MHF 4U Unit 0 – Get Ready – Outline

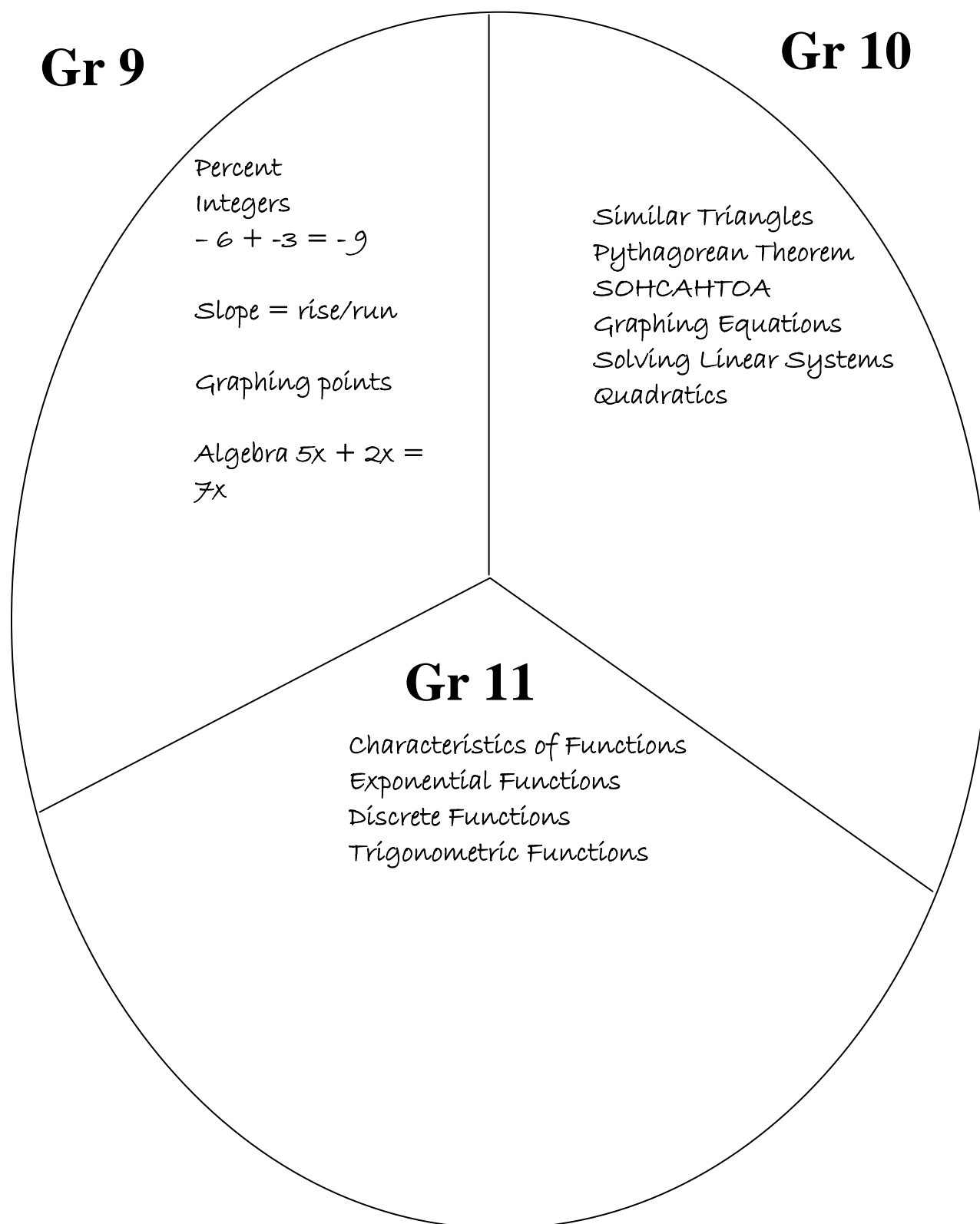| Day	Lesson Title	Specific Expectations
1 (Lesson Included)	Introduction to Grade 12	Grades 7 - 11
2 (Lesson Included)	Introduction to Grade 12	Grade 11 U expectations
3	Adding Precision to Graphical Models and Their Descriptions	D1.1, D1.1, D3.1, and setting up C1.2
4	Adding Precision to Graphical Models and Their Descriptions -Day 2	D1.1, D1.1, D3.1, and setting up C1.2
5	Transformations Across Function Types	Setting up C1.6, A2.3
6	Using Function Notation to Generalize Relationships	Setting up C1.6
7-8	JAZZ DAY	
9	SUMMATIVE	
TOTAL DAYS:		9

Unit 0: Day 1 Introduction to Grade 12		MHF4U
Minds On: 15 Action: 40 Consolidate:20 Total=75 min	**Learning Goal** • Review the mathematical concepts from previous years.	**Materials** BLM 0.1.1 BLM 0.1.2 BLM 0.1.3

