## Unit 2 Exploring Derivatives

### Lesson Outline

| Day | Lesson Title | Math Learning Goals | Expectations |
|---|---|---|---|
| 1 | *Key characteristics of instantaneous rates of change(Sample Lesson – TIPS4RM)* | • determine intervals in order to identify increasing, decreasing, and zero rates of change using graphical and numerical representations of polynomial functions<br>• Describe the behaviour of the instantaneous rate of change at and between local maxima and minima | A2.1 |
| 2 | *Patterns in the Derivative of Polynomial Functions (Sample Lesson – TIPS4RM)* | • Use numerical and graphical representations to define and explore the derivative function of a polynomial function with technology,<br>• Make connections between the graphs of the derivative function and the function | A2.2 |
| 3 | *Derivatives of Polynomial Functions (Sample Lesson Included)* | • Determine, using limits, the algebraic representation of the derivative of polynomial functions at any point | A2.3 |
| 4 | *Patterns in the Derivative of Sinusoidal Functions (Sample Lesson Included)* | • Use patterning and reasoning to investigate connections graphically and numerically between the graphs of $f(x) = sin(x)$, $f(x) = cos(x)$, and their derivatives using technology | A2.4 |
| 5 | *Patterns in the Derivative of Exponential Functions (Sample Lesson Included)* | • determine the graph of the derivative of $f(x) = a^x$ using technology<br>• Investigate connections between the graph of $f(x) = a^x$ and its derivative using technology | A2.5 |
| 6 | *Identify "e" (Sample Lesson Included)* | • investigate connections between an exponential function whose graph is the same as its derivative using technology and recognize the significance of this result | A2.6 |
| 7 | *Relating $f(x)= ln(x)$ and $f(x)=e^x$ (Sample Lesson Included)* | • Make connections between the natural logarithm function and the function $f(x)=e^x$<br>• Make connections between the inverse relation of $f(x) = ln(x)$ and $f(x)=e^x$ | A2.7 |
| 8 | *Verify derivatives of exponential functions (Sample Lesson Included)* | Verify the derivative of the exponential function $f(x)=a^x$ is $f'(x)=a^x \ln a$ for various values of $a$, using technology | A2.8 |

| Day | Lesson Title | Math Learning Goals | Expectations |
|---|---|---|---|
| 9 | Jazz Day/ Summative Assessment *(Sample Assessment Included)* | Summative Assessment | |
| 10, 11 | *Power Rule* *(Sample Lesson Included)* **** New – Jan 08*** | • Verify the power rule for functions of the form $f(x) = x^n$ (where n is a natural number)<br>Verify the power rule applies to functions with rational exponents<br>• Verify numerically and graphically, and read and interpret proofs involving limits, of the constant, constant multiple, sums, and difference rules | A3.1, A3.2<br>A3.4 |
| 12 | *Solve Problems Involving The Power Rule* | • determine the derivatives of polynomial functions algebraically, and use these to solve problems involving rates of change | A3.3 |
| 13, 14, 15 | *Explore and Apply the Product Rule and the Chain Rule* | • verify the chain rule and product rule<br>• Solve problems involving the Product Rule and Chain Rule and develop algebraic facility where appropriate | A3.4 A3.5 |
| 16, 17 | *Connections to Rational and Radical Functions* *(Sample Lesson Included)* | • Use the Product Rule and Chain Rule to determine derivatives of rational and radical functions<br>• Solve problems involving rates of change for rational and radical functions and develop algebraic facility where appropriate | A3.4<br>A3.5 |
| 18, 19 | Applications of Derivatives | • Pose and solve problems in context involving instantaneous rates of change | A3.5 |
| 20 | *Jazz Day* *(Sample Lesson Included)* | | |
| 21 | *Summative Assessment* | Added a day | |

