# MHF 4U Unit 1 –Polynomial Functions– Outline

| Day | Lesson Title | Specific Expectations |
|---|---|---|
| 1 | Average Rate of Change and Secants | D1.2, 1.6, both D1.1A's |
| 2-3 | Instantaneous Rate of Change and Tangents | D1.6, 1.4, 1.7, 1.5, both D1.1A's |
| 4 | Solving Problems Involving Average and Instantaneous Rate of Change Numerically and Graphically | D1.8, 1.1 |
| 5-6 | Characteristics of Polynomial Functions Through Numeric, Graphical, and Algebraic Representations | A1.1, 1.2, 1.3, 1.4 |
| 7<br>*(Lesson included)* | Using the Factored Form of a Polynomial Function to Sketch a Graph and Write Equations | A1.6, 1.8 |
| 8 | Transformations of $f(x) = x^3$ and $f(x) = x^4$ and Even and Odd Functions | A1.7, 1.9 |
| 9-10<br>*(Lessons included)* | Dividing Polynomials, The Remainder Theorem and The Factor Theorem | A2.1, A2.1A |
| 11-12 | The Zeros of a Polynomial Function Graphically and Algebraically with Applications to Curve Fitting | A2.2, 2.3 ,2 4, 2.6 |
| 13-14 | Solving Polynomials Inequalities Graphically, Numerically, and Algebraically | A3.1, 3.2 3.3 |
| 15-16 | JAZZ DAY | |
| 17 | SUMMATIVE ASSESSMENT | |
| TOTAL DAYS: | | 17 |

# Unit 1: Day 7: In Factored Form

| | | |
|---|---|---|
| Minds On: 15<br>Action: 40<br>Consolidate:20<br>Total=75 min | **Learning Goals**:<br>Make connections between a polynomial function in factored form and the x- intercepts of its graph<br>Sketch the graph of polynomial functions, expressed in factored form using the characteristics of polynomial functions<br>Determine the equation of a polynomial given a set of conditions (e.g. zeros, end behaviour) and recognize there may be more than one such function | **Materials**<br>• BLM 1.7.1<br>• BLM 1.7.2<br>• BLM 1.7.3<br>• Graphing calculators/ software/ sketch: **GSP_gr12_U1D7.gsp** |

**Assessment Opportunities**

## Minds On…

### Individual → Exploration

Recall with students that $(x-1)^2=(x-1)(x-1)$, $(x-1)^3=(x-1)(x-1)(x-1)$ etc.
Students will complete questions 1, 2 and 3 on BLM 1.7.1. Take up questions 2 and 3. Possible answer #2: the degree of the polynomial is equal to the number of factors that contain an "x". Possible answers #3: multiply the factored form; multiply the variable term of each factor and logically conclude that this is the degree of the polynomial.

*Assessment Opportunities:* Display sketches of yesterdays polynomial functions. Reference to finite differences and its connection to the degree would be beneficial

## Action!

### Individual → Investigation

Students observe the sketches of the graphs generated and displayed in "minds on" and answer questions #4, 5, 6. Teacher takes up questions with the class or students check their answers in pairs. Make connection between the x-intercept and its corresponding co-ordinates $(x_I,0)$. Introduce language to describe shape and post on word wall. Possible answer #4: the graph crosses the x axis when y = 0 in the polynomial function. This occurs when each factor is equal to zero. When these mini equations are "solved" the x-intercepts are evident.
Answers #6 respectively: The graph "bounces" at the intercept. The graph has an "inflection point" at the intercept.

### Pairs → Think Pair Share

Students complete BLM 1.7.2 in pairs.
**Curriculum Expectation/Observation/Mental Note**: Circulate to observe pairs are successful in representing the graph of a polynomial function given the equation in factored form. Make a mental note to consolidate misconceptions.

### Whole Class → Discussion then Practice

Present a graph of a polynomial function with 3 intercepts and ask: What is a possible equation of this polynomial? How do you know? What are some possible other equations? Do a few other examples including some with double roots and inflection points. Then state some characteristics of a polynomial function and ask for an equation that satisfies these characteristics. (e.g. What is a possible equation of the polynomial function of degree 3 that begins in quadrant 2, ends in quadrant 4, and has x intercepts of -1, ½, and 4.) Students complete BLM 1.7.3.

*Assessment Opportunities:*
Word wall: concave up/down, hills, valleys, inflection point (informal descriptions in students' own words)

If students look only at factors of the form (x-a) they may have the misconception that "the x intercept is the opposite of the "a" value. Use some factors of the form (ax-b) and have the students understand that the intercept can be found by solving ax-b=0

This assessment for learning will determine if students are ready to formulate an equation given the characteristics of a polynomial function.

## Consolidate Debrief

### Pairs to Whole Class → A answers B

Teacher asks the following questions: What are the advantages of a polynomial function being given in factored form? Why is it possible that 2 people sketching the same graph could have higher or lower "hills" and "valleys"? How could you get a more accurate idea of where these turning points are located? Explain why the graph the function $y = (x^2 – 4)(x^2 – 9)$ is easier or more difficult to graph than $y = (x^2 + 4)(x^2 + 9)$. How can you identify the y intercept of a function given algebraically and why is this helpful to know?

