Cubic functions provide all the features that are needed in the analysis of functions. This is the function of lowest degree that will display intercepts, turning points and points of inflection and allows us to use a CAS (computer algebra system) to analyze the function.

Resetting the Device

Before you do anything with the cubic function, reset the device so that you avoid any surprises left by the previous user. Press 2 followed by $\{$ in order to access the \downarrow menu.

Press
to begin the reset process. From the menu that pops up, choose •: Ram.

From the sub-menu that pops up, choose **•**: All Ram. You will have to press ÷ to confirm that you wish to proceed with clearing the memory and press ÷ a second time to proceed with the work that needs to be done.



You will begin by examining a cubic function graphically and estimating the features mentioned above. To enter a function, press ∞ followed by \Box to access the **Y**= window. Enter the function $2x^3 - 6x^2 - 6x - 12$ in Y1. You will need to use the Z key for exponents.

Press O and choose the Graphs application. Alternatively, you could press ∞ followed by \Box .

MEMORY RESET (PK 24 RAM Free Flash ROM fr Enter=0 RAD AUTO MEMORY ash kun 1 M<u>emor</u> RAM Free Flash ROM free Inter=Ok







The screen will automatically change to the graphing

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window and display the function. The graph will appear in the default window, which obviously needs to be changed.

F1+ F2+ F3 F4 F5+ F6+ F7+80 ToolsZoomTraceRe9raphMathDrawPen.c MAIN RAD AUTO FUNC

F1+ F2+ ToolsZoom

xscl=1 ymin= ymax=1 yscl=8 0.553.11

MAIN

×min=⁻5. xmax=5.

Press ∞ followed by \Box to change the window. One window that works well for this function is shown to the right.

With this window, the features of the function are more obvious. From the graph, estimate the coordinates of the y-intercept, the x-intercept(s), the local maximum, the local minimum and the point of inflection. You will return to the graph after the analysis done on the CAS in order to verify your results and check the estimates.

Defining the function

Press \forall to move to the Home application. The computer algebra system resides in this application. In order to clear the history area, press and choose option v: Clear Home.

You will be using the **Algebra** and **Calculus** menus extensively for the analysis of the function, but first you need to use the **Other** menu. Choose the Define option.





RAD AUTO

FUNC

The word "Define" appears on the entry line. Use φ followed by \subseteq to get the letter "f" for the function name. Complete the command by entering the polynomial. Press \div to execute the command. The phrase "*Done*" simply means that the function has been stored in memory and is available to you anytime that you need it.

Analysis of the Function

In order to find the *y*-intercept, you need to set the *x*-value of the function to 0. Enter f(0) and press \div .

To find the *x*-intercept(s), set the function equal to 0 and solve for *x*. On the device, you begin this by accessing the Solve command. Press \bigcirc . From the **Algebra** menu, choose Solve.

The Solve command has two inputs: the equation to be solved and the variable to be solved for. Once those have been entered, press ÷.

Due to the precision displayed on the screen, you are not able to see the third root and some of the digits in the second root. Move up by pressing X and then move to the right by pressing B.

F1+ F2+ ToolsAl9ebr	aCalcOther	FS F Pr9ml0C1ed	67 In Up
		-	
∎Define	f(x) =2	•× ³ -6	× ² −∙ Done
…ine f(: MANN	x)=2x^3- Rad Auto	-6x^2-6; Func	x=12 1/30



∎Define	f(x)=2·	× ³ -6·	ײ-•
			Done
∎f(0)			12
solve(f(x) = 0, :	x)	
× = -1.	52892 or	$\times = 1.1$	1674
solve(f	(x)=0,x)		
MAIN	RAD AUTO	FUNC	3/30

F1

$$3 \times 1^{5}$$
 5×1^{5}

 Tools
 3×1^{5}
 7^{5}

 • Define $f(x) = 2 \cdot x^{3} - 6 \cdot x^{2} - 1^{5}$
 Done

 • f(0)
 12

 • solve(f(x) = 0, x)
 12

 (x = 1.16745 or x = 3.36147)
 501ve(f(x)=0, x)

 MAIN
 RAD AUTO
 FUNC
 1/3

Define a new function f1(x) which will hold the first derivative of f(x). Press (menu). From the **Calculus** menu, choose Derivative.