# Unit 2: Day 10: The Power Rule

| | | |
|---|---|---|
| Minds On: 10<br>Action: 50<br>Consolidate: 15<br>Total=75 min | **Learning Goals**:<br>Students will<br>• Verify the power rule for functions of the form $f(x) = x^n$, (where *n* is a natural number)<br>• Verify that the power rule applies to functions with rational exponents | **Materials**<br>• graphing calculators<br>• BLM 2.10.1<br>• BLM 2.10.2<br>• BLM 2.10.3<br>• BLM 2.10.4<br>• BLM 2.10.5 |

| | | | |
|---|---|---|---|
| | | | **Assessment Opportunities** |
| **Minds On…** | **Pairs → Activity**<br>Students work in pairs to complete **BLM2.10.1.** | | |
| **Action!** | **Small Groups→ Guided Exploration**<br>Students work in groups of four to complete **BLM2.10.2** and **BLM2.10.3**.<br><br>**Curriculum Expectation/Observation/Mental Note**<br>Observe students and assess their understanding of polynomial functions and derivatives.<br><br>**Mathematical Process Focus:** Problem Solving; Communicating | | |
| **Consolidate Debrief** | **Whole Class→ Debrief**<br>Have students explain, in their own words, the relationship between the derivative, $f'(x)$, and a function of the form $f(x) = x^n$, (where *n* is a natural number).<br><br>**Whole Class→ Teacher Led Discussion**<br>Using **BLM2.10.4** as a guide, demonstrate how the power rule applies to functions with rational exponents. | | |
| | **Home Activity or Further Classroom Consolidation**<br>Complete **BLM2.10.5**. | | |

# BLM 2.10.1: Investigating Binomial Expressions

1. a) Complete the next two lines in PASCAL'S TRIANGLE:.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | | 1 | | | | |
| | | | 1 | | 1 | | | |
| | | 1 | | 2 | | 1 | | |
| | 1 | | 3 | | 3 | | 1 | |
| 1 | | 4 | | 6 | | 4 | | 1 |

b) Explain the pattern that you found in Pascal's Triangle in part a).

2. Use the pattern that you found in Pascal's Triangle to expand each of the following binomial expressions.

$(x + h)^2 =$

$(x + h)^3 =$

$(x + h)^4 =$

$(x + h)^5 =$

$(x + h)^6 =$

3. Find the derivative of each of the following polynomial functions using first principles (using difference quotient)

a) $f(x) = x^2 + 5x + 2$

b) $f(x) = -x^2 - 3x + 1$

# BLM 2.10.2: The Power Rule

1. Determine the derivative function of each function algebraically using $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

a) $f(x) = x$

b) $f(x) = x^2$

c) $f(x) = x^3$

d) $f(x) = x^4$

2. Explain how the derivative functions, $f'(x)$, of each polynomial function in 1) are related to each of the original polynomial functions, $f(x)$.

3. The derivative for the function, $f(x) = x^n$, (where *n* is a natural number) is $f'(x) =$ ____ .

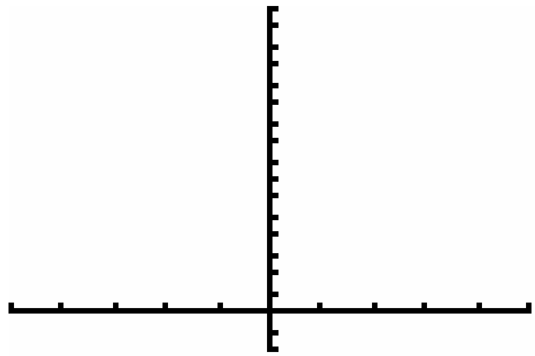
**This is called the Power Rule.**

# BLM 2.10.3: The Power Rule:  A Graphical Approach

1. a) Use a graphing calculator to complete the table of values and sketch the graph of the function $f(x) = x^2$.

$f(x) = x^2$

| *x* | *y* |
|---|---|
| −2 | |
| −1 | |
| 0 | |
| 1 | |
| 2 | |



b) Find the slope of the secant through the points where $x_1 = x$, and $x_2 = x + h$ on the curve $f(x) = x^2$.