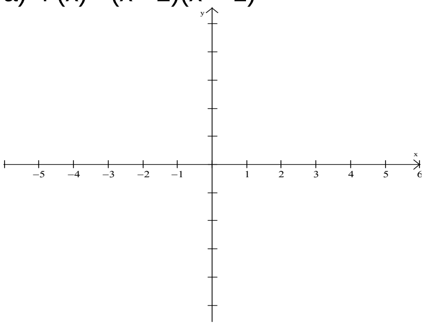
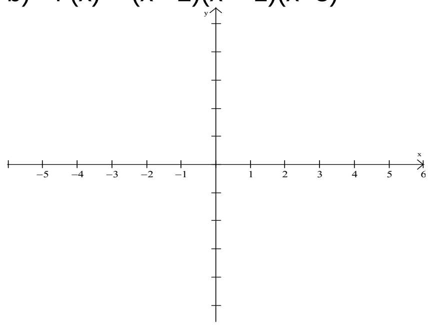
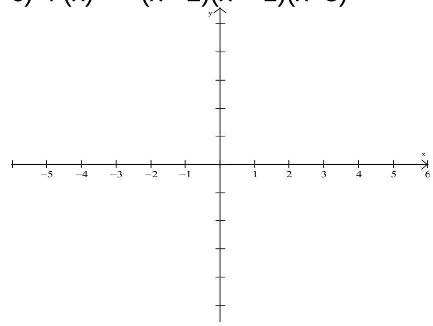
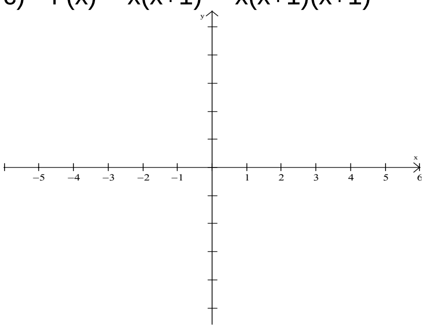
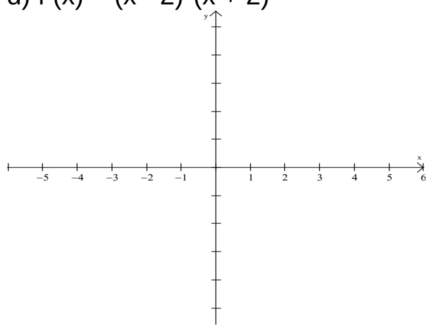
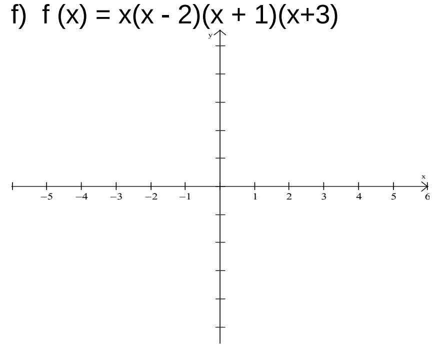
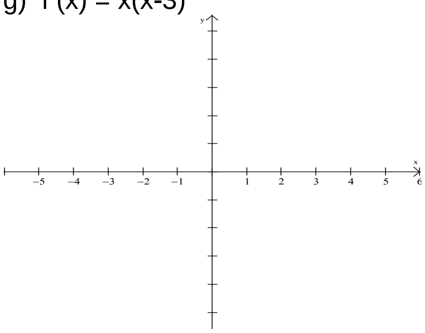
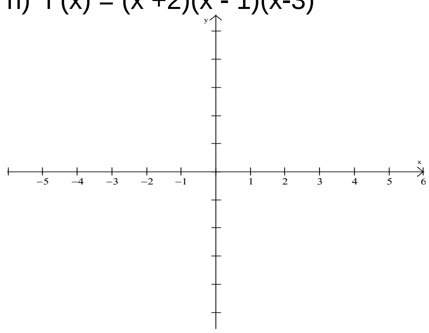
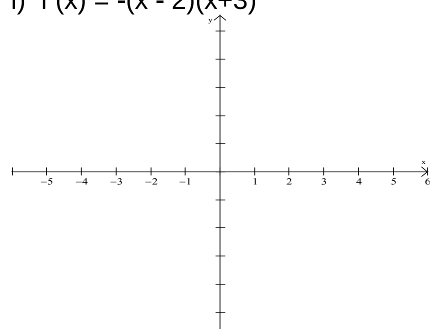
*Assessment Opportunities:* A answers B: Ask a series of questions one at a time, to be answered in pairs, then ask for one or more students to answer, alternating A and B partners

| | | | |
|---|---|---|---|
| *Concept Practice* | **Home Activity or Further Classroom Consolidation**<br>Assign questions similar to BLM 1.7.1 Ask the question: What information is needed to determine an exact equation for a graph if x intercepts are given?<br>Possible answer to be taken up the next day and followed through in lesson : Another point on the function. | | |

| A-W 11 | McG-HR 11 | H11 | A-W12 (MCT) | H12 | McG-HR 12 |
|---|---|---|---|---|---|
| | | | | 1.1 | 2.1 |

# 1.7.1 What Role Do Factors Play?

1. Use technology (graphing calculator, software, GSP_Gr12_U1D7) to determine the graph of each polynomial function. Sketch the graph, clearly identifying the x-intercepts.

