Unit 4: Day 1: What are Probability Distributions?

MDM4UI

Minds On: 5 Action: 50 Consolidate:20 Total=75 min	**Learning Goals**: • Review and recognize the concepts of probabilities used to represent the likelihood of a result of an experiment or a real-world event • Determine the theoretical probability of each outcome of a discrete sample space • Understand probability distributions for discrete random variables	**Materials** • Coins (or see alternative below) • BLM 4.1.1
		Assessment Opportunities
Minds On…	**Whole Class → Discussion** Review the concepts of 'experimental probability' (ex. coin flips, card draws, dice rolls, spinning spinners) and 'frequency distributions' from previous units. Remind the class about real-world events that require understanding of probability (ex. Chances of rain shown on TWN; lottery picks; defective products; etc.) Define 'probability distribution' by extending the concepts of 'probability' and 'distribution' from previous units. Discuss the meaning of the terms 'discrete', 'random', and 'variable' to introduce the concept of a discrete random variable	Add the 2 definitions to the word wall
Action!	**Small Groups → Experiment** Students in groups of 2-4 complete the BLM 4.2.1. Hand out (or have them take out) 5 coins *after* they have completed Step #1. **Expectations/Observation/Anecdotal comments:** Observe students as they complete the investigation. Make sure they are all engaged, and question them on their responses to Step #1. Check to see if the students are understanding the concept of probability distributions **Mathematical Process Focus: Reasoning & Proving** students will combine their intuition with data they have found to make conjectures and predictions about probabilities in coin flipping.	An alternative activity with the TI83+ is to use the ProbSim application (if new to it, see http://education.ti.com/guidebooks/apps/83probability_simulation/probsim.pdf).
Consolidate Debrief	**Small Groups → Jigsaw** Request that they reconfigure their groups to compare and discuss their results. **Whole Class → Discussion** Ask for a representative from each group to report their findings, or have them summarize their results on the board (especially experimental probabilities – does an average of results come closer to the theoretical ones?). Wrap up by showing how the experiment produced 3 types of distributions, and how they are related. Directed question: What is the sum of all the probabilities in a probability distribution?	Remind students that the concepts from this and tomorrow's lesson will be applied in the Games Fair Project later in the Unit.
Concept Practice *Exploration* *Application*	**Home Activity or Further Classroom Consolidation** • Complete the BLM 4.2.1, if you did not complete it in class. • Write a journal to summarize your understanding of probability distributions so far, with examples.	

4.1.1: Flipping Coins **MDM4UI**

A *discrete variable* is one that can only have certain values within a given range (for example, shoe sizes are *discrete* – they are always whole numbers, or ± 0.5 - while a person's height is *continuous* – ex. people could be 141cm, 91.2cm, 181.33cm, etc.).

A discrete *random* variable adds the element of unpredictability – you don't have any way of knowing what the next result is going to be, based on what has just happened. Classic examples of this are flipping a coin, drawing a card, or determining whether someone will have a girl or boy baby.

**Here's an experiment to determine how discrete random variables work, and how they can be used to produce frequency distributions and probability distributions. We will take 5 coins, flip them, record the results, and generate a chart, a table, and a graph.

1. In groups of 3-4, predict what you would expect to find when you flip 5 coins. How likely do you think the following results would be (try to put a probability to it – a value from 0 to 1, correct to 2 decimal places – record the group *consensus* value, or the range of guesses if there is no consensus (and don't change it after Step 2!)):

   a) How likely are 5 Heads out of the 5 flips (i.e. HHHHH)?________

   b) How about 4 Heads (ex. HHHHT or HHTHH)?________ c) 3 Heads?________

   d) 2 Heads?________ e) 1 Head?________ f) 0 Heads?________
   (HInt: what should the total of P(*a* to *f*) be equal to? ________)

2. Now obtain 5 coins, and start flipping. One person should record, while others flip 1 or 2 coins. Record your resulting # of *Heads* after each trial in the table. Do this 20 times.

