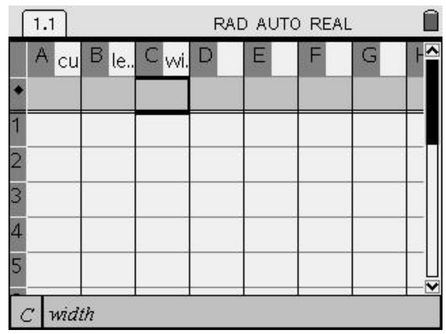
U4L4B – Box Problem - TI Nspire CAS™ Teacher Notes

You are provided with a sheet of metal that measures 80 cm by 60 cm. If you cut congruent squares from each corner, you are left with a rectangle in the centre and four flaps that can fold up to form the sides of a box. Find the size of the square that you need to cut in order to maximize the volume of the box.

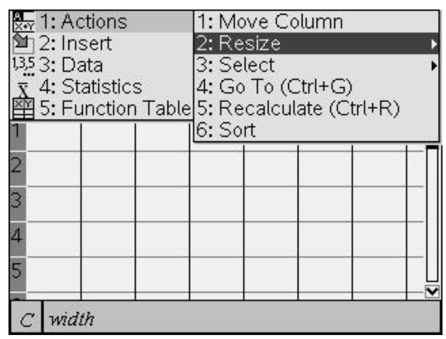
Part 1 – Numerical Investigation

In the first section, you will look at the spreadsheet to investigate the relationship between the cut size and the length and width.

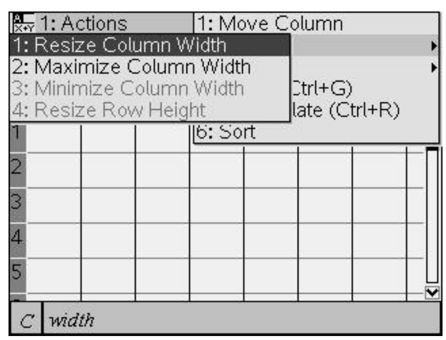
Open a new document and select the Lists & Spreadsheet application. Label columns A, B and C as “cut”, “length” and “width” respectively.



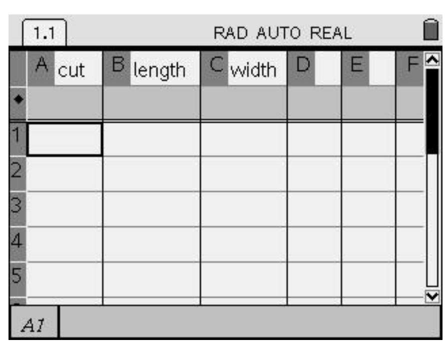
Since the full titles cannot be seen, you need to resize the column. From the **Actions** menu, choose **Resize**.



A sub-menu will pop up. Choose the **Resize Column Width** option.



Use the ▸ (cursor right) key on the Nav Pad to make the column wider. When you have the size that you need, press (enter). Repeat for each column.



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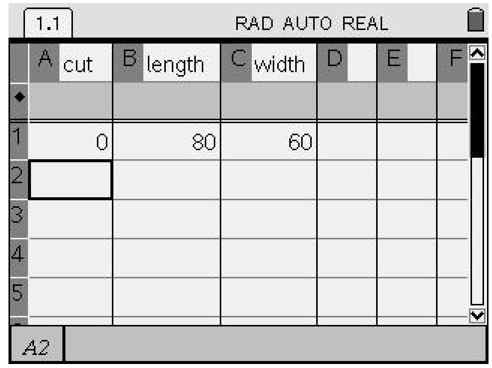
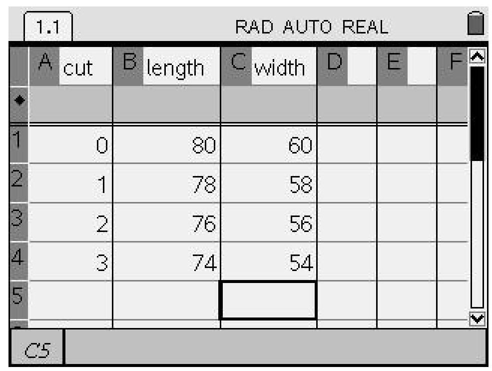
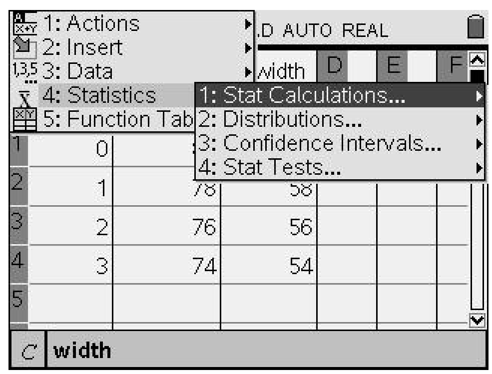
Enter the initial values for the three variables.