| | | |
|---|---|---|
| a) f (x)= (x - 2)(x + 1)<br><br>Degree of the function: _______<br>x-intercepts: ____________ | b) f (x) = (x - 2)(x + 1)(x+3)<br><br>Degree of the function: _______<br>x-intercepts: ____________ | c) f (x) = - (x - 2)(x + 1)(x+3)<br><br>Degree of the function: _______<br>x-intercepts: ____________ |
| c) f (x) = $x(x+1)^2$= x(x+1)(x+1)<br><br>Degree of the function: _______<br>x-intercepts: ____________ | d) f (x) = $(x - 2)^2(x + 2)^2$<br><br>Degree of the function: _______<br>x-intercepts: ____________ | f) f (x) = x(x - 2)(x + 1)(x+3)<br><br>Degree of the function: _______<br>x-intercepts: ____________ |
| g) f (x) = $x(x-3)^3$<br><br>Degree of the function: _______<br>x-intercepts: ____________ | h) f (x) = $(x +2)(x - 1)(x-3)^2$<br><br>Degree of the function: _______<br>x-intercepts: ____________ | i) f (x) = $-(x - 2)(x+3)^3$<br><br>Degree of the function: _______<br>x-intercepts: ____________ |

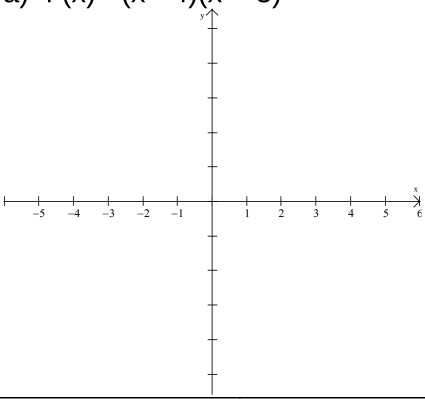
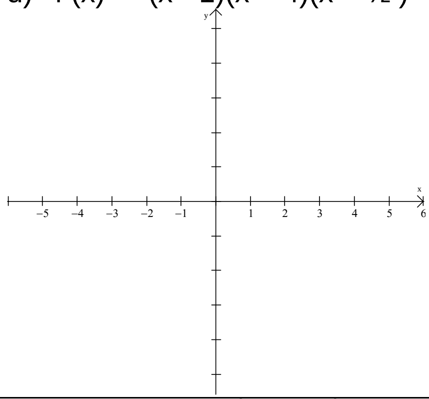
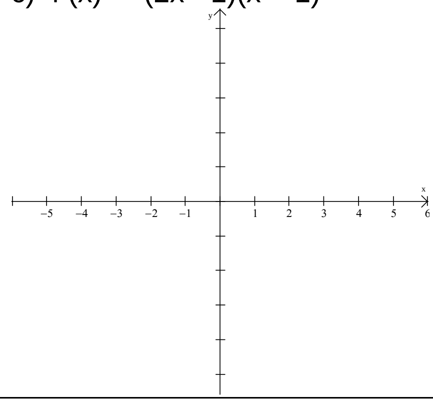
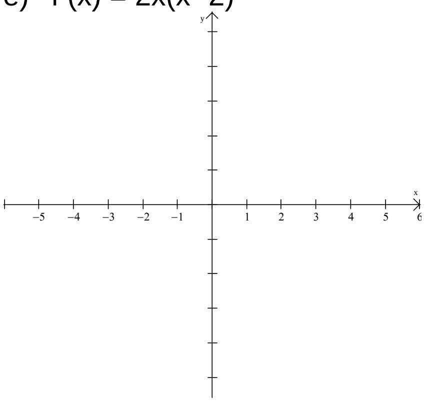
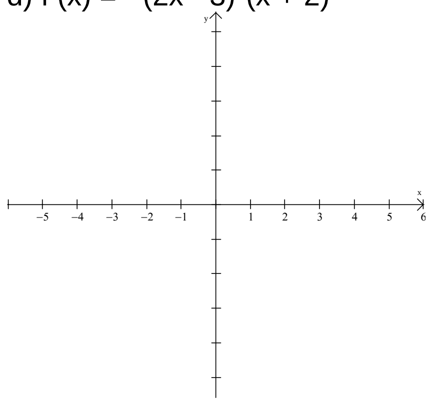
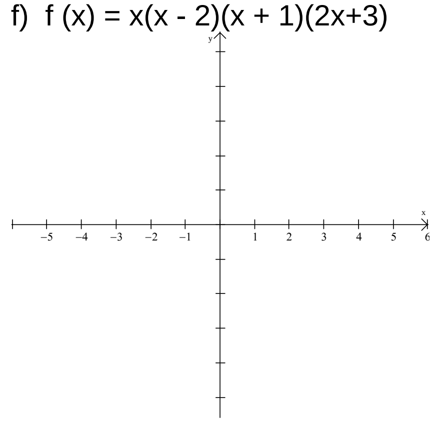
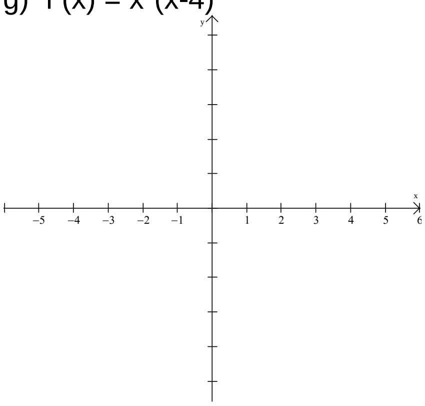
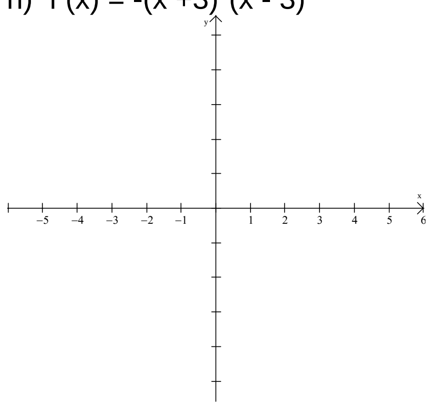
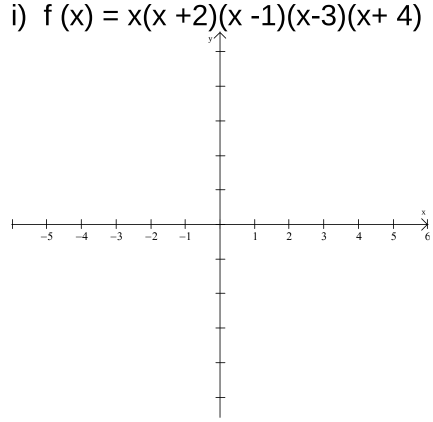
2. Compare your graphs with the graphs generated on the previous day and make a conclusion about the degree of a polynomial when it is given in factored form.
3. Explain how to determine the degree of a polynomial algebraically if given in factored form.
4. What connection do you observe between the factors of the polynomial function and the x-intercepts? Why does this make sense? (hint: all co-ordinates on the x axis have y = 0).
5. Use your conclusions from #4 to state the x-intercepts of each of the following. Check by graphing with technology, and correct if necessary.

| | | |
|---|---|---|
| f(x) = (x-3)(x+5)(x-1/2)<br>x-intercepts: ____________<br>does this check? ________ | f(x) = (x-3)(x+5)(2x-1)<br>x-intercepts: ___________<br>does this check? _______ | f(x) = (2x-3)(2x+5)(x-1)(3x-2)<br>x-intercepts: ____________<br>does this check? ________ |

6. What do you notice about the graph when the polynomial function has a factor that occurs twice? Three times?

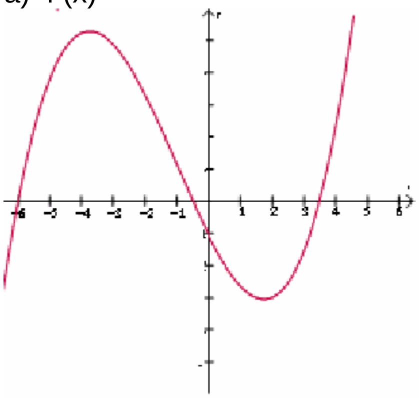
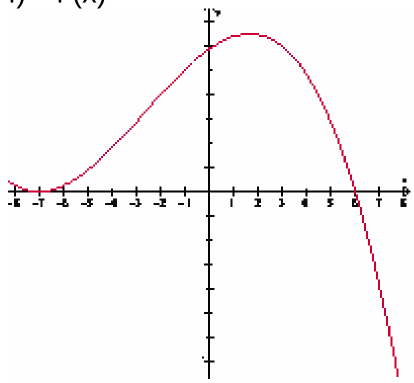
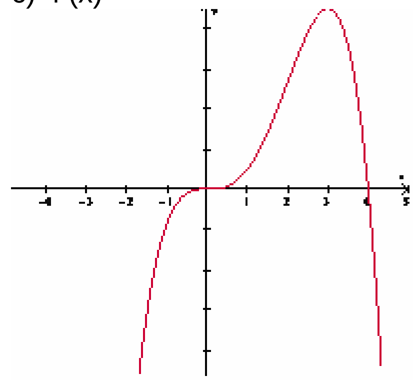
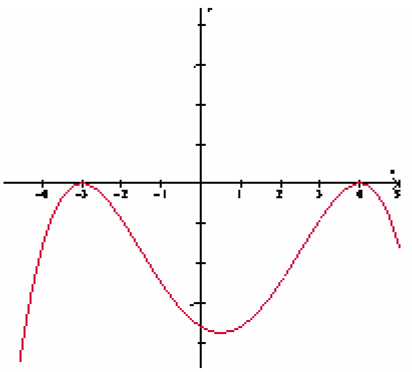
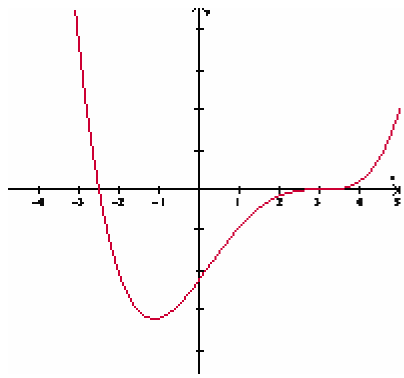
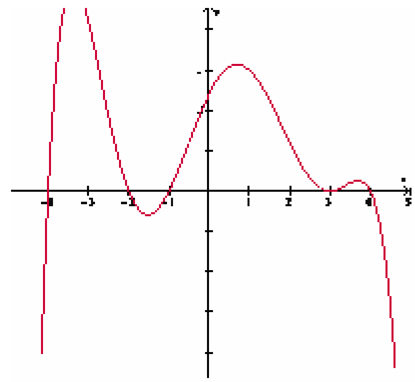
# 1.7.2 Factoring in our Graphs

Draw a sketch of each graph using the properties of polynomial functions.  After you complete each sketch, check with your partner, discuss your strategies and make any corrections needed.

| | | |
|---|---|---|
| a) $f(x) = (x - 4)(x + 3)$   | d) $f(x) = -(x - 1)(x + 4)(x - ½)$   | c) $f(x) = (2x - 1)(x + 1)^2$   |
| e) $f(x) = 2x(x - 2)^3$   | d) $f(x) = -(2x - 3)^2(x + 2)^2$   | f) $f(x) = x(x - 2)(x + 1)(2x+3)$   |
| g) $f(x) = x^3(x-4)$   | h) $f(x) = -(x + 3)^2(x - 3)^3$   | i) $f(x) = x(x + 2)(x - 1)(x-3)(x+ 4)$   |

# 1.7.3 What's My Polynomial Name?

1. Determine a possible equation for each polynomial function.

| a) f (x)= | f) f (x) = | c) f (x) = |
|---|---|---|
|  |  |  |
| g) f (x) = | d) f (x) = | f) f (x) = |
|  |  |  |

2. Determine an example of an equation for a function with the following characteristics:

a) Degree 3, a double root at 4, a root at -3 ____________________________

b) Degree 4, an inflection point at 2, a root at 5 ____________________________

c) Degree 3, roots at ½, ¾, -1 ______________________________

d) Degree 3, starting in quadrant 2, ending in quadrant 4, root at -2 and double root at 3

_______________________

e) Degree 4, starting in quadrant 3, ending in quadrant 4, double roots at -10 and 10

_____________________

# Unit 1: Day 9: Dividing Polynomials (Day 1)

MHF4U

| Minds On: 5<br>Action: 50<br>Consolidate:20<br>Total=75 min | **Learning Goals**:<br>Divide polynomials<br>Examine remainders of polynomial division and connect to the remainder theorem<br>Make connections between the polynomial function f(x), the divisor x –a, the remainder of the division f(x)/(x-a) and f(a) using technology<br>Identify the factor theorem as a special case of the remainder theorem<br>Factor polynomial expressions in one variable of degree greater than two. | **Materials**<br>BLM 1.9.1<br>BLM 1.9.2. |
|---|---|---|

| | | **Assessment Opportunities** |
|---|---|---|
| **Minds On…** | **Whole Class → Discussion**<br>Students should put away their calculators and attempt the following without the use of calculators – remind them of the long division methods for dividing:<br>Divide 789 by 7. result: 112 remainder 5<br>Divide 12546 by 6. result: 2091 remainder 0<br>Divide 32455 by 4. result: 8113 remainder 3<br>Include as many more as desired until students have a clear understanding of the method of long division. | |
| **Action!** | **Small Groups → Experiment**<br>Students will work either as a class or in pairs to complete the activities:<br>Activity 1: Read through the worked example on BLM 1.9.1 and then do the division questions on page 2 of BLM 1.9.1<br>Activity 2: Work time for homework exercises (BLM 1.9.2 or textbook work). | |
| **Consolidate Debrief** | **Small Groups → Interview**<br>Students will consolidate :<br>Activity 1: Teacher can choose to have students work in pairs or work through the first few questions as a class and then in pairs or individually for the remainder. As students finish, the teacher can have students complete questions on the board to aid those who might be having difficulty.<br>Activity 2: Complete exercises on BLM 1.9.2. | |
| *Exploration*<br>*Application* | **Home Activity or Further Classroom Consolidation**<br>BLM 1.9.2 or textbook work from outlined text below:<br>Addison Wesley 12 (MCT): Sections – 2.4 | |

| A-W 11 | McG-HR 11 | H11 | A-W12 (MCT) | H12 | McG-HR 12 |
|---|---|---|---|---|---|
| | | | | 1.3, 1.4, 2.1, 2.2 | 2.2, 2.3, 2.4 |