$f(x) = x^2$

| *x* | *y* |
|---|---|
| *x* | |
| *x* + *h* | |

*h* ≠ 0

The slope of the secant between the points where $x_1 = x$, and $x_2 = x + h$ on the curve $f(x) = x^2$

is $m = \dfrac{\Delta y}{\Delta x} = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{(x+h)^2 - x^2}{x + h - x} = \dfrac{x^2 + 2hx + h^2 - x^2}{h} = \dfrac{2hx + h^2}{h} = \dfrac{h(2x+h)}{h} =$ ______________

c) Find the slope of the tangent line to the function at the point P(*x*, *y*).

The slope of the tangent line at any point P(*x*, *y*) on the curve $f(x) = x^2$

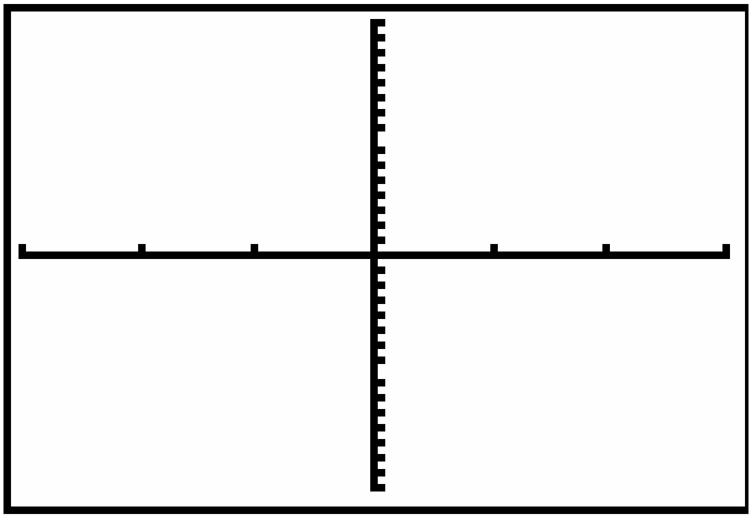
is $m = \lim\limits_{h \to 0} \dfrac{\Delta y}{\Delta x} = \lim\limits_{h \to 0}(2x + h) =$ ______________________

## BLM 2.10.3: The Power Rule:  A Graphical Approach (cont.)

3. a) Use a graphing calculator to complete the table of values and sketch the graph of the function $f(x) = x^3$.

$f(x) = x^3$

| *x* | *y* |
|---|---|
| −2 | |
| −1 | |
| 0 | |
| 1 | |
| 2 | |



b) Find the slope of the secant through the points where $x_1 = x$, and $x_2 = x + h$ on the curve $f(x) = x^3$.

$f(x) = x^3$

| *x* | *y* |
|---|---|
| *x* | |
| *x* + *h* | |

The slope of the secant line between the points where $x_1 = x$, and $x_2 = x + h$ on the curve $f(x) = x^3$

is $m = \dfrac{\Delta y}{\Delta x} = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{(x+h)^3 - x^3}{x + h - x} =$

c) Find the slope of the tangent line to the function at the point P(*x*, *y*).

The slope of the tangent line at any point P(*x*, *y*) on the curve $f(x) = x^3$

is $m = \lim\limits_{h \to 0} \dfrac{\Delta y}{\Delta x} =$

# BLM 2.10.3: The Power Rule: A Graphical Approach (cont.)

4. a) Use the results from 1, 2, and 3 to complete the following chart.

| Function, $f(x)$ | Slope of the tangent line at P($x$, $y$) | Derivative, $f'(x)$ |
|---|---|---|
| $f(x) = x$ | | |
| $f(x) = x^2$ | | |
| $f(x) = x^3$ | | |
| $f(x) = x^4$ | | |
| $f(x) = x^n$ | | |