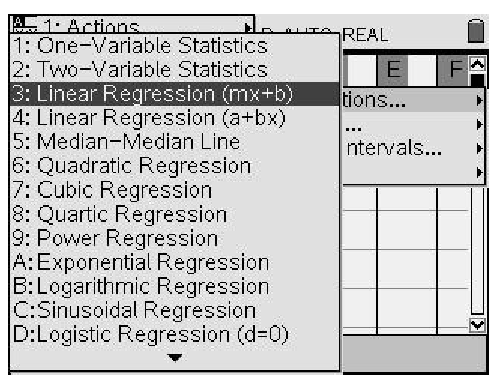
Let the cut size increase by 1 cm. As the size of the square increases by 1 cm, the length and width both decrease by 2 cm. Type the values into the columns as shown.



In order to find the relationship, you will perform a linear regression on length vs. cut and width vs. cut. From the **Statistics** menu, choose **Stat Calculations**.



A sub-menu pops up. Choose **the Linear Regression (mx + b)** option.



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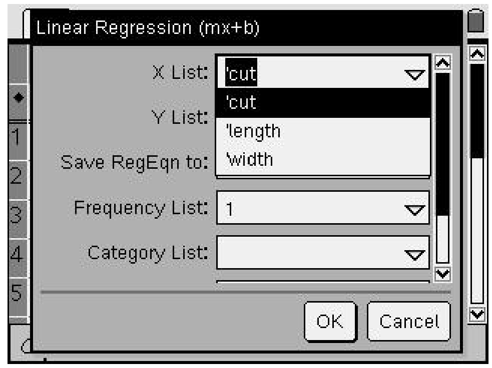
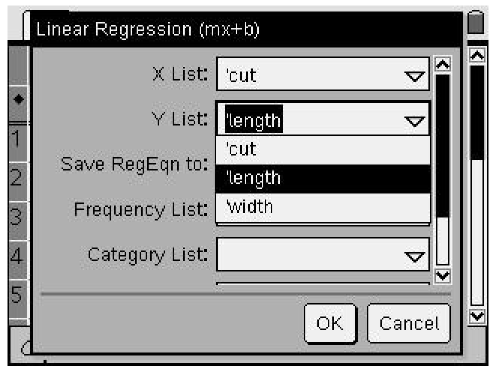
A dialog box will open. To choose the variable for the X List, press . Move to “cut” and press again.

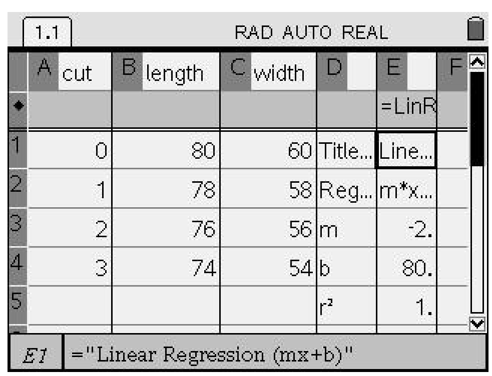


Press tab to move to the next field and choose “length” for the Y List.



Note that the third field indicates where the regression equation will be saved. By default, the device selects the first function that is not in use. Press enter to complete the calculation.

The results are displayed in columns D and E. From the slope and y-intercept fields, you can see that a formula for the length would be $l = 80 - 2x$.



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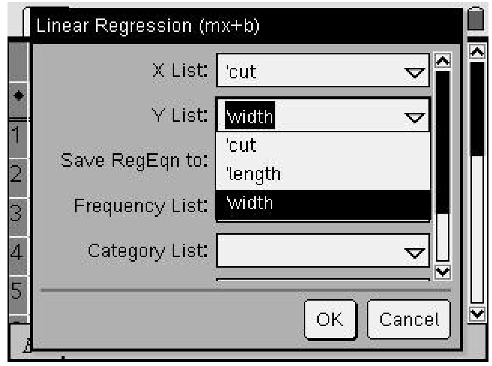
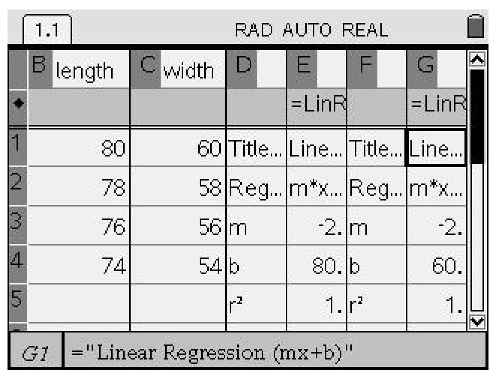
Repeat the regression calculation for the second relationship. Choose “cut” for the X List again and “width” for the Y List.



Note that the regression equation will be saved in *f2(x)*.

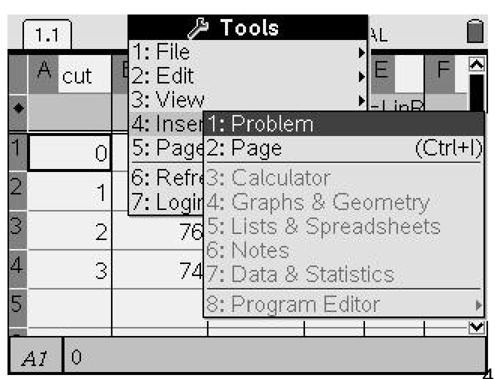
Looking at the results, you conclude that the width can be expressed as $w = 60 - 2x$.



Part 2 – The Algebraic Investigation

Now that you have a relationship between the length and cut size and also for width and cut size, you can use calculus to analyze the function and determine the cut size that will produce the maximum volume.

Press ctrl followed by home to open the **Tools** menu. From the **Insert** menu, choose **Problem**.



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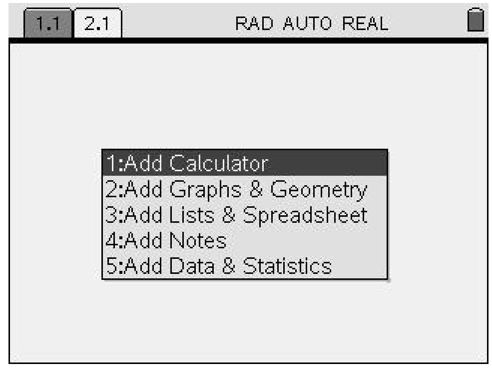
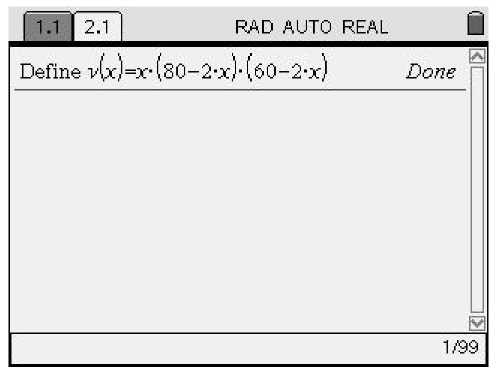
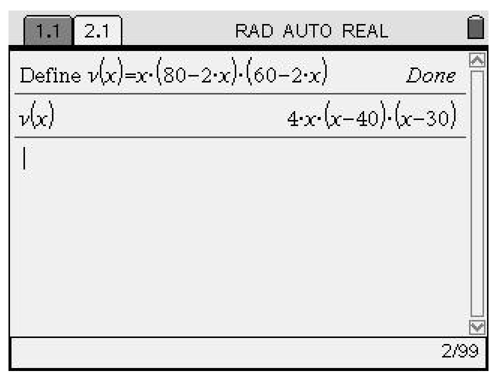
Choose the Calculator application.



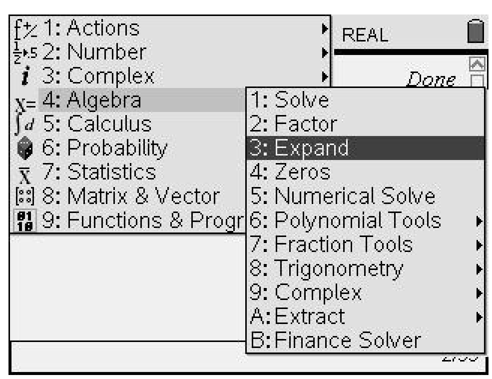
Define a new function, *v(x)*, as the product of the cut size, the length and the width.



Display the function. Note that the CAS does some "simplification".



From the algebra menu, choose Expand.



