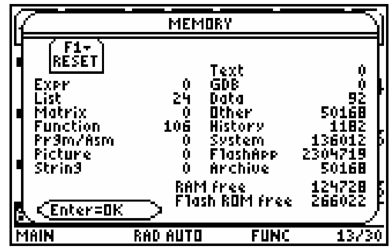
U3L5 - Curve Sketching TI-89 CAS™ Teacher Notes

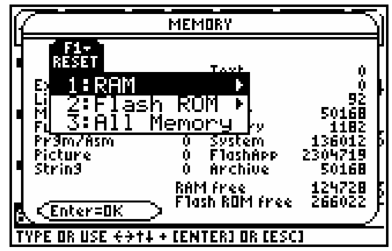
Cubic functions provide all the features that are needed in the analysis of functions. This is the function of lowest degree that will display intercepts, turning points and points of inflection and allows us to use a CAS (computer algebra system) to analyze the function.

Resetting the Device

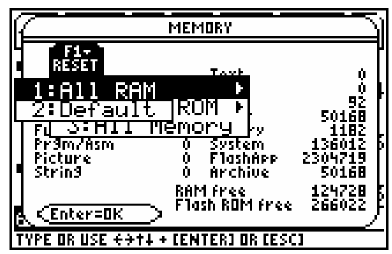
Before you do anything with the cubic function, reset the device so that you avoid any surprises left by the previous user. Press 2 followed by { in order to access the ↓ menu.



Press □ to begin the reset process. From the menu that pops up, choose ♦: Ram.

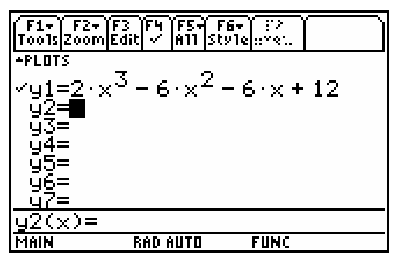


From the sub-menu that pops up, choose ♦: All Ram. You will have to press ÷ to confirm that you wish to proceed with clearing the memory and press ÷ a second time to proceed with the work that needs to be done.

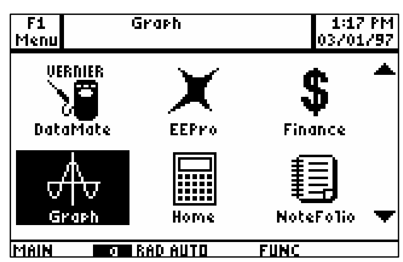


Graphing the Function

You will begin by examining a cubic function graphically and estimating the features mentioned above.To enter a function, press ∞ followed by □ to access the **Y=** window. Enter the function $2x^3 – 6x^2 – 6x – 12$ in Y1. You will need to use the Z key for exponents.



Press O and choose the **Graphs** application. Alternatively, you could press ∞ followed by □.



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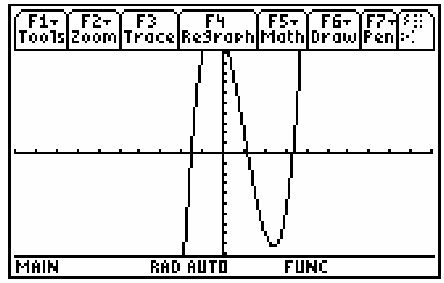
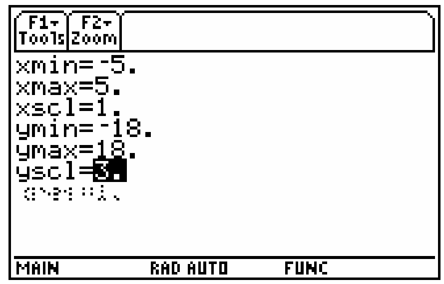
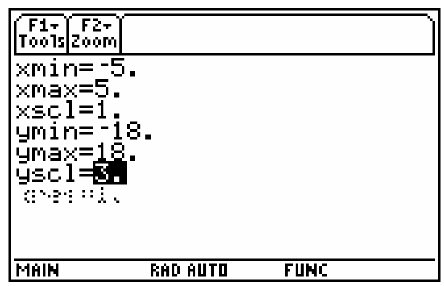
The screen will automatically change to the graphing window and display the function. The graph will appear in the default window, which obviously needs to be changed.