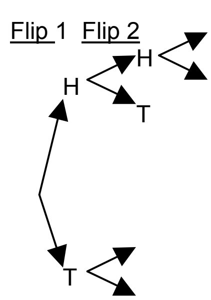
Trial#	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
#Heads																				

3. Calculate:
   a) the mean # of Heads:

   b) the *experimental* probabilities of each # of heads, to 2 decimal places:

#Heads(=x)	0	1	2	3	4	5
N(x)						
P(x) [=N(x)/20]						

4. Now we'll determine what the <u>actual</u> (or theoretical) probabilities are for flipping 5 coins.
   a) The tree chart below models the first 2 flips. Complete it for 5 flips.



4. b)Tabulate your results from the chart in #4a by adding up all of each type (ex. there is only 1 HHHHH, or 5H). What are the total number of possibilities? State the probability of each result as a fraction and decimal.

<u>Results</u>:		<u>Probability</u>: (fraction)	(decimal)
5H:	__1___	_______	_______
4H+1T:	_______	_______	_______
3H+2T:	_______	_______	_______
2H+3T:	_______	_______	_______
1H+4T:	_______	_______	_______
5T:	_______	_______	_______

4.1.1: Flipping Coins (cont) MDM4UI

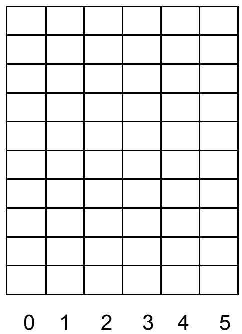
5. Graph the following on the grids provided. Supply scale, labels, and title for each.

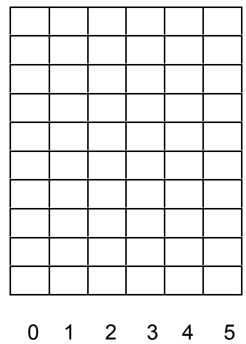
a) the frequency histogram (from #2). Fill in the table to summarize the results.

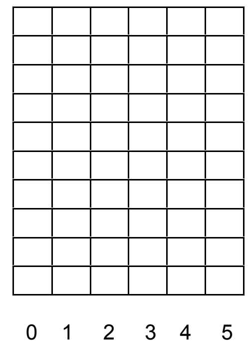
b) *experimental* probabilities (from #3b).

c) the *theoretical* probabilities (from #4b).

X	N(x)
0	
1	
2	
3	
4	
5	







6. The graph in #5a is often called the *frequency* histogram, while the one in #5b is the experimental or *empirical probability* histogram, and #5c the *theoretical probability* histogram. Remember that each represents a type of distribution.

   a) Define the 3 terms here (what makes them different? Similar?):

   b) Compare the shapes of the 3 graphs. How are they different? Similar?

   c) What are the totals of the area of all the 'bars' in each of the graphs?

   #5a) #5b) #5c)

7. Are the results what you expected? Compare your answers for #1, #3b, and #4b here.

8. Finally, summarize your results from #1-3 in comparison to other groups. Was there a consensus of predictions? Of results?

Unit 4 : Day 2 : More on Probability Distributions

MDM4UI

Minds On: 5 Action: 55 Consolidate:15 Total=75 min	**Description/Learning Goals** • Generate a probability distribution using technology • Compare a probability histogram to a frequency histogram • Recognize differences are to be expected between probability histograms and frequency histograms	**Materials** • Fathom • BLM 4.2.1

Assessment Opportunities

Minds On…	**Whole Class → Discussion** In computer lab, begin by reviewing the concept of probability distributions for discrete random variables, from previous lesson. Ensure that the students can distinguish between experimental and theoretical probabilities, and between frequency and probability distributions.	Note: the BLM is written with a more complete intro to Fathom than might be required.
Action!	**Individual → Investigation** Students work through the investigation on BLM 4.2.1. **Process expectations/ Observation /Anecdotal comments:** As students work, keep questioning them on their understanding of the difference between what they're finding in Fathom (empirical) and what they're calculating (theoretical). **Mathematical Process Focus: Reasoning & Proving** students will predict answers to questions given and refine their predictions based on experimental data found.	Note that students should demonstrate understanding of discrete random variables on the BLM before proceeding to Fathom. Refer to the fathom file MDM_U4FTM1 for an example .
Consolidate Debrief	**Whole Class → Discussion** If time allows, tabulate the class results on the board and have them individually prove that 'the more experimental results we have, the closer we get to the theoretical or expected results'.	
Application *Concept Practice* *Exploration* *Reflection*	**Home Activity or Further Classroom Consolidation** Complete BLM 4.2.1 to be handed in at the beginning of class next day. Write a journal entry to summarize the introductory concepts of this unit.	

4.2.1: The Die is Cast MDM4UI

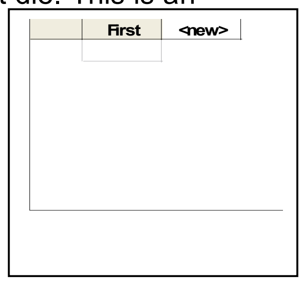
This is an activity that will use Fathom to copy or *simulate* the actual rolls of 1 or 2 dice. It uses a *random-number generator* to produce the rolls. You could get close to the same results by rolling the dice yourself (up to 500 times), but it would take you a lot longer!

**We'll start by rolling one die. The roll result will be a *discrete random variable*. Define this expression, and give another example of a discrete random variable:

____

____

1. Open **Fathom**, and make a new **Case Table** by dragging the appropriate icon onto your blank window (or you could use the **Insert** command, then **Case Table**).
2. In the case table, we'll use the word <new> to represent the first die. This is an *attribute.* Double-click on this word, and type **First**.
3. Now double-click on **Collection 1** and give it a different title (see ex.1).
4. A die has 6 numbers, so we need to have it randomly 'roll' one of these 6 numbers by right-clicking on the word F**irst**, then choosing **Edit Formula**, which causes the *formula editor* to appear.
5. In the (top) window of this *editor*, type **randomPick(1,2,3,4,5,6)** and then click OK. You won't see any change, because Fathom needs to know how many times to roll the die (the *cases*).



ex.1

6. Right-click on **First** again and choose **New Cases…** Type in **1**, then OK, and observe what happens. Press **ctrl-y** to repeat this step. Try it several times. What range of numbers are generated? From _____to _____
7. Let's get a few more cases. This will be the same as rolling the die, say, 20 times. Right-click on **First**, choose **New Cases…**, and type in 19. How many 'rolls' were counted in total? _____ Again, try **ctrl-y** a few times.
8. A graph is a good way to visually model the data. Drag a **New Graph** from the shelf into your Fathom window. In your Dice Rolls table, drag the word **First** to the horizontal axis of the graph. What is produced is a Dot Plot. Click on the graph's pull-down menu (top right) and choose **Histogram**. Now try **ctrl-y**. How does the graph change as you do this? Predict what you might find if there were many more rolls.

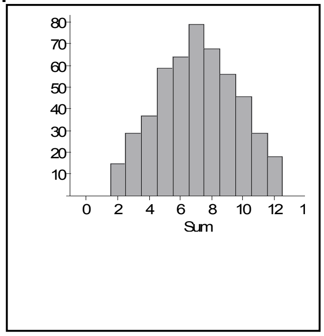
9. Now try the model with 500 rolls. How did you make this change? What do you observe about the histogram bars when you press ctrl-y repeatedly?

**Now we'll try it with 2 dice, and find their sum (just like in Monopoly, etc.). The *sum* of the dice is another example of a *discrete random variable*:

10. In the Dice Rolls table, click on **<new>** (to the right of **First**). Type **Second**, and give it the same attribute formula as **First**.
11. Make one more new attribute: **Sum**. Type a formula that adds the two dice's rolls together (ex. First + Second).

