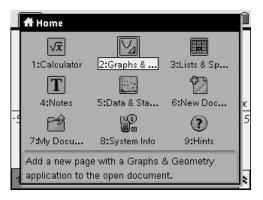
U3L5 - Curve Sketching TI Nspire CAS™ Teacher Notes

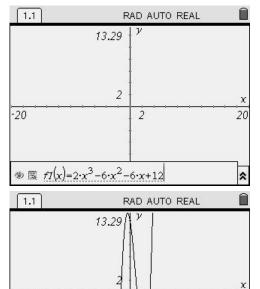
Cubic functions provide all the features that are needed in the analysis of functions. This is the function of lowest degree that will display intercepts, turning points and points of inflection and allows us to use a CAS (computer algebra system) to analyze the function.

Graphing the Function

On the TI Nspire CAS[™], open a new document. Choose the Graphs & Geometry application. You will begin by examining a cubic function graphically and estimating the features mentioned above.



The cursor will automatically appear in the entry line. Enter the expression for the function. Use the button to enter exponents. As you finish each power, press to exit exponent mode.



20

*

-20

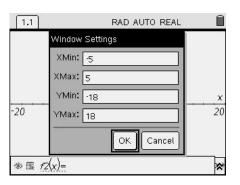
 $\circledast \equiv f_2(x) =$

The graph will appear in the default window, which obviously needs to be changed.

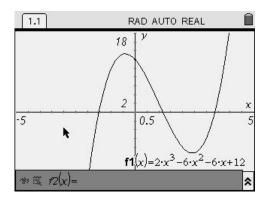
Press e. From the **Window** menu, choose Window Settings.

▶ 1: Actions ₩ 2: View	D AUTO REAL
4ि: 3: Graph Type ग्रें 4: Window	1: Window Settings
🏌 5: Trace	2: Zoom - Box
 6: Points & Line Ø 7: Measurement 	9 9: Zoom − In 9 4: Zoom − Out
	†⊯ 5: Zoom – Standard †⊯ 6: Zoom – Quadrant 1 †ज 7: Zoom – User
	(∯i 8: Zoom – Trig) ∰i 9: Zoom – Data Mi A: Zoom – Fit
$\circledast \equiv f_2(x) =$	IL® ¥.50011 – LIC

One window that works well for this function is shown to the right.

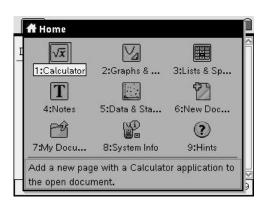


With this window, the features of the function are more obvious. From the graph, estimate the coordinates of the *y*-intercept, the *x*-intercept(s), the local maximum, the local minimum and the point of inflection. You will return to the graph after the analysis done on the CAS in order to verify our results and check the estimates.



Defining the function

Open a new document and choose the Calculator. The computer algebra system resides in this application.



Press . From the Actions menu, choose Define.

f½ 1: Actions	1: Define
¹₂ •s 2: Number	2: Recall Definition
<i>i</i> 3: Complex x= 4: Algebra ∫d 5: Calculus	3: Delete Variable 4: Clear a−z 5: Clear History
6: Probability	6: Insert Comment
x̄ 7: Statistics I≋ 8: Matrix & Vector	7: Library
9: Functions & Prog	grams 🔸
2 2	0/95

Enter the expression for the function using the button for exponents as before. Press to complete the operation. The phrase "Done" simply means that the function has been stored in memory and is available to you anytime that you need it.