# 1.9.1 Dividing Polynomials

Dividing a polynomial by another polynomial is similar to performing a division of numbers using long division. For example, divide the polynomial $x^3+13x^2+39x+46$ by $x+9$

**Solution**:

1)
$$\begin{array}{rl} & x^2 \\ x+9 & \overline{)\,x^3+13x^2+39x+46} \end{array}$$

first divide x into $x^3$ to get $x^2$

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2)
$$\begin{array}{rl} & x^2 \\ x+9 & \overline{)\,x^3+13x^2+39x+46} \\ & \underline{x^3+9x^2} \\ & \quad\;\; 4x^2 \end{array}$$

now multiply $x^2$ by $x + 9$ to get $x^3 + 9x^2$
then subtract $x^3 + 9x^2$ from $x^3 + 13x^2$ to get $4x^2$

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3)
$$\begin{array}{rl} & x^2+4x \\ x+9 & \overline{)\,x^3+13x^2+39x+46} \\ & \underline{x^3+9x^2}\;\downarrow \\ & \quad\;\; 4x^2+39x \end{array}$$

bring down the + 39x
divide $4x^2$ by x to get 4x

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4)
$$\begin{array}{rl} & x^2+4x \\ x+9 & \overline{)\,x^3+13x^2+39x+46} \\ & \underline{x^3+9x^2}\;\downarrow \\ & \quad\;\; 4x^2+39x \\ & \quad\;\; \underline{4x^2+36x} \\ & \qquad\qquad 3x \end{array}$$

now multiply 4x by x + 9 to get $4x^2 + 36x$
then subtract $4x^2 + 36x$ from $4x^2 + 39x$ to get 3x

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5)
$$\begin{array}{rl} & x^2+4x+3 \\ x+9 & \overline{)\,x^3+13x^2+39x+46} \\ & \underline{x^3+9x^2}\;\downarrow\quad\downarrow \\ & \quad\;\; 4x^2+39x\;\downarrow \\ & \quad\;\; \underline{4x^2+36x}\;\downarrow \\ & \qquad\qquad 3x+46 \\ & \qquad\qquad \underline{3x+27} \\ & \qquad\qquad\quad\; 19 \end{array}$$

bring down the + 46
divide 3x by x to get 3

multiply 3 by x + 9 to get 3x + 27
then subtract 3x + 27 from 3x + 46 to get 19

Since the remainder has a lower degree than the divisor, the division is now complete. The result can be written as:

$$x^3+13x^2+39x+46 = (x+9)(x^2+4x+3)+19$$

(**NOTE**: *You could check your answer by multiplying out the result*.)

## 1.9.1 Dividing Polynomials (Continued)

Using the previous example, complete the polynomial division questions below:

**1.** $x^3 - 5x^2 - x - 10$ by $x - 2$

**2.** $2y^3 + y^2 - 27y - 36$ by $y + 3$

**3.** $y^3 - 28y - 41$ by $y + 4$
[*note: $y^3 - 28y - 41 = y^3 + 0y^2 - 28y - 41$]

**4.** $2x^3 - 3x^2 - 8x - 3$ by $2x + 1$

**5.** $-6x^3 + 29x^2 + 7x - 13$ by $2x - 1$

**6.** $y^3 + 4y^2 - 3y - 12$ by $y + 4$

# 1.9.1 Dividing Polynomials (Answers)

**1.** $x^3 - 5x^2 - x - 10$ by $x - 2$

$$\begin{array}{r} x^2 - 3x - 7 \\ x-2\overline{)\,x^3 - 5x^2 - x - 10} \\ \underline{x^3 - 2x^2} \downarrow\ \downarrow \\ -3x^2 - x \downarrow \\ \underline{-3x^2 + 6x} \downarrow \\ -7x - 10 \\ \underline{-7x + 14} \\ -24 \end{array}$$

Result: $(x-2)(x^2 - 3x - 7) - 24$

**2.** $2y^3 + y^2 - 27y - 36$ by $y + 3$

$$\begin{array}{r} 2y^2 - 5y - 12 \\ y+3\overline{)\,2y^3 + y^2 - 27y - 36} \\ \underline{2y^3 + 6y^2} \downarrow\ \downarrow \\ -5y^2 - 27y \downarrow \\ \underline{-5y^2 - 15y} \downarrow \\ -12y - 36 \\ \underline{-12y - 36} \\ 0 \end{array}$$

