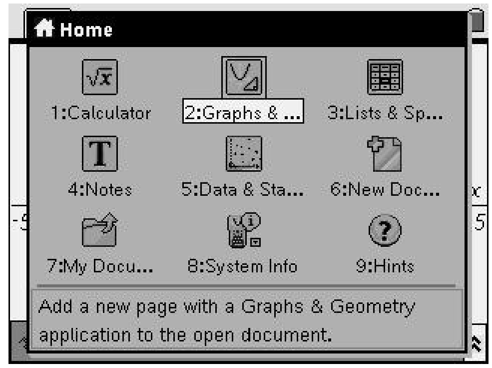
U3L5 - Curve Sketching TI Nspire CAS™ Teacher Notes

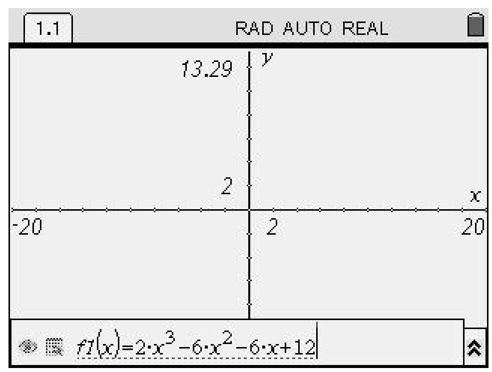
Cubic functions provide all the features that are needed in the analysis of functions. This is the function of lowest degree that will display intercepts, turning points and points of inflection and allows us to use a CAS (computer algebra system) to analyze the function.

Graphing the Function

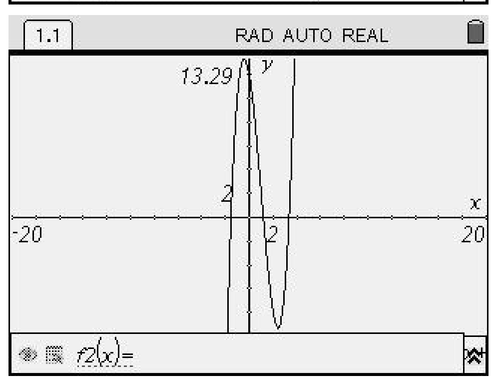
On the TI Nspire CAS™, open a new document. Choose the Graphs & Geometry application. You will begin by examining a cubic function graphically and estimating the features mentioned above.



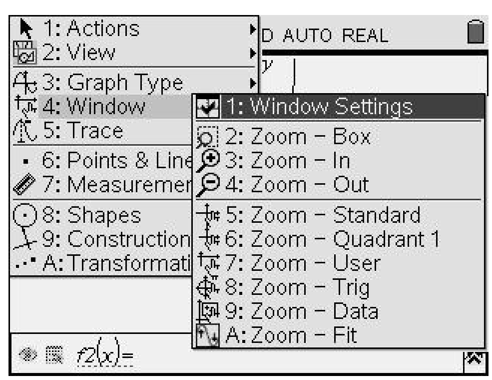
The cursor will automatically appear in the entry line. Enter the expression for the function. Use the button to enter exponents. As you finish each power, press tab to exit exponent mode.



The graph will appear in the default window, which obviously needs to be changed.



Press menu. From the **Window** menu, choose Window Settings.



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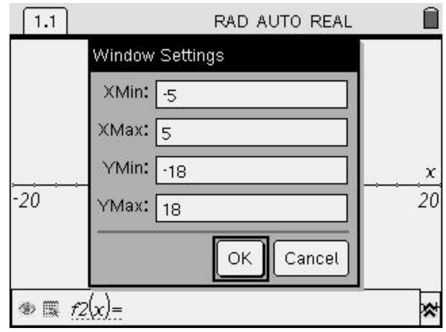
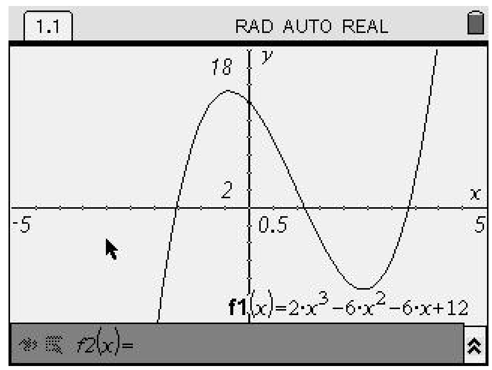
One window that works well for this function is shown to the right.