F1+ F2 Too1s A19el	+ F3+ F4+ F5 F6+ bra <mark>Calc</mark> OtherPr9mlOClean Up
∎Defir	1:d(differentiate 2:J(integrate 3:limit(
∎f(0)	4:Σ(sum 5:∏(product 6:fMin(
■solve 【×=1	7:fMax(8↓arcLen(
Define TYPE OR US	$\frac{f1(x)}{f(x)} = \frac{f1(x)}{f(x)}$

F1+ F2+ F3+ F4+ F5 F6+ Too1sA19ebraCa1cOtherPr9mIOC1ean UP

solve(f(x) = 0, x)

solve(f(x) = 0, x)

■ f(0)

MAIN

•f(0)

• Define $f(x) = 2 \cdot x^3 - 6 \cdot x^2 - 1$

∢x = 1.16745 or x = 3.36147

Define f1(x)=d(f(x),x) MAIN RAD AUTO FUNC

F1+ F2+ F3+ F4+ F5 F6+ ToolsAl9ebraCalcOtherPr9mlOClean Up

Done

12

3/30

12

The syntax for the derivative command is shown on the entry line. Two inputs are required – the function and variable for the derivative.

Press ÷ to execute the command. It's in	nteresting to
see how the result is displayed.	-

∢ × = 1	.16745 or	~`x=3.	36147
∎Defin	e f1(x)=-	$\frac{d}{d\times}(f(x))$)
			Done
Define	f1(x)=d(f(x), x)	
MAIN	RAD AUTO	FUNC	4/30

To see the expression for the first derivative, type the	Í
function name into the entry line.	١

F1+ F2+ ToolsAl9ebro	F3+ F4+ CalcOtherP	FS F6 r9ml0C1ea	i u ti
<pre>solve(+</pre>	"(x)=⊍,: 16745 or	<) ` x = 3.	36147
• Define	$f1(x) = -\frac{1}{6}$	$\frac{d}{l\times}(f(x))$)
			Done
•f1(x)	6.	× ² - 12	·× - 6
f1(x)			
MAIN	RAD AUTO	FUNC	5/30

When the first derivative is equal to 0, you get the x-coordinates of the local maxima and local minima, or turning points. Use the Solve command to find these values. The default setting of the device displays results in exact form where possible.

F1+ F2+ Tools Algebr	aCalcOtherP	FS F6 r9ml0[C1eal	n Up
∎Define	f1(x) = -0	$\frac{\tilde{v}}{\tilde{i}\times}(f(x))$)
		_	Done
■f1(x)	6.	× ² - 12	·× - 6
solve($F1(\underline{X}) = 0,$, x)	_
× =	-(or x=	12 + 1
solve(f)	(X)=0,X) FUNC	6/20
1. IULIA	NUC 4010	runc	8730

Notice that the previous command re-appears on the entry line. Press the ∞ key followed by \div to display the approximate values or the roots. If you choose to, you can set the defaults for the document to display fewer digits.

F1+ F2+ Too1sA19ebra	F3+ F4+ CalcOtherPi	FS FÉ r9ml0C1ea	it n Up
■f1(x)	6.	× ² - 12	·×-6
■solve(f	$1(\mathbf{x}) = 0,$	X)	_
× =	-(12 - 1)	or ×=	√2 + 1
■ solve(f	1(x) = 0,	×)	41421
solve(f1	(x)=0,x	<u>x-z.</u>	41421
MAIN	RAD AUTO	FUNC	7/30

F1+ F2+ F3+ F4+ F5 F6+ ToolsAlgebraCalcOtherPr9mIOClean Up

 $x = -(\sqrt{2} - 1)$ or $x = \sqrt{2} +$

solve(f1(x) = 0, x)

solve(f1(x) = 0, x)

x = -.414214 or x = 2

or x=2.414213

• f1(x)

309

 $6 \cdot x^2 - 12 \cdot x -$

To find the *y*-coordinates of the two turning points, you should substitute the *x*-coordinates back into the original function. Enter "f(" on the entry line and press \blacktriangle once to highlight the previous results. This will highlight both roots. Press \div and the values will be pasted onto the entry line.

