Grade 12 Math for College Technology

Unit 1: Exponential Functions (9 days + 1 jazz day + 1 summative evaluation day)

BIG Ideas:

- Graph exponential functions and solve exponential equations graphically and numerically
- Investigate patterns of exponential functions with different integral bases
- Explore and define logarithms with different bases
- Connect logarithms and exponents
- Solve problems arising from real-world contexts and college technology applications involving logarithms and exponents

DAY	Lesson Title & Description	Expectations		Teaching/Assessment Notes and Curriculum Sample Problems
1	• Activate prior experience with exponentials by graphing various exponential functions. • Determine through investigation with technology what happens when the base changes or the sign of the exponent changes. ***LESSON INCLUDED***	A1.1	determine, through investigation with technology, and describe the impact of changing the base and changing the sign of the exponent on the graph of an exponential function	
2	• Solve simple exponential equations both numerically and graphically.(guess and check, tracing graph, or using point of intersection). ***LESSON INCLUDED***	A1.2	solve simple exponential equations numerically and graphically, with technology (e.g., use systematic trial with a scientific calculator to determine the solution to the equation $1.05^x = 1,276$), and recognize that the solutions may not be exact	***Sample problem:*** Use the graph of $y = 3^x$ to solve the equation $3^x = 5$.
3	• Determine the point of intersection of two exponential functions graphically (eg.$y = 4^{-x}$ and $y = 8^{x+3}$). • Make connections between finding the intersection point and solving the corresponding exponential equation. (eg. $4^{-x} = 8^{x+3}$). ***LESSON INCLUDED***	A1.3	determine, through investigation using graphing technology, the point of intersection of the graphs of two exponential functions (e.g., $y = 4^{-x}$ and $y = 8^{x+3}$), recognize the x-coordinate of this point to be the solution to the corresponding exponential equation (e.g., $4^{-x} = 8^{x+3}$), and solve exponential equations graphically (e.g., solve $2^{x+2} = 2^x + 12$ by using the intersection of the graphs of $y = 2^{x+2}$ and $y = 2^x + 12$)	***Sample problem:*** Solve $0.5^x = 3^{x+3}$ graphically.
4	• Pose and solve real world application problems involving exponential functions graphically.	A1.4	pose problems based on real-world applications (e.g., compound interest, population growth) that can be modelled with exponential equations, and solve these and other such problems by using a given graph or a graph generated with technology from a table of	***Sample problem:*** A tire with a slow puncture loses pressure at the rate of 4%/min. If the tire's pressure is 300 kPa to begin with, what is its pressure after 1 min?

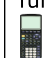
			values or from its equation	After 2 min? After 10 min? Use graphing technology to determine when the tire's pressure will be 200 kPa.
5	• Simplify and evaluate expressions using the laws of exponents in order to solve exponential equations with a common base.	A2.1 A2.2	simplify algebraic expressions containing integer and rational exponents using the laws of exponents (e.g., $x^3 \div x^{\frac{1}{2}}$, $\sqrt{x^6y^{12}}$ solve exponential equations in one variable by determining a common base (e.g., 2(x) = 32, 4(5x-1) = 2(2)(x+11), 3(5x+8) = 27(x))	***Sample problem:*** Simplify $\frac{a^3b^2c^3}{\sqrt{a^2b^4}}$ and then evaluate for a = 4, b = 9, and c =−3. Verify your answer by evaluating the expression without simplifying first. Which method for evaluating the expression do you prefer? Explain. ***Sample problem:*** Solve $3^{5x+8} = 27^x$ by determining a common base, verify by substitution, and investigate connections to the intersection of $y = 3^{5x+8}$ and $y = 27^x$) using graphing technology.
6	• Solve problems in bases other than 10 graphically or by systematic trial and error. ***LESSON INCLUDED***	A2.4	determine, with technology, the approximate logarithm of a number to any base, including base 10 [e.g., by recognizing that $\log_{10}(0.372)$ can be determined using the LOG key on a calculator; by reasoning that $\log_3 29$ is between 3 and 4 and using systematic trial to determine that $\log_3 29$ is approximately 3.07]	
7	• Define a logarithm. • Explore change of bases. • Make connections between related logarithms and exponential equations. ***LESSON INCLUDED***	A2.3 A2.4 A2.5	recognize the logarithm of a number to a given base as the exponent to which the base must be raised to get the number, recognize the operation of finding the logarithm to be the inverse operation (i.e., the undoing or reversing) of exponentiation, and evaluate simple logarithmic expressions determine, with technology, the approximate logarithm of a number to any base, including base 10 [e.g., by recognizing that $\log_{10}(0.372)$ can be determined using the LOG key on a calculator; by reasoning that $\log_3 29$ is between 3 and 4 and using systematic trial to determine that $\log_3 29$ is approximately 3.07] make connections between related logarithmic and exponential equations (e.g., $\log_5 125 = 3$ can also be expressed as $5^3 = 125$), and solve simple exponential equations by rewriting them in logarithmic form (e.g.,	***Sample problem:*** Why is it possible to determine $\log_{10}(100)$ but not $\log_{10}(0)$ or l $\log_{10}(-100)$? Explain your reasoning.