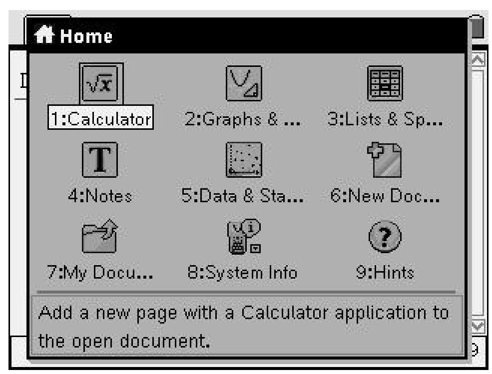


With this window, the features of the function are more obvious. From the graph, estimate the coordinates of the *y*-intercept, the *x*-intercept(s), the local maximum, the local minimum and the point of inflection. You will return to the graph after the analysis done on the CAS in order to verify our results and check the estimates.

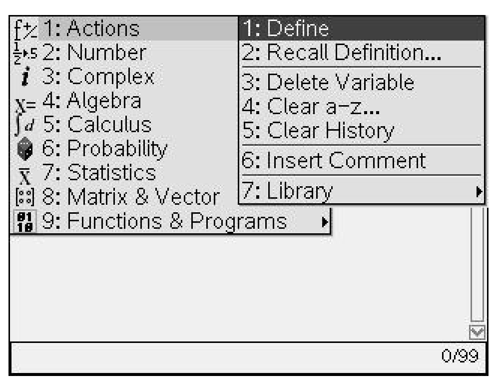


Defining the function

Open a new document and choose the Calculator. The computer algebra system resides in this application.



Press (menu). From the **Actions** menu, choose Define.



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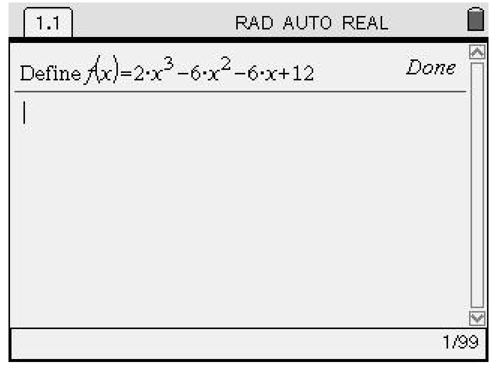
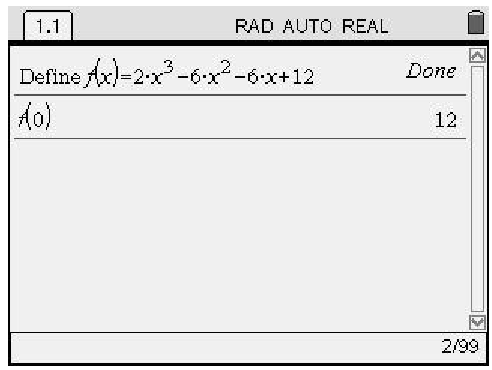
Enter the expression for the function using the ^ button for exponents as before. Press enter to complete the operation. The phrase "*Done*" simply means that the function has been stored in memory and is available to you anytime that you need it.

