MHF 4U Unit 4 –Polynomial Functions– Outline

Day	Lesson Title	Specific Expectations
1	Transforming Trigonometric Functions	B2.4, 2.5, 3.1
2	Transforming Sinusoidal Functions	B2.4, 2.5, 3.1
3	Transforming Sinusoidal Functions - continued	B2.4, 2.5, 3.1
4	Writing an Equation of a Trigonometric Function	B2.6, 3.1
5	Real World Applications of Sinusoidal Functions	B2.7, 3.1
6	Real World Applications of Sinusoidal Functions Day 2	B2.7, 3.1
7	Compound Angle Formulae	B 3.1. 3.2
8 (Lesson included)	Proving Trigonometric Identities	B3.3
9 (Lesson included)	Solving Linear Trigonometric Equations	B3.4
10 (Lesson included)	Solving Quadratic Trigonometric Equations	B3.4
11-12	JAZZ DAY	
13	SUMMATIVE ASSESSMENT	
TOTAL DAYS:		13

Unit 4: Day 8: Proving Trigonometric Identities			**MHF4U**
Minds On: 10 Action: 55 Consolidate:10 Total=75 min		**Learning Goals**: Demonstrate an understanding that an identity holds true for any value of the independent variable (graph left side and right side of the equation as functions and compare) Apply a variety of techniques to prove identities	**Materials** BLM 4.8.1 BLM 4.8.2 BLM 4.8.3 BLM 4.8.4
			Assessment Opportunities
	Minds On…	**Whole Class → Investigation** Using BLM 4.8.2 the teacher introduces the idea of proof… trying to show something, but following a set of rules by doing it.	The "trickledown" puzzle has two rules: you may only change one letter at a time, and each change must still result in a rule. Trig proofs are similar: you must use only valid "substitutions" and you must only deal with one side at a time.
	Action!	**Whole Class → Discussion** The teacher introduces the students to the idea of trigonometric proofs (using the trickledown puzzle as inspiration. The teacher goes through several examples with students.	
	Consolidate Debrief	**Small Groups → Activity** Using BLM 4.8.3 students perform the "complete the proof" activity. The "Labels" go on envelopes and inside each envelope students get a cut-up version of the proof which they can put in order. When they are finished they can trade with another group. Also, this could be done individually, or as a kind of race/competition.	
Exploration *Application*		**Home Activity or Further Classroom Consolidation** Complete BLM 4.8.4	

A-W 11	**McG-HR 11**	**H11**	**A-W12 (MCT)**	**H12**	**McG-HR 12**
5.9	5.7	9.2		Appendix p.390-395	

4.8.1 Proving Trigonometric Identities (Teacher Notes)

To prove an identity, the RHS and LHS should be dealt with separately. In general there are certain "rules" or guidelines to help:

1. Use algebra or previous identities to transform one side to another.
2. Write the entire equation in terms of one trig function.
3. Express everything in terms of sine and cosines
4. Transform both LHS and RHS to the same expression, thus proving the identity.

Known identities:
2 quotient identities
reciprocal identities
Pythagorean identities
Compound Angle Formulae

Example 1: cot x sin x = cos x

$$\begin{aligned} LS &= \cot x \sin x \\ &= \left(\frac{\cos x}{\sin x}\right)\sin x \\ &= \left(\frac{\cos x}{\cancel{\sin x}}\right)\cancel{\sin x} \\ &= \cos x \\ &= RS \end{aligned}$$

Example 2: $(1 - \cos^2 x)(\csc x) = \sin x$

$$\begin{aligned} LS &= (1-\cos^2 x)(\csc x) \\ &= \sin^2 x \csc x \\ &= \sin^2 x\left(\frac{1}{\sin x}\right) \\ &= \sin^{\cancel{2}} x\left(\frac{1}{\cancel{\sin x}}\right) \\ &= \sin x \\ &= RS \end{aligned}$$