Press ∞ followed by □ to change the window. One window that works well for this function is shown to the right.

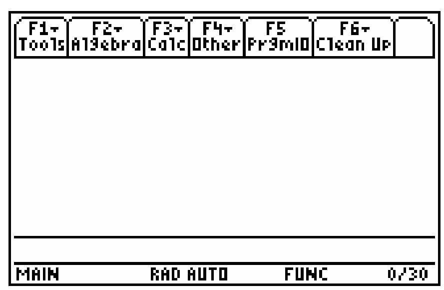


With this window, the features of the function are more obvious. From the graph, estimate the coordinates of the *y*-intercept, the *x*-intercept(s), the local maximum, the local minimum and the point of inflection. You will return to the graph after the analysis done on the CAS in order to verify your results and check the estimates.

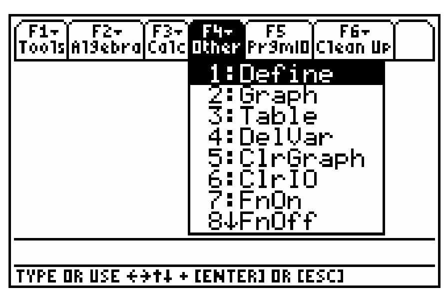


Defining the function

Press ∀ to move to the Home application. The computer algebra system resides in this application. In order to clear the history area, press □ and choose option ν: Clear Home.



You will be using the **Algebra** and **Calculus** menus extensively for the analysis of the function, but first you need to use the **Other** menu. Choose the Define option.



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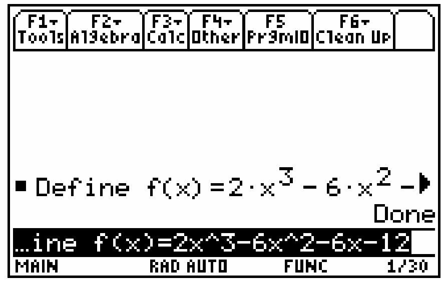
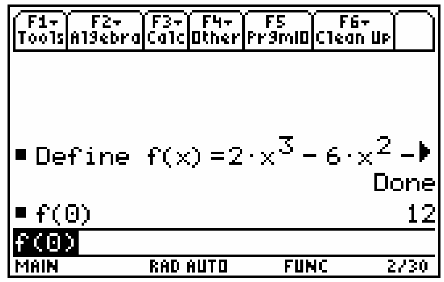
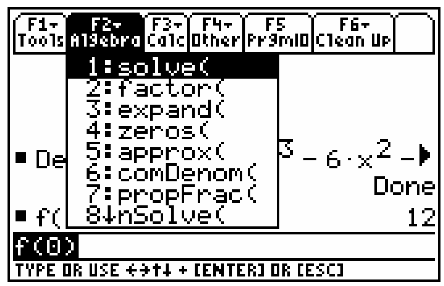


The word “Define” appears on the entry line. Use φ followed by ⊆ to get the letter “f” for the function name. Complete the command by entering the polynomial. Press ÷ to execute the command. The phrase “*Done*” simply means that the function has been stored in memory and is available to you anytime that you need it.

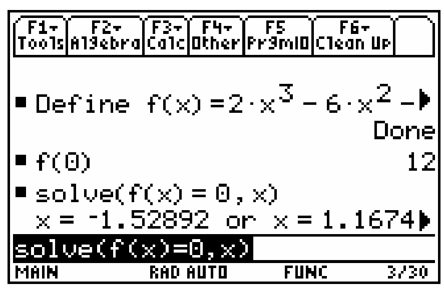
Analysis of the Function



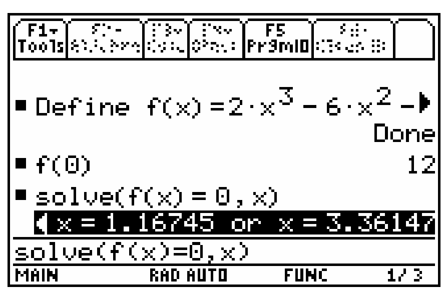
In order to find the *y*-intercept, you need to set the *x*-value of the function to 0. Enter *f*(0) and press ÷.