			Assessment Opportunities
■	**Minds On…**	**Pairs →Think/Pair** Students fill in the math they remember from previous grades on BLM 0.1.1 ("Math I Know"). Allow 4 or 5 minutes individually, then an additional 5 minutes in pairs.	**Process Expectation: Reflecting:** The students are working to recall all the math they know from high school.
■	**Action!**	**Individual → Practice** Students solve as many of the questions from BLM 0.1.2 "Find Someone Who" as they can. **Whole Class →Exploration** Students circulate around the classroom, introduce themselves to others in the class at the same time finding students who can help them fill out the rest of their sheet. Students record persons name as well as the full solution to the question. Take up all questions on the overhead or board to be certain the correct answers are all found.	
■	**Consolidate Debrief**	**Small Groups →Extending** Add to BLM 0.1.1 "Math I Know" Worksheets. **Whole Class → Discussion** Share their responses to BLM 0.1.1.	
		Home Activity or Further Classroom Consolidation Students complete BLM 0.1.3.	

0.1.1 Math I Know!

Gr 9

Gr 10

Gr 11

Gr 9

Gr 10

Percent
Integers
$-6 + -3 = -9$

Slope = rise/run

Graphing points

Algebra $5x + 2x = 7x$

Similar Triangles
Pythagorean Theorem
SOHCAHTOA
Graphing Equations
Solving Linear Systems
Quadratics

Gr 11

Characteristics of Functions
Exponential Functions
Discrete Functions
Trigonometric Functions
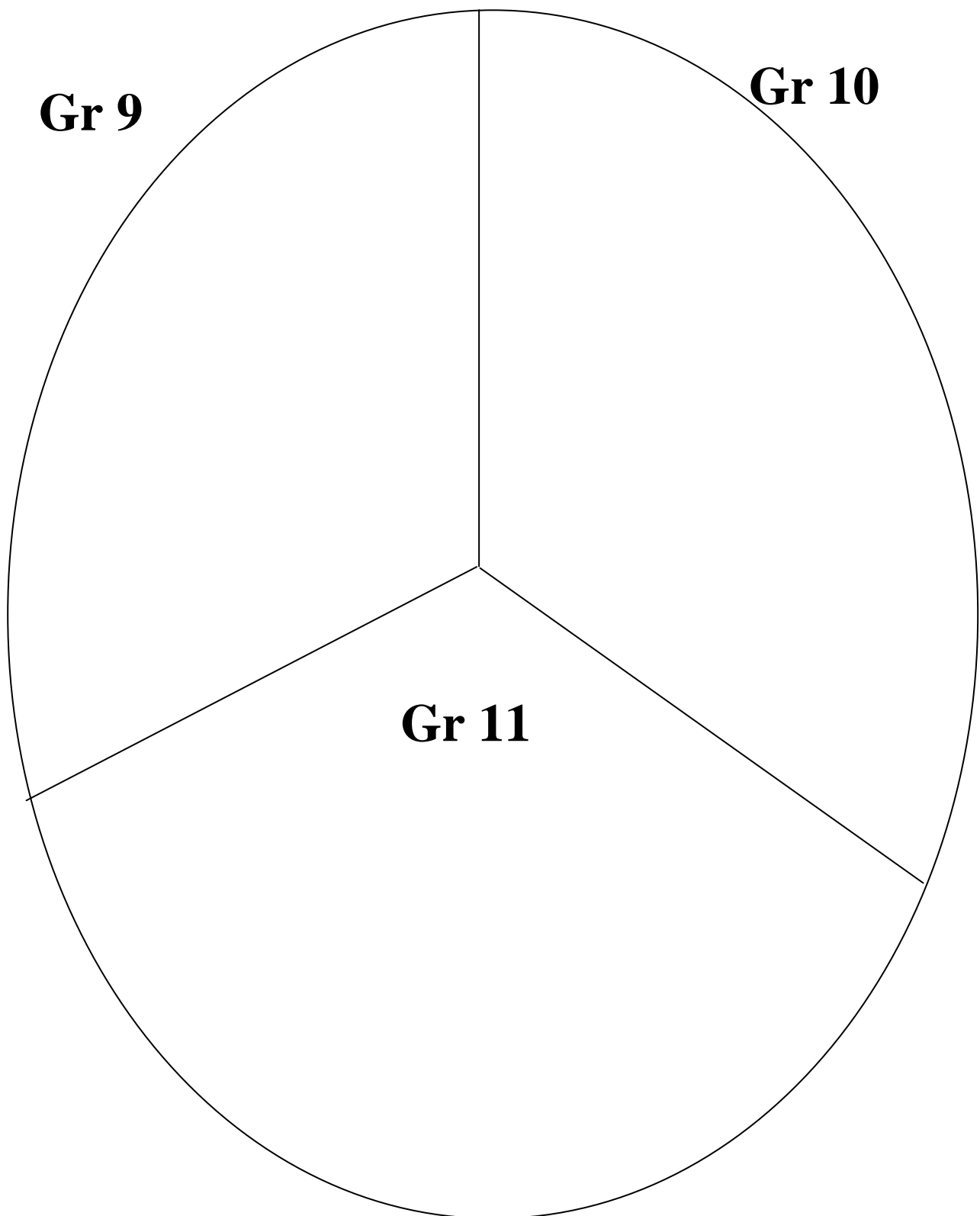
0.1.2 Find Someone Who ….