1.1	RAD AUTO R	EAL
Define $f(x)=2$	$x^3 - 6 \cdot x^2 - 6 \cdot x + 12$	Done
1		
		1/9

Analyzing the Function

In order to find the *y*-intercept, you need to set the *x*-value of the function to 0. Enter f(0) and press (3).

	x ³ -6·x ² -6·x+12	Done
Define $f(x)=2$.	$x^{-}-0 \cdot x^{-}-0 \cdot x+12$	20000
(o)		12

To find the *x*-intercept(s), set the function equal to 0 and solve for *x*. On the device, you begin this by accessing the Solve command. Press \bigcirc . From the **Algebra** menu, choose Solve.

f½1:Actions	REAL
i 3: Complex	Done
x= 4: Algebra	1: Solve
$\int d 5: Calculus 6: Probability \overline{X} 7: Statistics[#] 8: Matrix & Vector$	2: Factor 3: Expand 4: Zeros 5: Numerical Solve
9: Functions & Prog	6: Polynomial Tools 7: Fraction Tools
	8: Trigonometry 9: Complex A:Extract
	B:Finance Solver

The Solve command has two inputs: the equation to be solved and the variable to be solved for. Once that is entered, press $\langle \overline{a} \rangle$.

1.1 RAD AUTO REAL		EAL 🗎
Define $f(x)=2$	$2 \cdot x^3 - 6 \cdot x^2 - 6 \cdot x + 12$	Done
A(0)		12
solve(Ax)=0; x=-1.5	x) 2892 or x=1.16745 or :	c=3.36147
		₩ 3/99

Define a new function f1(x) which will hold the first derivative of f(x). Press (m). From the **Calculus** menu, choose Derivative.

ft 1: Actions		REAL	
¹ / ₂ ×52: Number i 3: Complex		2	
v- 4. Algebra	1: Derivative	N 7188	2
4.5. Calculus	2: Integral		
📦 6. Prohahili	3: Limit		
x 7: Statistic: ₿ 8: Matrix &	5: Product		
9: Function	5: Function Minir	num	
Define $f_1(x) = $	7: Function Maxi	mum	
Detme (177(2))	3: Arc Length 9: Series		
100	A:Differential Eq	uation Solver	1
	B:Implicit Differe		
	C:Numerical Cal		►

The derivative template will be pasted into the entry line.

1.1	RAD AUTO R	EAL
Define $f(x)=2$	$x^3 - 6 \cdot x^2 - 6 \cdot x + 12$	Done 🛛
A(0)		12
	2892 or x=1.16745 or .	x=3.36147
Define <i>f1(x)=</i>	$\frac{d}{d\mathbb{Q}}(\mathbb{Q})$	
		3/99

Complete each field of the template as shown. Use (a) to move from one field to the next. Press (a) when the command is complete.

1.1	RAD AUTO R	EAL
Define $f(x)=2$.	$x^{3}-6 \cdot x^{2}-6 \cdot x+12$	Done
/ (0)		12
solve $(f(x)=0,x)$ x=-1.52) :892 or x=1.16745 or :	x=3.36147
Define $fI(x) = \frac{1}{2}$	$\frac{d}{dx}(f(x))$	Done
		•
		4/99

To see the expression for the first derivative, type the function name into the entry line.

1.1	RAD AUTO REAL
A(0)	12
solve(f(x)=0,x)	
x=-1.5289	2 or x=1.16745 or x=3.36147
Define $fI(x) = \frac{d}{dx}$	(f(x)) Done
f1(x)	6·x ² -12·x-6
1	×
	5/99

When the first derivative is equal to 0, you get the *x*-coordinates of the local maxima and local minima, or turning points. Use the Solve command to find these values. The default setting of the device displays results in exact form where possible.

1.1	0	.D AUTO REAL
solve	f(x)=0,x)	
	x=-1.52892 or x=1	.16745 or x=3.36147
Defin	$e_{fI}(x) = \frac{d}{dx}(f(x))$	Done
+1(x)		$6 \cdot x^2 - 12 \cdot x - 6$
solve	$f_{f_{1}}(x)=0,x$	$=-(\sqrt{2}-1)$ or $x=\sqrt{2}+1$
T		
		6/99

To see approximate (or decimal) values for the roots, press the ▲ button on the Navpad twice. This will highlight the previous Solve command. Press → to move this command to the entry line. Press → followed by → to display the approximate values. If you choose to, you can set the defaults for the document to display fewer digits.

1.1	RAD AUTO REAL
x=-1.528	392 or x=1.16745 or x=3.36147
Define $fI(x) = \frac{d}{dx}$	$\frac{l}{x}(f(x))$ Done
f](x)	$6 \cdot x^2 - 12 \cdot x - 6$
solve(fi(x)=0,x)) $x=-(\sqrt{2}-1) \text{ or } x=\sqrt{2}+1$
solve(fi(x)=0,x)) $x=414214$ or $x=2.41421$
1	
8- 2-	7/99

To find the *y*-coordinates of the two turning points, you should substitute the *x*-coordinates back into the original function. Enter "f(" on the entry line and press \blacktriangle once to highlight the previous results. This will highlight both roots. Press \triangleright or \triangleleft to remove the highlighting. Move the cursor to a point just before the last root. Press and hold the T key while pressing the \triangleright key. Continue this until only the one root is highlighted.

1.1	RAD AUT	
x=-1	.52892 or x=1.16745	or x=3.36147
Define <i>f1</i> (x	$=\frac{d}{dx}(A_x)$	Done
+1(x)		$6 \cdot x^2 - 12 \cdot x - 6$
solve(fi(x))	$=0,x$ $x=-(\sqrt{2}$	-1) or $x=\sqrt{2}+1$
$\frac{\operatorname{solve}(fI(x))}{\operatorname{solve}(fI(x))}$	=0,x) <u>x=414214</u>	4 or x = 2.41421
f ()		<u> </u>
÷.		1//

Press $\stackrel{\sim}{\twoheadrightarrow}$ and the value will be inserted into the function.

1.1	RAD AUTO REAL
x=-1.528	92 or x=1.16745 or x=3.36147
Define $fI(x) = \frac{d}{dx}$	$\frac{1}{x}(f(x))$ Done
+1(x)	6·x ² -12·x-6
$\operatorname{solve}(fI(x)=0,x)$	$x=-(\sqrt{2}-1) \text{ or } x=\sqrt{2}+1$
$\operatorname{solve}(fI(x)=0,x)$) x=414214 or x=2.41421
f (2.41421)	
	7/99

Press (a) to evaluate the function at this value. Repeat this process for the other root.

1.1	RAD AUTO REAL	
Define $fI(x) = \frac{d}{dx}$	-(f(x)) Done	
+1(x)	6·x ² -12·x-6	
$\operatorname{solve}(fI(x)=0,x)$	$x=-(\sqrt{2}-1)$ or $x=\sqrt{2}+1$	
$\operatorname{solve}(f_{i}(x)=0,x)$	x=.414214 or x=2.41421	
A(2.41421)	-9.31371	
f ()		~
	2/	8

Press (a) to complete the computation for the second root.

1.1	RAD AUTO REAL
	6·x ² -12·x-6
solve(f1(x)=0,x) $x=-(\sqrt{2}-1) \text{ or } x=\sqrt{2}+1$
$\operatorname{solve}(fI(x)=0,x)$) $x=414214$ or $x=2.41421$
A2.41421)	-9.31371
A(414214)	13.3137

The first derivative test requires that the values of the first derivative be found on either side of the *x*-coordinate of a turning point. For the smaller root, try x = -0.5 (as a value to the left of the root) and x = -0.4 (as a value to the right of the root). Since the sign of f1(x) changes from positive to negative, you conclude that the turning point at (-0.41, 13.31) is a local maximum.

1.1	RAD AUTO REAL
solve(71(x)=0,x)	$x = (\sqrt{2} - 1) \text{ or } x = \sqrt{2} + 1$
solve(fi(x)=0,x)	x=414214 or x=2.41421
A2.41421)	-9.31371
A(414214)	13.3137
<i>f1</i> (5)	1.5
<i>f1</i> (4)	24
	11/99

The other root is x = 2. 41. Test values in the derivative on either side, such as 2.4 and 2.5 Since the sign of the derivative changes from negative to positive, you conclude that the turning point (2.41, -9.31) is a local minimum.