4.8.1 Proving Trigonometric Identities (Teacher Notes continued)

Example 3: (1 + sec x)/ (tan x + sin x) = csc x

$$
\begin{aligned}
LS &= \frac{1+\sec x}{\tan x+\sin x} \\
&= \frac{1+\frac{1}{\cos x}}{\frac{\sin x}{\cos x}+\sin x} \\
&= \frac{1+\frac{1}{\cos x}}{\frac{\sin x}{\cos x}+\sin x} \bullet \frac{\cos x}{\cos x} \\
&= \frac{\cos x+1}{\sin x+\sin x\cos x} \\
&= \frac{\cos x+1}{\sin x(1+\cos x)} \\
&= \frac{\cancel{\cos x+1}}{\sin x\cancel{(1+\cos x)}} \\
&= \frac{1}{\sin x} \\
&= \csc x \\
&= RS
\end{aligned}
$$

Example 4: 2cos x cos y = cos(x + y) + cos(x – y)

$$
\begin{aligned}
RS &= \cos(x+y)+\cos(x-y) \\
&= \cos x\cos y-\sin y\sin x+\cos x\cos y+\sin y\sin x \\
&= \cos x\cos y+\cos x\cos y \\
&= 2\cos x\cos y \\
&= LS
\end{aligned}
$$

4.8.2 The Trickledown Puzzle

You goal is to change the top word into the bottom word in the space allowed. The trickledown puzzle has two simple rules:

1. You may only change one letter at a time;
2. Each new line must make a new word.

COAT	PLUG	SLANG
_______	_______	_______
_______	_______	_______
_______	_______	_______
VASE	_______	_______
	STAY	TWINE

Proving The Trigonometric Identity

Your goal is to show that the two sides of the equation are equal. You may only do this by:

1. Substituting valid identities
2. Working with each side of the equation separately

$$[1 + \cos(x)][1 - \cos(x)] = \sin^2(x)$$

4.8.3 Trigonometric Proofs!

Cut the following labels and place each one on an envelope.

Label: $\frac{\sin^2 x}{1-\cos x}=1+\cos x$	Label: $\tan x(\cot x+\tan x)=\sec^2 x$
Label: $(1+\sin x)(1-\sin x)=\frac{1}{\sec^2 x}$	Label: $\sin^2 x(\csc^2 x+\sec^2 x)=\sec^2 x$
Label: $\tan x(\cot x+\tan x)=\sec^2 x$	Label: $\tan^2 x\cos^2 x=1-\cos^2 x$
Label: $\csc x\sec x=\cot x+\tan x$	Label: $\cos^2 x+\tan^2 x\cos^2 x=1$
Label: $\tan x\sin x=\frac{1-\cos^2 x}{\cos x}$	Label: $\sin x(\csc x-\sin x)=\cos^2 x$

4.8.3 Trigonometric Proofs! (Continued)

For each of the following proofs cut each line of the proof into a separate slip of paper. Place all the strips for a proof in the envelope with the appropriate label.

$\frac{\sin^2 x}{1-\cos x} = 1+\cos x$	$\tan x(\cot x + \tan x) = \sec^2 x$
$\frac{1-\cos^2 x}{1-\cos x} = 1+\cos x$	$\tan x\left(\frac{1}{\tan x} + \tan x\right) = \sec^2 x$
$\frac{(1+\cos x)(1-\cos x)}{(1-\cos x)} = 1+\cos x$	$1+\tan^2 x = \sec^2 x$
$1+\cos x = 1+\cos x$	$1+\frac{\sin^2 x}{\cos^2 x} = \sec^2 x$
$LS = RS$	$\frac{\cos^2 x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x} = \sec^2 x$
	$\frac{1}{\cos^2 x} = \sec^2 x$
	$\sec^2 x = \sec^2 x$
	$LS = RS$

4.8.3 Trigonometric Proofs! (Continued)

$(1+\sin x)(1-\sin x)=\dfrac{1}{\sec^2 x}$ $1-\sin^2 x=\dfrac{1}{\sec^2 x}$ $\cos^2 x=\dfrac{1}{\sec^2 x}$ $\dfrac{1}{\sec^2 x}=\dfrac{1}{\sec^2 x}$ $LS=RS$	$\sin^2 x(\csc^2 x+\sec^2 x)=\sec^2 x$ $\sin^2 x\csc^2 x+\sin^2 x\sec^2 x=\sec^2 x$ $\sin^2 x\left(\dfrac{1}{\sin^2 x}\right)+\sin^2 x\left(\dfrac{1}{\cos^2 x}\right)=\sec^2 x$ $1+\dfrac{\sin^2 x}{\cos^2 x}=\sec^2 x$ $\dfrac{\cos^2 x}{\cos^2 x}+\dfrac{\sin^2 x}{\cos^2 x}=\sec^2 x$ $\dfrac{1}{\cos^2 x}=\sec^2 x$ $\sec^2 x=\sec^2 x$ $LS=RS$
$\cos^2 x+\tan^2 x\cos^2 x=1$ $\cos^2 x+\left(\dfrac{\sin^2 x}{\cos^2 x}\right)\cos^2 x=1$ $\cos^2 x+\sin^2 x=1$ $1=1$ $LS=RS$	$\tan^2 x\cos^2 x=1-\cos^2 x$ $\tan^2 x\left(1-\sin^2 x\right)=1-\cos^2 x$ $\dfrac{\sin^2 x}{\cos^2 x}(\cos^2 x)=1-\cos^2 x$ $\sin^2 x=1-\cos^2 x$ $1-\cos^2 x=1-\cos^2 x$ $LS=RS$

4.8.3 Trigonometric Proofs! (Continued)

$\csc x \sec x = \cot x + \tan x$ $\csc x \sec x = \left(\frac{\cos x}{\sin x}\right) + \left(\frac{\sin x}{\cos x}\right)$ $\csc x \sec x = \left(\frac{\cos^2 x}{\sin x \cos x}\right) + \left(\frac{\sin^2 x}{\sin x \cos x}\right)$ $\csc x \sec x = \frac{1}{\sin x \cos x}$ $\csc x \sec x = \left(\frac{1}{\sin x}\right)\left(\frac{1}{\cos x}\right)$ $\csc x \sec x = \csc x \sec x$ $LS = RS$	$\tan x(\cot x + \tan x) = \sec^2 x$ $\tan x\left(\frac{1}{\tan x} + \tan x\right) = \sec^2 x$ $1 + \tan^2 x = \sec^2 x$ $1 + \frac{\sin^2 x}{\cos^2 x} = \sec^2 x$ $\frac{\cos^2 x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x} = \sec^2 x$ $\frac{1}{\cos^2 x} = \sec^2 x$ $\sec^2 x = \sec^2 x$ $LS = RS$
$\tan x \sin x = \frac{1 - \cos^2 x}{\cos x}$ $\tan x \sin x = \frac{\sin^2 x}{\cos x}$ $\tan x \sin x = \left(\frac{\sin x}{\cos x}\right)\left(\frac{\sin x}{1}\right)$ $\tan x \sin x = \tan x \sin x$ $LS = RS$	$\sin x(\csc x - \sin x) = \cos^2 x$ $\sin x \csc x - \sin^2 x = \cos^2 x$ $\sin x\left(\frac{1}{\sin x}\right) - \sin^2 x = \cos^2 x$ $1 - \sin^2 x = \cos^2 x$ $\cos^2 x = \cos^2 x$ $LS = RS$

4.8.4 Proving Trigonometric Identities: Practice

Knowledge

1. Prove the following identities:
   (a) $\tan x \cos x = \sin x$ (b) $\cos x \sec x = 1$

   (c) $(\tan x)/(\sec x) = \sin x$

2. Prove the identity:
   (a) $\sin^2 x(\cot x + 1)^2 = \cos^2 x(\tan x + 1)^2$

   (b) $\sin 2x – \tan 2x = -\sin 2x \tan 2x$

   (c) $(\cos 2x – 1)(\tan 2x + 1) = -\tan 2x$

   (d) $\cos^4 x – \sin^4 x = \cos^2 x – \sin^2 x$

3. Prove the identity
   (a) $\cos(x – y)/[\sin x \cos y] = \cot x + \tan y$

   (b) $\sin(x + y)/[\sin(x – y)] = [\tan x + \tan y]/[\tan x – \tan y]$

4. Prove the identity:
   (a) $\sec x / \csc x + \sin x / \cos x = 2 \tan x$

   (b) $[\sec x + \csc x]/[1 + \tan x] = \csc x$

   (c) $1/[\csc x – \sin x] = \sec x \tan x$

Application

5. Half of a trigonometric identity is given. Graph this half in a viewing window on $[-2\pi, 2\pi]$ and write a conjecture as to what the right side of the identity is. Then prove your conjecture.