Move the cursor to a point just after the second root. Press the 0 key repeatedly to erase characters to the left of the cursor position. Clean up the first root by removing "x=". Complete the command by adding a right bracket at the end of the command.

Press ÷ to evaluate the function.

Press (to ev	aluate the functior	at this value.
Repeat this pro	ocess for the other	root.

F1+ F2+ F3+ F4+ F5 F6+ ToolsAl9ebraCalcOtherPr9mIOClean UP
<pre>solve(f1(x) = 0, x)</pre>
x =414214 or $x = 2.41421$
f(41421356237309)
13.3137
f(2.4142135623731)
-9.31371
f(2.4142135623731)
MAIN RAD AUTO FUNC 9/30

F1+ F2+ ToolsA19eb	raCalcOtherP	FS FI r9mi0C1ea	67 In Up
•f1(×)	6.	× ² - 12	2·×−6
■ solve((f1(x) = 0),	,×)	_
X÷	= -(√2 - 1)	or ×=	12+1
solve	(f1(x)=0)	, x)	
×=	<u>414214 or</u>	<u>~ ×=2.</u>	41421
f(414	21356237	309)	
MAIN	RAD AUTO	FUNC	7/30

F1+ F2+ ToolsAl9ebr	aCalcOtherP	FS F r9ml0C1e	67 311 UP
solve(F1(x)=0,	, x)	
× =	-(12 - 1)	or x=	= 12 + 1
solve()	f1(x) = 0	, x)	
×=4	14214 or	×=2.	41421
∎f(41	42135623	7309)	
		13	3.3137
f(414)	21356237	309)	
MAIN	RAD AUTO	FUNC	8/30

To see the expression for the second derivative, enter the function name and press $\langle \tilde{\bar{m}} \rangle$.

U3L5 - Curve Sketching TI-89 CAS[™] Teacher Notes (cont.) ۲۰ The first derivative test requires that the values of the first derivative be found on either side of the .

x-coordinate of a turning point. For the smaller root, try x = -0.5 (as a value to the left of the root) and x = -0.4 (as a value to the right of the root). Since the sign of f1(x) changes from positive to negative, you conclude that the turning point at (-0.41, 13.31) is a local maximum.

The other root is x = 2. 41. Test values in the derivative on either side, such as 2.4 and 2.5 Since the sign of the derivative changes from negative to positive, you conclude that the turning point (2.41, -9.31) is a local minimum.

Define the second derivative in function $f_2(x)$. One of the options in the derivative command is to add a third input to indicate the order of the derivative.

Alternatively, you could have defined $f_2(x)$ as the first	
derivative of f1(x).	

ache	NOLES (com.)	
F1+ F2 Too1s A19e	t+ F3+ F4+ braCa1cOtherF	FS FE r3ml0Clea	i-
•f(4	142135623	7309)	
		13	.3137
∎f(2.4	142135623	731)	
		-9.	31371
∎f1(.5)		1.5
■f1(.4)		24
f1(4	2		
MAIN	RAD AUTO	FUNC	11/30

F1+ F2+ Too1sA19ebi	raCalcOther	FS F6 Pr9mIOC1ear	, UP
∎f(2.41	42135623	731)	
		-9.	31371
■f1(5	5)		1.5
∎f1(4	Ð		24
∎f1(2.4	•)		24
∎f1(2.5	0		1.5
f1(2.5)			
MAIN	RAD AUTO	FUNC	13/30

F1+ F2+ ToolsA19ebr	F3+ F4+ aCalcOther	FS FI Pr9mIDClea	67 In UP
■ f(2.414	4213562.	5731) -9.	31371
■f1(5))		1.5
■f1(4))		24
•f1(2.4))		24
f1(2.5))		1.5
Define f	^2(x)=d((f(x), x)	,2)
MAIN	RAD AUTO	FUNC	13/30

F1+ F2+ F3+ F4+ F5 F1 ToolsAl9ebraCalcOtherPr9mIOClea	it n Up
■ f1(2.4)	24
■ f1(2.5)	1.5
• Define $f_2(x) = \frac{d^2}{dx^2} (f(x))$	x))
	Done
Define f2(x)=d(f(x),x, MAIN RADAUTO FUNC	2) 14/30

F1- T0015A	F2+ 19ebra	F3+ Calc	F4+ Other	F5 Pr9mi0	F6+ Clean Up	\square
•f1(2.5)					1.5
■ Def	ine	f2(×) =	$\frac{d^2}{d\times^2}$	(f(x))	
					D	one
•f2(X)				12·×-	- 12
f2(×)					
MAIN		RAD	AUTO	FUN	IC 1	5730

To find points of inflection, set the second derivative equal to 0 and solve for x. This can be accomplished using the Solve command.