4.2.1: The Die is Cast (cont.) MDM4UI

12. Drag down a *second* graph, and drag **Sum** to the horizontal axis. Make sure it looks similar to ex.2.
13. Type your name on your Fathom window (drag down a **New Text** box).
14. **Print Preview** your work (make sure it's all on one page), then **Print** it. Attach it to the back of this handout, and answer the following, using the example (ex.2) of 500 rolls shown at right.
   a) According to the graph shown in ex.2, about how many rolls totalled 7? Why is 7 usually the most common sum of 2 dice?



ex.2

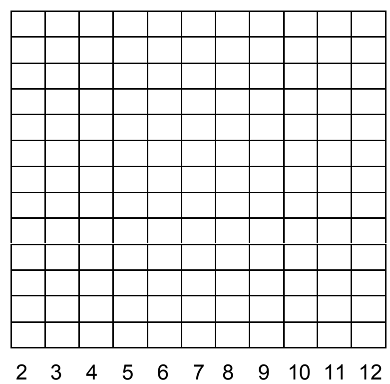
   b) Write the probability of rolling a 7 according to *this* graph. This is the *experimental* (or *empirical*) probability. Write it as a fraction, a decimal, and a %:

   c) What is the *theoretical* probability of rolling a 7 with two dice? (If needed, check the Note at bottom.) How close was your answer in (b) to this value?

15. Draw a histogram that reveals the *theoretical* probability distribution for the sum of the rolling of 2 dice. First show the probability values in the table provided. Round answers to 2 decimal places. Provide labels and title for the graph.

Sum	2	3	4	5	6	7	8	9	10	11	12
Probability											

(See Note at bottom)



16. Compare the two graphs on this page, and conjecture why there are differences. If the number of cases was increased to 5000, would you expect the two graphs to be more similar, less, or not much different from before? How about 5 000 000? Why?

<u>Note</u>: use this space for calculations for #15, if desired. Here is an example, for a sum of 5: P(5) = P(1+4)+P(4+1)+P(2+3)+P(3+2) = $\frac{1}{6}*\frac{1}{6}*2+\frac{1}{6}*\frac{1}{6}*2=\frac{4}{36}=\frac{1}{9}$ (or about 0.11).

Unit 4: Day 5: Intro to The Binomial Distribution

MDM4UI

Minds On: 15 Action: 40 Consolidate:20 Total=75 min	**Learning Goals**: • Recognize conditions that produce discrete random variables which follow a binomial probability distribution • Calculate the probabilities associated with all values of a binomial distribution • Generalize the algebraic representation of the binomial distribution • Calculate the expected value for a probability (binomial) distribution	**Materials** • Chart paper • Binomial Distribution.ppt • Computer & Data projector • BLM 4.5.1

Assessment Opportunities

Minds On…

Whole Class → Guided Discussion

Recall and discuss the 'flipping coins' experiment from Unit 4 Day 1, plus the discussion of 'discrete random variables'. (Refer to BLM 4.1.1). Compare this to the dice activity (BLM 4.2.1) – draw out the similarities in the distributions and graphs, as well as the differences.

Engage students in a discussion of the idea of 'heads/tails' as similar to 'success(eg. heads)/failure(eg. tails)', and extend this to the dice, in which eg. a roll of 'one' is a success, while any other roll is a failure.

Add the terms "binomial distribution" "Bernoulli trial" and "expected value" to the Word Wall.

Small Groups → Brainstorming

Introduce students to the binomial distribution and have them brainstorm on chart paper other 'games' that produce this distribution.

While Class → Discussion

Have students post their chart paper and discuss the ideas as a class. Discuss and emphasize the 'real-world' ideas like 'defective bulbs' or 'probability of getting a hit'.

Review the idea of Expected Value from Day 4.

Action!

Whole Class → PowerPoint Presentation

Present the BinomialDistribution.ppt. Instruct student to take notes and answer the questions posed in the ppt as they view this presentation.

Electronic Resource: MDM_U4L5PPT1.ppt

Small Group → Application

Handout one question from BLM 4.5.1 to each group. Instruct the groups to complete their solution on chart paper and post it. Each group should then present their results in a math workshop format.

The last 4 slides cover the hypergeometric distribution, for Day 8, if desired

Curriculum Expectation/Presentation/Checklist: Assess how the students apply the b. d. function to solve problems.

Mathematical Process Focus: Connecting – students will connect real world probabilities to the mathematics of the binomial distribution .

As an alternative, if time is short the groups could post their answers on a Gallery Board, for later comparison.

Consolidate Debrief

Whole Class → Connections

Ask the class if the histograms generated in BLM 4.1.1 (#5bc) are examples of binomial distributions [they are!]. Review the comparison of the two (#6). Discuss the connection between the types of experiments done in the Unit so far (coin flips, dice rolls, Lucky Aces, etc.) and draw out of the students the differences between binomial and other probability distributions. Ask them to list the characteristics of a b.d.:
*independent trials of a discrete random variable *the only outcomes are *success* or *failure*

Application
Concept Practice
Skill Drill

Home Activity or Further Classroom Consolidation

1. Write a journal to summarize your understanding of the binomial distribution, with examples.
2. Complete assigned questions

Provide appropriate practice questions for the students.

4.5.1: Binomial Distributions–Sample Problems MDM4UI

Cut out each box below and hand one out to each group after the presentation.

1. The realtor in the presentation now says he has a success rate of 30% (does this seem more reasonable?). What is:
   a) the probability of 'closing the deal' on 2 out of 8 houses? On 3 of 8?
   b) the expected number of 'closes' out of 8 houses (to 1 decimal place)?

2. The realtor in the presentation now says he has a success rate of 20% (does this seem more reasonable?). What is:
   a) the probability of 'closing the deal' on 2 out of 8 houses? On 3 of 8?
   b) the expected number of 'closes' out of 8 houses (to 1 decimal place)?

3. The realtor in the presentation now says he has a success rate of 10% (does this seem more reasonable?). What is:
   a) the probability of 'closing the deal' on 2 out of 8 houses? On 3 of 8?
   b) the expected number of 'closes' out of 8 houses (to 1 decimal place)?

4. The realtor in the presentation now says he has a success rate of 50% (does this seem more reasonable?). What is:
   a) the probability of 'closing the deal' on 2 out of 8 houses? On 3 of 8?
   b) the expected number of 'closes' out of 8 houses (to 1 decimal place)?

5. Alex Rios is now hitting 0.320. He has 5 at bats in today's ball game. What is:
   a) the probability that he'll have 1 hit? 2 hits?
   c) the expected number of hits (to 1 decimal place)?

6. Alex Rios is still hitting 0.310. He has 5 at bats in today's ball game. What is:
   a) the probability that he'll have 1 hit? 2 hits?
   c) the expected number of hits (to 1 decimal place)?

7. Alex Rios is now hitting 0.300. He has 5 at bats in today's ball game. What is:
   a) the probability that he'll have 1 hit? 2 hits?
   c) the expected number of hits (to 1 decimal place)?

8. Alex Rios is now hitting 0.290. He has 5 at bats in today's ball game. What is:
   a) the probability that he'll have 1 hit? 2 hits?
   c) the expected number of hits(to 1 decimal place)?

<table>
<tr><td colspan="3">Unit 4 : Day 6 : More on Binomial Distributions</td><td>MDM4UI</td></tr>
<tr><td>Minds On: 10
Action: 55
Consolidate:10
Total=75 min</td><td colspan="2"><u>Description/Learning Goals</u>
• Calculate the probabilities associated with all values of a binomial distribution
• Calculate, interpret, and apply Expected Value
• Represent a binomial distribution numerically using a table and graphically using a probability histogram, and make connections to the algebraic representation</td><td><u>Materials</u>
• Graphing calculators
• BLM 4.6.1
• Dice</td></tr>
<tr><td colspan="3"></td><td>Assessment Opportunities</td></tr>
<tr><td>Minds On…</td><td><u>Whole Class → Discussion</u>
Do a brief review of the concepts from the PowerPoint presentation - the word wall might be handy here – and take up the assigned homework questions.</td><td></td><td rowspan="3">On BLM 4.6.1 instructions are given for both the TI83+ and the TInspire – delete the instructions you do not need before distributing.</td></tr>
<tr><td>Action!</td><td><u>Pairs → Investigation</u>
Students complete the BLM 4.6.1, using first a paper + pencil approach, and then a TI-83+/89/nSpire calculator.
Tools & