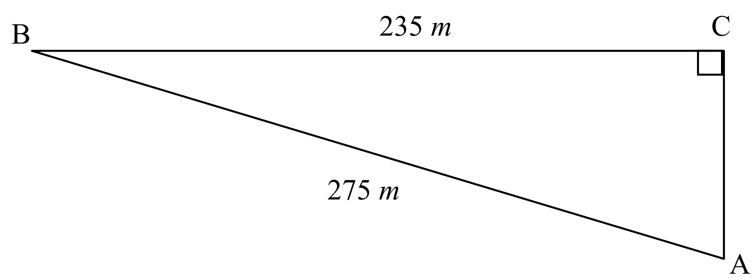
(c) a = 212 *m*, b = 100 *m* $\rightarrow$ find $\angle$B

(d) a = 30 *cm*, b = 285 *cm* $\rightarrow$ find $\angle$B



3. Using the diagram below on the *left* solve for the measure of the missing angles.





4. Using the diagram above on the *right* solve for both the measure of the missing angles and the length of the missing side.

Solutions:
1. (a) $\angle$A = 28 ° (b) $\angle$A = 70 ° (c) $\angle$B = 17 ° (d) $\angle$B = 79 °
2. (a) $\angle$A = 22 ° (b) $\angle$A = 74 ° (c) $\angle$B = 25 ° (d) $\angle$B = 84 °
3. $\angle$A = 28 °, $\angle$B = 62 ° 4. $\angle$A = 59 °, $\angle$B = 31 °, b = 142.8*m*

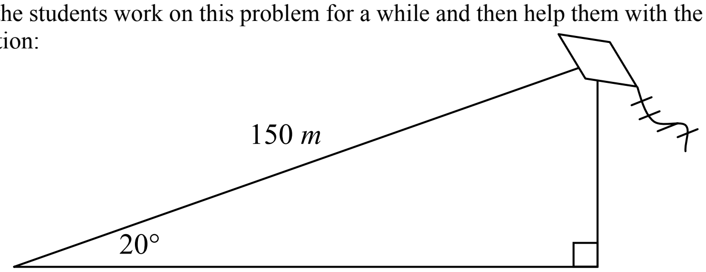
Unit 2 Day 3: Trigonometry – Applications		MBF 3C
	Description This lesson continues the use of Trigonometric Ratios from the last two days but applies them to real world problems.	Materials Chart paper markers
		Assessment Opportunities
Minds On…	**Pairs → Practice**	Students refer to word wall of the trig ratios as needed.

Pairs → Practice

Write the following problem on the board:

You're out in a field flying your kite. You have just let out all 150 m of your kite string. You estimate that the kite is at an angle of elevation from you of about 20°. Can you calculate the height of your kite above the ground?
(*Hint*: try drawing a diagram.)

Let the students work on this problem for a while and then help them with the solution:



Draw the sketch on the board. Examining the triangle, we see that we have an angle and the hypotenuse, so we need to find the side opposite the given angle. This sounds like the sin trigonometric ratio.

$$\sin A = \frac{opposite\ side\ to\ \angle A}{hypotenuse}$$

$$\sin A = \frac{length\ of\ side\ a}{length\ of\ side\ c}$$

$$\sin 20° = \frac{k}{150}$$ ← k is the height of the kite we want to find.

$$0.34202014 = \frac{k}{150}$$

Solving for k by multiplying both sides by 150 gives.

k = 51.3030215

∴ the kite is approximately 51 m above the ground.

Today's lesson will include more real world problems using trigonometric ratios.

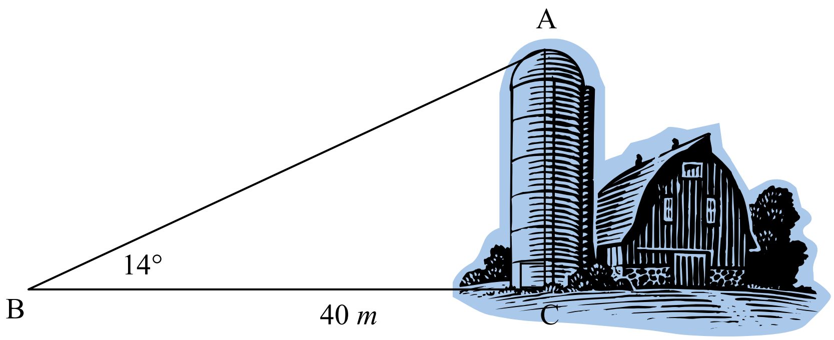
Action!	**Small Groups → Placemat**		

Small Groups → Placemat

Provide each group with one of the following problems (BLM2.3.1) to complete on a placemat or chart paper.

Problem 1:

While walking to school you pass a barn with a silo. Looking up to the top of the silo you estimate the angle of elevation to the top of the silo to be about 14°. You continue walking and find that you were around 40 m from the silo. Using this information and your knowledge of trigonometric ratios calculate the height of the silo.



Possible Solution: $\tan B = \dfrac{\textit{opposite side to } \angle B}{\textit{adjacent side to } \angle B}$

$$\tan B = \frac{\textit{length of side } b}{\textit{length of side } a}$$

$$\tan 14° = \frac{s}{40}$$ ← s represents the height of the silo

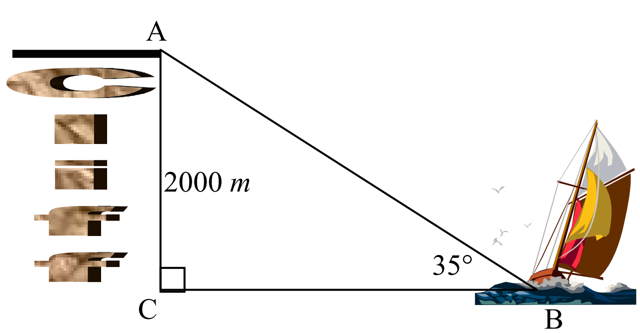
$$0.249328003 = \frac{s}{40}$$

$$s = 9.97312011$$

∴ the silo is approximately 10 m high.

Problem 2:

A sailboat is approaching a cliff. The angle of elevation from the sailboat to the top of the cliff is 35°. The height of the cliff is known to be about 2000 m. How far is the sailboat away from the base of the cliff?



Possible solution

$$\tan B = \frac{opposite\ side\ to\ \angle B}{adjacent\ side\ to\ \angle B}$$

$$\tan B = \frac{length\ of\ side\ b}{length\ of\ side\ a}$$

$$\tan 35° = \frac{2000}{s}$$ ← s represents the distance between the sailboat and the cliff.

$$0.700207538 = \frac{2000}{s}$$

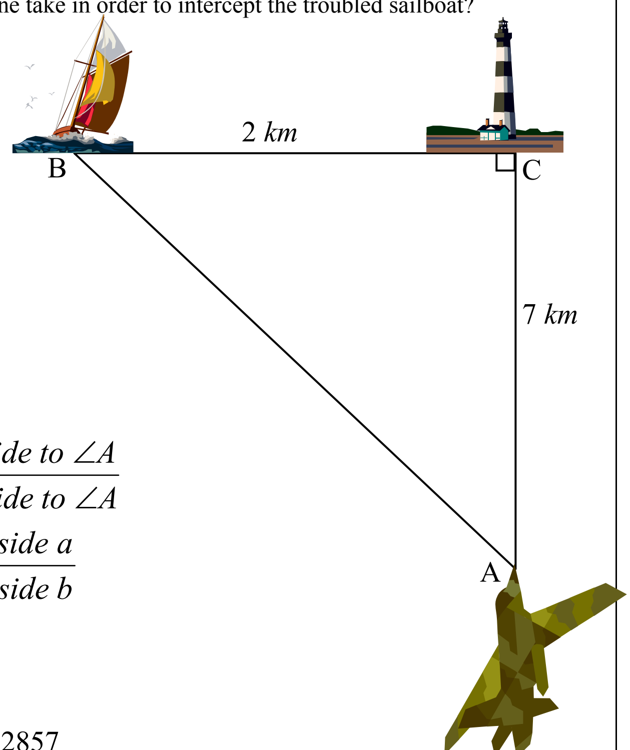
$$0.700207538\ s = 2000$$

$$s = 2856.2960143$$

∴ the sailboat is approximately 2856 m away from the cliff. (or almost 3 km)

Problem 3:

A sailboat that is 2 km due west of a lighthouse sends a signal to the lighthouse that it is in distress. The lighthouse quickly signals a rescue plane that is 7 km due south of the lighthouse. What heading from due north should the plane take in order to intercept the troubled sailboat?



Possible Solution:

$$\tan A = \frac{opposite\ side\ to\ \angle A}{adjacent\ side\ to\ \angle A}$$

$$\tan A = \frac{length\ of\ side\ a}{length\ of\ side\ b}$$

$$\tan A = \frac{2}{7}$$

$$\tan A = 0.2857142857$$
$$\angle A = 15.9453959$$

∴ the plane should take a heading of about 16° west of north to intercept and rescue the sailboat.