1.1	RAD AUTO REAL	
A2.41421/	-9.31371	
<u>(414214</u>)	13.3137	
<u>f1</u> (5)	1.5	
£7(4)	24	
f1(2.4)	24	
f1(2.5)	1.5	
1		-
	13/9	9

Define the second derivative in function f2(x). To call up the second derivative, press (eff) followed by (mer). Choose the option labeled Math Templates.

1.1 A2.41421)	RAD AUTO RI	-9.31371
	_1:Cut	13.3137
f1(5)	2:Copy	1.5
f1(4)	3:Paste 4:Delete	24
f1(2.4)	5:Variables	24
f1(2.5)	6:Symbols 7:Math Templates	1.5
Define <i>f2</i> (x)		
		13/9

A set of templates appear on the screen in a popup window. Find the *nth* derivative template.

1.	1					RAD) AL	то	REA	L		Î
A2.	414:	21)							2	9.3	1371	<u> </u>
A4	142	214)								13.	3137	
f](-	.5)										1.5	
믐		√ū	%	e	logO	{0,0 0,0	{ 8 8	{:	{ B		24	
0	011	[83]	[00]			Σ̈́□	<u>∎</u> ⊓	4°D	뿖		24	
]åda	040	lim¤ ¤≁¤									5	
Def											.	-
Der	ine;	72\X	/=								13/	▼ 99

The template appears on the screen with the cursor in the variable position. Enter "x" and press (a).

1.1	RAD AUTO REAL
A(414214)	13.3137
<i>f1</i> (5)	1.5
f1(4)	24
f1(2.4)	24
f1(2.5)	1.5
Define $f^2(x) = \frac{d^{[1]}}{d^{[1]}}([1])$	
	13/99

1.1	RAD AUTO REAL
π414214)	13.3137
<i>f1</i> (5)	1.5
<i>f1</i> (4)	24
f1(2.4)	24
<i>f1</i> (2.5)	1.5
Define $f^2(x) = \frac{a^{2 }}{dx^2}([])$	~
	13/99

The cursor will jump to the power. Enter 2. The digit "2" in the denominator automatically appears. Press (1) again.

Complete the command by entering the function name, f(x) and press (3). Alternatively, you could have defined f2(x) as the first derivative of f1(x).

1.1	RAD AUTO REAL
77(5)	1.5
f1(4)	24
f1(2.4)	24
<i>f1</i> (2.5)	1.5
Define $f^2(x) = \frac{d^2}{dx^2} (f(x))$)) Done
	№ 14/99

To see the expression for the second derivative, enter the function name and press .

1.1	RAD AUTO REAL
77(4)	24
f1(2.4)	24
f1(2.5)	1.5
Define $f^2(x) = \frac{d^2}{dx^2} (f(x))$) Done
f2(x)	12·x-12
1	
	15/99

To find points of inflection, set the second derivative equal to 0 and solve for *x*. This can be accomplished using the Solve command.

1.1	RAD AUTO REAL
/1\2.4/	.24
f1(2.5)	1.5
Define <i>f2</i> (x)=	$-\frac{d^2}{dx^2}(f_{(X)}))$ Done
f2(x)	12·x-12
solve(f2(x)=0),x) x=1
	E
	16/99

To find the *y*-coordinate of the point of inflection, substitute the root into the original function.

1.1 RAD AUTO		UTO REAL 🛛 🗍
11(2.3)		<u> </u>
Define $f^{2}(x) =$	$\frac{d^2}{dx^2}(A_x))$	Done
12(x)		12·x-12
$\operatorname{solve}(f_2(x)=0$,x)	x=1
<i>⊧</i> (1)		2
		17/99

To verify that this is a point of inflection, substitute values on either side of the root into the second derivative. In this case, 0.9 and 1.1 have been used. Since the sign of the second derivative changes, you can conclude that the point (1,2) is a point of inflection.