For a particular sequence, $t_1 = 5$ and $t_n = 2t_{n-1} + 3$. Find t_5.

Name_____

Given $f(x) = x^2 + 5x + 20$, find $f(6)$.

Name _____

$1500 is invested at 4%/year compounded quarterly for 5 years. The value of the investment at the end of the term is _____

Name: _____

Can expand $3(x + 2)$

Name_____

Can solve for p:
$P - 4 = 13$

Name: _____

Can round to the nearest hundred
a) $459 =$ _____
b) $32926 =$ _____

Name _____

Simplify $\dfrac{x^2 - 4}{x^2 - 5x + 6}$

Name: _____

Can represent slope in two ways

Name_____

Can round to the nearest tenths
a) $7.22 =$ _____
b) $3.024 =$ ____
c) $2.56 =$ ____

Name _____

Can solve: If you build a skateboard ramp whose ratio of height to base must be 2:3, what is the base if the height is 4.5 m?

Name _____

Simplify $\dfrac{m^2 + m - 20}{m^2 - 11m + 28}$ and state restrictions

Name_____

Can expand and simplify $(x + 1)^2$

Name _____

Can simplify
$$\dfrac{3a}{a+3} - \dfrac{(a-4)}{a-2}$$

Name: _____

Can use the Pythagorean theorem to solve for c given a=3 and b=4

Name _____

0.1.2 Find Someone Who …. (Teacher)

For a particular sequence, $t_1 = 5$ and $t_n = 2t_{n-1} + 3$. Find t_5.
$t_5 = 125$

Given $f(x) = x^2 + 5x + 20$, find $f(6)$.
F(6) = 86

$1500 is invested at 4% /year compounded quarterly for 5 years. The value of the investment at the end of the term is $ 1830 29

Can solve for p:
P – 4 = 13
P -4 + 4 = 13 + 4
P = 17

Can expand 3(x + 2)
3x + 6

Simplify $\dfrac{x^2 - 4}{x^2 - 5x + 6}$

= (x+2) / (x- 3)

Can round to the nearest hundred
a) 459 = __500__
b) 32926 = __33000_

$\dfrac{m^2 + m - 20}{m^2 - 11m + 28}$ and state restrictions
m ≠ 4 or 7
= (m+5)/(m – 7)

Can round to the nearest tenth
a) 7.22 = __7.2__
b) 3.024 = _3.0___
c) 2. 56 = ___2.6_

Can solve: If you build a skateboard ramp whose ratio of height to base must be 2:3, what is the base if the height is 4.5 m?
2/3 = 4.5/x
2x = 13.5
x = 6.75m

Can expand and simplify
(x + 1)(x – 1)
= x² – 1x + 1x – 1
= x² - 1

Can expand and simplify (x + 1)²
= (x + 1)(x + 1)
= x² + 1x + 1x + 1
= x² + 2x + 1

Can simplify

$\dfrac{3a}{a+3} - \dfrac{(a-4)}{a-2}$

Can use the Pythagorean theorem to solve for c given a=3 and b=4

C = 5

$= \dfrac{2a^2 - 5a + 12}{(a+3)(a-2)}$ **

Name: _____

Date: _____

Match the letter from Column B with the most appropriate number in Column A.
<u>Be certain to **show ALL work**</u>