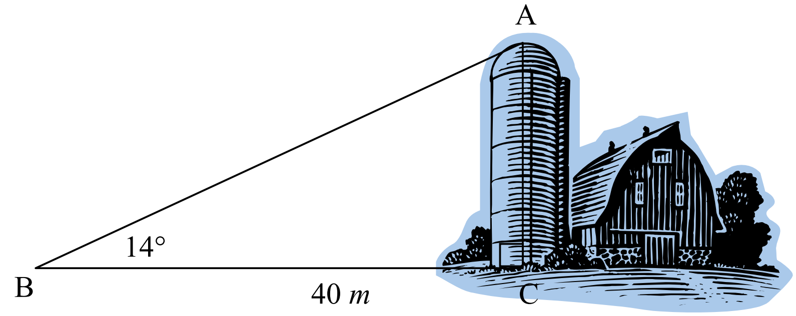
Consolidate Debrief	**Small Groups → Presentation** Place the small groups together that have the same problem to check each others work. Each of the groups prepares for the presentation. Randomly select one of the groups for each problem to present to the class.		
Application	**Home Activity or Further Classroom Consolidation** Students complete BLM2.3.2		

MBF3C
BLM2.3.1

Trigonometry Application Problems

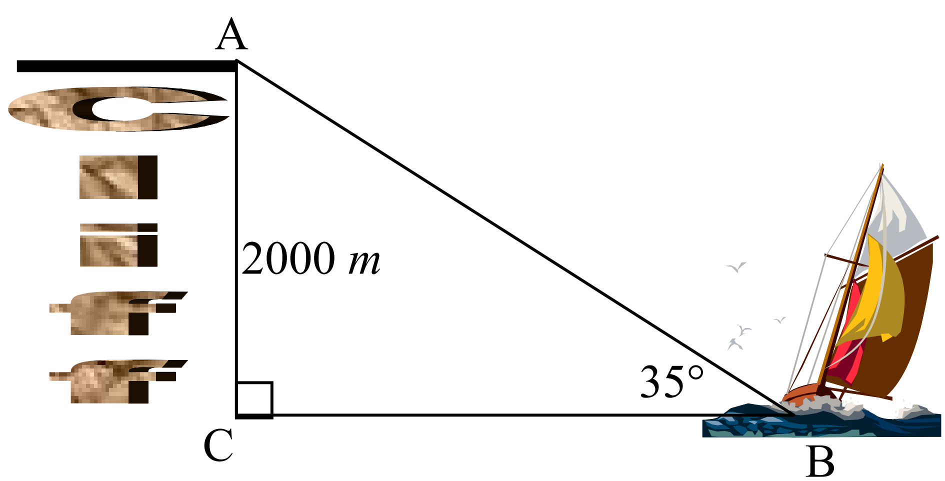
Problem 1:

While walking to school you pass a barn with a silo. Looking up to the top of the silo you estimate the angle of elevation to the top of the silo to be about 14°. You continue walking and find that you were around 40 m from the silo. Using this information and your knowledge of trigonometric ratios calculate the height of the silo.



Problem 2:

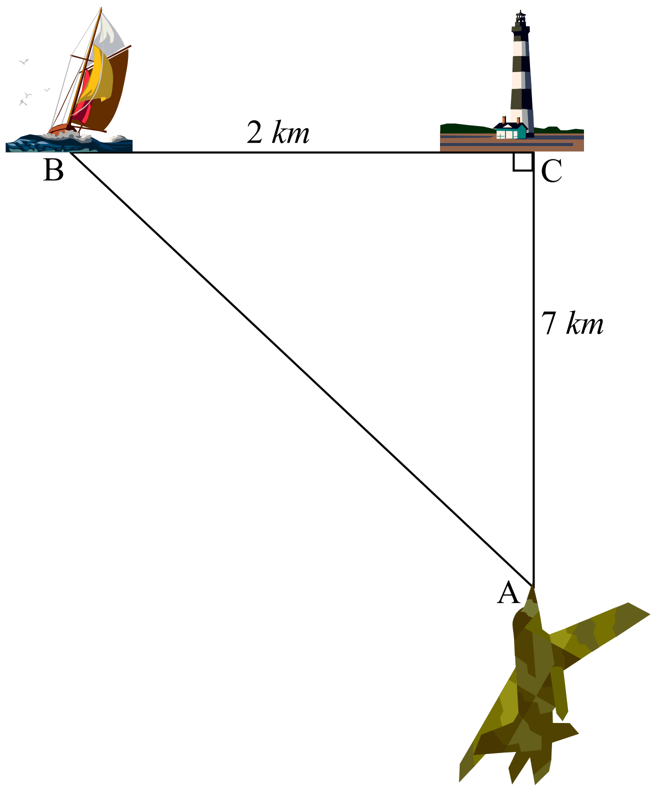
A sailboat is approaching a cliff. The angle of elevation from the sailboat to the top of the cliff is 35^0. The height of the cliff is known to be about 2000 m. How far is the sailboat away from the base of the cliff?



Trigonometry Application Problems (Continued)

Problem 3:

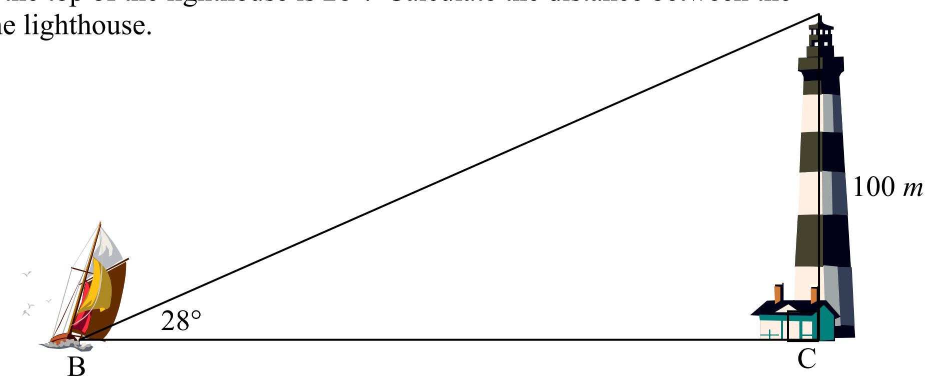
A sailboat that is 2 km due west of a lighthouse sends a signal to the lighthouse that it is in distress. The lighthouse quickly signals a rescue plane that is 7 km due south of the lighthouse. What heading from due north should the plane take in order to intercept the troubled sailboat?



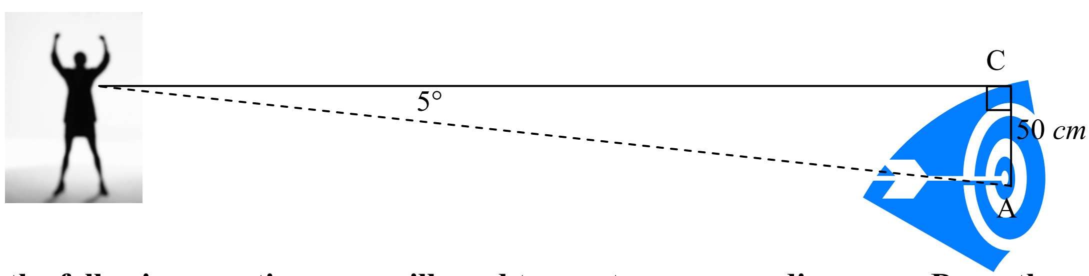
MBF3C Name:
BLM2.3.2 Date:

Round ∠'s to whole degrees. Length answers should be rounded to 1 decimal place and include units.

1. The top of a lighthouse is 100 m above sea level. The angle of elevation from the deck of the sailboat to the top of the lighthouse is 28°. Calculate the distance between the sailboat and the lighthouse.



2. An archer shoots and gets a bulls-eye on the target. From the archer's eye level the angle of depression to the bulls-eye is 5°. The arrow is in the target 50 cm below the archer's eye level. Calculate the distance the arrow flew to hit the target (*the dotted line*).



For the following questions you will need to create your own diagrams. Draw them carefully and refer to the written description to ensure you find the correct solution.

3. Two islands **A** and **B** are 3 km apart. A third island **C** is located due south of **A** and due west of **B**. From island **B** the angle between islands **A** and **C** is 33°. Calculate how far island **C** is from island **A** and from island **B**.

4. The foot (*bottom*) of a ladder is placed 1.5 m from a wall. The ladder makes a 70° angle with the level ground. Find the length of the ladder. (Round to one decimal place.)

5. A tow truck raises the front end of a car 0.75 m above the ground. If the car is 2.8 m long what angle does the car make with the ground?

6. A construction engineer determines that a straight road must rise vertically 45 m over a 250 m distance measured along the surface of the road (this represents the hypotenuse of the right triangle). Calculate the angle of elevation of the road.

Solutions:
1. 188.1 *m* 2. 573.7 *cm* 3. Distance A to C: 1.6 *km* Distance B to C: 2.5 *km*
4. 4.4 *m* 5. 16° 6. 10°

Unit 2 Day 4: Trigonometry – Sine Law	MBF 3C

Description

This lesson introduces the Sine Law to the students.