Result: $(y+3)(2y^2 - 5y - 12)$

*extra*→ $(y+3)(2y+3)(y-4)$

**3.** $y^3 - 28y - 41$ by $y + 4$

$$\begin{array}{r} y^2 - 4y - 12 \\ y+4\overline{)\,y^3 \qquad - 28y - 41} \\ \underline{y^3 + 4y^2} \downarrow\ \downarrow \\ -4y^2 - 28y \downarrow \\ \underline{-4y^2 - 16y} \downarrow \\ -12y - 41 \\ \underline{-12y - 48} \\ 7 \end{array}$$

Result: $(y+4)(y^2 - 4y - 12) + 7$

*extra*→ $(y+4)(y+2)(y-6) + 7$

**4.** $2x^3 - 3x^2 - 8x - 3$ by $2x + 1$

$$\begin{array}{r} x^2 - 2x - 3 \\ 2x+1\overline{)\,2x^3 - 3x^2 - 8x - 3} \\ \underline{2x^3 + x^2} \downarrow\ \downarrow \\ -4x^2 - 8x \downarrow \\ \underline{-4x^2 - 2x} \downarrow \\ -6x - 3 \\ \underline{-6x - 3} \\ 0 \end{array}$$

Result: $(2x+1)(x^2 - 2x - 3)$

*extra*→ $(2x+1)(x+1)(x-3)$

**5.** $-6x^3 + 29x^2 + 7x - 13$ by $2x - 1$

$$\begin{array}{r} -3x^2 + 13x + 10 \\ 2x-1\overline{)\,-6x^3 + 29x^2 + 7x - 13} \\ \underline{-6x^3 + 3x^2} \downarrow\ \downarrow \\ 26x^2 + 7x \downarrow \\ \underline{26x^2 - 13x} \downarrow \\ 20x - 13 \\ \underline{20x - 10} \\ -3 \end{array}$$

Result: $(2x-1)(-3x^2 + 13x + 10) - 3$

*extra*→ $(2x-1)(-3x-2)(x-5) - 3$

**6.** $y^3 + 4y^2 - 3y - 12$ by $y + 4$

$$\begin{array}{r} y^2 \qquad - 3 \\ y+4\overline{)\,y^3 + 4y^2 - 3y - 12} \\ \underline{y^3 + 4y^2} \downarrow\ \downarrow \\ 0 \quad -3y - 12 \\ \underline{-3y - 12} \\ 0 \end{array}$$

Result: $(y+4)(y^2 - 3)$

*extra*→ $(y+4)(y+\sqrt{3})(y-\sqrt{3})$

## 1.9.2 Dividing Polynomials

Complete the exercises below:

1. Find each quotient and remainder:
   (a) $(x^2+6x+15)\div(x+3)$ (b) $(x^2-4x+13)\div(x-2)$

   (c) $(x^2-x+3)\div(x+2)$ (d) $(2x^3+x^2-24x-32)\div(x-4)$

2. When a certain polynomial is divided by $x+3$, the quotient is $x^2-3x+5$ and the remainder is $6$. What is the polynomial?

3. When a certain polynomial is divided by $x-2$, the quotient is $x^2+4x-7$ and the remainder is $-4$. What is the polynomial?

4. Divide:
   (a) $(x^3+3x^2-4x-12)\div(x-2)$ (b) $(3x^3+2x^2-11x-12)\div(x+1)$

   (c) $(2x^3+x^2-24x-32)\div(x-4)$ (d) $(2x^3+3x^2-14x-13)\div(x-3)$

## Unit 1: Day 10: Dividing Polynomials (Day 2)

**MHF4U**

| | |
|---|---|
| Minds On: 5 | **Learning Goal**: |
| Action: 50 | Divide polynomials<br>Examine remainders of polynomial division and connect to the remainder theorem<br>Make connections between the polynomial function f(x), the divisor x –a, the remainder of the division f(x)/(x-a) and f(a) using technology<br>Identify the factor theorem as a special case of the remainder theorem<br>Factor polynomial expressions in one variable of degree greater than two. |
| Consolidate:20 | |
| Total=75 min | |

**Materials**
BLM 1.10.1
BLM 1.10.2.

**Assessment Opportunities**

**Minds On…**

**Whole Class → Discussion**
Students should take out the BLM 1.9.1 from last day:
Examine the results from the left hand (odd) questions and the right hand (even) questions. Using the original polynomial try substituting the indicated values for the variable in each question:
1. x= 2 2. y = -3 (y = -3/2 and y = 4)
3. y = -4 (*y = -2 and y = 6*) 4. x = -½ (x = -1 and x = 3)
5. x= ½ (*x = -2/3 and x = 5*) 6. y = -4 (y = $-\sqrt{3}$ and y = $\sqrt{3}$)

Note: at this point (if not previously completed) it might be suggested to the students that they further factor their previous answers for questions 2 through 6 and then try the numbers given in parenthesis above

After examining the results try and come to a generalization of the results → this is essentially the Remainder Theorem (*odd questions*) and Factor Theorem (*even questions*)

**Action!**

**Small Groups → Experiment**
Students will work either as a class or in pairs to complete the activities:
Activity 1: Complete the minds on and then work on the BLM 1.10.1.
Activity 2: Work time for homework exercises – BLM 1.10.2 or textbook work.