Column A

_____ 1.   Simplify

$$\frac{3a}{a+3} - \frac{(a-4)}{a-2}$$

_____ 2.

Simplify $\dfrac{3m}{m+5} + \dfrac{2m}{m-3}$

_____ 3.

Simplify $\dfrac{a}{a-1} + \dfrac{2a}{a+3}$ *

_____ 4.

Simplify $\dfrac{m-2}{m^2-7m+10} + \dfrac{m+2}{m^2-4m-5}$ *

_____ 5.   Expand and simplify
$3x(x-2) - 4x(x+1)$

_____ 6.   Expand and simplify
$(m+2)(m^2+3m-5)$

_____ 7.   Expand and simplify
$(-3m^2+5m-3) + (m^2-6m+1) - (-3m^2-2m-4)$

_____ 8.   Expand and
simplify
$(3x-6)(x+3)^2 - (3x-1)(2x+1)^2$

Column B

A. $m^3 + 5m^2 + m - 10$

B. $\dfrac{2m+3}{(m-5)(m+1)}$

C. $\dfrac{2a^2 \ -7a\ -12}{(a+3)(a-2)}$

D. $-9x^3 - 4x^2 - 16x - 57$

E. $-x^2 - 2x$

F. m(5m+1)/(m+5)(m-3)

G. $\dfrac{a(3a+1)}{(a-1)(a+3)}$

H. $m^2 - 3m - 6$

I. $\dfrac{2a^2-5a+12}{(a+3)(a-2)}$

J. $-x^2 - 10x$

K. $m^2 + m + 2$

L. $-9x^3 + 4x^2 - 8x - 53$

0.1.3 Math Matching (Teacher)

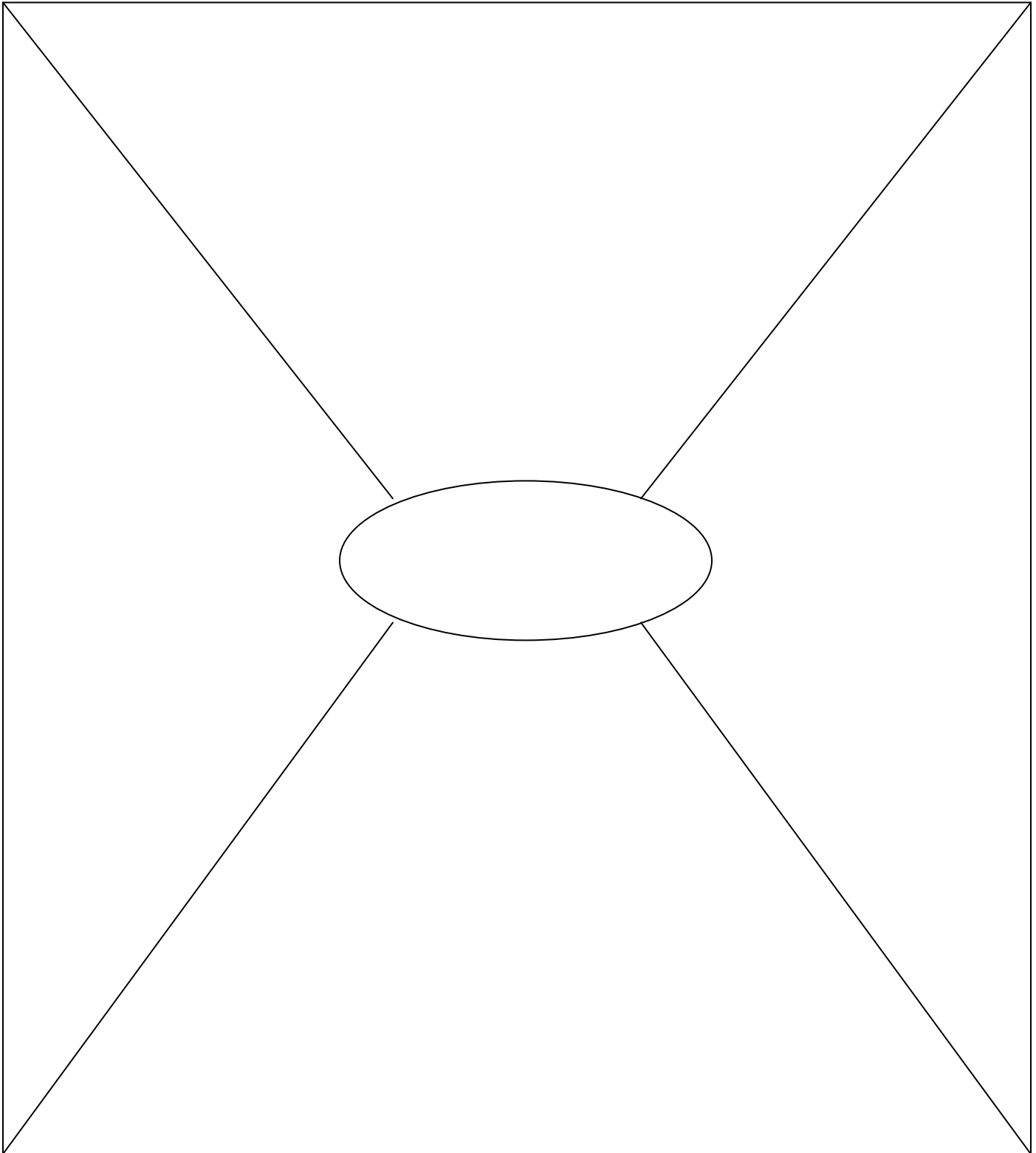Match the letter from Column B with the most appropriate number in Column A.
<u>Be certain to **show ALL work**</u>