Materials

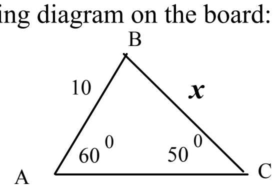
Computer Lab with Geometer's SketchPad.
BLM2.4.1, 2.4.2

Assessment Opportunities

Minds On…

Whole Class → Discussion

Draw the following diagram on the board:

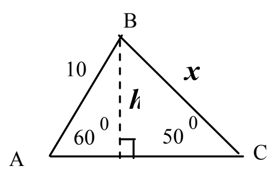


Pose the following question to the students:
With what we know from the diagram, can we find the value of ***x***?

Intended Response: No, too hard for us!

Ask the students, if more information was provided like the following diagram if that would help them solve for ***x***? Lead students through the following solution:

The students are discovering the sine law using numerical values.



$$\frac{h}{10} = \sin 60^0$$

$$\therefore h = 8.66cm$$

$$\text{Now, } \frac{h}{x} = \sin 50^0$$

$$x = \frac{8.66}{\sin 50^0}$$

$$x = 11.3 \text{ cm}$$

Observation: $h = 10\sin 60^0$

and $h = 11.3\sin 50^0$

in this case $\dfrac{a}{\sin A} = \dfrac{c}{\sin C}$

Pose the question to them: Is this true all the time?

Action!

Pairs → Investigation

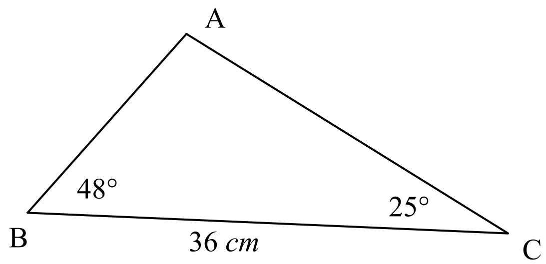
Students use Geometer's Sketchpad to explore the sine law following BLM2.4.1.

If the school uses LanSchool or some broadcast capability for the teacher then the teacher could broadcast the steps first and let the students try it afterward.

Whole Class → Guided Instruction

Demonstrate to students how to use the Sine Law to solve triangles.

Example 1: In ΔABC, given that $\angle B = 48°$, $\angle C = 25°$, and side *a* (*named as **BC***) BC = 36 cm. Find the length of **AB** and **AC** correct to 1 decimal place. (*Help by drawing the diagram included below*.)



According to the Sine Law we need a ratio of the sine of an angle and its corresponding side, currently we don't have this, however, we do have $\angle B$ and $\angle C$ so we are able to solve for $\angle A$.

$\angle A = 180° - (48° + 25°)$
$\therefore \angle A = 43°$

We have **BC** = 36 cm and we need to find **AB** and **AC**.

So using the Sine Law $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$ we have:

$$\frac{36}{\sin 43°} = \frac{\mathbf{AB}}{\sin 48°} = \frac{\mathbf{AC}}{\sin 25°}$$

Solving separately:

$$\frac{36}{\sin 43°} = \frac{\mathbf{AB}}{\sin 48°} \quad \text{and} \quad \frac{36}{\sin 43°} = \frac{\mathbf{AC}}{\sin 25°}$$

Here, multiply both sides by sin 48° and here, multiply both sides by sin 25°

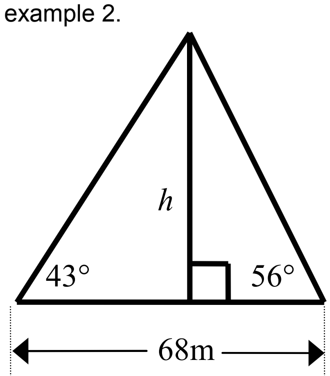
Gives: $\sin 48° \cdot \dfrac{36}{\sin 43°} = \mathbf{AB}$ and $\sin 25° \cdot \dfrac{36}{\sin 43°} = \mathbf{AC}$

Finishing off using a calculator:

$\therefore$ **AB** = 39.2 cm and **AC** = 22.3 cm

Have students working with a partner solve example 2.

Example 2: Solve for the value of h in the following diagram:



Possible Solution:
Let's label the diagram A at the peak and BC on the base. Thus, in

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad \text{We get: } \frac{68}{\sin 81°} = \frac{b}{\sin 43°} = \frac{c}{\sin 56°}$$

We can solve for either of b or c and then use the primary trigonometric ratios to complete the solution for h.

$$\frac{68}{\sin 81°} = \frac{b}{\sin 43°} \quad \textbf{or} \quad \frac{68}{\sin 81°} = \frac{c}{\sin 56°}$$

Solving for the chosen side following the same steps as above we get:

$\therefore$ $b = 47.0\ m$ or $c = 57.1\ m$

Finally to solve for *h* use the sine trigonometric ratio.

$\text{Sin } 56° = \frac{47}{h}$ or $\sin 43° = \frac{57}{h}$

$\therefore$ $h = 38.9$ m or $h = 38.9$ m

Consolidate Debrief

Whole Class → Discussion

Summarize when to use the Sine Law.

The Sine Law:
Using the same triangle above you could construct another perpendicular line from B or C to the opposite side and create a similar expression for sin $\angle$A and its corresponding side a. In general the Sine Law takes the form:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \quad \text{or} \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Emphasize that the two equal signs constitutes three equations.

Concept Practice

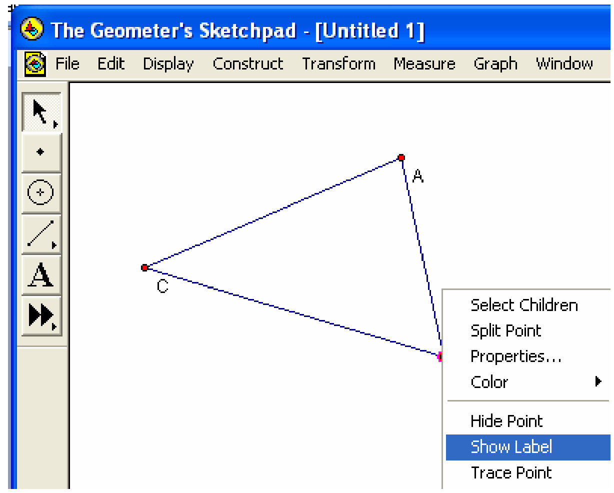
Home Activity or Further Classroom Consolidation

Students complete BLM2.4.2.

MBF 3C
BLM 2.4.1

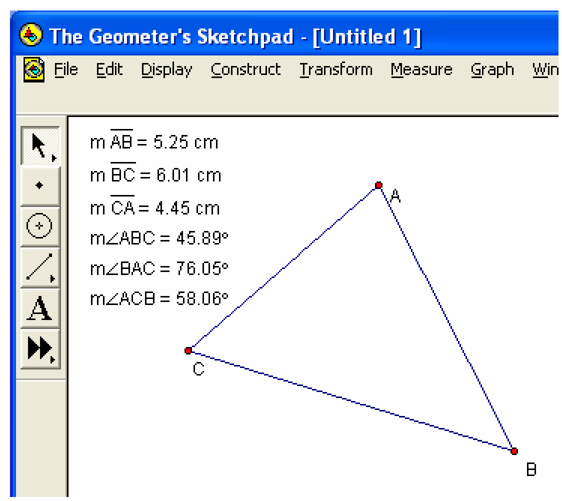
Name: ____________
Date: ____________

Investigation: Sine Law – Geometer's Sketchpad



1. Load Geometer's Sketchpad.
2. Start with a new document (*default*).
3. Select the Straightedge Tool (*4th button down the toolbar*)
4. Draw three lines – making sure that each new line starts from a previous line and that the last point returns to the first completing the triangle. (*shown right*)
5. Switch to the selection tool (*1st button on the toolbar*)
6. Select and right-click on each vertex and from the short-cut menu select "Show Label" (*also shown right*)
7. Next select any line and from the Measure menu (*or from the right-click short-cut*) select "Length".

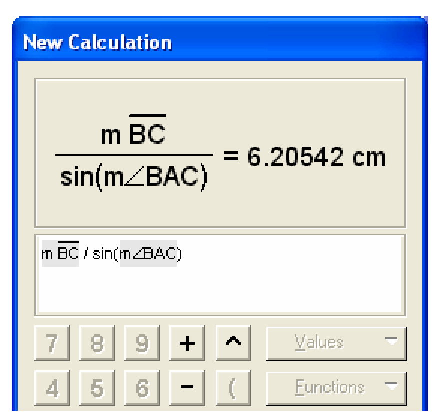
   This should display $\mathbf{m}\ \overline{\mathbf{AB}}$ (*shown*)
8. Repeat **Step 6** for the other lines, making sure to unselect before selecting a new line. (*If anything else is selected length may not appear on the menu.*)
9. Next select in the following order the vertices: A, B then C – then click the Measure menu and choose "Angle". This should display **m** ∠ **ABC** and the measure of that angle.
10. Now repeat **Step 8** but for angles ∠ BAC and ∠ ACB. (*shown*)



11. If you select any point you can drag the point to a new location and all of the measurements update automatically. (*You can also select and move an entire line.*)
12. Try this and adjust the position of the triangle to leave more room below our measurements.
13. We will now add some calculations namely the values for the Sine Law:

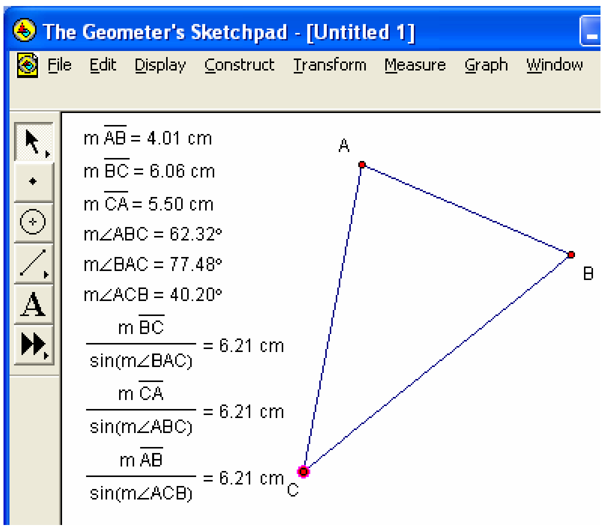
$$\frac{a}{\sin \angle A} = \frac{b}{\sin \angle B} = \frac{c}{\sin \angle C}$$

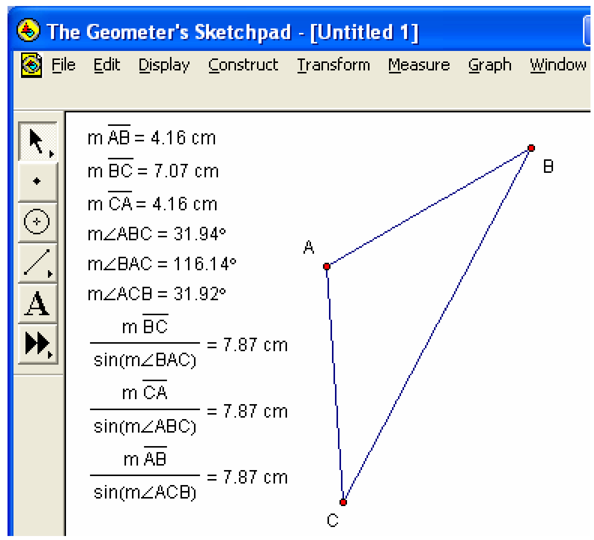
14. To do this select the Measure menu and select "Calculate…". A new dialogue box appears (*shown right*) where we will enter our calculation.
15. First click on the measurement for side *a* (*in this case it is* $\mathbf{m}\ \overline{\mathbf{BC}}$), then click on the division sign



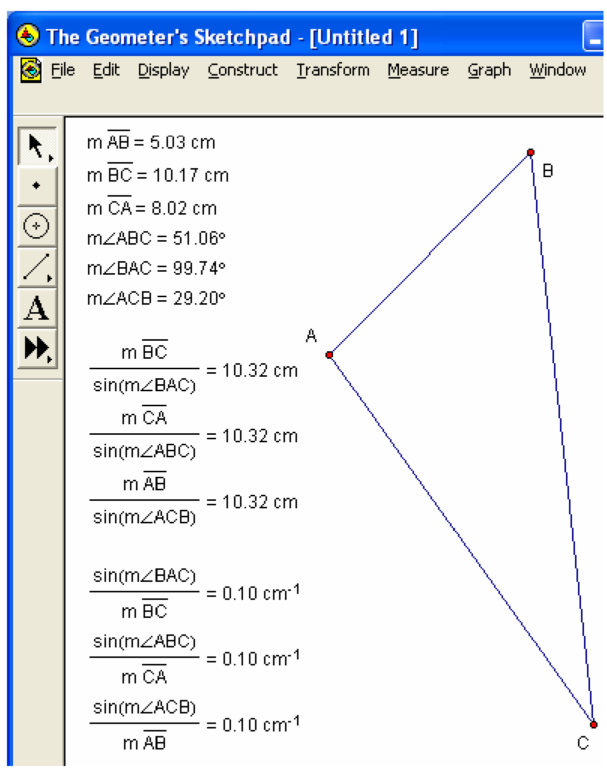
and type “si” for the sine function, next click on the measurement for $\angle$A (*in this case it is* **m$\angle$BAC** (*depending on the size of your triangle you will see different results.*) Click **OK**

16. This will add a new measurement to your document, repeat step 15 for side b and side c. For side b use **m $\overline{CA}$** and sin(**m$\angle$ABC)** for side c use **m $\overline{AB}$** and sin(**m$\angle$ACB**). Calculations are shown in the bottom diagrams.
17. Now change the position of your vertices; this will change the lengths and angles in your triangle – make note of what happens to all three of the calculation boxes for the Sine Law: $\dfrac{a}{\sin \angle A} = \dfrac{b}{\sin \angle B} = \dfrac{c}{\sin \angle C}$. (*two variations shown below*)





18. Next create three more calculations for the other version of the Sine Law: $\dfrac{\sin \angle A}{a} = \dfrac{\sin \angle B}{b} = \dfrac{\sin \angle C}{c}$ (*shown right*)



19. Experiment with more positions of the triangle vertices.

20. Notice that the set of three values in either version of the Sine Law remain the same. This shows that the ratio of any side to the sine of the corresponding angle in a triangle remains equal to the ratio of any other side to the sine of the corresponding angle. Either

$$\frac{\sin \angle A}{a} = \frac{\sin \angle B}{b} = \frac{\sin \angle C}{c}$$

or

$$\frac{a}{\sin \angle A} = \frac{b}{\sin \angle B} = \frac{c}{\sin \angle C}$$

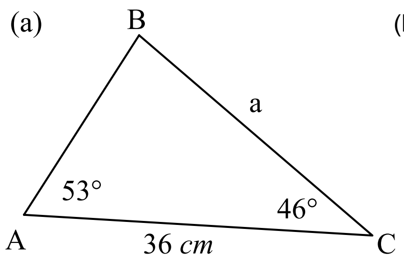
MBF3C Name:
BLM2.4.2 Date:

1. Solve for the given variable (correct to 1 decimal place) in each of the following:

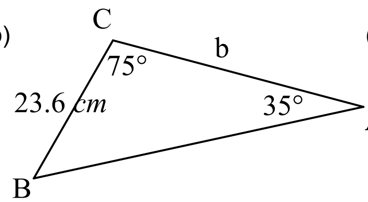
(a) $\dfrac{a}{\sin 35°} = \dfrac{10}{\sin 40°}$ (b) $\dfrac{65}{\sin 75°} = \dfrac{b}{\sin 48°}$ (c) $\dfrac{75}{\sin 55°} = \dfrac{c}{\sin 80°}$

2. For each of the following diagrams write the equation you would use to solve for the indicated variable:

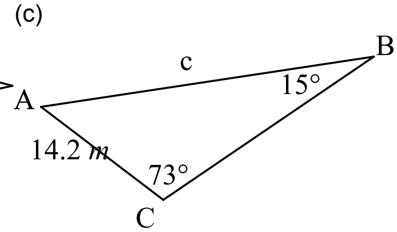
(a)



(b)



(c)



3. Solve for each of the required variables from Question #2.

4. For each of the following triangle descriptions you should make a sketch and then find the indicated side rounded correctly to one decimal place.

(a) In ΔABC, given that ∠A = 57°, ∠B = 73°, and AB = 24 cm. Find the length of AC

(b) In ΔABC, given that ∠B = 38°, ∠C = 56°, and BC = 63 cm. Find the length of AB

(c) In ΔABC, given that ∠A = 50°, ∠B = 50°, and AC = 27 m. Find the length of AB

(d) In ΔABC, given that ∠A = 23°, ∠C = 78°, and AB = 15 cm. Find the length of BC

(e) In ΔABC, given that ∠A = 55°, ∠B = 32°, and BC = 77 cm. Find the length of AC

(f) In ΔABC, given that ∠B = 14°, ∠C = 78°, and AC = 36 m. Find the length of BC

Solutions:

1. (a) 8.9 *units* (b) 50.0 *units* (c) 90.2 *units*
2. (a) $\dfrac{a}{\sin 53°} = \dfrac{36}{\sin 81°}$ (b) $\dfrac{23.6}{\sin 35°} = \dfrac{b}{\sin 70°}$ (c) $\dfrac{14.2}{\sin 15°} = \dfrac{c}{\sin 73°}$
3. (a) 29.1 *cm* (b) 38.7 *cm* (c) 52.5 *m*
4. (a) 30.0 *cm* (b) 52.4 *cm* (c) 34.7 *m* (d) 6.0 *cm* (e) 49.8 *cm* (f) 148.7 *m*

Unit 2 Day 5 - Trigonometry - Cosine Law

MBF 3C

Description

This lesson introduces the Cosine Law to the students.