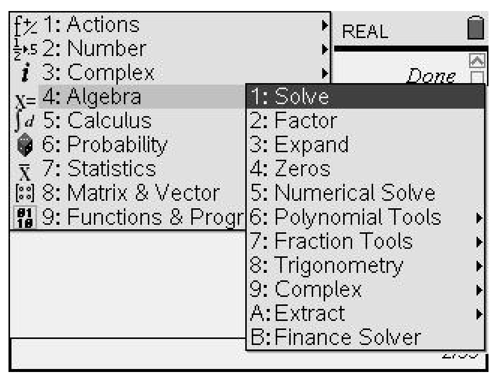


Analyzing the Function

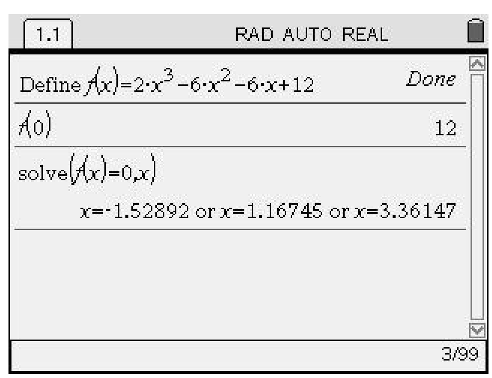
In order to find the *y*-intercept, you need to set the *x*-value of the function to 0. Enter *f*(0) and press enter.



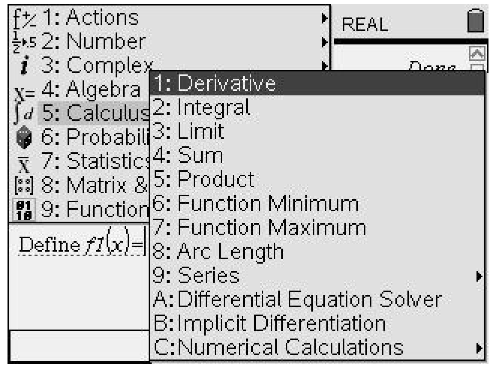
To find the *x*-intercept(s), set the function equal to 0 and solve for *x*. On the device, you begin this by accessing the Solve command. Press menu. From the **Algebra** menu, choose Solve.



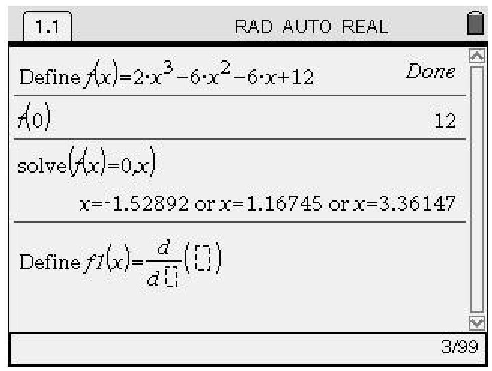
The Solve command has two inputs: the equation to be solved and the variable to be solved for. Once that is entered, press enter.



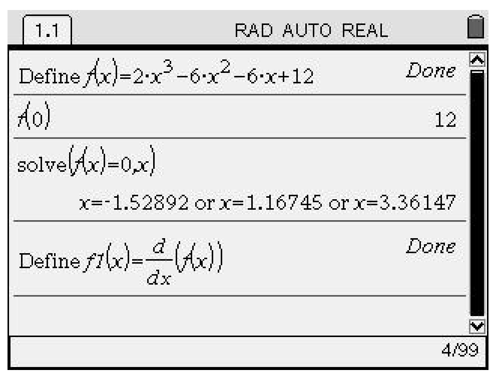
Define a new function *f1(x)* which will hold the first derivative of *f(x)*. Press menu. From the **Calculus** menu, choose Derivative.



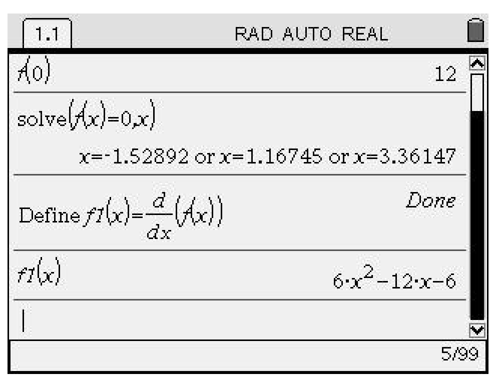
The derivative template will be pasted into the entry line.



Complete each field of the template as shown. Use tab to move from one field to the next. Press enter when the command is complete.



To see the expression for the first derivative, type the function name into the entry line.