To find the *x*-intercept(s), set the function equal to 0 and solve for *x*. On the device, you begin this by accessing the Solve command. Press (menu). From the **Algebra** menu, choose Solve.



The Solve command has two inputs: the equation to be solved and the variable to be solved for. Once those have been entered, press ÷.



Due to the precision displayed on the screen, you are not able to see the third root and some of the digits in the second root. Move up by pressing X and then move to the right by pressing B.

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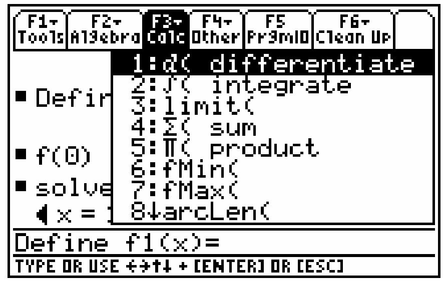
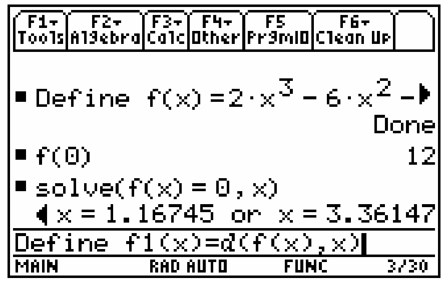
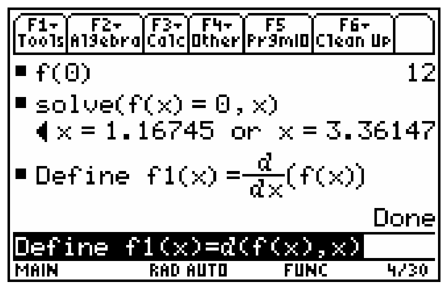
Define a new function f1(x) which will hold the first derivative of f(x). Press (menu). From the **Calculus** menu, choose Derivative.



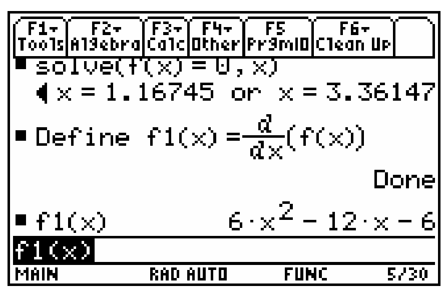
The syntax for the derivative command is shown on the entry line. Two inputs are required – the function and variable for the derivative.



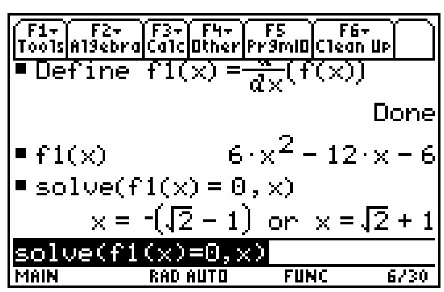
Press ÷ to execute the command. It's interesting to see how the result is displayed.



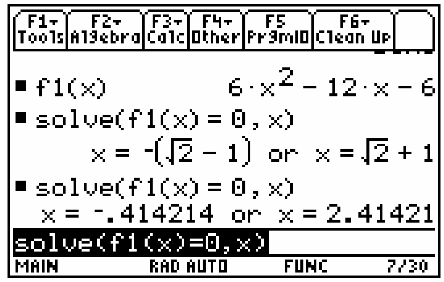
To see the expression for the first derivative, type the function name into the entry line.



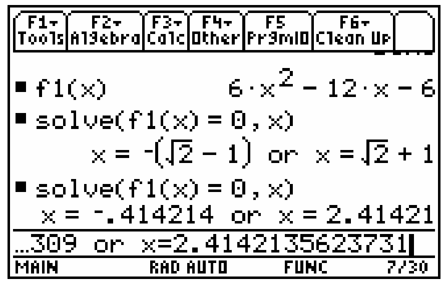
When the first derivative is equal to 0, you get the *x*-coordinates of the local maxima and local minima, or turning points. Use the Solve command to find these values. The default setting of the device displays results in exact form where possible.