# BLM 2.10.4: Applying the Power Rule

1. Use the power rule to differentiate each of the following functions.
[LEAVE YOUR ANSWERS IN UNSIMPLIFIED FORM.]

a) $f(x) = x^8$

b) $f(x) = x^{12}$

a) $f(x) = x^{-2}$

b) $f(x) = x^{-5}$

c) $f(x) = x^{\frac{1}{4}}$

d) $f(x) = x^{-\frac{1}{5}}$

2. Express each of the following expressions with fraction exponents.

a) $\sqrt{x}$

b) $\sqrt[3]{x}$

c) $\left(\sqrt[5]{x}\right)^2$

d) $\left(\sqrt[2]{x}\right)^7$

3. Use the power rule to differentiate each of the following functions.

a) $f(x) = \sqrt{x}$

b) $f(x) = \sqrt[3]{x}$

c) $f(x) = \left(\sqrt[5]{x}\right)^2$

d) $f(x) = \left(\sqrt[2]{x}\right)^7$

# BLM 2.10.5: Using the Power Rule

1. Use the power rule to differentiate each of the following functions.
   [LEAVE YOUR ANSWERS IN UNSIMPLIFIED FORM.]

| Function, $f(x)$ | Derivative, $f'(x)$ |
|---|---|
| a) $f(x)=x^{10}$ | |
| b) $f(x)=x^{13}$ | |
| c) $f(x)=x^{-8}$ | |
| d) $f(x)=x^{-4}$ | |
| e) $f(x)=x^{\frac{1}{5}}$ | |
| f) $f(x)=x^{\frac{1}{9}}$ | |
| g) $f(x)=x^{-\frac{1}{4}}$ | |
| h) $f(x)=x^{-\frac{1}{3}}$ | |
| i) $f(x)=\left(\sqrt[5]{x}\right)^3$<br>$f(x)=$ | |
| j) $f(x)=\left(\sqrt[4]{x}\right)^7$<br>$f(x)=$ | |

2. Use the power rule and show the steps to determine that the derivative, $f'(x)$, of the function $f(x)=\sqrt{x}$ is $f'(x)=\frac{1}{2\sqrt{x}}$.

| **Unit 2: Day 11: Differentiation: Operations on Functions** | **MCV4U** |
|---|---|
| Minds On: 10<br>Action: 45<br>Consolidate:20<br>Total=75 min<br>**Learning Goals**:<br>Students will<br>• Verify numerically and graphically, and read and interpret proofs involving limits of the constant, constant multiple, sums, and difference rules | **Materials**<br>• graphing calculators<br>• BLM2.11.1<br>• BLM2.11.2<br>• BLM2.11.3<br>• BLM2.11.4<br>• BLM2.11.5 |

| | | | **Assessment Opportunities** |
|---|---|---|---|
| **Minds On…** | **Pairs → Activity**<br>Students work in pairs to complete **BLM2.11.1**.<br><br>**Curriculum Expectations/Observation/Mental Note:**<br>Observe students and assess their understanding of polynomial functions. | | |
| **Action!** | **Small Groups→ Guided Investigation**<br>Students will work in small groups to complete **BLM2.11.2, BLM2.11.3**, and **BLM2.11.4** in order to verify numerically and graphically and algebraically using $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ the constant, constant multiple, sum and difference rules.<br><br>**Mathematical Process Focus:** Selecting Tools and Computational Strategies | | |
| **Consolidate Debrief** | **Whole Class→ Teacher Led Discussion**<br>Have students share their findings and demonstrate the proofs involving limits algebraically using $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ of the constant, constant multiple, sum and difference rules.<br>Students will write a summary of the constant, constant multiple, sum and difference rules in their mathematics journals.<br><br>**Curriculum Expectations/Journal/Rubric**:<br>Collect student journals and assess their understanding of the curriculum content. | | |
| *Practice* | **Home Activity or Further Classroom Consolidation**<br>Complete **BLM2.11.5**. | | |