**Consolidate Debrief**

**Small Groups → Interview**
Students will consolidate :
Activity 1: Teacher can choose to have students work in pairs or work through the first couple of questions as a class and then either in pairs or individually for the remainder. As students finish, the teacher can have students complete questions on the front board to aid those who might be having difficulty.
Activity 2: Complete exercises on BLM 1.10.2.

*Exploration*
*Application*

**Home Activity or Further Classroom Consolidation**
BLM 1.10.2 or textbook work from outlined text below:
Addison Wesley 12 (MCT): Sections – 2.5

| A-W 11 | McG-HR 11 | H11 | A-W12 (MCT) | H12 | McG-HR 12 |
|---|---|---|---|---|---|
| | | | | 1.3, 1.4, 2.1, 2.2 | 2.2, 2.3, 2.4 |

# 1.10.1 Remainder Theorem and Factor Theorem

**Remainder Theorem:**
When a polynomial $f(x)$ is divided by $x-a$, the remainder is $f(a)$

1. Find the remainder when $2x^3+3x^2-17x-30$ is divided by each of the following:

   (a) $x-1$ (b) $x-2$ (c) $x-3$

   (d) $x+1$ (e) $x+2$ (f) $x+3$

**Factor Theorem**:
If $x=a$ is substituted into a polynomial for $x$, and the remainder is 0, then $x-a$ is a factor of the polynomial.

2. Using the above Theorem and your results from question 1 which of the given binomials are factors of $2x^3+3x^2-17x-30$?

3. Using the binomials you determined were factors of $2x^3+3x^2-17x-30$, complete the division (i.e. divide $2x^3+3x^2-17x-30$ by your chosen ( $x-a$ ) and remember to fully factor your result in each case.

# 1.10.1 Remainder Theorem and Factor Theorem (Answers)

1. Find the remainder when $2x^3+3x^2-17x-30$ is divided by each of the following:

(a) $x-1$

$\therefore a=1$

$f(1)=2(1)^3+3(1)^2-17(1)-30$

$f(1)=2+3-17-30$

$f(1)=-42$

(b) $x-2$

$a=2$

$f(a)=-36$

(c) $x-3$

$a=3$

$f(a)=0$

(d) $x+1$

$a=-1$

$f(a)=-12$

(e) $x+2$

$a=-2$

$f(a)=0$

(f) $x+3$

$a=-3$

$f(a)=-6$

2. Using the above Theorem and your results from question 1 which of the given binomials are factors of $2x^3+3x^2-17x-30$?

From results → (c) $x-3$ and (e) $x+2$ are factors

3. Using the binomials you determined were factors of $2x^3+3x^2-17x-30$ complete the division (i.e. divide $2x^3+3x^2-17x-30$ by your chosen $x-a$) and remember to fully factor your result in each case.

(c) $x-3$

$$\begin{array}{r} 2x^2+9x+10 \\ x-3\overline{)2x^3+3x^2-17x-30} \\ \underline{2x^3-6x^2} \downarrow \quad \downarrow \\ 9x^2-17x \downarrow \\ \underline{9x^2-27x} \downarrow \\ 10x-30 \\ \underline{10x-30} \\ 0 \end{array}$$

**Result**: $(x-3)(2x^2+9x+10)$

$(x-3)(2x+5)(x+2)$

(e) $x+2$

$$\begin{array}{r} 2x^2-x-15 \\ x+2\overline{)2x^3+3x^2-17x-30} \\ \underline{2x^3+4x^2} \downarrow \quad \downarrow \\ -x^2-17x \downarrow \\ \underline{-x^2-2x} \downarrow \\ -15x-30 \\ \underline{-15x-30} \\ 0 \end{array}$$

**Result**: $(x+2)(2x^2-x-15)$

$(x+2)(2x+5)(x-3)$

***(Note:*** The results are the same just rearranged.)

## 1.10.2 Dividing Polynomials Practice

Complete the polynomial divisions below:

1. Without using long division, find each remainder:
   (a) $(2x^2+6x+8)\div(x+1)$ (b) $(x^2+4x+12)\div(x-4)$

   (c) $(x^3+6x^2-4x+3)\div(x+2)$ (d) $(3x^3+7x^2-2x-11)\div(x-2)$

2. Find each remainder:
   (a) $(2x^2+x-6)\div(x+2)$ (b) $(x^3+6x^2-4x+2)\div(x+1)$

   (c) $(x^3+x^2-12x-13)\div(x-2)$ (d) $(x^4-x^3-3x^2+4x+2)\div(x+2)$

3. When $x^3+kx^2-4x+2$ is divided by $x+2$ the remainder is 26, find *k*.

4. When $2x^3-3x^2+kx-1$ is divided by $x-1$ the remainder is 2, find *k*.

**ANSWERS**:

1. (a) 4 (b) 44 (c) 27 (d) 37
2. (a) 0 (b) 11 (c) -25 (d) 6
3. 6
4. 4