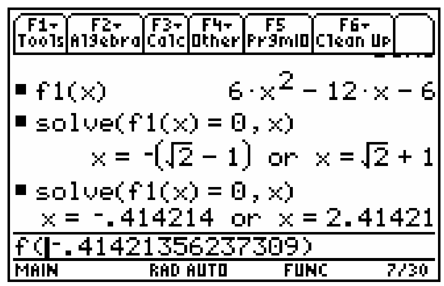
Notice that the previous command re-appears on the entry line. Press the ∞ key followed by ÷ to display the approximate values or the roots. If you choose to, you can set the defaults for the document to display fewer digits.



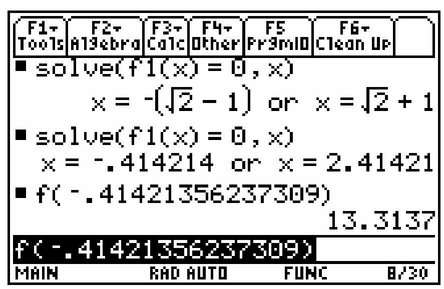
To find the *y*-coordinates of the two turning points, you should substitute the *x*-coordinates back into the original function. Enter "f(" on the entry line and press ▲ once to highlight the previous results. This will highlight both roots. Press ÷ and the values will be pasted onto the entry line.



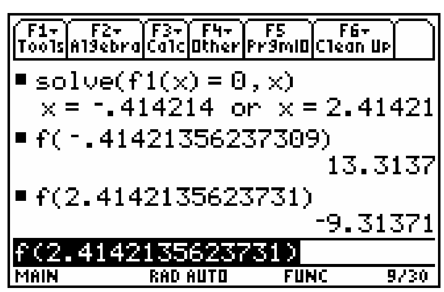
Move the cursor to a point just after the second root. Press the 0 key repeatedly to erase characters to the left of the cursor position. Clean up the first root by removing "x=". Complete the command by adding a right bracket at the end of the command.



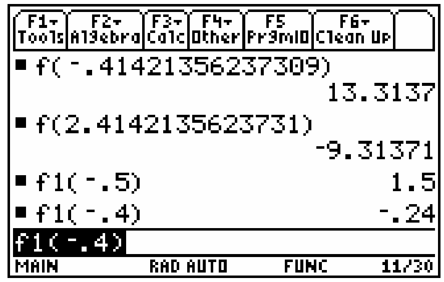
Press ÷ to evaluate the function.



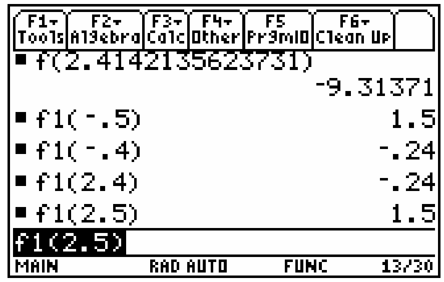
Press (enter) to evaluate the function at this value. Repeat this process for the other root.



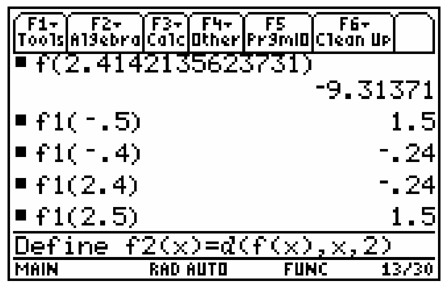
The first derivative test requires that the values of the first derivative be found on either side of the *x*-coordinate of a turning point. For the smaller root, try *x* = -0.5 (as a value to the left of the root) and *x* = -0.4 (as a value to the right of the root). Since the sign of *f1(x)* changes from positive to negative, you conclude that the turning point at (-0.41, 13.31) is a local maximum.



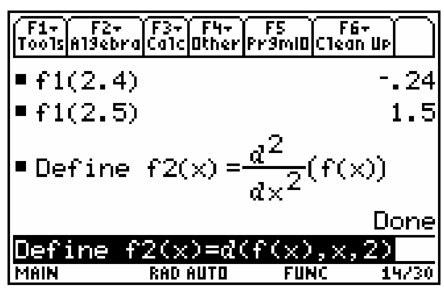
The other root is *x* = 2. 41. Test values in the derivative on either side, such as 2.4 and 2.5 Since the sign of the derivative changes from negative to positive, you conclude that the turning point (2.41, -9.31) is a local minimum.



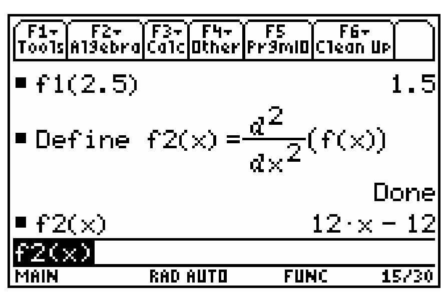
Define the second derivative in function *f2(x)*. One of the options in the derivative command is to add a third input to indicate the order of the derivative.



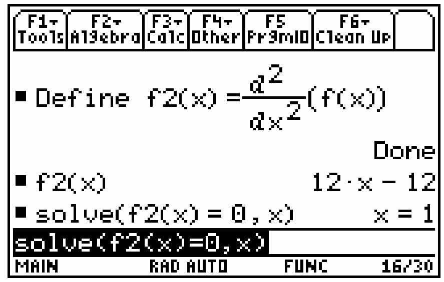
Alternatively, you could have defined *f2(x)* as the first derivative of *f1(x)*.



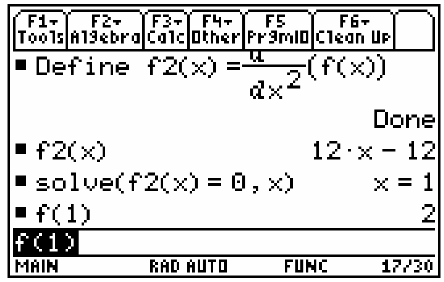
To see the expression for the second derivative, enter the function name and press enter.



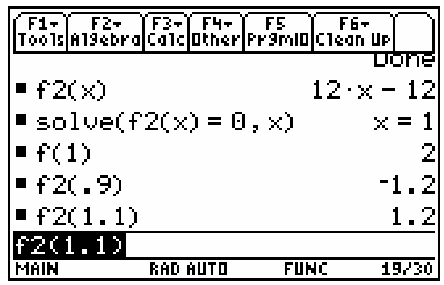
To find points of inflection, set the second derivative equal to 0 and solve for *x*. This can be accomplished using the Solve command.



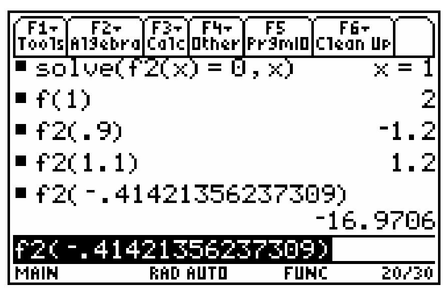
To find the *y*-coordinate of the point of inflection, substitute the root into the original function.



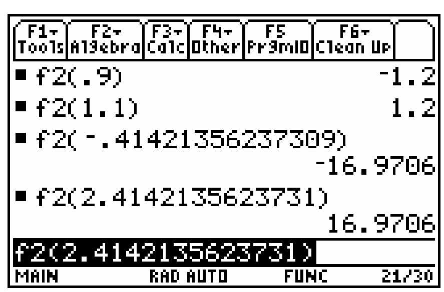
To verify that this is a point of inflection, substitute values on either side of the root into the second derivative. In this case, 0.9 and 1.1 have been used. Since the sign of the second derivative changes, you can conclude that the point (1,2) is a point of inflection.



The second derivative can also be used to test the roots found when the first derivative was set equal to 0. Substitute each of these values into the second derivative. In this case, since the sign of the second derivative is negative, the point (-0.41, 13.31) is a local maximum.

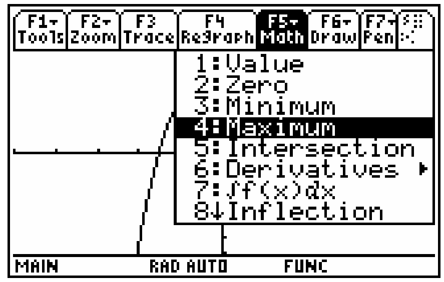


In the same way, since the second derivative for x = 2.41 is positive, the point (2.41, -9.31) is a local minimum.

