

Unit 5
Representing Vectors

Calculus and Vectors

Lesson Outline

<u>BIG PICTURE</u>			
Students will:			
<ul style="list-style-type: none"> • Introduce vectors in two-space and three-space • Represent vectors geometrically and algebraically • Determine vector operations and properties • Solve problems involving vectors including those arising from real-world applications 			
Day	Lesson Title	Math Learning Goals	Expectations
1	<i>What's the Connection?</i>	<ul style="list-style-type: none"> • Explore connections between calculus and vectors 	
2	<i>What's your Vector Vector?</i> <i>(Sample Lesson Included)</i>	<ul style="list-style-type: none"> • Represent vectors geometrically and algebraically in two-space. • Develop an understanding of equivalent vectors • Use geometric vectors to interpret information arising from real- world applications <p>(Use applets described in Appendix A)</p>	C1.1, 1.2
3	<i>Back and Forth with Vectors</i>	<ul style="list-style-type: none"> • Determine methods for changing from geometric (directed line segment) to algebraic (Cartesian) forms of a vector in two-space and vice versa. 	C1.3
4	<i>Operating with Vectors</i> <i>(Sample Lesson Included)</i> <i>* New – Jan 08</i>	<ul style="list-style-type: none"> • Add, subtract, and multiply vectors by a scalar in two-space, both geometrically and algebraically • Solve problems including problems arising from real-world applications involving vector operations in two-space 	C2.1, 2.3
5	<i>The Dot Product</i>	<ul style="list-style-type: none"> • Determine the dot product of vectors in two-space geometrically and algebraically • Describe applications in two-space of the dot-product including projections 	C2.4
6		<ul style="list-style-type: none"> • Jazz Day (Use applets described in Appendix A) 	C1.4, 2.1, 2.3
7		<ul style="list-style-type: none"> • Summative Assessment 	
8	<i>Let's Go 3D</i> <i>(Sample Lesson Included)</i> <i>* New – Jan 08</i>	<ul style="list-style-type: none"> • Represent both points and vectors algebraically in three-space • Determine the distance between points and the magnitude of vectors in three-space both geometrically and algebraically • Solve problems including problems arising from real-world applications involving vector operations in three-space 	C1.4, 2.1, 2.3
9	<i>The Laws of Vectors</i> <i>(Sample Lesson Included)</i> <i>* New – Jan 08</i>	<ul style="list-style-type: none"> • Investigate, with and without technology, the commutative, associative and distributive properties of the operations of addition, subtraction and multiplication by a scalar in two and three-space (Use Vector Laws applet described in Appendix A) 	C2.2

10	<i>3D Dot Product</i>	<ul style="list-style-type: none"> Determine the dot product of vectors in three-space geometrically and algebraically Describe applications in three-space of the dot-product including projections 	C2.4
11	<i>More on Dot Product</i>	<ul style="list-style-type: none"> Determine through investigation the properties of dot product in two and three space 	C2.5
12	<i>The Cross Product</i>	<ul style="list-style-type: none"> Determine the cross product of vectors in three-space algebraically including magnitude and describe applications 	C2.6
13	<i>More on Cross Product</i>	<ul style="list-style-type: none"> Through investigation, determine properties of the cross product of vectors 	C2.7
14	<i>Putting it All Together</i>	<ul style="list-style-type: none"> Solve problems arising from real-world applications that involve the use of dot products, cross products, including projections 	C2.8
15	<i>Jazz Day</i>		
16	Summative Assessment		

Unit 5: Day 4: Operating with Vectors		
Minds On: 5	Learning Goal: <ul style="list-style-type: none"> Add, subtract, and multiply vectors by a scalar in two-space, both geometrically and algebraically Solve problems including problems arising from real-world applications involving vector operations in two-space 	Materials <ul style="list-style-type: none"> GSP BLM 5.4.1 Chart Paper
Action: 50		
Consolidate:20		
Total=75 min		
Assessment Opportunities		
Minds On...	Whole Class → Activity Introduction Explain to students that they will be working individually on an activity using Geometer's Sketchpad. Review construction of vectors in GSP using the following example as well as how the sum and difference of the two vectors may be represented geometrically. Example: Construct $x=[1, 4]$ and $y=[1, 3]$ on the same graph using the Gettothe point sketch Alternatively use the included link to demonstrate the sum and difference of two vectors.	Use the following link as a visual aid to help students understand geometrically and algebraically how to complete various operations involving vectors. www.frontiernet.net/~imaging/vector_calculator.html
Action!	Individual → Investigation Students complete BLM 5.4.1. Curriculum Expectations/Written Work/Rubric: Assess student's demonstration of their learning, using a rubric. Mathematical Process: Connecting, Problem Solving	
Consolidate Debrief	Whole Class → Class Sharing Ask students in groups of three to discuss the key points of the investigation. Call on groups to share responses using a Graffiti strategy. Each group of students records the responses on chart paper. Students can then compare and contrast their group responses with those of other groups.	
<i>Exploration Application</i>	Home Activity or Further Classroom Consolidation Research using the Internet where the knowledge in this lesson is applied, listing examples and providing at least one detailed example. Provide up to three areas of application of this knowledge. Hand in your research next class.	

BLM.5.4.1: Investigating Vector Operations Using *The Geometer's Sketchpad*

Part A: Scalar multiplication

1. Use the Gettothepoint sketch to construct the vector $\mathbf{x} = [4, 3]$
2. Construct each of the following vectors on the same grid.
 $2\mathbf{x}$, $-3\mathbf{x}$ and $-\frac{1}{2}\mathbf{x}$.
3. What do you observe? State any relationships.
4. Construct other vectors like those in 2).
5. Generalize your observations. Use direction, dilates (expands) and shrinks (contracts) in your generalization.

Complete the following statement:

Scalar Multiplication of a Vector

In two dimensions the scalar multiple of a vector $c\mathbf{x}$ produces a vector in the same _____ as \mathbf{x} when c is _____ and opposite to \mathbf{x} when c is _____

BLM.5.4.1: Investigating Vector Operations Using *The Geometer's Sketchpad* (cont.)

Part B: Vector Addition

1. Use the Gettothepoint sketch to construct $\mathbf{x} = [1, 3]$ and $\mathbf{y} = [2, 2]$
2. Construct $\mathbf{x} + \mathbf{y}$.
3. Add $\mathbf{x} + \mathbf{y}$ on the sketch.
4. Construct two other pairs of vectors and sketch their sums.
5. What can you say about the sum of two vectors geometrically?
6. How is this sum determined algebraically?
7. What is an important prerequisite for vector addition? For example, can any two-dimensional vectors be added?

BLM.5.4.1: Investigating Vector Operations Using *The Geometer's Sketchpad* (cont.)

Part C: Vector Subtraction

1. Construct and then subtract the vectors in Part B, 1). Include the difference in your sketch.
2. Subtraction of vectors is written as $\mathbf{x} - \mathbf{y}$. In two dimensions $\mathbf{x} - \mathbf{y} = \mathbf{x} + (-\mathbf{y})$ where $-\mathbf{y}$ represents the scalar multiple $(-1)\mathbf{y}$.
3. Construct and subtract two other pairs of vectors. Where does the difference appear on the plane?
4. How is the difference determined algebraically?
5. What can you say in general about the subtraction of vectors in \mathfrak{R}^2 ?
6. State a prerequisite for vector subtraction.


Unit 5: Day 8: Let's Go 3D		
Minds On: 5	Learning Goals: <ul style="list-style-type: none"> • Represent both points and vectors algebraically in three-space. • Determine the distance between points and the magnitude of vectors in three-space both geometrically and algebraically. • Solve problems including problems arising from real world applications involving vector operations in three-dimensional space. 	Materials <ul style="list-style-type: none"> • Graph paper • BLM 5.8.1 • Chart Paper
Action: 50		
Consolidate:20		
Total=75 min		
Assessment Opportunities		
Minds On...	Whole Class → Discussion Review the algebraic representation of vectors in two-space Discuss that the relationship between the coordinates of the point and a geometrical vector on a line is the algebraic representation of a geometrical vector and that this finding will be extended to three-space. Have students identify the formula for determining magnitude. Probe them to think about how this formula would be adapted for vectors in three-space. Ensure that they understand the use of this formula.	Access link below for excellent description for determining distance and magnitude of vectors in 3D. www.netcomuk.co.uk/~jenolive/vect5.html
Action!	Pairs → Exploration Students will complete BLM 5.8.1. Curriculum Expectations/Written Work/Rubric: Assess student's demonstration of their understanding of vectors, using a rubric.	
Consolidate Debrief	Whole Class → Class Sharing Students share any generalizations they have made in the investigation.	
<i>Exploration Reflection</i>	Home Activity or Further Classroom Consolidation Work in groups of three and use the Internet or Physics courses to find three different problems using the subject matter of this lesson. Examples include calculating the work done by a force moving in a particular direction of a vector in three-space. Your group will present one of the problems to the class tomorrow.	