# BLM 2.11.1: From Power to Sum

1. Differentiate using the power rule.

a) $f(x) = x^4$

b) $f(x) = x^{13}$

c) $f(x) = x^{-3}$

d) $f(x) = x^{\frac{2}{3}}$

2. Simplify.

a) $5(2x)$

b) $3(5x^4)$

c) $-4(-3x^{-4})$

d) $-3(-x^{-2})$

3. Given the functions: $f(x) = x^3$ and $g(x) = 2x$

a) Determine $p(x) = f(x) + g(x)$

b) Determine $q(x) = f(x) - g(x)$

## BLM 2.11.2: The Constant Rule

1. a) Graph the function $f(x) = -2$ using a graphing calculator. Use **ZOOM 4** to set the parameters for the **WINDOW**.

b) Is the function that you graphed in part a) a horizontal line, or a vertical line?

c) Use the **TRACE** function and the **CALC** function to determine the derivative, $f'(x) = \dfrac{dy}{dx}$, of the function $f(x) = -2$ for the *x*-values −3,−2, −1, 0, 1, 2, 3.

The function $f(x) = -2$ is a _________________ line. The slope of this line is *m* = __________.

| $x$ | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $\dfrac{dy}{dx}$ | = | = | = | = | = | = | = |

d) Repeat part a), part b), and part c) for the functions $f(x) = -1$, $f(x) = 0$, $f(x) = 1$, and $f(x) = 2$.

e) Summarize your findings.

| For the function $f(x) = c$ (where *c* is a constant), the derivative is $f'(x) =$ |
|---|

2. Find the derivatives, $f'(x)$, of each of the following functions algebraically using $\lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

a) $f(x) = -2$

b) $f(x) = 0$

c) $f(x) = 2$

# BLM 2.11.3: The Constant Multiple Rule

**PART A**

1. a) Graph the function $f(x) = -2x$ using a graphing calculator. Use **ZOOM 4** to set the parameters for the **WINDOW**.

b) Use the **TRACE** function and the **CALC** function to determine the derivative, $f'(x) = \frac{dy}{dx}$, of the function $f(x) = -2x$ for the *x*-values −3,−2, −1, 0, 1, 2, 3. Complete the table

The function $f(x) = -2x$ is a _________________ function. The slope of this line is *m* = __________.

| $x$ | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $\frac{dy}{dx}$ | =<br>= −2( ) | =<br>= −2( ) | =<br>= −2( ) | =<br>= −2( ) | =<br>= −2( ) | =<br>= −2( ) | =<br>= −2( ) |

c) Repeat parts a), and b for the functions $f(x) = -x$, $f(x) = x$, and $f(x) = 2x$.

2. Complete the following statement.

| For the function $f(x) = kx$, the derivative is $f'(x) = k($ $\quad$ $) =$ |
|---|

3. Find the derivatives of each of the following functions algebraically using $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$.

a) $f(x) = -2x$

b) $f(x) = x$

c) $f(x) = 2x$

# BLM 2.11.3: The Constant Multiple Rule (cont.)

**PART B**

1. a) Graph the function $f(x) = -2x^2$ using a graphing calculator. Use **ZOOM 4** to set the parameters for the **WINDOW**.

b) Is the function that you graphed in part **a)** a parabola opening upward or a parabola opening downward?

c) Use the **TRACE** function and the **CALC** function to determine the derivative, $f'(x) = \frac{dy}{dx}$, of the function $f(x) = -2x^2$ for the *x*-values −3,−2, −1, 0, 1, 2, 3.

| *x* | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $\frac{dy}{dx}$ | =<br>= −2( )<br>= −2(2( )) | =<br>= −2( )<br>= −2(2( )) | =<br>= −2( )<br>= −2(2( )) | =<br>= −2( )<br>= −2(2( )) | =<br>= −2( )<br>= −2(2( )) | =<br>= −2( )<br>= −2(2( )) | =<br>= −2( )<br>= −2(2( )) |

d) Repeat part a), part b), and part c**)** for the functions $f(x) = -x^2$,, and $f(x) = 2x^2$.

e) **C**omplete the following statement.