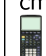
			solving $3^x = 10$ by rewriting the equation as $\log_3 10 = x$)	
8	• Solve problems arising from real-world contexts and college technology applications using logarithms.	A2.6	pose problems based on real-world applications that can be modelled with given exponential equations, and solve these and other such problems algebraically by rewriting them in logarithmic form	***Sample problem:*** When a potato whose temperature is 20°C is placed in an oven maintained at 200°C, the relationship between the core temperature of the potato T, in degrees Celsius, and the cooking time t, in minutes, is modelled by the equation $200 - T = 180(0.96)^t$. Use logarithms to determine the time when the potato's core temperature reaches 160°C.
9	• Collect data that behaves exponentially or logarithmically. • Solve problems based on the data collected. ***LESSON INCLUDED***	A2.6	pose problems based on real-world applications that can be modelled with given exponential equations, and solve these and other such problems algebraically by rewriting them in logarithmic form	
10	Review Day (JAZZ DAY)			
11	Summative Unit Evaluation ***LESSON & ASSESSMENT INCLUDED***			

Unit 2: Polynomial Functions (10 days + 2 jazz days + 1 summative evaluation day)

BIG Ideas:

- Describe key features of graphs of cubic and quartic functions
- Solve problems using graphs of cubic and quartic functions arising from a variety of applications
- Connect domain and range to contexts in problems
- Make connections between numeric, graphical and algebraic representations of polynomial functions

DAY	Lesson Title & Description	Expectations		Teaching/Assessment Notes and Curriculum Sample Problems
1	• Activate prior knowledge: Review features of linear and quadratic – what does it look like, how do you describe it? Is it a function? How do you know? • Consolidate understanding of domain and range • Introduce end behaviour terminology and leading coefficient. ***LESSON INCLUDED***	B1.1	recognize a polynomial expression (i.e., a series of terms where each term is the product of a constant and a power of x with a nonnegative integral exponent, such as $x^3 - 5x^2 + 2x - 1$); recognize the equation of a polynomial function and give reasons why it is a function, and identify linear and quadratic functions as examples of polynomial functions	Computer with LCD projector needed
2-3	• Investigate cubic and quartic functions and explain why they are functions. • Graph the equations of cubic and quartic functions and investigate end behaviours, domain and range. • Describe end behaviours and the impact of the leading coefficient (positive and negative values) • Describe the shape of each function with the maximum number of zeros. ***LESSONS INCLUDED***	B1.2 B1.3	compare, through investigation using graphing technology, the graphical and algebraic representations of polynomial (i.e., linear, quadratic, cubic, quartic) functions (e.g., investigate the effect of the degree of a polynomial function on the shape of its graph and the maximum number of x-intercepts; investigate the effect of varying the sign of the leading coefficient on the end behaviour of the function for very large positive or negative x-values) describe key features of the graphs of polynomial functions (e.g., the domain and range, the shape of the graphs, the end behaviour of the functions for very large positive or negative x-values)	***Sample problem:*** Investigate the maximum number of x-intercepts for linear, quadratic, cubic, and quartic functions using graphing technology.  ***Sample problem:*** Describe and compare the key features of the graphs of the functions $f(x) = x$, $f(x) = x^2$, $f(x) = x^3$, and $f(x) = x^4$.
4	• Consolidate understanding of properties of cubic and quartic functions.	B1.4	distinguish polynomial functions from sinusoidal and exponential functions [e.g., $f(x) = \sin x$, $f(x) = 2^x$], and compare and contrast the graphs of various polynomial functions with the graphs of	