BLM.5.8.1: Let's Go 3D

1. Draw a coordinate system in three-space as follows.
 - Pick a point as the origin and draw three mutually perpendicular lines through this point. The z-axis will be the vertical line and the x-axis the line pointing towards you.
 - Each point in the plane is an ordered triple of real numbers (a, b, c).
 - To plot each point in space, move a units from the origin in the direction of x, b units in the direction of y and c units in the direction of z.
2. Plot the vectors \hat{i} , \hat{j} , and \hat{k} , where $\hat{i} = (1, 0, 0)$, $\hat{j} = (0, 1, 0)$, and $\hat{k} = (0, 0, 1)$. These vectors are called **unit vectors**.
3. a) Locate the following points:
 $G(-5, 3, 4)$ and $\overline{OG} = 2\hat{i} - 3\hat{j} + 4\hat{k}$

b) Sketch the position vector \overline{OG} in three dimensions, and calculate its magnitude $|\underline{P}|$. (Remember: $|\underline{P}| = \sqrt{x^2 + y^2 + z^2}$)
4. Repeat 3) using two other examples of your own for each part. What can you conclude about how a vector can be written in three-dimensional space?
5. What can you conclude about how its magnitude is calculated?

As in two-dimensional space, when a vector has its initial point at the origin, its tip will be an ordered triple of real numbers as mentioned above which can be used to calculate its magnitude and direction. The ordered triple represents an algebraic vector in three-space.

Unit 5: Day 9: The Laws of Vectors		
Minds On: 5	Learning Goals: <ul style="list-style-type: none"> Investigate without technology, the commutative, associative and distributive properties of the operations of addition, subtraction and multiplication by a scalar in two and three dimensional -space. 	Materials <ul style="list-style-type: none"> BLM 5.9.1 Chart Paper
Action: 50		
Consolidate:20		
Total=75 min		
Assessment Opportunities		
Minds On...	Whole Class → Discussion Ask students to define a law in mathematics. Tell them that the investigation will allow them to discover the laws of vectors. Demonstrate one algebraic “proof” using general component vectors if students have not seen one yet.	
Action!	Pairs Exploration → Assessment Activity Students will complete the investigation. Curriculum Expectations/Written Work/Rubric: Assess students’ demonstration of their learning, using a rubric.	
Consolidate Debrief	Whole Class → Class Sharing As students share their summaries, key points should be clustered in order to generate the key points included on BLM 5.9.1 Teacher Notes.	
<i>Application</i>	Home Activity or Further Classroom Consolidation Exchange your Summary of Vector Laws with a partner and peer-edit each other’s writing.	

BLM.5.9.1: Vector Laws

Name:

Date:

Use $\mathbf{u}=[a, b]$, $\mathbf{v}=[c, d]$ and $\mathbf{w}=[e, f]$ for this investigation.

1. Determine $\mathbf{u} + \mathbf{v}$ algebraically. Determine $\mathbf{v} + \mathbf{u}$. What do you observe?
2. Generalize your observation in 1).
3. Determine $(\mathbf{u} + \mathbf{v}) + \mathbf{w}$ and $\mathbf{u} + (\mathbf{v} + \mathbf{w})$ algebraically. What can you conclude?

Let r and p be real numbers.

4. Find $(rp)\mathbf{u}$ and compare it to $r(p\mathbf{u})$. Write your observations.
5. a) Find $r(\mathbf{u} + \mathbf{v})$ and compare it to $r\mathbf{u} + r\mathbf{v}$.

b) Find $(r + p)\mathbf{u}$. Rewrite the expression to obtain the same result.

c) Write your conclusions for 5a) and b).

BLM.5.9. 1: Vector Laws (continued)

6. What is the result when the vector $\mathbf{0}$ is added to any vector? Demonstrate your reasoning.

7. What is the result when the negative of a vector is added to itself?

8. Write a summary about vector laws based on your findings in this investigation.

Summary:

BLM.5.9. 1: Vector Laws (Teacher's Notes)

Summary

- **Properties of Vector Addition**

$$\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$$

Commutative Law

$$(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{w} + (\mathbf{u} + \mathbf{v})$$

Associative Law

- **Properties of Scalar Multiplication**

$$(rp)\mathbf{u} = r(p\mathbf{u})$$

Associative Law

$$r(\mathbf{u} + \mathbf{v}) = r\mathbf{u} + r\mathbf{v}$$

Distributive Law

$$(r + p)\mathbf{u} = r\mathbf{u} + p\mathbf{u}$$

- **Properties of the zero vector: $\mathbf{0}$**

$$\mathbf{u} + \mathbf{0} = \mathbf{u}$$

Every vector has a negative that satisfies the following condition.

$$\mathbf{u} + (-\mathbf{u}) = \mathbf{0}$$

The laws state that order is unimportant in vector addition and factoring and expanding occur as usual.