**The Constant Multiple Rule:**

| |
|---|
| For the function $f(x) = kx^2$, the derivative is $f'(x) = k$(2( )) = |

2**.** Find the derivatives of each of the following functions algebraically using $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$.

a) $f(x) = -2x^2$

b**)** $f(x) = x^2$

c**)** $f(x) = 2x^2$

# BLM 2.11.4: The Sum Rule

1. Given the functions: $f(x)=x^2$ and $g(x)=3x$

a) Determine the derivatives of $f(x)$ and $g(x)$.

$f(x)=x^2$
$f'(x)=$

$g(x)=3x$
$g'(x)=$

b) Determine $f'(x)+g'(x)$.

c) Given that $h(x)=f(x)+g(x)$, determine $h'(x)$.

d) Hypothesize a relationship between $h'(x)$ and $f'(x)+g'(x)$.

f) Graph the functions $h'(x)$ and $f'(x)+g'(x)$ from part b**)** and part c) in the same viewing screen on a graphing calculator to check your hypothesis.

g) Find the derivative $h'(x)$ of the function $h(x)$ in part c) using $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

**The Sum Rule**

| For the function $h(x)=f(x)+g(x)$, the derivative is $h'(x)=$ |
|---|

# BLM 2.11.4: The Difference Rule

1. Given the functions: $f(x) = x^2$ and $g(x) = 4x$
   a) Determine $f'(x) - g'(x)$.

   b) Write an expression for the function $h(x) = f(x) - g(x)$

   c) Determine $h'(x)$.

   d) Hypothesize a relationship between $h'(x)$ and $f'(x) - g'(x)$.

   e) Graph the functions $h'(x)$ and $f'(x) - g'(x)$ on the same viewing screen on a graphing calculator to check your hypothesis.

   f) Find the derivative $h'(x)$ of the function $h(x)$ in part c) using $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

2. Complete the steps in 1) for the functions: $f(x) = 4x^2$ and $g(x) = 3x$.

Complete the following statement.

**The Difference Rule:**

| |
|---|
| For the function $h(x) = f(x) - g(x)$, the derivative is $h'(x) =$ |

# BLM 2.11.5: Home Activity: Differentiation Practice

1. Use the constant, constant multiple, sum and difference rules to differentiate each of the following functions.

   [LEAVE YOUR ANSWERS IN UNSIMPLIFIED FORM.]

| **Function,** $f(x)$ | **Derivative,** $f'(x)$ |
|---|---|
| a) $f(x)=4x^2+3x$ | |
| b) $f(x)=8x^5-5x^3$ | |
| c) $f(x)=3x^2+7x+2$ | |
| d) $f(x)=x^3-5x^2-4x$ | |
| e) $f(x)=9$ | |
| f) $f(x)=\frac{2}{3}x^7+\frac{3}{5}x^3$ | |
| f) $f(x)=5x^{-3}-4x^{-2}$ | |
| g) $f(x)=3x^{\frac{1}{4}}+2x^{\frac{3}{5}}$ | |
| h) $f(x)=4x^{-\frac{1}{5}}-7x^{-\frac{2}{7}}$ | |
| i) $f(x)=\frac{4}{5}x^{-\frac{1}{5}}+\frac{5}{6}x^{-\frac{2}{7}}$ | |
| j) $f(x)=\left(\sqrt[4]{x}\right)^3+\sqrt{x}$<br>$f(x)=$ | |
| k)<br>$f(x)=\frac{2}{3}\left(\sqrt[4]{x}\right)^7-\frac{3}{5}\left(\sqrt[3]{x}\right)^2$<br>$f(x)=$ | |
| l) $f(x)=\left(\sqrt[4]{x}\right)^7$<br>$f(x)=$ $f(x)=$ | |