	• Identify various curves as cubic, quartic, exponential, sinusoidal, linear and quadratic. ***LESSON INCLUDED***		other types of functions	
5-6	• Answer questions about graphs in contexts taken from real-world application and have students answer questions related to the graph. • Connect restrictions on domain and range to the application problems.	B1.6 B1.7	pose problems based on real-world applications that can be modelled with polynomial functions, and solve these and other such problems by using a given graph or a graph generated with technology from a table of values or from its equation recognize, using graphs, the limitations of modelling a real-world relationship using a polynomial function, and identify and explain any restrictions on the domain and range (e.g., restrictions on the height and time for a polynomial function that models the relationship between height above the ground and time for a falling object)	***Sample problem:*** The forces acting on a horizontal support beam in a house cause it to sag by d centimetres, x metres from one end of the beam. The relationship between d and x can be represented by the polynomial function $d(x) = \frac{1}{1850}x(1000 - 20x^2 + x^3)$. Graph the function, using technology, and determine the domain over which the function models the relationship between d and x. Determine the length of the beam using the graph, and explain your reasoning.
7	• Review function notation in order to find a specific point on the graph. • Compare point on graph with the answer found by substitution. • Connect restrictions on domain and range. • Substitute into and evaluate polynomial functions. ***LESSON INCLUDED***	B1.5	substitute into and evaluate polynomial functions expressed in function notation, including functions arising from real-world applications	***Sample problem:*** A box with no top is being made out of a 20-cm by 30-cm piece of cardboard by cutting equal squares of side length x from the corners and folding up the sides. The volume of the box is V = x(20 – 2x)(30 – 2x). Determine the volume if the side length of each square is 6 cm. Use the graph of the polynomial function V(x)to determine the size of square that should be cut from the corners if the required volume of the box is 1000 cm^3. 
8-9	• Review finding zeros with quadratics and observing the need for factored form. • Find zeros of cubic and quartic	B2.2	make connections, through investigation using graphing technology (e.g., dynamic geometry software), between a polynomial function given in factored form [e.g., f(x) = x(x – 1)(x + 1)] and the x-intercepts of its graph, and sketch the graph	***Sample problem:*** Sketch the graphs of f(x) = –(x – 1)(x + 2)(x – 4) and g(x) = –(x – 1)(x + 2)(x + 2) and compare their shapes and the number of x-intercepts.

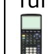
	functions in standard form that can only be factored using common factoring, difference of squares, and trinomial factoring. (**Note: NO factor theorem.)** • Verify the zeros are correct by graphing with technology. • Write the factored form in standard form to verify the functions are the same. ***LESSONS INCLUDED***	 B2.3	of a polynomial function given in factored form using its key features (e.g., by determining intercepts and end behaviour; by locating positive and negative regions using test values between and on either side of the x-intercepts) determine, through investigation using technology (e.g., graphing calculator, computer algebra systems), and describe the connection between the real roots of a polynomial equation and the x-intercepts of the graph of the corresponding polynomial function [e.g., the real roots of the equation $x^4 - 13x^2 + 36 = 0$ are the x-intercepts of the graph of $f(x) = x^4 - 13x^2 + 36$]	***Sample problem:*** Describe the relationship between the x-intercepts of the graphs of linear and quadratic functions and the real roots of the corresponding equations. Investigate, using technology, whether this relationship exists for polynomial functions of higher degree. ***Note***: *A GSP sketch has been included that was created for the MHF 4U course. The expectations overlap with the ones presented on these two days.*
10	• Students pose and solve problems based on real world applications. ***LESSON INCLUDED***	B1.6 B1.7	pose problems based on real-world applications that can be modelled with polynomial functions, and solve these and other such problems by using a given graph or a graph generated with technology from a table of values or from its equation recognize, using graphs, the limitations of modelling a real-world relationship using a polynomial function, and identify and explain any restrictions on the domain and range (e.g., restrictions on the height and time for a polynomial function that models the relationship between height above the ground and time for a falling object)	
11-12	Review Day (JAZZ DAY)			
13	Summative Unit Evaluation			

Grade 12 Math for College Technology

Unit 3: Polynomial Equations (8 days + 2 jazz days + 1 summative evaluation day)

BIG Ideas:

- Solve equations up to degree four by factoring
- Develop facility in working with formulae appropriate to college technology
- Focus on applications appropriate to college technology

DAY	Lesson Title & Description	Expectations		Teaching/Assessment Notes and Curriculum Sample Problems
1	• Make connections between polynomial functions in factored form, graphical form, and numeric form • Consolidate concept that the zeros of the function correspond to the solutions or roots of the corresponding equation when f(x) is equal to zero. ***LESSON INCLUDED***	B2.2	make connections, through investigation using graphing technology (e.g., dynamic geometry software), between a polynomial function given in factored form [e.g., $f(x) = x(x - 1)(x + 1)$] and the x-intercepts of its graph, and sketch the graph of a polynomial function given in factored form using its key features (e.g., by determining intercepts and end behaviour; by locating positive and negative regions using test values between and on either side of the x-intercepts)	***Sample problem:*** Sketch the graphs of $f(x) = -(x - 1)(x + 2)(x - 4)$ and $g(x) = -(x - 1)(x + 2)(x + 2)$ and compare their shapes and the number of x-intercepts.
		B2.3	determine, through investigation using technology (e.g., graphing calculator, computer algebra systems), and describe the connection between the real roots of a polynomial equation and the x-intercepts of the graph of the corresponding polynomial function [e.g., the real roots of the equation $x^4 - 13x^2 + 36 = 0$ are the x-intercepts of the graph of $f(x) = x^4 - 13x^2 + 36$]	***Sample problem:*** Describe the relationship between the x-intercepts of the graphs of linear and quadratic functions and the real roots of the corresponding equations. Investigate, using technology, whether this relationship exists for polynomial functions of higher degree. 
2	• Review and then extend knowledge of factoring to factor cubic and quartic expressions that can be factored using common factoring, difference of squares, trinomial factoring and grouping. ***LESSON INCLUDED***	B2.1	factor polynomial expressions in one variable, of degree no higher than four, by selecting and applying strategies (i.e., common factoring, difference of squares, trinomial factoring)	Sample problem: Factor: $x^4 - 16$; $x^3 - 2x^2 - 8x$.
3	• Solve equations, of degree no higher	B3.1	solve polynomial equations in one variable, of degree no higher than four (e.g., $x^2 - 4x = 0$, $x^4 - 16 = 0$,	Sample problem: Solve $x^3 - 2x^2 - 8x = 0$.

	than four, and verify the solutions using technology. [e.g. Cubic and quartic functions in standard form that can only be factored using common factoring, difference of squares, trinomial factoring and/or the quadratic formula.] (**Note: NO factor theorem.)** • Solve equations of the form $x^n = a$ to compare to polynomials ***LESSON INCLUDED***	 B3.5	$3x^2 + 5x + 2 = 0$), by selecting and applying strategies (i.e., common factoring; difference of squares; trinomial factoring), and verify solutions using technology (e.g., using computer algebra systems to determine the roots of the equation; using graphing technology to determine the x-intercepts of the corresponding polynomial function) solve equations of the form $x^n = a$ using rational exponents (e.g., solve $x^3 = 7$ by raising both sides to the exponent 1/3)	 Computer with LCD projector needed
4	• Expand and simplify polynomial expressions	B3.4	expand and simplify polynomial expressions involving more than one variable [e.g., simplify $-2xy(3x^2y^3 - 5x^3y^2)$], including expressions arising from real-world applications	Sample problem: Expand and simplify the expression $\pi(R + r)(R - r)$ to explain why it represents the area of a ring. Draw a diagram of the ring and identify R and r.
5-6	• Solve problems arising from a real world application. • Revisit the box problem from previous unit – construct a box of a specific volume this time. • Solve multi step problems from real world applications	B3.2 B3.8	solve problems algebraically that involve polynomial functions and equations of degree no higher than four, including those arising from real-world applications solve multi-step problems requiring formulas arising from real-world applications (e.g., determining the cost of two coats of paint for a large cylindrical tank)	
7	• Rearranging an equation for a given variable, then substituting in to find the value. • Make connections between the formula and the variables to determine what type of function it is.	B3.6 B3.7	3.6 determine the value of a variable of degree no higher than three, using a formula drawn from an application, by first substituting known values and then solving for the variable, and by first isolating the variable and then substituting known values make connections between formulas and linear, quadratic, and exponential functions [e.g., recognize that the compound interest formula, $A = P(1 + i)^n$, is an example of an exponential function A(n) when P and i are constant, and of a linear function A(P) when i and n are constant], using a variety of tools and	***Sample problem:*** The formula $s = ut + \frac{1}{2}at^2$ relates the distance, s, travelled by an object to its initial velocity, u, acceleration, a, and the elapsed time, t. Determine the acceleration of a dragster that travels 500 m from rest in 15 s, by first isolating a, and then by first substituting known values. Compare and evaluate the two methods. ***Sample problem:*** Which variable(s) in the formula $V = \pi r^2h$ would you need to set as a constant to generate a linear equation? A quadratic equation?

			strategies (e.g., comparing the graphs generated with technology when different variables in a formula are set as constants)	
8	• Investigate applications of mathematical modelling in occupations. ***LESSON INCLUDED***	B3.9	gather, interpret, and describe information about applications of mathematical modelling in occupations, and about college programs that explore these applications	
9-10	Review Day (JAZZ DAY)			
11	Summative Unit Evaluation			

Grade 12 Math for College Technology

Unit 4: Trigonometric Functions (14 days + 1 jazz day + 2 summative evaluation day)				
BIG Ideas: • Connect sine and cosine ratios to sine and cosine functions • Investigate and describe roles of the parameters in the graphs of y=a sin(k(x-d))+c or y=a cos(k(x-d))+c • Sketch the graphs of y=sinx and y=cosx and apply transformations to these graphs • Identify and discuss amplitude, period, phase shift, domain and range with respect to sinusoidal functions • Represent sinusoidal functions algebraically given its graph or its properties				
DAY	**Lesson Title & Description**	**Expectations**		**Teaching/Assessment Notes and Curriculum Sample Problems**
1-3	• Determine the primary trigonometric ratios for angles less than 90º. • Use the special angles, less than 90º to obtain their coordinates on the unit circle for quadrant 1, ie. (x, y) = (cosθ, sinθ) • Use the coordinates from quadrant 1 to generate the coordinates on the unit circle for the related rotation angles (quadrants 2,3 and 4) • Use the coordinates on the unit circle generated from the related rotation angles, 90° ≤ θ ≤ 360°, to make connections between the sine ratio and the sine function and between the cosine ratio and the cosine functions by graphing the relationship between angles from 0º to 360º and the corresponding sine ratios or cosine ratios, with or without technology • Determine the measures of two angles from 0º to 360º for which the value of a given trigonometric ratio is	C1.1 C1.2 C1.3 C2.1	determine the exact values of the sine, cosine, and tangent of the special angles 0°, 30°, 45°, 60°, 90°, and their multiples determine the values of the sine, cosine, and tangent of angles from 0° to 360°, through investigation using a variety of tools (e.g., dynamic geometry software, graphing tools) and strategies (e.g., applying the unit circle; examining angles related to the special angles) determine the measures of two angles from 0° to 360° for which the value of a given trigonometric ratio is the same (e.g., determine one angle using a calculator and infer the other angle) make connections between the sine ratio and the sine function and between the cosine ratio and the cosine function by graphing the relationship between angles from 0° to 360° and the corresponding sine ratios or cosine ratios, with or without technology (e.g., by generating a table of values using a calculator; by unwrapping the unit circle), defining this relationship as the function f(x) = sin x or f(x) = cos x, and explaining why the relationship is a function	***Sample problem:*** Determine the approximate measures of the angles from 0° to 360° for which the sine is 0.3423.

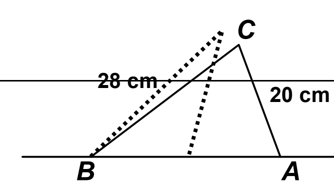
	the same			
4-5	• Sketch the graphs of f(x) = sinx and f(x) = cos x for angle measures expressed in degrees • Determine and describe key properties of both functions	C2.2	sketch the graphs of f(x) = sin x and f(x) = cos x for angle measures expressed in degrees, and determine and describe their key properties (i.e., cycle, domain, range, intercepts, amplitude, period, maximum and minimum values, increasing/decreasing intervals)	***Sample problem:*** Describe and compare the key properties of the graphs of f(x) = sin x and f(x) = cos x. Make some connections between the key properties of the graphs and your understanding of the sine and cosine ratios.
6-7	• Determine, through investigation using technology, and describe the roles of the parameters d and c in functions of the form y = sin(x-d) + c and y = cos(x-d) + c for angles expressed in degrees	C2.3	determine, through investigation using technology, the roles of the parameters d and c in functions of the form y = sin(x − d) + c and y = cos(x − d) + c, and describe these roles in terms of transformations on the graphs of f(x) = sin x and f(x) = cos x with angles expressed in degrees (i.e., vertical and horizontal translations)	***Sample problem:*** Investigate the graph f(x) = 2sin(x − d) + 10 for various values of d, using technology, and describe the effects of changing d in terms of a transformation.
8	• Determine, through investigation using technology, and describethe roles of the parameters a and k in functions of the form y =asinkx and y = acoskx for angles expressed in degrees	C2.4	determine, through investigation using technology, the roles of the parameters a and k in functions of the form y = a sin kx and y = a cos kx, and describe these roles in terms of transformations on the graphs of f(x) = sin x and f(x) = cos x with angles expressed in degrees (i.e., reflections in the axes; vertical and horizontal stretches and compressions to and from the x- and y-axes)	***Sample problem:*** Investigate the graph f(x) = 2sin kx for various values of k, using technology, and describe the effects of changing k in terms of transformations.
9-10	• Determine the amplitude, period, phase shift, domain and range of sinusoidal functions whose equations are given by y = asin(k(xd))+c or y = acos (k(x-d))+c	C2.5	determine the amplitude, period, and phase shift of sinusoidal functions whose equations are given in the form f(x) = a sin(k(x − d)) + c or f(x) = a cos(k(x − d)) + c, and sketch graphs of y = a sin(k(x − d)) + c and y = a cos(k(x − d)) + c by applying transformations to the graphs of f(x) = sin x and f(x) = cos x	***Sample problem:*** Transform the graph of f(x) = cos x to sketch g(x) = 3cos(x + 90°) and h(x) = cos(2x) − 1, and state the amplitude, period, and phase shift of each function.
11	• Sketch graphs of y = asin(k(x-d))+c or y = acos (k(x-d))+c by applying transformations to the graphs y = sinx or y = cosx • Discuss the domain and range of the transformed function	C2.5	determine the amplitude, period, and phase shift of sinusoidal functions whose equations are given in the form f(x) = a sin(k(x − d)) + c or f(x) = a cos(k(x − d)) + c, and sketch graphs of y = a sin(k(x − d)) + c and y = a cos(k(x − d)) + c by applying transformations to the graphs of f(x) = sin x and f(x) = cos x	
12	• Represent a sinusoidal function with an equation given a graph or its properties • Pose and solve problems based on applications involving sinusoidal functions using graphs and graphing technology	C2.6	represent a sinusoidal function with an equation, given its graph or its properties	***Sample problem:*** A sinusoidal function has an amplitude of 2 units, a period of 180°, and a maximum at (0,3). Represent the function with an equation in two different ways, using first the sine function and then the cosine function.

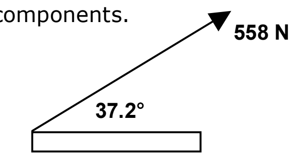
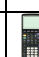
	LESSON INCLUDED	C3.3	pose problems based on applications involving a sinusoidal function, and solve these and other such problems by using a given graph or a graph generated with technology, in degree mode, from a table of values or from its equation	***Sample problem:*** The height above the ground of a rider on a Ferris wheel can be modelled by the sinusoidal function h(t) = 25cos(3(t – 60)) + 27, where h(t) is the height in metres and t is the time in seconds. Graph the function, using graphing technology in degree mode, and determine the maximum and minimum heights of the rider, the height after 30 s, and the time required to complete one revolution.
13-14	• Collect data that would show sinusoidal behaviour and model with sinusoidal functions with and without technology • Describe how sinusoidal graphs change given changes in the context ***LESSONS INCLUDED***	C3.1 C3.2	collect data that can be modelled as a sinusoidal function (e.g., voltage in an AC circuit, pressure in sound waves, the height of a tack on a bicycle wheel that is rotating at a fixed speed), through investigation with and without technology, from primary sources, using a variety of tools (e.g., concrete materials, measurement tools such as motion sensors), or from secondary sources (e.g., websites such as Statistics Canada, E-STAT), and graph the data identify periodic and sinusoidal functions, including those that arise from real-world applications involving periodic phenomena, given various representations (i.e., tables of values, graphs, equations), and explain any restrictions that the context places on the domain and range	***Sample problem:*** Measure and record distance-time data for a swinging pendulum, using a motion sensor or other measurement tools, and graph the data. Describe how the graph would change if you moved the pendulum further away from the motion sensor. What would you do to generate a graph with a smaller amplitude? ***Sample problem:*** The depth, w metres, of water in a lake can be modelled by the function w = 5sin(31.5n + 63) + 12, where n is the number of months since January 1, 1995. Identify and explain the restrictions on the domain and range of this function.
15	Review Day (JAZZ DAY)			
16-17	Summative Unit Evaluation ***LESSONS INCLUDED***			

Unit 5: Applications of Trigonometric Ratios and Vectors (10 days + 2 jazz days + 1 summative evaluation day)

BIG Ideas:
- Solve problems arising from real-world applications using primary trigonometric ratios, the sine law and the cosine law
- Investigate conditions leading to the ambiguous case and solve problem involving oblique triangles
- Represent geometric vectors as directed line segments and find their sum and differences.
- Solve vector problems arising from real-world applications

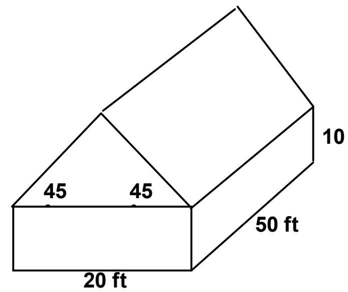
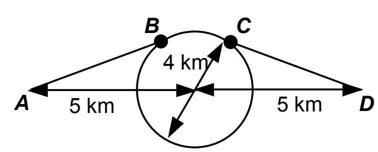
DAY	Lesson Title & Description	Expectations		Teaching/Assessment Notes and Curriculum Sample Problems
1-2	• Solve problems that will activate prior knowledge about primary trigonometric ratios, the sine law and the cosine law	C1.4	solve multi-step problems in two and three dimensions, including those that arise from real-world applications (e.g., surveying, navigation), by determining the measures of the sides and angles of right triangles using the primary trigonometric ratios	***Sample problem:*** Explain how you could find the height of an inaccessible antenna on top of a tall building, using a measuring tape, a clinometer, and trigonometry. What would you measure, and how would you use the data to calculate the height of the antenna?
3	• Solve multi-step problems in two and three dimensions, including those that arise from real-world applications by determining the measures of the sides and angles of right triangles using the primary trigonometric ratios	C1.4	solve multi-step problems in two and three dimensions, including those that arise from real-world applications (e.g., surveying, navigation), by determining the measures of the sides and angles of right triangles using the primary trigonometric ratios	
4-5	• Make connections between the graphical solution of $\sin\theta = k$, $0 < k < 1$, for $0 < \theta < 180$, and the geometric representation of ambiguous case using a variety of tools and strategies (e.g. dynamic geometry software, graphing calculator, graph paper and string) • Solve problems involving oblique triangles, including those that arise from real-world applications, using	C1.5	solve problems involving oblique triangles, including those that arise from real-world applications, using the sine law (including the ambiguous case) and the cosine law	***Sample problem:*** The following diagram represents a mechanism in which point B is fixed, point C is a pivot, and a slider A can move horizontally as angle B changes. The minimum value of angle B is 35°. How far is it from the extreme left position to the extreme right position of slider A?