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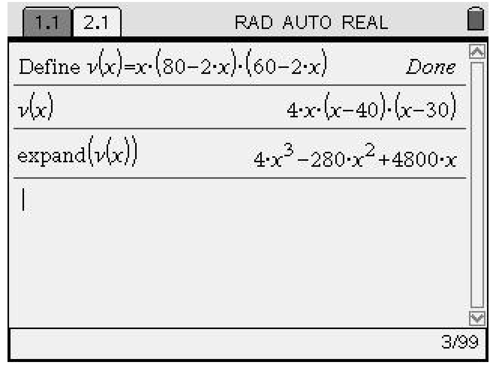
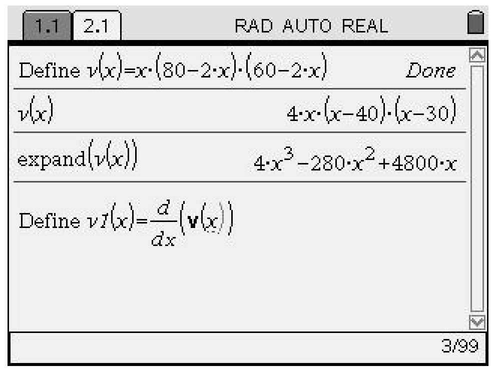
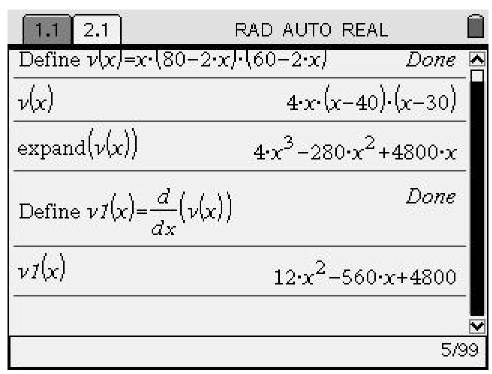
Complete the command as shown. The polynomial form of the function will be displayed.



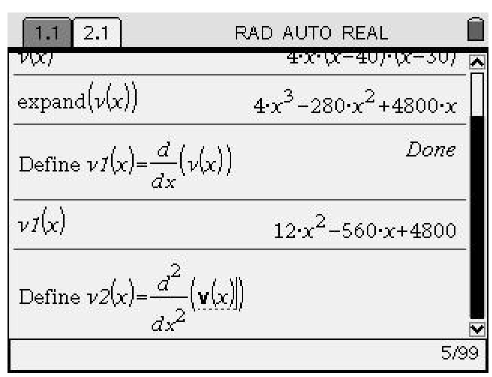
Define a new function, *v1(x),* as the derivative of *v(x)*.



Display the new function.



Before solving for the *x*-coordinates of the turning points, define another new function, v2(x), to hold the second derivative.



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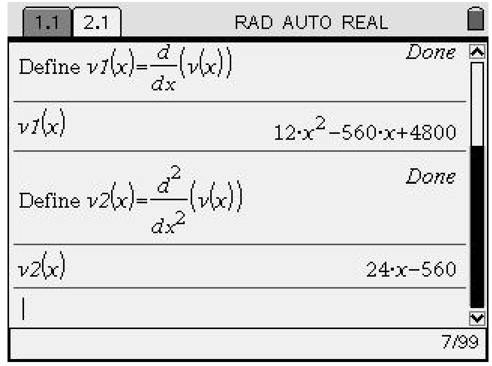
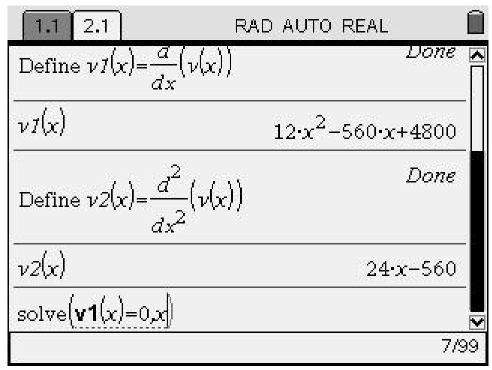
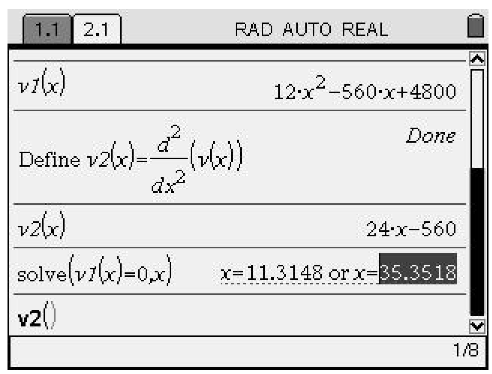
Display the new function.