   (a) $1 – (\sin^2 x / [1 + \cos x]) = ?$

   (b) $(\sin x + \cos x)(\sec x + \csc x) – \cot x – 2 = ?$

4.8.4 Proving Trigonometric Identities (Continued)

6. Prove the identity:
   (a) [1 – sin x] / sec x = cos3x / [1 + sin x]

   (b) –tan x tan y(cot x – cot y) = tan x – tan y

7. Prove the identity:

   cos x cot x / [cot x – cos x] = [cot x + cos x] / cos x cot x

8. Prove the identity:

   (cos x – sin y) / (cos y – sin x) = (cos y + sin x) / (cos x + sin y)

Thinking

9. Prove the "double angle formulae" shown below:

$$\sin 2x = 2 \sin x \cos x$$
$$\cos 2x = \cos^2 x - \sin^2 x$$
$$\tan 2x = 2 \tan x / [1 - \tan^2 x]$$

Hint: 2x = x + x

Answers:

All identities in #1 - 9 can be proven.

1. (a) conjecture: cos(x)
   (b) conjecture: tan(x)

Unit 4: Day 9: Solving Linear Trigonometric Equations

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Minds On: 20 Action: 45 Consolidate:10 Total=75 min	**Learning Goals**: Solve linear and quadratic trigonometric equations with and without graphing technology, for real values in the domain from 0 to 2π Make connections between graphical and algebraic solutions	**Materials** • BLM 4.9.1 • BLM 4.9.2 • BLM 4.1.1

Assessment Opportunities

Minds On…	**Small Groups → Activity** Put up posters around the class and divides the class into small groups (as many as there are posters) Groups go and write down their thoughts on their assigned poster for 5 minutes, then they are allowed to tour the other posters (1 min each) and write new comments. Once the "tour" is done, the posters are brought to the front and the comments are discussed The Poster titles are: CAST RULE, SOLVE sinx = 0, SOLVE cosx = 1, SOLVE sinx = 0.5, SOLVE cosx = 0.5, SOLVE 2tanx = 0	BLM 4.8.1 contains teacher notes to review solving trigonometric equations.
Action!	**Whole Class → Discussion** The teacher reviews solving linear trigonometric equations. The teacher should use different schemes to describe the solution and its meaning (i.e., each student can have a graphing calculator to use and graph equations, the unit circle can be discussed, etc).	
Consolidate Debrief	**Whole Class → Investigation** Teacher reintroduces the Unit Problem again (using BLM 4.1.1) and discusses how to solve the problems involving finding solutions to the equation. "When is the ball m high?" etc. This reviews solving an absolute value equation as well.	**Process Expectation: Reflecting:** Student reflect on the unit problem and all of the ways they have been able to tackle the problem.
Exploration *Application*	**Home Activity or Further Classroom Consolidation** Complete BLM 4.9.2	

A-W 11	McG-HR 11	H11	A-W12 (MCT)	H12	McG-HR 12
5.7	5.8	9.3			8.1, 8.2

4.9.1 Trigonometric Equations Review (Teacher Notes)

Quick review of CAST rule, special angles, and graphs of primary trig functions

Example 1:

Solve

$$\sin x = -\frac{\sqrt{3}}{2}, \quad 0 \le x \le 2\pi$$

$$\sin\alpha = \frac{\sqrt{3}}{2}$$

Here, α is the reference angle (or related acute angle).

$$\alpha = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$= \frac{\pi}{3}$$

$$\therefore x = \pi + \frac{\pi}{3} + 2\pi n, 2\pi - \frac{\pi}{3} + 2\pi n$$

Let n belong to the set of integers. Find all values of n such that x is in desired interval.

$$x = \frac{4\pi}{3}, \frac{5\pi}{3}$$

Example 2:

Solve

$$2\cos\left(x - \frac{\pi}{6}\right) = 1, \quad 0 \le x \le 2\pi$$

$$\cos\left(x - \frac{\pi}{6}\right) = \frac{1}{2}$$

Here, α is the reference angle (or related acute angle).

$$\cos\alpha = \frac{1}{2}$$

$$\alpha = \cos^{-1}\left(\frac{1}{2}\right)$$

$$= \frac{\pi}{3}$$

$$x - \frac{\pi}{6} = \alpha + 2\pi n, \ x - \frac{\pi}{6} = 2\pi - \alpha + 2\pi n$$

Let n belong to the set of integers. Find all values of n such that x is in desired interval.

$$x - \frac{\pi}{6} = \frac{\pi}{3} + 2\pi n, \ x - \frac{\pi}{6} = 2\pi - \frac{\pi}{3} + 2\pi n$$

$$x = \frac{\pi}{2} + 2\pi n, \quad x = \frac{11\pi}{6} + 2\pi n$$

$$x = \frac{\pi}{2}, \quad x = \frac{11\pi}{6}$$

4.9.1 Trigonometric Equations Review (Teacher Notes Continued)

Example 3:

Solve

$$1-\tan\left(2x+\frac{\pi}{2}\right)=0, \quad 0\le x\le 2\pi$$

$$-\tan\left(2x+\frac{\pi}{2}\right)=-1$$

$$\tan\left(2x+\frac{\pi}{2}\right)=1$$

Here, α is the reference angle (or related acute angle).

$$\tan\alpha=1$$

$$\alpha=\tan^{-1}(1)$$

$$=\frac{\pi}{4}$$

$$2x+\frac{\pi}{2}=\alpha+2\pi n\,,\ 2x+\frac{\pi}{4}=\pi+\alpha+2\pi n$$

$$2x+\frac{\pi}{2}=\frac{\pi}{4}+2\pi n,\ 2x+\frac{\pi}{2}=\pi+\frac{\pi}{4}+2\pi n$$

Let n belong to the set of integers. Find all values of n such that x is in desired interval.

$$2x=-\frac{\pi}{4}+2\pi n, \quad 2x=\frac{3\pi}{4}+2\pi n$$

$$x=-\frac{\pi}{8}+\pi n, \quad x=\frac{3\pi}{8}+\pi n$$

$$x=\frac{7\pi}{8}, \quad x=\frac{3\pi}{8}, \quad x=\frac{15\pi}{8}, \quad x=\frac{11\pi}{8}$$

The n-notation is important for students to realize the infinite number of solutions and how we are simply taking the ones that lie in the interval given.

4.9.2 Solving Linear Trigonometric Equations

Knowledge

1. Find the exact solutions:

$$(a)\ \sin x = \frac{\sqrt{3}}{2} \quad (b)\ \tan x = -\sqrt{3} \quad (c)\ 2\cos x = -\sqrt{3}$$

2. Find all solutions of each equation:

$$(a)\ \cot x = 2.3 \quad (b)\ 2\cos\frac{x}{2} = \sqrt{2} \quad (c)\ 5\cos 3x = -3$$

3. Find solutions on the interval [0, 2π]:

$$(a)\ \tan x = -0.237 \quad (b)\ \csc x = \sqrt{2} \quad (c)\ 2\sin x - 1 = -2$$

Application

Use the following information for questions 4 and 5.

When a beam of light passes from one medium to another (for example, from air to glass), it changes both its speed and direction. According to Snell's Law of Refraction,

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2}$$

Where v_1 and v_2 are the speeds of light in mediums 1 and 2, and θ_1 and θ_2 are the angle of incidence and angle of refraction, respectively. The number v_1/v_2 is called the index of refraction.

4. The index of refraction of light passing from air to water is 1.33. If the angle of incidence is 38°, find the angle of refraction.

5. The index of refraction of light passing from air to dense glass is 1.66. If the angle of incidence is 24°, find the angle of refraction.

6. A weight hanging from a spring is set into motion moving up and down. Its distance d (in cm) above or below its "rest" position is described by

$$d = 5\left(\sin 6t - 4\cos 6t\right)$$

At what times during the first 2 seconds is the weight at the "rest" position (d = 0)?

4.9.2 Solving Linear Trigonometric Equations (continued)

7. When a projectile leaves a starting point at an angle of elevation of θ with a velocity v, the horizontal distance it travels is determined by

$$d = \frac{v^2}{32}\sin 2\theta$$

Where d is measured in feet and v in feet per second.

An outfielder throws the ball at a speed of 75 miles per hour to the catcher who it 200 feet away. At what angle of elevation was the ball thrown?

8. Use a trigonometric identity to solve the following:

$$\sin x \cos\frac{\pi}{6} + \sin\frac{\pi}{6}\cos x = \frac{1}{\sqrt{2}}$$

Thinking

9. Let n be a fixed positive integer. Describe all solutions of the equation

$$\sin qx = \frac{1}{2}$$

Answers:

1. $(a)\ x = \frac{\pi}{3} + 2n\pi \ or\ \frac{2\pi}{3} + 2n\pi \quad (b)\ x = -\frac{\pi}{3} + n\pi \quad (c) x = \pm\frac{5\pi}{6} + 2n\pi$
2. (a) x = 0.4101 + kπ, (b) $x = \pm\frac{\pi}{2} + 4n\pi$ (c) $x = \pm 0.7381 + \frac{2n\pi}{3}$
3. (a) x = 2.9089, 6.05048 (b) $x = \frac{\pi}{4}, \frac{3\pi}{4}$ (c) $x = \frac{11\pi}{6}, \frac{7\pi}{6}$
4. 27.57° 5. 14.18° 6. $t = \frac{\tan^{-1} 4}{6} + \frac{n\pi}{6} \approx$ 1.2682, 0.7466, 0.2210, 1.7918
7. 16.0° or 74.0°
8. $\sin(x + \frac{\pi}{6}) = \frac{1}{\sqrt{2}}$

$$x = \frac{\pi}{12} + 2n\pi \quad or \quad \frac{7\pi}{12} + 2n\pi$$

9. $x = \frac{\pi}{6q} + \frac{2n\pi}{q}, \quad x = \frac{5\pi}{6q} + \frac{2n\pi}{q}$

Unit 4: Day 10: Solving Quadratic Trigonometric Equations		**MHF4U**
Minds On: 10 Action: 55 Consolidate:10 Total=75 min	**Learning Goal**: Solve linear and quadratic trigonometric equations with and without graphing technology, for real values in the domain from 0 to 2π Make connections between graphical and algebraic solutions	**Materials** BLM 4.10.1) BLM 4.10.2 BLM 4.10.3 Graphing Calculators
		Assessment Opportunities
Minds On…	**Pairs → Matching Activity** Using BLM 4.10.1 the teacher gives pairs of students cards on which they are to find similar statements. (they are trying to match a trigonometric equation with a polynomial equation) On their card they must justify why their equations are similar. Teacher can give hints about the validity of their reasons. The reasons for the "matches" are then discussed using an overhead copy of BLM 4.10.1 (Answers)	
Action!	**Individual Students → Discussion/Investigation** The teacher goes through solutions to the different cards. Then using that as a basis students work through BLM 4.10.2 to develop strategies and skills in solving quadratic trigonometric equations.	
Consolidate Debrief	**Small Groups → Graphing Calculators** Students are then put into small groups to discuss their answers to 4.10.2 and to use graphing calculators to see the graphs of the functions and determine if their solutions are correct.	
Exploration *Application*	**Home Activity or Further Classroom Consolidation** Complete BLM 4.10.3	

A-W 11	McG-HR 11	H11	A-W12 (MCT)	H12	McG-HR 12
5.7	5.8	9.3			8.1, 8.2

4.10.1 Trigonometric Equations: Matching Cards

✂

$x^2 = \frac{1}{2}$	$\sin^2 x = \frac{1}{2}$
$x^2 - x = 0$	$\tan^2 x - \tan x = 0$
$x^2 + 3x + 2 = 0$	$\cos^2 x + 3\cos x + 2 = 0$
$y + 2xy = 0$	$\sin x + 2\cos x \sin x = 0$
$xy = x$	$\cot x \cos x = \cot x$
$xy = 0$	$\cos x \sin x = 0$

4.10.1 Trigonometric Equations: Matching (Solutions)

Use the following notes to help explain the process of solving quadratic trig equations (and their similarity to polynomial equations)

$x^2 = \frac{1}{2}$ $x = \pm\sqrt{\frac{1}{2}}$ $x = \pm\frac{1}{\sqrt{2}}$	$\sin^2 x = \frac{1}{2}$ $\sin x = \pm\sqrt{\frac{1}{2}}$ $\sin x = \pm\frac{1}{\sqrt{2}}$ $x = \frac{\pi}{4} + \frac{n\pi}{2}$ equation has solutions in all quadrants
$x^2 - x = 0$ $x(x-1) = 0$ $x = 0 \quad or \quad x = 1$	$\tan^2 x - \tan x = 0$ $\tan x(\tan x - 1) = 0$ $\tan x = 0 \quad or \quad \tan x = 1$ $x = n\pi \quad or \quad x = \frac{\pi}{4} + n\pi$ equation has solution on x-axis, or on y = x