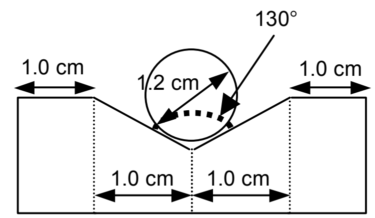


	the sine law (including the ambiguous case) and cosine law relate to the values of sine and cosine for angles $90 < \theta < 180$			
6	• Recognize the properties of a vector (magnitude, direction). • Represent a vector as a directed line segment geometrically. • Identify, gather, and interpret information about real-world applications of vectors. ***LESSON INCLUDED***	D1.1 D1.2	recognize a vector as a quantity with both magnitude and direction, and identify, gather, and interpret information about real-world applications of vectors (e.g., displacement; forces involved in structural design; simple animation of computer graphics; velocity determined using GPS) represent a vector as a directed line segment, with directions expressed in different ways (e.g., 320°; N 40°W), and recognize vectors with the same magnitude and direction but different positions as equal vectors	Computer with LCD projector needed ***Sample problem:*** Position is represented using vectors. Explain why knowing that someone is 69 km from Lindsay, Ontario, is not sufficient to identify their exact position.
7-8	• Understand the equality of vectors • Resolve a vector represented as a directed line segment into its vertical and horizontal components in context. • Represent a vector as a directed line segment, given its vertical and horizontal components. ***LESSONS INCLUDED***	D1.2 D1.3 D1.4	represent a vector as a directed line segment, with directions expressed in different ways (e.g., 320°; N 40°W), and recognize vectors with the same magnitude and direction but different positions as equal vectors resolve a vector represented as a directed line segment into its vertical and horizontal components represent a vector as a directed line segment, given its vertical and horizontal components (e.g., the displacement of a ship that travels 3 km east and 4 km north can be represented by the vector with a magnitude of 5 km and a direction of N 36.9°E)	Computer with LCD projector and computer lab optional ***Sample problem:*** A cable exerts a force of 558 N at an angle of 37.2° with the horizontal. Resolve this force into its vertical and horizontal components. 
9-10	• Determine, through investigation using a variety of tools and strategies, the sum and difference of two vectors. • Solve problems involving the addition and subtraction of vectors, including problems arising from real-world applications. ***LESSONS INCLUDED***	D1.5 D1.6	determine, through investigation using a variety of tools (e.g., graph paper, technology) and strategies (i.e., head-to-tail method; parallelogram method; resolving vectors into their vertical and horizontal components), the sum (i.e., resultant) or difference of two vectors solve problems involving the addition and subtraction of vectors, including problems arising from real-world applications (e.g., surveying, statics, orienteering)	 Computer with LCD projector needed ***Sample problem:*** Two people pull on ropes to haul a truck out of some mud. The first person pulls directly forward with a force of 400 N, while the other person pulls with a force of 600 N at a 50° angle to the first person along the horizontal

				plane. What is the resultant force used on the truck?
11-12	Review Day (JAZZ DAY)			
13	Summative Unit Evaluation			