Materials

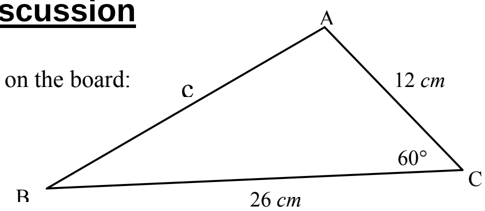
BLM2.5.1
Scientific calculator

Assessment Opportunities

Minds On…

Whole Class → Discussion

Draw the following diagram on the board:



Ask students to solve for c in the triangle above using the Sine Law:

Results:

$$\frac{26}{\sin \angle A} = \frac{12}{\sin \angle B} = \frac{c}{\sin 60°}$$

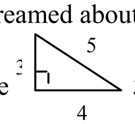
The solution is stalled at this point since each part of the ratio has some missing information. We cannot solve the triangle. We need to develop a new formula – this formula is called the **Cosine Law**

Action!

Whole Class → Guided Instruction

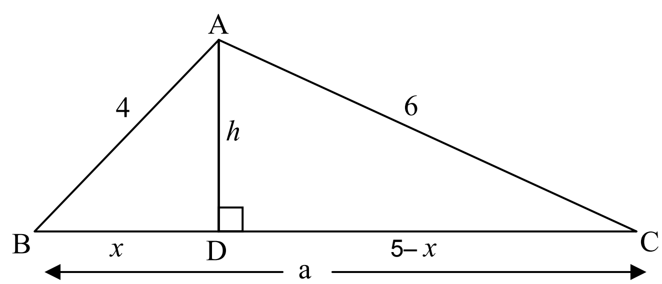
Pose to the students: Haven't you always dreamed about using the Pythagorean Theorem for all triangles?

Recall the most famous Pythagorean triangle



$5^2 = 3^2 + 4^2$

For Δ ABC,



$$h^2 + x^2 = 4^2 \qquad \text{and} \qquad h^2 + (5-x)^2 = 6^2$$

$$h^2 = 4^2 - x^2 \quad (1) \qquad\qquad h^2 = 6^2 - (5-x)^2 \quad (2)$$

From (1) and (2)

$$4^2 - x^2 = 6^2 - (5-x)^2$$
$$4^2 - x^2 = 6^2 - (5^2 - 10x + x^2)$$
$$4^2 - x^2 + 5^2 + x^2 - 10x = 6^2$$
$$4^2 + 5^2 - 10(4\cos A) = 6^2$$
$$c^2 + b^2 - 2bc\cos A = a^2$$

Is this true for all triangles?

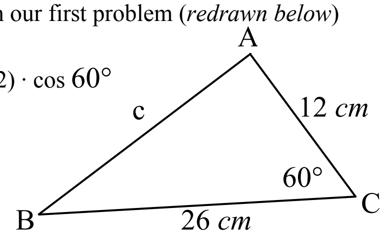
cosA = $\frac{x}{4}$

$\therefore 4\cos A = x$

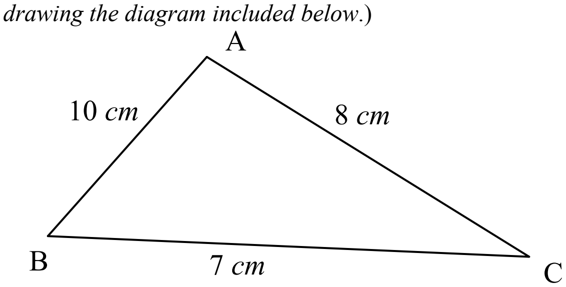
Guide students through example 1 and 2 to solve for a missing side and a missing angle using the Cosine Law.

Example 1: Now we can solve for c from our first problem (*redrawn below*)

$$c^2 = a^2 + b^2 - 2ab \cdot \cos \angle C$$
$$c^2 = (26)^2 + (12)^2 - 2\,(26)\,(12) \cdot \cos 60°$$
$$c^2 = 676 + 144 - 624 \cdot (0.5)$$
$$c^2 = 676 + 144 - 312$$
$$c^2 = 508$$
$$\sqrt{c^2} = \sqrt{508}$$
$$\therefore c = 22.54 \text{ cm}$$



Example 2: In ΔABC, given **BC** = 7 cm, **AC** = 8 cm and **AB** = 10 cm. Find the measure of $\angle A$ to the nearest degree.
(*Help by drawing the diagram included below.*)



We are asked to find the measure of $\angle A$ to the nearest degree so we should use the Cosine Law formula that is most appropriate:

$$a^2 = b^2 + c^2 - 2bc \cdot \cos \angle A$$

Filling in what we know gives:

$$(7)^2 = (8)^2 + (10)^2 - 2(8)(10) \cdot \cos \angle A$$
$$49 = 64 + 100 - 160 \cdot \cos \angle A$$

Rearranging:

$$160 \cdot \cos \angle A = 64 + 100 - 49$$
$$160 \cdot \cos \angle A = 64 + 100 - 49$$
$$160 \cdot \cos \angle A = 115$$

Finally:

$$\cos \angle A = \frac{115}{160}$$
$$\cos \angle A = 0.71875$$

Use your calculator to solve for $\angle A$

$$\angle A = 44.048625674°$$

$\therefore$ $\angle A$ to the nearest degree is 44°

Consolidate Debrief

Whole Class → Discussion

Ask students to set up the equations using the Cosine Law for questions #1a, and #1b on BLM2.5.1. Verify that they have the correct set up.

Concept Practice

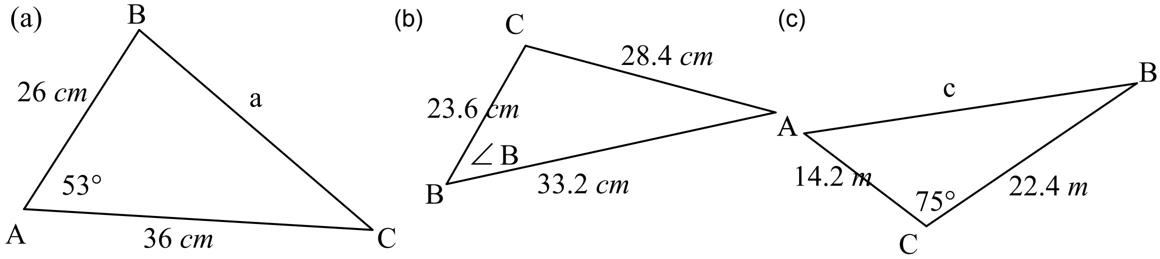
Home Activity or Further Classroom Consolidation

Students complete BLM2.5.1.

MBF3C Name:
BLM2.5.1 Date:

1. For each of the following diagrams write the equation you would use to solve for the indicated variable:

(a) (b) (c)



2. Solve for each of the required variables from Question #1.

3. For each of the following triangle descriptions you should make a sketch and then find the indicated value.

(a) In ΔABC, given that AB = 24 cm, AC = 34 cm, and ∠A = 67°. Find the length of BC

(b) In ΔABC, given that AB = 15 m, BC = 8 m, and ∠B = 24°. Find the length of AC

(c) In ΔABC, given that AC = 10 cm, BC = 9 cm, and ∠C = 48°. Find the length of AB

(d) In ΔABC, given that ∠A = 24°, AB = 18.6 m, and AC = 13.2 m. Find the length of BC

(e) In ΔABC, given that AB = 9 m, AC = 12 m, and BC = 15 m. Find the measure of ∠B.

(f) In ΔABC, given that AB = 18.4 m, BC = 9.6 m, and AC = 10.8 m. Find the measure of ∠A.

Solutions:

1. (a) $a^2 = (36)^2 + (26)^2 - 2(36)(26) \cdot \cos 53°$

(b) $(28.4)^2 = (23.6)^2 + (33.2)^2 - 2(23.6)(33.2) \cdot \cos \angle B$

(c) $c^2 = (22.4)^2 + (14.2)^2 - 2(22.4)(14.2) \cdot \cos 75°$

2. (a) 29.1 *cm* (b) 57° (c) 23.2 *m*

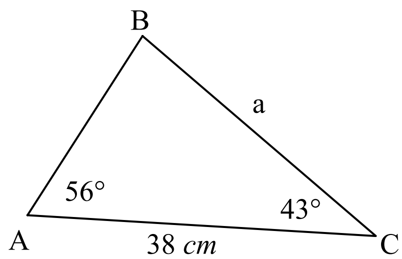
3. (a) 33.1 *cm* (b) 8.4 *m* (c) 7.8 cm (d) 8.5 *m* (e) 53° (f) 24°