4.10.1 Trigonometric Equations: Matching (Teacher Notes)

$x^2+3x+2=0$ $(x+2)(x+1)=0$ $x=-2 \quad or \quad x=-1$	$\cos^2 x+3\cos x+2=0$ $(\cos x+2)(\cos x+1)=0$ $\cos x=-2 \quad or \quad \cos x=-1$ $(no\ solution) \quad or \quad x=\pi+2n\pi$
$y+2xy=0$ $y(1+2x)=0$ $y=0 \quad or \quad 1+2x=0$ $x=-\frac{1}{2}$	$\sin x+2\cos x\sin x=0$ $\sin x(1+2\cos x)=0$ $\sin x=0 \quad or \quad 1+2\cos x=0$ $\cos x=-\frac{1}{2}$ $x=n\pi \quad or \quad x=\frac{2\pi}{3}+2n\pi, \quad x=\frac{4\pi}{3}+2n\pi$
$xy=x$ $xy-x=0$ $x(y-1)=0$ $y-1=0 \quad or \quad x=0$ $y=1$	$\cot x\cos x=\cot x$ $\cot x\cos x-\cot x=0$ $\cot x(\cos x-1)=0$ $\cot x=0 \quad or \quad \cos x-1=0$ $\cos x=1$ $x=\frac{\pi}{2}+n\pi \quad or \quad x=2n\pi$

4.10.1 Trigonometric Equations: Matching **(Teacher Notes)**

$xy = 0$ $x = 0 \quad or \quad y = 0$	$\cos x \sin x = 0$ $\cos x = 0 \quad or \quad \sin x = 0$ $x = \frac{\pi}{2} + n\pi \quad or \quad x = n\pi$

4.10.2 Solving Quadratic Trigonometric Equations

In the following 4 examples, you will deal with 4 different methods to deal with quadratic trigonometric equations.

Method 1: Common Factor

$$Solve\ \ for\ \theta,\ \ 0 \le \theta \le 2\pi$$

$$\tan\theta\cos^2\theta = \tan\theta$$

> Move all terms to LS and factor out the common factor.

Method 2: Trinomial Factor

$$Solve\ \ for\ x,\ \ 0 \le x \le 2\pi$$

$$3\sin^2 x - \sin x - 2 = 0$$

> Replace sin(x) with q…
>
> Can you factor $3q^2 - q - 2$?

4.10.2 Solving Quadratic Trigonometric Equations
(Continued)

Method 3: Identities and Factoring

$$Solve\ \ for\ x,\ \ 0 \le x \le 2\pi$$

$$-10\cos^2 x - 3\sin x + 9 = 0$$

Use an identity here:

$\cos^2 x$ = ????

Method 4: Identities and Quadratic Formula

$$Solve\ \ for\ x,\ \ 0 \le x \le 2\pi$$

$$\sec^2 x + 5\tan x = -2$$

Use an identity here:

$\sec^2 x$ = ????

4.10.2 Solving Quadratic Trigonometric Equations
(Solutions)

In the following 4 examples, you will deal with 4 different methods to deal with quadratic trigonometric equations.