Unit 6: Solving Problems Involving Geometry (11 days + 1 jazz day + 1 summative evaluation day)				
BIG Ideas: • Solve problems involving 2D shapes and 3D figures and arising from real world applications • Determine circular properties and solve related problems, including those arising from real world applications.				
DAY	Lesson Title & Description	Expectations		Teaching/Assessment Notes and Curriculum Sample Problems
1	• Gather and interpret information about real world applications of geometric shapes in a variety of contexts in technology related fields (eg. Product design, architecture), and explain these applications (eg. one reason that sewer covers are round is to prevent them from falling into the sewer during removal and replacement) (sample problem: Explain why rectangular prisms are used to package many products)	D2.1	gather and interpret information about real-world applications of geometric shapes and figures in a variety of contexts in technology-related fields (e.g., product design, architecture), and explain these applications (e.g., one reason that sewer covers are round is to prevent them from falling into the sewer during removal and replacement)	Sample problem: Explain why rectangular prisms are often used for packaging.
2	• Perform required conversions between the imperial system and the metric system using a variety of tools, as necessary within applications	D2.2	perform required conversions between the imperial system and the metric system using a variety of tools (e.g., tables, calculators, online conversion tools), as necessary within applications	
3	• Solve problems involving the areas of rectangles, parallelograms, trapezoids, triangles, and circles, and of related composite shapes, in situations arising from real-world applications	D2.3	solve problems involving the areas of rectangles, parallelograms, trapezoids, triangles, and circles, and of related composite shapes, in situations arising from real-world applications	Sample problem: Your company supplies circular cover plates for pipes. How many plates with a 1-ft radius can be made from a 4-ft by 8-ft sheet of stainless steel? What percentage of the steel will be available for recycling?
4	• Solve problems involving the volumes and surface areas of spheres, right prisms, and cylinders, and of related composite figures, in situations arising from real-world applications	D2.4	solve problems involving the volumes and surface areas of spheres, right prisms, and cylinders, and of related composite figures, in situations arising from real-world applications	Sample problem: For the small factory shown in the following diagram, design specifications require that the air be exchanged every 30 min. Would a ventilation system that exchanges air at a rate of 400 ft /min satisfy the

				specifications? Explain. 
5-6	• Solve problems involving 2D shapes and 3D figures and arising from real world applications ***LESSONS INCLUDED***	D2.3 D2.4	solve problems involving the areas of rectangles, parallelograms, trapezoids, triangles, and circles, and of related composite shapes, in situations arising from real-world applications solve problems involving the volumes and surface areas of spheres, right prisms, and cylinders, and of related composite figures, in situations arising from real-world applications	
7	• Recognize and describe arcs, tangents, secants, chords, segments, sectors, central angles, and inscribed angles of circles ***LESSON INCLUDED***	D3.1	recognize and describe (i.e., using diagrams and words) arcs, tangents, secants, chords, segments, sectors, central angles, and inscribed angles of circles, and some of their real-world applications (e.g., construction of a medicine wheel)	
8	• Determine the length of an arc and the area of a sector or segment of a circle, and solve related problems	D3.2	determine the length of an arc and the area of a sector or segment of a circle, and solve related problems	Sample problem: A circular lake has a diameter of 4 km. Points A and D are on opposite sides of the lake and lie on a straight line through the centre of the lake, with each point 5 km from the centre. In the route ABCD, AB and CD are tangents to the lake and BC is an arc along the shore of the lake. How long is this route? 

9-10	• Determine, through investigation using a variety of tools, properties of the circle associated with chords, central angles, inscribed angles, and tangents ***LESSONS INCLUDED***	D3.3	determine, through investigation using a variety of tools (e.g., dynamic geometry software), properties of the circle associated with chords, central angles, inscribed angles, and tangents (e.g., equal chords or equal arcs subtend equal central angles and equal inscribed angles; a radius is perpendicular to a tangent at the point of tangency defined by the radius, and to a chord that the radius bisects)	Computer with LCD projector and computer lab needed Sample problem: Investigate, using dynamic geometry software, the relationship between the lengths of two tangents drawn to a circle from a point outside the circle.
11	• Solve problems involving properties of circles, including problems arising from real-world applications. ***LESSONS INCLUDED***	D3.4	solve problems involving properties of circles, including problems arising from real-world applications	Sample problem: A cylindrical metal rod with a diameter of 1.2 cm is supported by a wooden block, as shown in the following diagram. Determine the distance from the top of the block to the top of the rod. 
12	Review Day (JAZZ DAY)			
13	Summative Unit Evaluation			

Grade 12 Math for College Technology

Unit 7: Course Summative Performance Task and Final Exam (4 days)				
BIG Ideas: • Students will model data using exponential and trigonometric models around an environmental context. • Graphing technology is optional for the tasks. • A pencil and paper exam is provided to address expectations not addressed by the task and that are more skill based.				
DAY	**Lesson Title & Description**	**Expectations**		**Teaching/Assessment Notes and Curriculum Sample Problems**
1	• Are you "eco"-logical? ***LESSON INCLUDED***		This lesson provides the context for the next two days of the performance task.	
2	• Save Your Energy ***LESSON AND TASK INCLUDED***	C2.2 C2.6 C3.1 C3.2 C3.3		(Optional)
3	• Save Your Energy ***LESSON AND TASK INCLUDED***	A1.2 A1.4 A2.6		(Optional)
4	Final Exam ***EXAM INCLUDED***	Expectations not addressed by the performance task will be covered by the final exam.		***Note***: A formula sheet should be provided to students for measurement questions. The EQAO Grade 9 Assessment formula sheet is adequate for the expectations covered.