Column A   Column B

__I__ 1.   Simplify
$$\frac{3a}{a+3} - \frac{(a-4)}{a-2}$$

A. $m^3 + 5m^2 + m - 10$

__F__ 2.

Simplify $\dfrac{3m}{m+5} + \dfrac{2m}{m-3}$

B. $\dfrac{2m+3}{(m-5)(m+1)}$

__G__ 3.

Simplify $\dfrac{a}{a-1} + \dfrac{2a}{a+3}$

C. $\dfrac{2a^2\ -7a\ -12}{(a+3)(a-2)}$

__B__ 4.

Simplify $\dfrac{m-2}{m^2-7m+10} + \dfrac{m+2}{m^2-4m-5}$

D. $-9x^3 - 4x^2 - 16x - 57$

__J__ 5.

Expand and simplify $3x(x-2) - 4x(x+1)$

E. $-x^2 - 2x$

__A__ 6.

Expand and simplify $(m+2)(m^2+3m-5)$

F. $m(5m+1)/(m+5)(m-3)$

__K__ 7.

Expand and simplify
$(-3m^2+5m-3)+(m^2-6m+1)-(-3m^2-2m-4)$

G. $\dfrac{a(3a+1)}{(a-1)(a+3)}$

__L__ 8.   Expand and
simplify $(3x-6)(x+3)^2 - (3x-1)(2x+1)^2$

H. $m^2 - 3m - 6$

I. $\dfrac{2a^2-5a+12}{(a+3)(a-2)}$

J. $-x^2 - 10x$

K. $m^2 + m + 2$

L. $-9x^3 + 4x^2 - 8x - 53$

Unit 0: Day 2 Introduction to Grade 12 – Review of Previous Grades		MHF4U
Minds On: 5 Action: 65 Consolidate: 5 Total=75 min	**Learning Goal** • Review the mathematical concepts from previous years.	**Materials** BLM 0.2.1 BLM 0.2.2

		Assessment Opportunities
Minds On…	**Whole Class → Discussion** Ask the students "Why", "Why are they taking this course". A short discussion can ensue about the different pathways and needs of this course.	
Action!	**Groups of 4 → Placemat Activity** Students will write in their own quadrant (BLM 0.2.1) their solution to the question Stephanie and Malcolm have 24 m. of fencing to enclose a garden at the back of their house. What are the dimensions of the largest rectangular garden they could enclose with this length of fencing? Garden House In the centre of the placemat students should summarize The different strategies used to find the answer (Answer: the sides would be 6m, 6m, and 12m) **Pairs → Activity** Students work on the sample grade 11 U exam BLM 0.2.2	**Process Expectation: Choosing Tools and Strategies:** the students will pick the strategies that work best for them when solving for the garden example.
Consolidate Debrief	**Whole Class → Discussion** Take up any questions the students have on the 11U exam. Stress that this is a pre-calculus course (not the Calculus course) and that all of the numerical methods they have added to their tool boxes in the past 11 years will be of great help to them.	
	Home Activity Students complete BLM 0.2.2	

0.2.1 Placemat

0.2.2 MCR3U Review: Examination

Part A: **Multiple Choice [30 marks]. Show <u>all</u> of your Work…**

You MUST use a PENCIL for this.

1. For a particular sequence, $t_1 = 3$ and $t_n = -2t_{n-1} + 1$, then t_5 equals:
   [A] 7   [B] 21   [C] 11   [D] 43

2. The general term t_n of the sequence 6, 2, -2, -6, ….. is
   [A] $10 - 4n$ [B] $10 + 4n$ [C] $4n - 10$ [D] $-4n - 10$

3. $1800 is invested at 6% per year compounded quarterly. The number of conversion periods is:

   [A] 2   [B] 4   [C] 6   [D] 8

4. $1800 is invested at 6% per year compounded quarterly. The amount obtained after 2 years is:

   [A] $2272.46   [B] $1910.45   [C] $2027.69   [D]$2868.93

5. The present value of an investment that will be worth $1523 in 4 years at 5% per year compounded semi-annually is:

   [A] $1250   [B] $1200   [C] $1500   [D] $1520

6. If $5^{3x} + 25 = 15600$, then x is equal to:

   [A] 1   [B] 2   [C] 3   [D] 4

7. The sum of the first 16 terms of the series 2 – 2 + 2 – 2 + ….. is:

   [A] 2   [B] -2   [C] 0   [D] 4

8. For the geometric series defined by $t_1 = 1$ and $t_n = 5^{n-1}$ the number of terms to yield a sum of 19531 is :
   [A] 7   [B] 6   [C] 2   [D] 0

9. For the geometric series defined by 16 + 8 + 4 + 2 + …. The value of S_{11} is:

   [A] $32\frac{3}{8}$ [B] $31\frac{5}{6}$ [C] $32\frac{5}{8}$ [D] $31\frac{63}{64}$

10. The relation that is NOT a function is:

    [A] $y = \sqrt{x+2}$ [B] $y = \frac{-3}{x}$ [C] $y = \frac{2}{x^2}$ [D] $x^2 + y^2 = 36$

0.2.2 MCR3U Review: Examination (Continued)

11. The relation that is NOT a function is:
    [A] {(5,2),(3,1),(-6,7),(8,2)}
    [B] {(3,1),(5,2),(8,2),(5,7)}
    [C] {(3,7),(5,1),(-6,8),(7,2)}
    [D] {(5,3),(2,-1),(-6,8),(8,5)}

12. Given $h(x) = -2x^2 + 5x - 4$ then

    [A] $h(-2) = -22$ [B] $h(-2) = 9$ [C] $h(-2) = 17$ [D] $h(-2) = -17$

13. The domain that best describes the function $g(x) = \dfrac{2}{x-3}$ is :
    [A] $\{x > 0, x \in R\}$ [B] $\{x > 3, x \in R\}$ [C] $\{x < 3, x \in R\}$ [D] $\{x \neq 3, x \in R\}$

14. The range that best describes the function $g(x) = \dfrac{2}{x-3}$ is:
    [A] $\{y > 0, y \in R\}$ [B] $\{y > 3, y \in R\}$ [C] $\{y < 3, y \in R\}$ [D] $\{y \neq 0, y \in R\}$

15. Which of the following would NOT represent the inverse of $f(x) = 3x^2 - 2$?

    [A] $y = \sqrt{\dfrac{-(x+2)}{3}}$ [B] $y = \sqrt{\dfrac{x+2}{3}}$ [C] $y = -\sqrt{\dfrac{x+2}{3}}$ [D] $y = \pm\sqrt{\dfrac{x+2}{3}}$

16. Which of the following profit functions has break-even points at x=3 and x=11?
    [A] $P(x) = -2(x^2 - 28x + 66)$
    [B] $P(x) = -2(x+3)(x+11)$
    [C] $P(x) = -2(x-3)(x+11)$
    [D] $P(x) = -2(x^2 - 14x + 33)$

17. $\dfrac{3x-1}{x(3x+1)}$ is the simplified form of:

    [A] $\dfrac{3x^2 - 6x + 7}{3x^2 + 1}$ [B] $\dfrac{9x^2 - 1}{9x^3 + 6x^2 + x}$ [C] $\dfrac{9x^2 + 1}{9x^3 + 6x^2 + 1}$ [D] $\dfrac{9x^2 - 1}{9x^2 + 6x + 1}$

18. Simplify and state the restrictions for $\dfrac{3x^2-5x-2}{3x^2+13x+4} \div \dfrac{x^2-x-2}{x^2+3x-4}$

[A] $\dfrac{x-1}{x+1}, x \neq -\frac{1}{3}, -4, -1, 1, 2$

[B] $\dfrac{x+1}{x-1}, x \neq -\frac{1}{3}, -4, -1, 1, 2$

[C] $\dfrac{x-1}{x+1}, x \neq \frac{1}{3}, 4, 1, -1, -2$

[D] $\dfrac{x+1}{x-1}, x \neq \frac{1}{3}, 4, 1, -1, -2$

19. $\dfrac{x}{3x-6} - \dfrac{3}{2x-4}$ is equivalent to:

[A] $\dfrac{2x-9}{6(x-2)}$

[B] $\dfrac{2x^2-4x-3}{(3x-6)(x-2)}$

[C] $\dfrac{2x^2-13x+18}{(3x-6)(2x-4)}$

[D] none of the above

20. Simplify $(2x^4-2x^2-7)-(3x^4-x^3+x^2-4)+(x^4-x^3+2x^2-1)$

   [A] $-x^2-4$ [B] x^2-4 [C] x^2+4 [D] $-x^2+4$

21. Simplify $(x^2-2x-1)+(3x^2+x+2)$

   [A] $4x^2-x-1$ [B] $-2x^2-3x-3$ [C] $4x^2-3x-3$ [D] $4x^2-x+1$

22. Simplify $-2x(x+1)^2+3x(x-4)$

[A] $-2x^3-x^2-14x$ [B] $x^2-2x-12$ [C] $x^2-4x+12$ [D] none of the above

23. The period of the function $y=2\cos 3(x+45°)$ is:

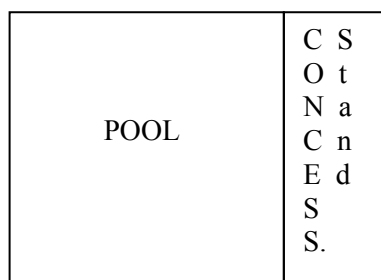   [A] 2   [B] 3   [C] 45   [D] 120

24. The amplitude of the function $y = 2\cos 3(x + 45°)$ is:

    [A] 2   [B] 3   [C] 45   [D] 120

25. The phase shift of the function $y = 2\cos 3(x + 45°)$ is

    [A] 3° to the right   [B] 3° to the left   [C] 45° to the right [D] 45° to the left

26. An angle that is coterminal with 75° is:

    [A] 105°   [B] 435°   [C] 453°   [D] 615°

27. One approximate solution to the equation $\sin x = 0.8192$ for $0 \le x \le 360°$

    [A] -55°   [B] 125°   [C] 305°   [D]no solution

28. The solution to the equation $\sqrt{2}\cos x = 1$ for $0 \le x \le 360°$ is:

    [A] 45° [B] 315°   [C] both A and B   [D] none of the above

29. In an acute $\triangle ABC$, the condition for one triangle to exist is:

    [A] a = bsinA   [B] a < bsinA   [C] bsinA < a <b   [D] a ≥ b

30. The identity $\dfrac{1}{\sin x} - \sin x$ is equivalent to:

    [A] $\dfrac{\cos x}{\sin x}$ [B] $\dfrac{\tan x}{\cos x}$ [C] $\dfrac{\sin x}{\cos x}$ [D] 1

Part B: Short Answer

1. Write an expression, in simplified form, for the general term of the sequence in which the first term is 14 and each term is 5 less than the term preceding it.

2. In an arithmetic sequence, $t_{15} = -27$ and $t_{10} = -12$. Find the value of the first term and the common difference.

3. Write an expression, in simplified form, for the general term of the sequence in which the first term is 16 and each succeeding term is ½ the term before it.

4. Solve the following exponential equation: $3^{2-x} = 27^{x-2}$

5. If $f(x) = x^2 - 2$ and $g(x) = 3x + 2$, find:

    a. f(2) – g(3)

    b. $\dfrac{f(3)}{g(-1)}$

6. For the function $f(x) = (x + 3)^2 + 4$, find $f^{-1}(x)$. Is the inverse a function?

7. Vlad must fence in a pool area next to a concession stand, as shown, with 25m of fencing. What are the dimensions which will maximize the pool area?

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┌──────────────┬─────────┐
│              │ C  S    │
│              │ O  t    │
│              │ N  a    │
│    POOL      │ C  n    │
│              │ E  d    │
│              │ S       │
│              │ S.      │
└──────────────┴─────────┘
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8. The revenue generated by a new product can be modeled as $R(x) = -5x^2 + 21x$. The cost function is $C(x) = 4x + 14$. How many items are required to maximize the profit? What is the maximum profit?

9. Rowing at 5 times the speed of the current, Jacob's team can travel 24km downstream in the same time it takes them to travel 16km upstream. What is Jacob's team's rate and the rate of the water?

10. Find 2 angles, one positive and one negative, that are coterminal with 105º.

11. Solve $2\sin x - \sqrt{3} = 0$ for $0° \le x \le 360°$

12. From 2 different tracking stations, a weather balloon was spotted from 2 angles of elevation, 57º and 83º, respectively. The altitude of the balloon is 28.5km. How far apart are the tracking stations?
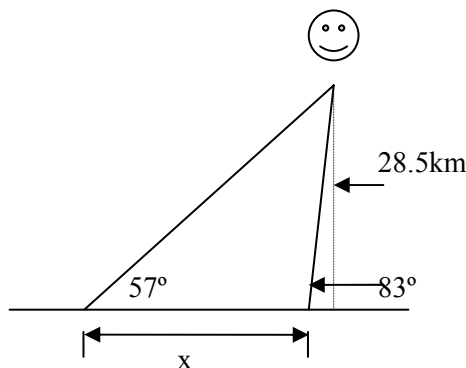
13. Two hot air balloons are moored directly over a level road. Based on the given diagram, how far apart are the balloons?
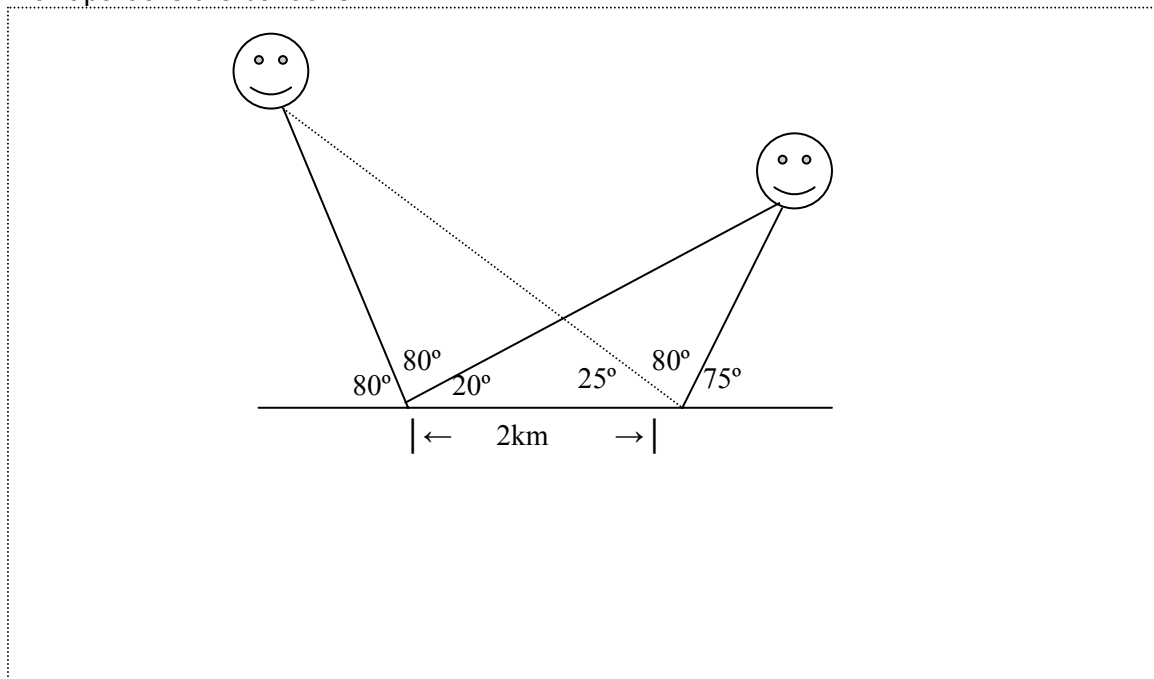
14. Prove the identity $\cos^2 x = (1 + \sin x)(1 - \sin x)$ *