Method 1: Common Factor

$$Solve\ for\ \theta,\ 0 \le \theta \le 2\pi$$

$$\tan\theta\cos^2\theta = \tan\theta$$

$$\tan\theta\cos^2\theta - \tan\theta = 0$$

$$\tan\theta(\cos^2\theta - 1) = 0$$

$$\cos^2\theta - 1 = 0 \quad or \quad \tan\theta = 0$$

$$\cos^2\theta = 1$$

$$\cos\theta = \pm 1$$

$$\theta = n\pi \quad or \quad \theta = n\pi$$

$$\therefore \theta = 0, \pi, 2\pi$$

Method 2: Trinomial Factor

$$Solve\ for\ x,\ 0 \le x \le 2\pi$$

$$3\sin^2 x - \sin x - 2 = 0$$

$$(3\sin x + 2)(\sin x - 1) = 0$$

$$3\sin x + 2 = 0 \quad or \quad \sin x - 1 = 0$$

$$\sin x = -\frac{2}{3} \quad or \quad \sin x = 1$$

$$x = 3.8713 \quad or \quad x = 5.5535 \quad or \quad x = \frac{\pi}{2}$$

4.10.2 Solving Quadratic Trigonometric Equations
(Solutions continued)

Method 3: Identities and Factoring

$$Solve\ \ for\ x,\ \ 0 \le x \le 2\pi$$

$$-10\cos^2 x - 3\sin x + 9 = 0$$

$$-10(1-\sin^2 x) - 3\sin x + 9 = 0$$

$$10\sin^2 x - 3\sin x - 1 = 0$$

$$2\sin x - 1 = 0 \quad or \quad 5\sin x + 1 = 0$$

$$\sin x = \frac{1}{2} \quad or \quad \sin x = -\frac{1}{5}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6} \quad or \quad x = 3.3430 \quad or \quad x = 6.0818$$

Method 4: Identities and Quadratic Formula

$$Solve\ \ for\ x,\ \ 0 \le x \le 2\pi$$

$$\sec^2 x + 5\tan x = -2$$

$$\sec^2 x + 5\tan x + 2 = 0$$

$$(1+\tan^2 x) + 5\tan x + 2 = 0$$

$$\tan^2 x + 5\tan x + 3 = 0$$

$$From\ Quadratic\ Formula\ \tan x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\tan x = -0.6972 \quad or \quad \tan x = -4.3028$$

$$x = 2.5327 \quad or \quad x = 5.6743 \quad or \quad x = 1.7992 \quad or \quad x = 4.9408$$

4.10.3 Solving Quadratic Trigonometric Equations: Practice

Knowledge

10. Find the solutions on the interval [0, 2π]:

$$(a)\ 3\sin^2 x - 8\sin x - 3 = 0 \quad (b)\ 5\cos^2 x + 6\cos x = 8$$

11. Find the solutions on the interval [0, 2π]:

$$(a)\ \cot x \cos x = \cos x \quad (b)\ \sin^2 x + 2\sin x - 2 = 0$$

12. Find solutions on the interval [0, 2π]:

$$(a)\ \cos x \csc x = 2\cos x \quad (b)\ 4\sin x \tan x - 3\tan x + 20\sin x - 15 = 0$$

Hint: in part (b) one factor is tanx + 5

13. Use an identity to find solutions on the interval [0, 2π]:

$$(a)\ \sec^2 x - 2\tan^2 x = 0 \quad (b)\ \sec^2 x + \tan x = 3$$

Application

14. Another model for a bouncing ball is

$$h(t) = 4\sin^2(8\pi t)$$

where h is height measured in metres and t is time measured in seconds.

When is the ball at a height of 2m?

15. Use a trigonometric identity to solve the following on the interval [0, 2π]::

$$\sin 2x = \cos x$$

4.10.3: Solving Quadratic Trigonometric Equations: Practice (Continued)

Communication

16. Compare and contrast the approach to solving a linear trigonometric equation and a quadratic trigonometric equation.

17. What is wrong with this "solution"?

$$\sin x \tan x = \sin x$$
$$\cancel{\sin x} \tan x = \cancel{\sin x}$$
$$\tan x = 1$$
$$x = \frac{\pi}{4} \quad or \quad x = \frac{5\pi}{4}$$

Thinking

18. Find all solutions, if possible, of the below equation on the interval [0, 2π]:

$$\sin^2 x + 3\cos^2 x = 0$$

Answers:

1. $(a)\ x = 3.4814, 5.9433 \quad (b)\ x = \frac{3\pi}{4}, \frac{7\pi}{4}, 2.1588, 5.3004$
2. (a) $x = \frac{\pi}{4}, \frac{\pi}{2}, \frac{5\pi}{4}, \frac{3\pi}{2}$ (b) $x = 0.8213, 2.3203$
3. (a) $x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$ (b) $x = 0.8481, 1.7682, 2.2935, 4.9098$
4. (a) $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$ (b) $x = \frac{\pi}{4}, \frac{5\pi}{4}$
5. $t = \frac{1}{32} + \frac{n}{16}, \quad n = 0, 1, 2, ...$
6. $x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2};$
7. Answers will vary
8. You "divide" one source of your roots. Sin(x) should be brought to LS and then factored
9. No solution