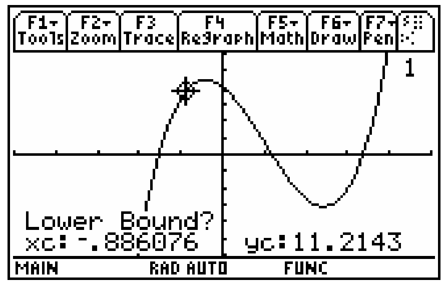


Checking the function

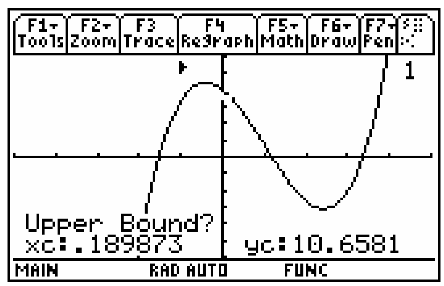
Press ∞ followed by □ to move back to the screen where the function was graphed. Press □ to bring up the Math menu. Choose ψ for a local maximum. This will place a trace point on the function.



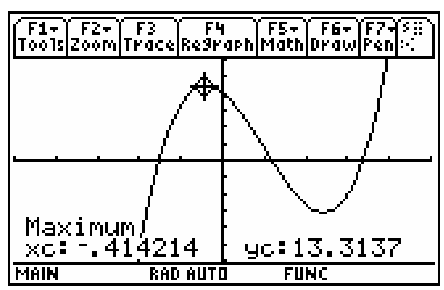
Move to a point on the curve that is to the left of the local maximum. Press ÷ to mark the point. You will see a mark appear on the screen above the point and under the menu bar.



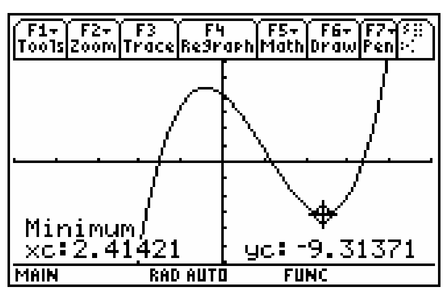
Move the cursor to a point to the right of the local maximum. Press ÷ to mark this point.



The cursor will disappear for a few seconds and re-appear at the local maximum. The coordinates of the local maximum point will appear at the bottom of the screen.



A similar result appears when the Minimum option is selected from the Math menu.



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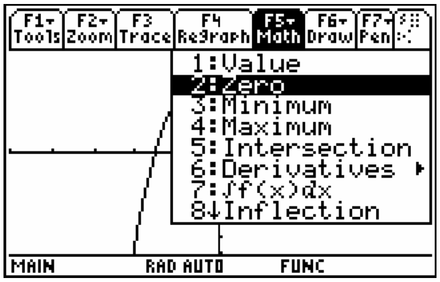
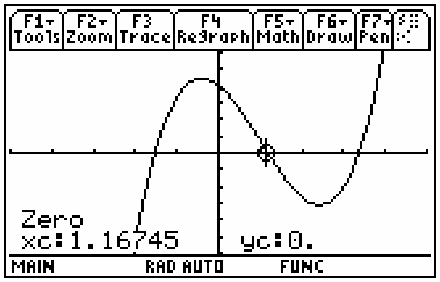
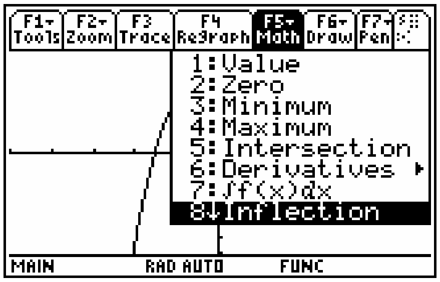
Another option in the math menu is the Zero feature. This will find an *x*-intercept of the graph using the same approach used for the local maximum and the local minimum.



The *x*-intercept will be displayed. You will need to repeat this for the other two x-intercepts.



Finally, one last feature is the Inflection option.



Using the same approach, the exact location of the point of inflection will be displayed on the screen.