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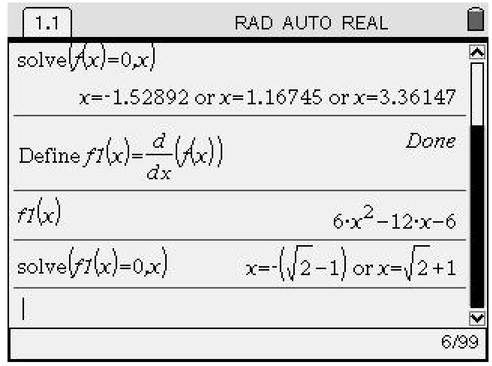
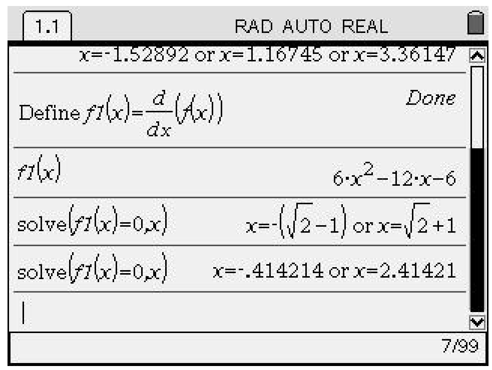
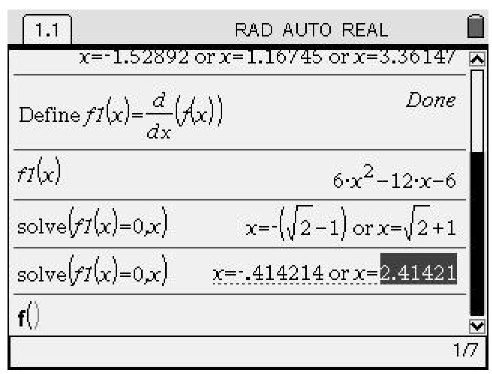
When the first derivative is equal to 0, you get the *x*-coordinates of the local maxima and local minima, or turning points. Use the Solve command to find these values. The default setting of the device displays results in exact form where possible.



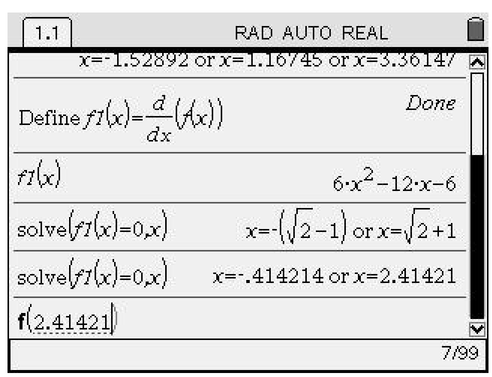
To see approximate (or decimal) values for the roots, press the ▲ button on the Navpad twice. This will highlight the previous Solve command. Press enter to move this command to the entry line. Press ctrl followed by enter to display the approximate values. If you choose to, you can set the defaults for the document to display fewer digits.



To find the *y*-coordinates of the two turning points, you should substitute the *x*-coordinates back into the original function. Enter "f(" on the entry line and press ▲ once to highlight the previous results. This will highlight both roots. Press ▶ or ◀ to remove the highlighting. Move the cursor to a point just before the last root. Press and hold the shift key while pressing the ▶ key. Continue this until only the one root is highlighted.



Press enter and the value will be inserted into the function.



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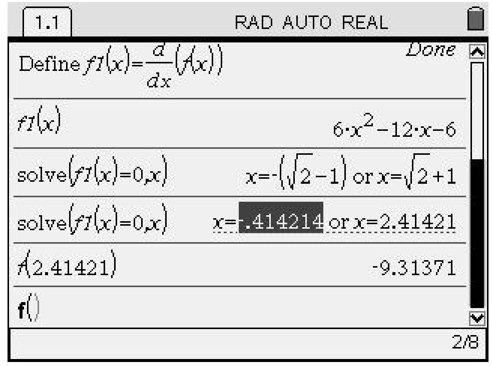
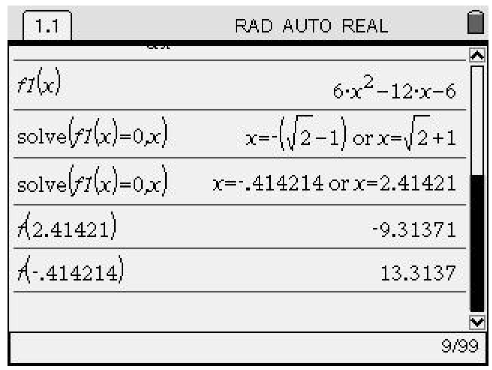
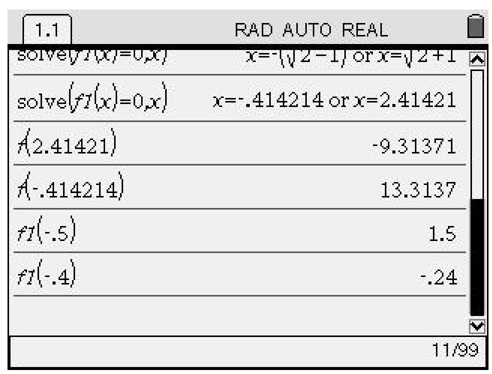
Press enter to evaluate the function at this value. Repeat this process for the other root.



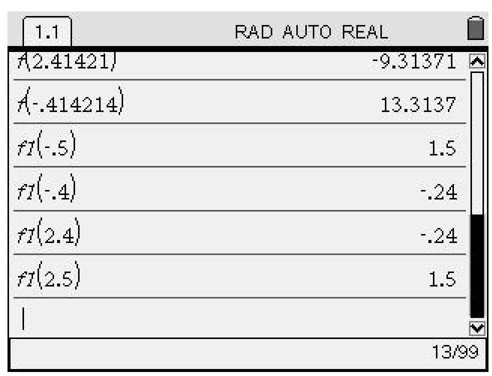
Press enter to complete the computation for the second root.



The first derivative test requires that the values of the first derivative be found on either side of the *x*-coordinate of a turning point. For the smaller root, try x = -0.5 (as a value to the left of the root) and x = -0.4 (as a value to the right of the root). Since the sign of *f1(x)* changes from positive to negative, you conclude that the turning point at (-0.41, 13.31) is a local maximum.



The other root is x = 2. 41. Test values in the derivative on either side, such as 2.4 and 2.5 Since the sign of the derivative changes from negative to positive, you conclude that the turning point (2.41, -9.31) is a local minimum.



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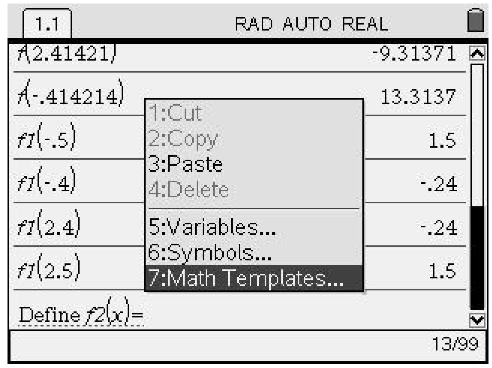
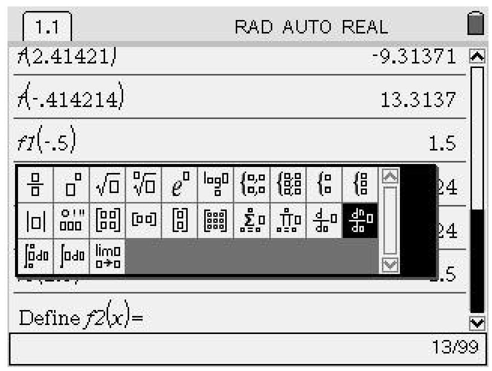
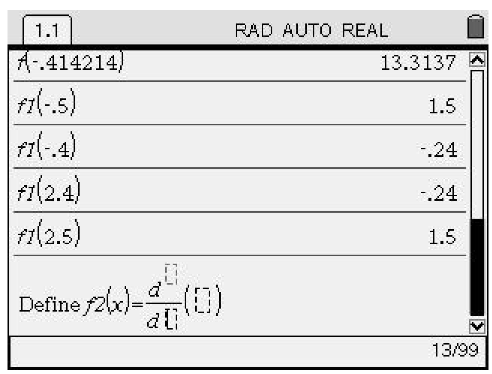
Define the second derivative in function *f2(x)*. To call up the second derivative, press ctrl followed by menu. Choose the option labeled Math Templates.



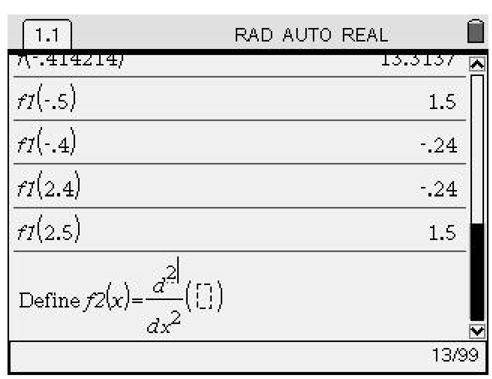
A set of templates appear on the screen in a pop-up window. Find the *nth* derivative template.



The template appears on the screen with the cursor in the variable position. Enter "*x*" and press tab.



The cursor will jump to the power. Enter 2. The digit "2" in the denominator automatically appears. Press tab again.



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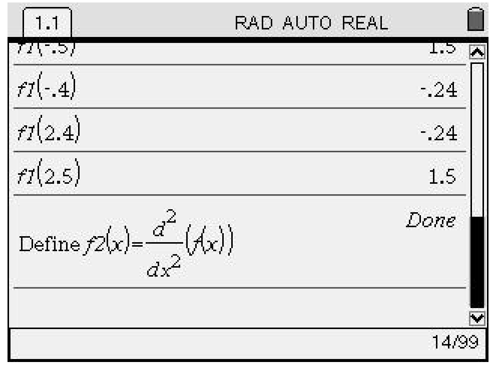
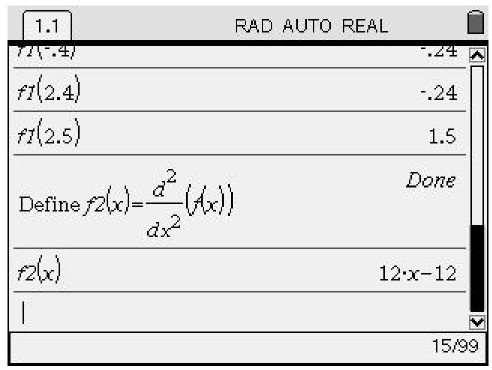
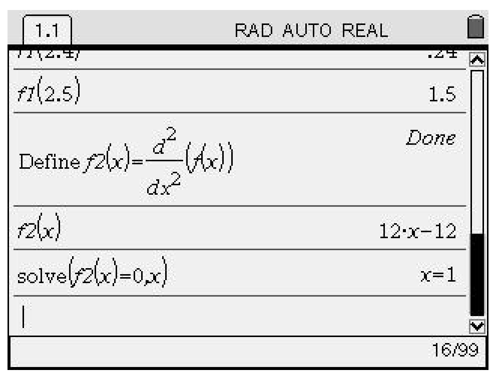
Complete the command by entering the function name, *f(x)* and press enter. Alternatively, you could have defined *f2(x)* as the first derivative of *f1(x)*.



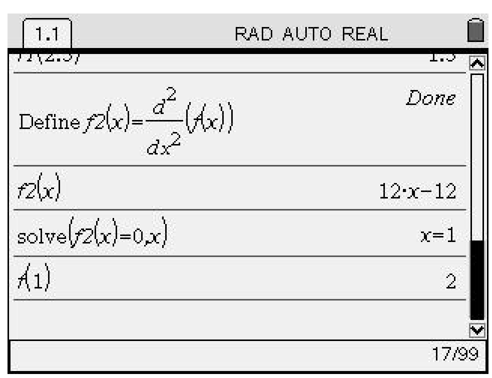
To see the expression for the second derivative, enter the function name and press enter.



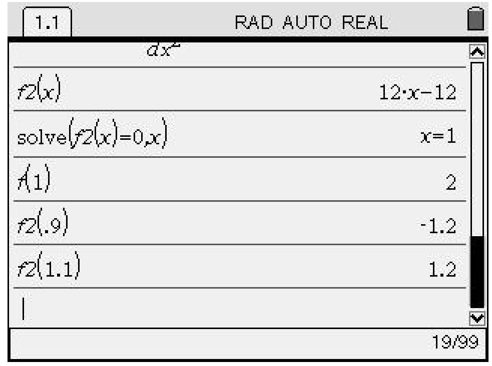
To find points of inflection, set the second derivative equal to 0 and solve for *x*. This can be accomplished using the Solve command.



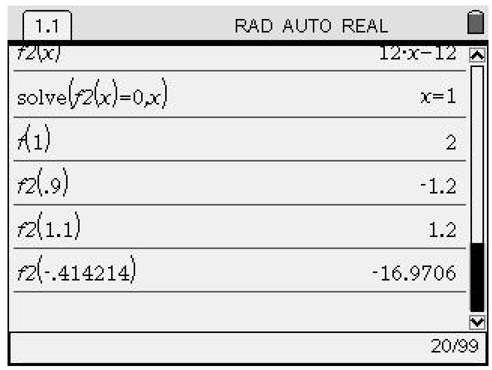
To find the *y*-coordinate of the point of inflection, substitute the root into the original function.



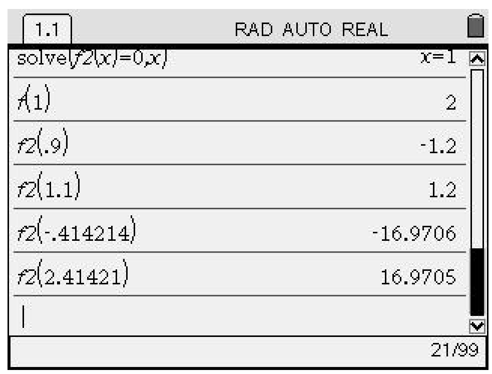
To verify that this is a point of inflection, substitute values on either side of the root into the second derivative. In this case, 0.9 and 1.1 have been used. Since the sign of the second derivative changes, you can conclude that the point (1,2) is a point of inflection.



The second derivative can also be used to test the roots found when the first derivative was set equal to 0. Substitute each of these values into the second derivative. In this case, since the sign of the second derivative is negative, the point (-0.41, 13.31) is a local maximum.

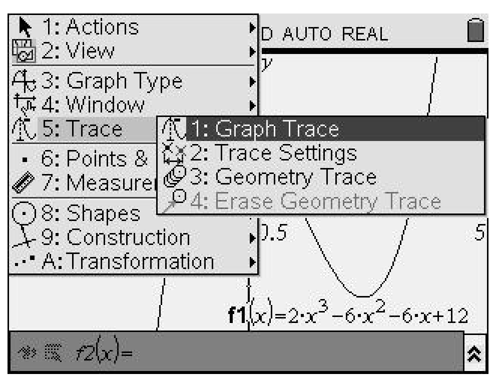


In the same way, since the second derivative for *x* = 2.41 is positive, the point (2.41, -9.31) is a local minimum.

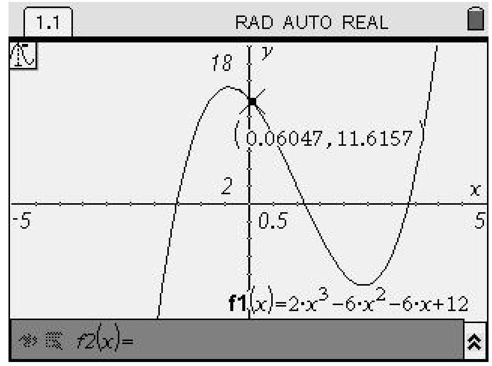


Checking with the Graph

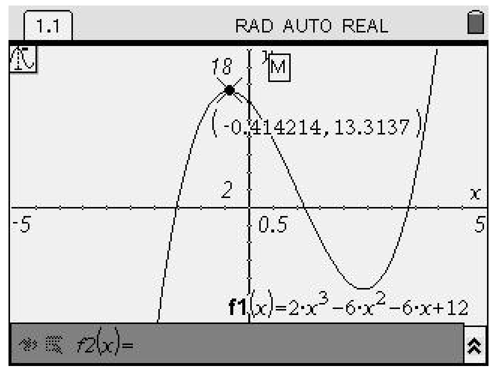
Move back to the screen or document where the function was graphed in a Graphs & Geometry window. A Trace Point will be used to verify some of the results. Press (menu). From the Trace menu, choose Graph Trace.



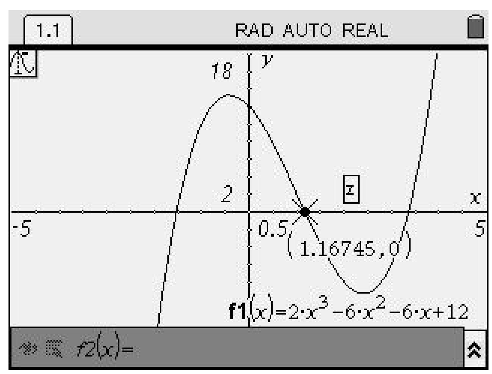
Move the point along the function and press enter. This will secure a point on the curve. Press esc to get out of Trace mode. The point will still be on the curve.



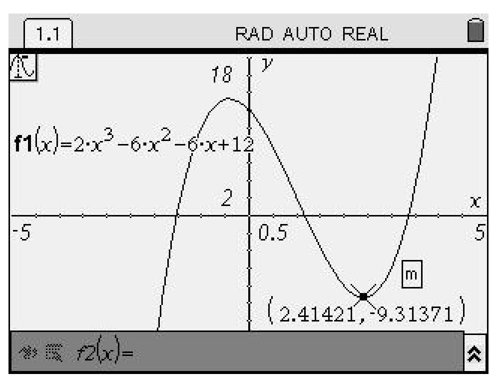
Move towards the local maximum on the curve. When the point is in the neighbourhood of the local maximum, the cursor will jump to the turning point and identify it using a caption box containing an upper case **M**.



A similar result appears when the point is dragged to a location near an *x*-intercept. This time the caption box contains a lower case **z**.



Finally, when the point is dragged to a location near the local minimum, a caption box displaying a lower case **m** will appear.



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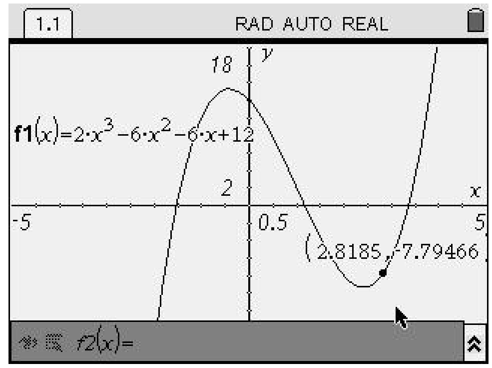
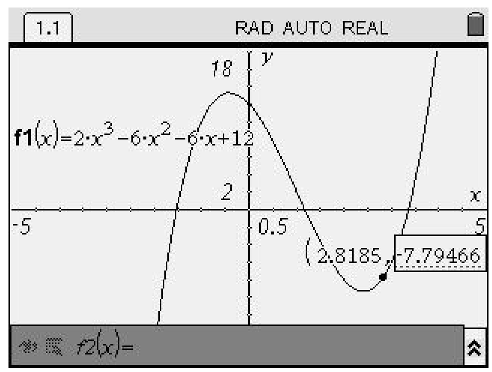
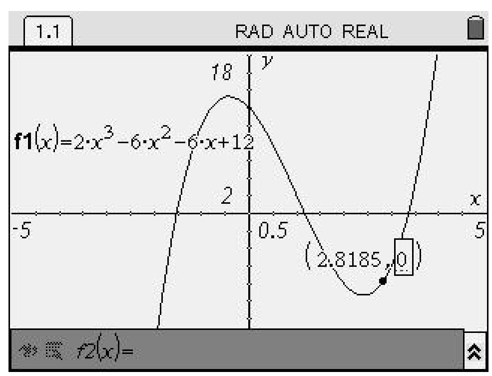
An alternative method of finding an *x*-intercept is to edit the coordinates. Move the point anywhere on the curve.



Double-click on the *y*-coordinate of the point. An edit box will appear around the coordinate.



Use the clear key to remove the characters in the y-coordinate and replace them with the number 0.



The cursor will jump to the nearest *x*-intercept and the coordinates of the *x*-intercept will be displayed.