Unit 2 Day 6 Trigonometry – Applying the Sine and Cosine Law			MBF 3C
	Description This lesson has students solving triangles by choosing between the Sine Law and the Cosine Law.		**Materials** BLM2.6.1,2.6.2
			Assessment Opportunities
Minds On…	**Whole Class → Four Corners** Post four signs, one in each corner labelled – Sine Law, Cosine Law, Pythagorean Theorem, Trigonometric Ratio (SOHCAHTOA). Provide each of the students with one of the triangles on BLM2.6.1. Instruct the students to make a decision as to which method they would use to solve the missing angle or side and to stand in the corner where it is labelled. Once students are all placed have the students discuss amongst themselves to confer that they have selected an appropriate method. Allow students to move to a different location after discussion. Ask one representative from each corner to explain why their triangle(s) would be best solved using that particular method. Ask students to complete a Frayer model (BLM2.6.2) for their method. Add to word wall or class math dictionary.		BLM2.6.1 Answers: 1.Sine Law 2.Cosine Law 3.Cosine Law 4.Sine Law 5.Pythagorean Theorem 6.Trig Ratio Support students with the characteristics. May want to include items like: 3 sides → you need to use Cosine Law to solve for angles. 2 sides and 1 angle → it depends on where the angle is located if the angle is between the two sides (*contained*) then you need to use the Cosine Law – usually to find last side. ii) if the angle is not contained then the Sine Law can be used usually to find another angle. 1 side and 2 angles → then you need to use the Sine Law, usually to find a side.
Action!	**Pairs → Practice** Ask students to individually solve the triangle they were given for the previous activity. Then with an elbow partner check each others work. Possible solutions for the triangles on teachers copy of the Four Corners.		
Consolidate Debrief	**Whole Class → Discussion** Clarify any problems from the pairs solving for their missing side or angle. Highlight what the phrase "solving a triangle" means (solve for all sides and angles in a triangle). **Pairs → Practice** Ask pairs to trade questions and solve for the remaining parts of their triangle.		
Application	**Home Activity or Further Classroom Consolidation** Students complete BLM2.6.3		

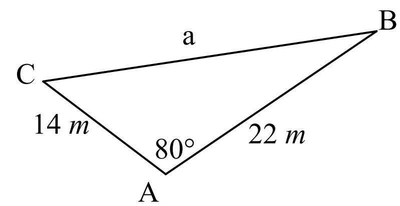
MBF3C
BLM2.6.1

Four Corners – Triangles

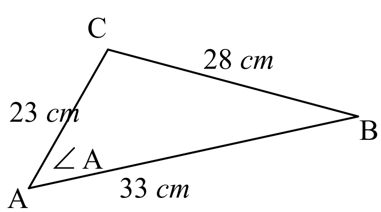
1)



2)



3)



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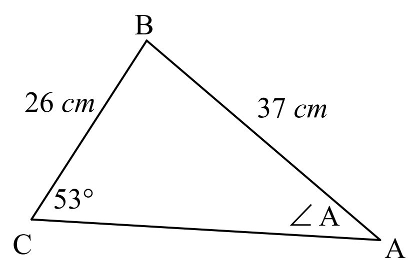
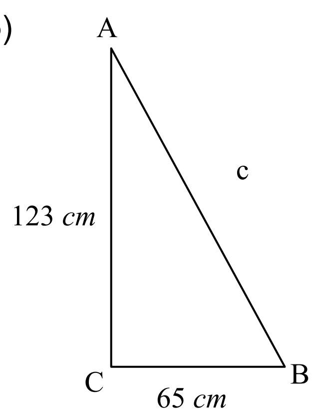


5)



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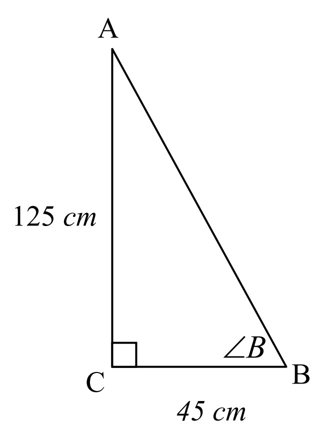
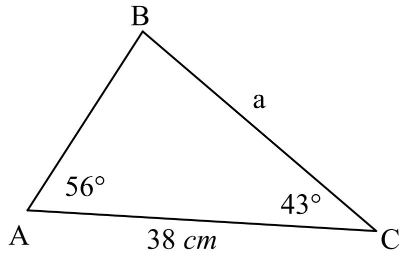


MBF3C
BLM2.6.1

Four Corners – Triangles (Teacher)

1)



First we need to find $\angle B$ we can do this using the sum of the angles in a triangle. $\angle B = 180° - 56° - 43°$ Therefore, $\angle B = 81°$

Now we can solve for a using the Sine Law

$$\frac{a}{\sin \angle A} = \frac{b}{\sin \angle B}$$

$$\frac{a}{\sin 56°} = \frac{38}{\sin 81°}$$

$$a = \frac{38}{\sin 81°} \cdot \sin 56°$$

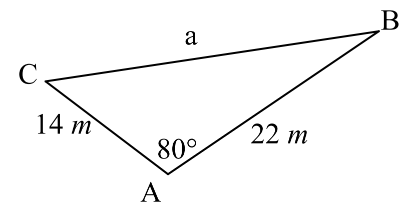
$$a = \frac{38}{0.98768834} \cdot 0.82903757$$

$\therefore$ a = 31.9 cm

Solving for the other sides:

Sine Law vs.	**Cosine Law**
$\frac{b}{\sin B} = \frac{c}{\sin C}$	$c^2 = a^2 + b^2 - 2ab \cdot \cos C$
$\frac{38}{\sin 81°} = \frac{c}{\sin 43°}$	$c^2 = (31.9)^2 + (38)^2 - 2(31.9)(38) \cos 43°$
$c = \frac{38}{\sin 81°} \cdot \sin 43°$	$c^2 = 1017.61 + 1444 - 2424.4(0.7313537)$
$c = \frac{38}{0.98768834} \cdot 0.68199836$	$c^2 = 688.5160858$
c = 26.2 cm	c = 26.2 cm

2)



$a^2 = b^2 + c^2 - 2bc \cdot \cos A$

$a^2 = (14)^2 + (22)^2 - 2(14)(22) \cdot \cos 80°$

$a^2 = 196 + 484 - 616 \cdot 0.173648178$

$a^2 = 196 + 484 - 106.96727744$

$a^2 = 573.03272256$

$\sqrt{a^2} = \sqrt{573.03272256}$

$\therefore$ a = 23.9 m

Solving for the other angles.

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin 80°}{23.9} = \frac{\sin B}{14}$$

$$\frac{\sin 80°}{23.9} \cdot 14 = \sin B$$

$$\sin B = \frac{0.98480775}{23.9} \cdot 14$$

sin B = 0.57687483

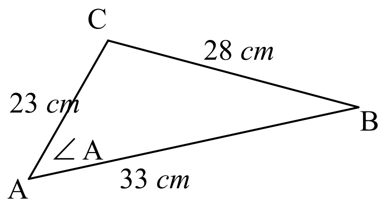
$\angle B = 35.231034°$

$\therefore$ $\angle B = 35°$ → this gives us $\angle C = 65°$
(i.e. 180° - 80° - 35°)

MBF3C
BLM2.6.1

Four Corners – Triangles (Teacher)

3)



$a^2 = b^2 + c^2 - 2bc \cdot \cos A$

$(28)^2 = (23)^2 + (33)^2 - 2(23)(33) \cdot \cos A$

$784 = 529 + 1089 - 1518 \cdot \cos A$

Rearranging:

$1518 \cdot \cos A = 529 + 1089 - 784$

$1518 \cdot \cos A = 834$

$\cos A = \dfrac{834}{1518}$

$\cos A = 0.54940711$

$\angle A = 56.67365194$

$\therefore \quad \angle A = 57°$

Solving for the other angles:

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin 57°}{28} = \frac{\sin B}{23}$$

$$\frac{\sin 57°}{28} \cdot 23 = \sin B$$

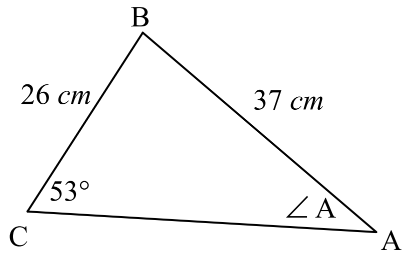
$$\sin B = \frac{0.83867057}{28} \cdot 23$$

$\sin B = 0.688907967$

$\angle B = 43.543727°$

$\therefore \angle B = 44°$ → this gives us $\angle C = 79°$ (i.e. $180° - 57° - 44°$)

4)



$$\frac{\sin A}{a} = \frac{\sin C}{c}$$

$$\frac{\sin A}{26} = \frac{\sin 53°}{37}$$

$$\sin A = \frac{\sin 53°}{37} \cdot 26$$

$$\sin A = \frac{0.79863551}{37} \cdot 26$$

$\sin A = 0.56120333$

$\angle A = 34.13905704$

$\therefore \quad \angle A = 34°$

Solving for the other sides

$\angle B = 180° - 34° - 53°$

$\therefore \angle B = 93°$

Sine Law **vs.** **Cosine Law**

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{b}{\sin 93°} = \frac{37}{\sin 53°}$$

$$b = \frac{37}{\sin 53°} \cdot \sin 93°$$

$$b = \frac{37}{0.79863551} \cdot 0.998629535$$

b = 46.3 cm

$b^2 = a^2 + c^2 - 2ac \cos B$

$b^2 = (26)^2 + (37)^2 - 2(26)(37) \cos 93°$

$b^2 = 676 + 1369 - 1924 (-0.05233596)$

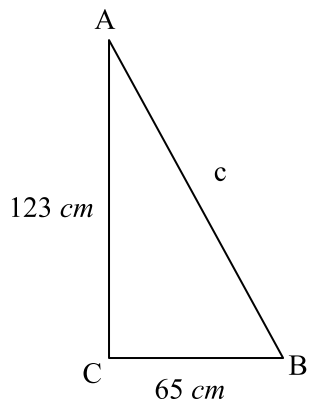
$b^2 = 2145.69437981$

b = 46.3 cm

MBF3C
BLM2.6.1

Four Corners – Triangles (Teacher)

5)

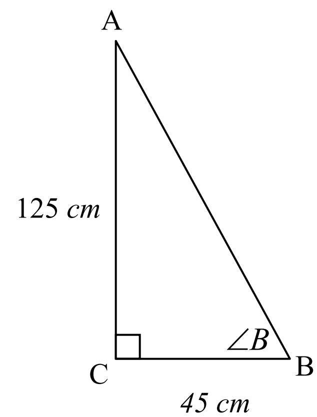


$c^2 = 123^2 + 65^2$
$c^2 = 15\ 129 + 4\ 225$
$c^2 = 19\ 354$
$c = \sqrt{19354}$
$c = 139.1$

Solving for angles:

$\tan B = \frac{123}{65}$
$\tan B = 1.8923$
$\angle B = 62^0$
$\therefore \angle A = 28^0$

6)



$\tan B = \frac{125}{45}$
$\tan B = 2.7778$
$\angle B = 70^0$

Solving for the other angle and side.

$c^2 = 125^2 + 45^2$
$= 15\ 625 + 2025$
$= 17\ 650$
$c = 132.85$ cm

MBF3C
BLM2.6.2

Frayer Model

Name:
Date:

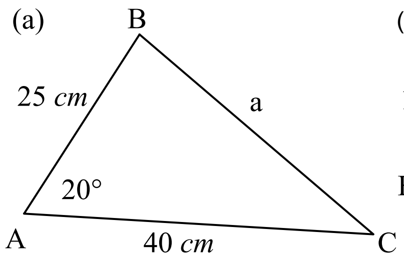
Definition	**Characteristics**
Examples	**Non-examples**

MBF3C Name:
BLM2.6.3 Date:

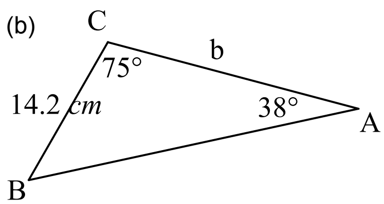
Round ∠'s to whole degrees; length answers should be rounded to 1 decimal place and include units.

1. For each of the following diagrams write the equation you would use to solve for the indicated variable:

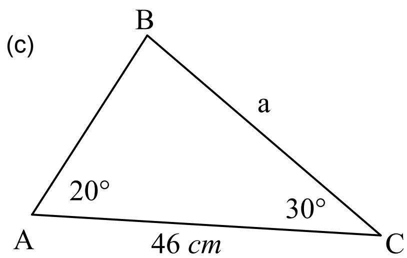
(a)



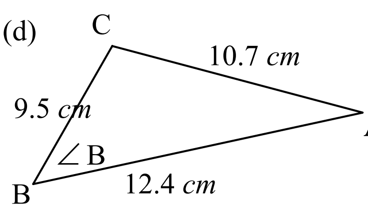
(b)



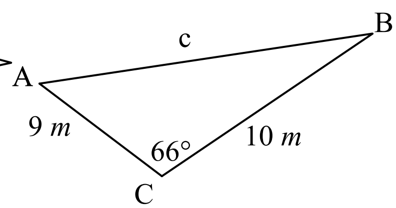
(c)



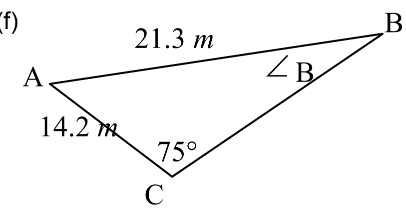
(d)



(e)



(f)



2. Solve for each of the required variables from Question #1.

3. For each of the following triangle descriptions you should make a sketch and then completely solve each triangle.

(a) In ΔABC, given that ∠A = 38°, ∠C = 85°, and c = 32 cm.
(b) In ΔABC, given that ∠A = 24°, b = 12.5 m, and c = 13.2 m.
(c) In ΔABC, given that a = 17 m, b = 18 m, and c = 26 m.
(d) In ΔABC, given that ∠A = 52°, ∠B = 47°, and a = 25 m.
(e) In ΔABC, given that ∠B = 43°, ∠C = 73°, and b = 19 m.
(f) In ΔABC, given that a = 32 m, b = 30 m, and c = 28 m.

Solutions: 1. (a) $a^2 = (40)^2 + (25)^2 - 2(40)(25) \cdot \cos 20°$ **(b)** $\frac{14.2}{\sin 38°} = \frac{b}{\sin 67°}$

(c) $\frac{a}{\sin 20°} = \frac{46}{\sin 130°}$ **(d)** $(10.7)^2 = (9.5)^2 + (12.4)^2 - 2(9.5)(12.4) \cdot \cos \angle B$

(e) $c^2 = (10)^2 + (9)^2 - 2(10)(9) \cdot \cos 66°$ **(f)** $\frac{\sin \angle B}{14.2} = \frac{\sin 75°}{21.3}$

2. (a) 18.6 *cm* (b) 21.2 *cm* (c) 20.5 cm (d) 57° (e) 10.4 *m* (f) 40°

(Note: the following answers have been listed in the optimal order for solving the triangle. If you did not solve your triangle in this order your angle measurements should be within 1° due to rounding differences; side length values should be accurate.**)**

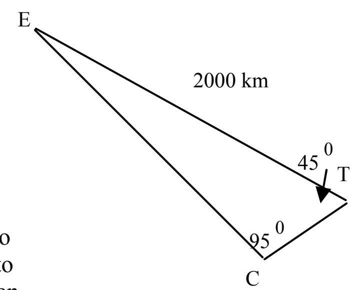
3. (a) ∠**B = 57°, *a* = 19.8 *cm*, *b* = 26.9 *cm*** **(b) *a* = 5.4 *m*, ∠B = 70°, ∠C = 86°**
(c) ∠A = 41°, ∠B = 43°, ∠C = 96° **(d) ∠C = 81°, *b* = 23.2 *m*, *c* = 31.3 *m***
(e) ∠A = 64°, *a* = 25.0 *m*, *c* = 26.6 *m* **(f) ∠A = 67°, ∠B = 60°, ∠C = 53°**

Unit 2 Day 7: Trigonometry – Solving Real-World Problems		**MBF 3C**
	Description This lesson has students solving real-world problems using the Sine Law and the Cosine Law.	Materials BLM2.7.1
		Assessment Opportunities

Minds On…

Pairs → Interpreting a Problem

Write the following example on the board:
Ask students to develop a diagram to illustrate Problem 1:.
David wants to go to Toronto from Edmonton, but he took the wrong road and ended up in Chicago instead. Upon realizing his directional mistake, David drove from Chicago to Toronto. If the angle at Toronto is 45°, the angle at Chicago is 95°, and the distance from Edmonton to Toronto is 2000 km, how much further did David drive than necessary?
Take up the diagram with the class.



Action!

Pairs → Practice

Ask students to finish solving the above problem and Problem 2.
Solution:
The $\angle$ E can be found using the sum of the angles in a triangle: $\angle$ E = 40°

Now use Sine Law to find the remaining two sides:

$$\frac{e}{\sin \angle E} = \frac{t}{\sin \angle T} = \frac{c}{\sin \angle C}$$

$$\frac{e}{\sin 40°} = \frac{t}{\sin 45°} = \frac{2000}{\sin 95°}$$

Break this into two separate equations and solve for e and t.

$$\frac{e}{\sin 40°} = \frac{2000}{\sin 95°}$$
$$e = \frac{2000}{\sin 95°} \cdot \sin 40°$$
$$e = \frac{2000}{0.9961947} \cdot 0.64278761$$

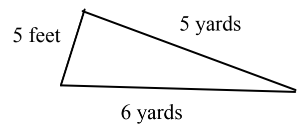
$$\frac{t}{\sin 45°} = \frac{2000}{\sin 95°}$$
$$t = \frac{2000}{\sin 95°} \cdot \sin 45°$$
$$t = \frac{2000}{0.9961947} \cdot 0.707107$$

e = 1290.4859 km t = 1419.6156 km

So David drove a total of 2710.1 km. ∴ he drove 710.1 km more than was necessary.

Problem 2: Jill and her friends built an outdoor hockey rink. Their hockey goal line is 5 feet wide. Jill shoots a puck from a point where the puck is 5 yards from one goal post and 6 yards from the other goal post. Within what angle must Jill make her shot to hit the net?

Solution:



If we make the position where Jill is standing A and the goalposts B and C then in this case we can use the Cosine Law to solve for the angle.

$a^2 = b^2 + c^2 - 2bc \cdot \cos A$

$(5)^2 = (15)^2 + (18)^2 - 2(15)(18) \cdot \cos A$

$25 = 225 + 324 - 540 \cdot \cos A$

$25 - 225 - 324 = -540\cos A$

$-524 = -540\cos A$

$.9703703704 = \cos A$

$\cos A = 0.99038462$

$\angle A = 13.98$

$\therefore$ Jill must shoot within an angle of about 14° to hit the net.

Consolidate Debrief

Pair/Group→ Share

Ask a pair of students to share their solution with another pair. As a whole class discuss any problems and share model solutions..

Application

Home Activity or Further Classroom Consolidation

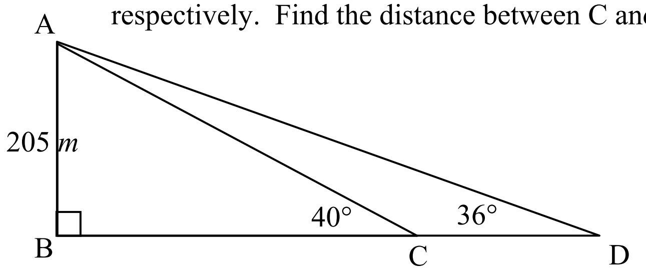
Students complete BLM2.7.1

MBF3C Name:
BLM2.7.1 Date:

If diagrams are not included in any of the following questions it is advisable to sketch a diagram to aid in your solution to the problem. Round $\angle$'s to a whole degrees; length answers should be rounded to 1 decimal place and include units.

1. A squash player hits the ball 2.3 m to the side wall. The ball rebounds at an angle of 100^{o} and travels 3.1 m to the front wall. How far is the ball from the player when it hits the front wall? (*Assume the player does not move after the shot*.)

2. A smokestack, **AB**, is 205m high. From two points C and D on the **same side** of the smokestack's base B, the angles of elevation to the top of the smokestack are 40^{o} and 36^{o} respectively. Find the distance between C and D. (*Diagram included*.)



3. Trina and Mazaheer are standing on the same side of a Red Maple tree. The angle of elevation from Mazaheer to the tree top is 67° and the angle of elevation from Trina to the tree top is 53°. If Mazaheer and Trina are 9.3 feet apart and Mazaheer is closer to the tree than Trina, how tall is the tree?

4. Two roads separate from a village at an angle of 37°. Two cyclists leave the village at the same time. One travels 7.5 km/h on one road and the other travels 10.0 km/h on the other road. How far apart are the cyclists after 2 hours?

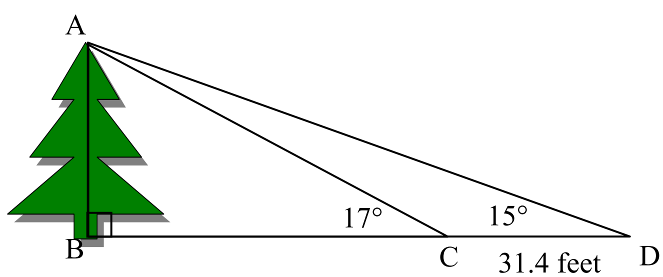
5. A pilot is flying from Thunder Bay, Ontario to Dryden, Ontario, a distance of approximately 320 km. As the plane leaves Thunder Bay, it flies 20° off-course for exactly 80 km.

(a) After flying off-course, how far is the plane from Dryden?

(b) By what angle must the pilot change her course to correct the error?

Solutions:

1. 4.2 *m*   2. 37.8 *m*   3. 28.3 *feet*   4. 12.1 *km*
5. (a) 246.4 *km* (b) approximately 26° turn towards Dryden.

Unit 2 Day 8 - Trigonometry – Solving Real World Problems Continued		MBF 3C
	Description This lesson has students solving more real-world problems using the Sine Law and the Cosine Law.	Materials
		Assessment Opportunities
Minds On…	**Individual → Creating Diagram** Write the following example on the board. Ask students to create a picture for the following problem. Clarify the term angle of elevation. Jillian stood at a distance admiring a magnificent Douglas Fir. Jillian measured the angle of elevation to the top of the tree and found it to be 15°. Jillian then walked 31.4 feet closer to the tree. This time the angle of elevation to the top of the tree was 17°. Calculate the height of the tree to the nearest tenth of a metre.	Word wall: Angle of elevation
Action!	**Whole Class → Sharing** Ask for students to share their pictures to the class. Once diagram is established, discuss strategies to solve the problem. **Individual → Practice** Ask students to individually solve the problem.  This problem will require a few steps to complete. Examining the problem we note that there are two main triangles: ΔABC and ΔACD. We need to find the height of the tree (***AB** in ΔABC*) but to find the height of the tree we need more information about ΔABC. So we need to work in ΔACD first. We can use the information we have to find $\angle ACD$ since BD makes a straight line (180°) and we have $\angle ACB$ = 17° we can find that $\angle ACD$ = 163° (180° - 17°) and finally $\angle DAC$ = 2°. So in order to find some information about ΔABC in this case the hypotenuse **AC** we will use the Sine Law to solve for **AC**. In ΔACD we have: $\frac{a}{\sin A} = \frac{d}{\sin D}$ $\frac{31.4}{\sin 2^\circ} = \frac{d}{\sin 15^\circ}$	

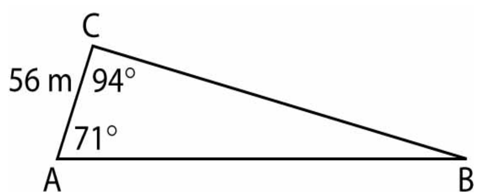
	$d = \dfrac{31.4}{\sin 2°} \cdot \sin 15°$ $d = \dfrac{31.4}{0.0348995} \cdot 0.25881905$ $d = 232.8663386$ feet This value is the length of AC – the hypotenuse of ΔABC. So now we can use the primary trigonometric ratio for Sine to solve for the height of the tree. In ΔABC we have: $\sin C = \dfrac{opposite\ side\ to\ \angle C}{hypotenuse}$ $\sin C = \dfrac{AB}{AC}$ $\sin 17° = \dfrac{tree}{232.9}$ $tree = 232.9 \text{ x } \sin 17°$ $tree = 232.9 \text{ x } 0.2923717$ $tree = 68.0835284$ $\therefore$ The tree is about 68 feet tall.		
Consolidate Debrief	**Whole Class → Review** Any other examples from the last day's homework could be done to help students before giving out today's homework. Also any extra time available could be used for a Quiz or review of any other homework from previous lessons in this unit.		
Application Concept Practice	**Home Activity or Further Classroom Consolidation** Students complete BLM 2.8.1.		

MBF3C Name:
BLM2.8.1 Date:

If diagrams are not included in any of the following questions it is advisable to sketch a diagram to aid in your solution to the problem. Round $\angle$'s to whole degrees; length answers should be rounded to 1 decimal place and include units.

1. To calculate the height of a tree, Marie measures the angle of elevation from a point A to be 34°. She then walks 10 feet directly toward the tree, and finds the angle of elevation from the new point B to be 41°. What is the height of the tree?

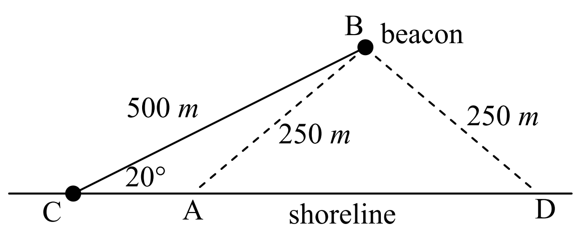
2. To measure the distance from a point A to an inaccessible point B, a surveyor picks out a point C and measures $\angle$BAC to be 71°. He moves to point C, a distance of 56 m from point A, and measures $\angle$BCA to be 94° How far is it from A to B? (*Diagram below.*)



3. A radar tracking station locates an oil tanker at a distance of 7.8 km, and a sailboat at a distance of 5.6 km. At the station, the angle between the two ships is 95°. How far apart are the ships?

4. Two islands **A** and **B** are 5 km apart. A person took a vacation from island **B** and travelled 7 km to a third island **C**. At island **B** the angle separating island **A** and island **C** was 34°. While on this vacation the person decided to visit island **A**. Calculate how far the person will have to travel to get to island **A** from island **C**.

5. The light from a rotating offshore beacon can illuminate effectively up to a distance of 250 m. From a point on the shore that is 500 m from the beacon, the sight line to the beacon makes an angle of 20° with the shoreline. What length of shoreline is effectively illuminated by the beacon? (*i.e. solve for the length of AD in the diagram below.*)



Solutions:

1. 30.1 *feet* 2. 215.8 *m* 3. 10.0 *km* 4. 4.0 *km*

5. HINT: When you solved for $\angle$CAB the angle 43.2° actually is the value for angle(s) $\angle$ADB and $\angle$DAB (*ΔABD is isosceles since AB* = DB ∴ $\angle$ADB and $\angle$DAB) and the result 43.2° is too small for ΔABC's $\angle$CAB (*which is actually 136.8°* ← check sin 136.8° vs sin 43.2°) so the length of shoreline that is effectively illuminated by the beacon 364.5 *m*.