To find the *y*-coordinate of the point of inflection, substitute the root into the original function.

To verify that this is a point of inflection, substitute values on either side of the root into the second derivative. In this case, 0.9 and 1.1 have been used. Since the sign of the second derivative changes, you can conclude that the point (1,2) is a point of inflection.

The second derivative can also be used to test the roots found when the first derivative was set equal to 0. Substitute each of these values into the second derivative. In this case, since the sign of the second derivative is negative, the point (-0.41, 13.31) is a local maximum.

In the same way, since the second derivative for x = 2.41 is positive, the point (2.41, -9.31) is a local minimum.

F1+ F2+ Too1sA19ebi	raCalcOther	F5 F9MIDC1ea	iτ n Up
∎Define	e f2(x)=-	$\frac{u}{d^{2}}(f(x))$	x))
	·	*^	Done
■f2(x)		12.	× - 12
solve($f_{2(x)} = 0$,×)	× = 1
■f(1)			2
f(1)			
MAIN	RAD AUTO	FUNC	17/30

F1+ F2+ ToolsAl9ebro	F3+ F4+ CalcOther	F5 F Pr9mi0Clev	10 10 10 10 10 10 10 10 10 10 10 10 10 1
			Done
∎f2(x)		12	× - 12
∎solve(f	2(x) = 0	I, X)	× = 1
∎f(1)			2
∎f2(.9)			-1.2
•f2(1.1)			1.2
f2(1.1)			
MAIN	RAD AUTO	FUNC	19/30

F1+ F2+ ToolsAl9ebro	F3+ F4+ CalcOther	F5 F6 Pr9mIDC1ear	, III)
■solve(f	°2(x) = 0	,×)	× = 1
■f(1)			2
∎f2(.9)			-1.2
■f2(1.1)			1.2
■f2(4)	14213562	237309)	
_		-16	.9706
f2(414	2135623	(7309)	
MAIN	RAD AUTO	FUNC	20/30

F1+ F2 Too1sA19el	+ F3+ F4+ braCalcOtherF	F5 F6 r9ml0C1ear	, nb
∎f2(.9)		-1.2
■f2(1.	1)		1.2
■f2(414213562	27309)	
-		-16	.9706
∎f2(2.	414213562	3731)	
		16	.9706
f2(2.4	142135623	731)	
MAIN	RAD AUTO	FUNC	21/30

Checking the function

Press ∞ followed by \Box to move back to the screen where the function was graphed. Press \Box to bring up the Math menu. Choose ψ for a local maximum. This will place a trace point on the function.



F1+ F2+ F3 F4 F5+ F6+ F7+% ToolsZoomTraceRegraphMathDrawPen

Lower Bound? xc: -.886076

RAD AUTO

F1+ F2+ F3 F4 F5+ F6+ F7+%; Too1sZoomTraceRe9raphMathDrawPen::

MAIN

1

1

yc:11.2143

yc:10.6581

FUNC

Move to a point on the curve that is to the left of the local maximum. Press ÷ to mark the point. You will see a mark appear on the screen above the point and under the menu bar.

Move the cursor to a point to the right of the local	
maximum. Press ÷ to mark this point.	

The cursor will disappear for a few seconds and reappear at the local maximum. The coordinates of the local maximum point will appear at the bottom of the screen.

A similar result appears when the Minimum option is selected from the Math menu.







Another option in the math menu is the Zero feature. This will find an *x*-intercept of the graph using the same approach used for the local maximum and the local minimum.



The *x*-intercept will be displayed. You will need to repeat this for the other two *x*-intercepts.



F1+ F2+ F3 F4 F5+ F6+ F7+% ToolsZoomTraceRe9raph Math DrawPen+

Finally, one last feature is the Inflection option.



Using the same approach, the exact location of the point of inflection will be displayed on the screen.