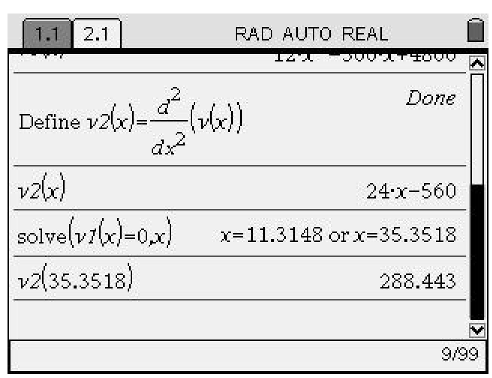
To find the *x*-coordinates of the turning points, use the Solve command to find the roots of the first derivative. In order to get decimal solutions, press ctrl followed by enter.



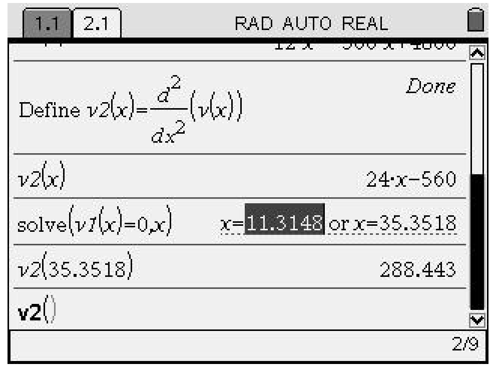
Two solutions are shown which you will test with the second derivative to determine if the value(s) found are appropriate to the context of the problem. On a new line, enter *v2*(), move up to the solutions and highlight the larger root. Press enter and the value will be pasted into the entry line.



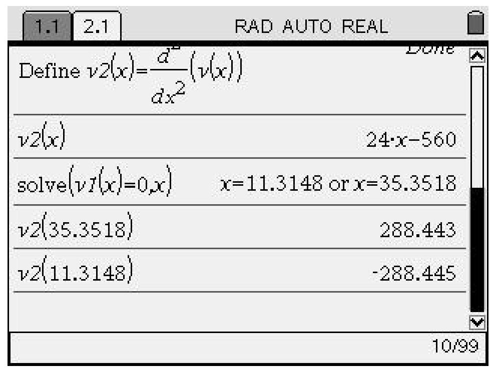
Since the value of the second derivative is positive, you conclude that this value represents the *x*-coordinate of a local minimum. In the context of the problem, you can also exclude this root since it produces a width that is negative.



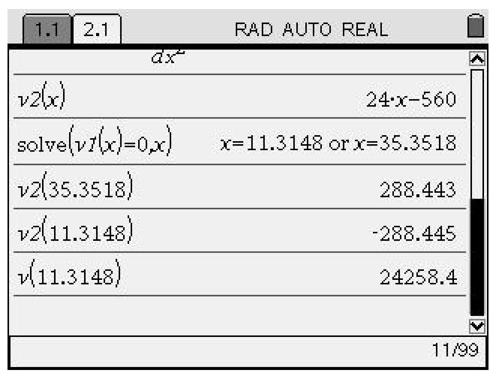
Substitute the smaller root into the second derivative in the same way.



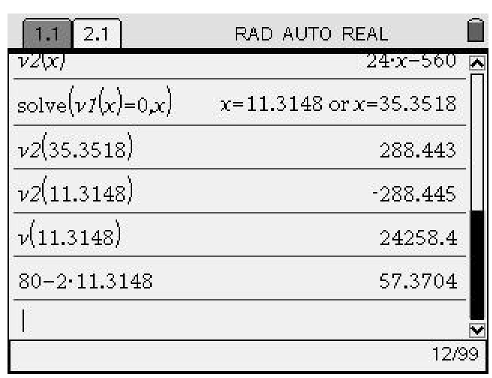
Since the value of the second derivative is negative, you conclude that the root represents the *x*-coordinate of a local maximum. Therefore, a cut size of 11.3 cm produces the maximum volume.



If you substitute this value into the function, you get a value that represents the maximum volume.



If you substitute this value into the expressions for length and width, you will get the other dimensions of the box of maximum volume.



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So, a cut size of 11.3 cm produces a box with a length of about 57.4 cm and a width of about 37.4 cm.