ANSWERS:

Part A: **Multiple Choice [30 marks]**

1. D	6. B	11. B	16. D	21. D	26. B
2. A	7. C	12. A	17. B	22. A	27. B
3. B	8. A	13. D	18. A	23. D	28. C
4. C	9. D	14. D	19. A	24. A	29. A
5. A	10. D	15. A	20. A	25. D	30. A

Part B: Short Answer

1. $t_n = 14 + (n-1)(-5)$

$t_n = -5n + 19$

2. $d = -3$

$-12 = a + 9(-3)$

$15 = a$

3. $t_n = 16\left(\dfrac{1}{2}\right)^{n-1}$

4. $3^{2-x} = (3^3)^{x-2}$

$3^{2-x} = 3^{3x-6}$

$2 - x = 3x - 6$

$8 = 4x$

$2 = x$

5. a. f(2) – g(3)= $4 - 2 - (9 + 2) = 2 - 11 = -9$

    b. $\dfrac{f(3)}{g(-1)} = \dfrac{7}{-1} = -7$

6. $y = (x+3)^2 + 4$

$x = (y+3)^2 + 4$

$x - 4 = (y+3)^2$

$\pm\sqrt{x-4} = y + 3$

$-3 \pm \sqrt{x-4} = y$ It is NOT a function

7. $2l + w = 25$

$w = 25 - 2l$

Maximize Area = $l \times w = l(25 - 2l) = -2l^2 + 25l$

$-2(l^2 - 12.5l + 39.0625) - 39.0625(-2) = -2(l - 6.25)^2 + 78.125$

This means that a maximum area of 78.125m^2 is achieved with a length of 6.25m and a width of 12.5m.

8. $P(x) = R(x) - C(x) = -5x^2 + 21x - (4x + 14) = -5x^2 + 17x - 14$ Maximize!

$$-5x^2 + 17x - 14 = -5\left(x^2 - \frac{17}{5}x + 2.89\right) - 14 - 2.89(-5)$$

$$-5\left(x - \frac{17}{10}\right)^2 + .45$$ This means that a maximum profit of 0.45 occurs with 1.7 items.

9. Rate of the water is any value r, Rate of Jacob's rowing is 5r
   True for any current rate as long as Jacob's team rows at 5 times the speed

10. $105° + 360° = 465°$ and $105° - 360° = -255°$

11. $2\sin x = \sqrt{3}$

$\sin x = \dfrac{\sqrt{3}}{2}$ Sine x is positive in quadrants I and II. Therefore,

$\sin^{-1}\dfrac{\sqrt{3}}{2}$ yields   x = 60° and 120°

12. $\sin 83 = \dfrac{28.5}{s}$

$s = 28.714$

$\dfrac{x}{\sin 26} = \dfrac{28.714}{\sin 57}$

$x = 15$

13. Using the 20-25-135 Triangle: $\dfrac{2}{\sin 135} = \dfrac{R}{\sin 20}$ Therefore $R = .9674$

And $\dfrac{2}{\sin 135} = \dfrac{L}{\sin 25}$ Therefore $L = 1.1953$

Using the 80-45-55 Triangle: $\dfrac{x}{\sin 80} = \dfrac{1.1953}{\sin 55}$ Therefore $x = 1.4370$

Using the 45-80-55 Triangle: $\dfrac{y}{\sin 80} = \dfrac{0.8351}{\sin 45}$ Therefore $y = 1.1631$

Using the 135 Triangle with sides of 1.4370 and 1.1631:

$$d^2 = 1.1631^2 + 1.4370^2 - 2(1.1631)(1.4370)(\cos 135)$$
$$d^2 = 5.781451370$$
$$d = 2.4045$$

14. $\cos^2 x = 1 - \sin^2 x$
   $\cos^2 x = \cos^2 x$
   $LS = RS$