1.1	RAD AUTO REAL
	dx ⁴
$f_2(x)$	12•x-12
$\operatorname{solve}(f_2(x)=0,$	x) x=1
A(1)	2
<i>f2</i> (.9)	-1.2
$f_2(1.1)$	1.2
	19/99

The second derivative can also be used to test the roots found when the first derivative was set equal to 0. Substitute each of these values into the second derivative. In this case, since the sign of the second derivative is negative, the point (-0.41, 13.31) is a local maximum.

1.1	RAD AUTO REAL
72(x)	12·x-12
solve(f2(x)=0,x)	x=1
A(1)	2
f2(.9)	-1.2
f2(1.1)	1.2
f2(414214)	-16.9706
	20/99

In the same way, since the second derivative for x = 2.41 is positive, the point (2.41, -9.31) is a local minimum.

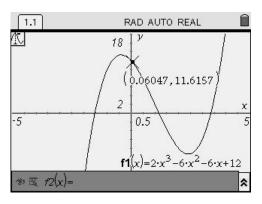
1.1	RAD AUTO REAL
solve(f2(x)=0,x)	x=1 2
ત (1)	2
f2(.9)	-1.2
$f_2(1.1)$	1.2
<i>f2</i> (414214)	-16.9706
<i>f2</i> (2.41421)	16.9705
1	
	21/99

Checking with the Graph

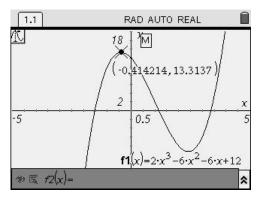
Move back to the screen or document where the function was graphed in a Graphs & Geometry window. A Trace Point will be used to verify some of the results. Press *mem*. From the Trace menu, choose Graph Trace.

▶ 1: Actions ₩ 2: View	D AUTO REAL	
4: 3: Graph Type 标 4: Window	ν •	
/K.5: Trace /K.1: C	Graph Trace	
	race Settings Geometry Trace Erase Geometry Trace	
↓9: Construction ·• A: Transformation).5	5
	$f1(x) = 2 \cdot x^3 - 6 \cdot x^2 - 6 \cdot x + 12$	2
*# ∰ <i>f</i> 2(x)=		*

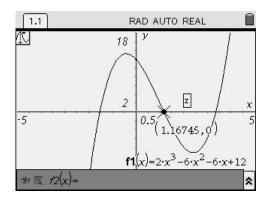
Move the point along the function and press . This will secure a point on the curve. Press . get out of Trace mode. The point will still be on the curve.



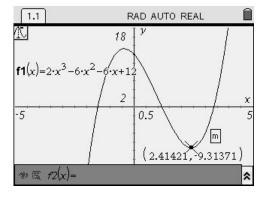
Move towards the local maximum on the curve. When the point is in the neighbourhood of the local maximum, the cursor will jump to the turning point and identify it using a caption box containing an upper case \mathbf{M} .



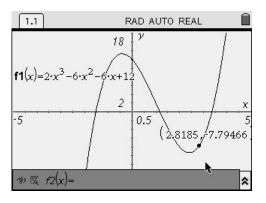
A similar result appears when the point is dragged to a location near an *x*-intercept. This time the caption box contains a lower case z.



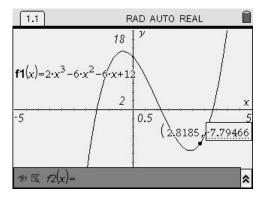
Finally, when the point is dragged to a location near the local minimum, a caption box displaying a lower case \mathbf{m} will appear.



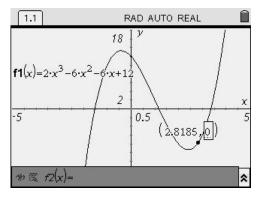
An alternative method of finding an *x*-intercept is to edit the coordinates. Move the point anywhere on the curve.



Double-click on the *y*-coordinate of the point. An edit box will appear around the coordinate.



Use the key to remove the characters in the y-coordinate and replace them with the number 0.



The cursor will jump to the nearest *x*-intercept and the coordinates of the *x*-intercept will be displayed.

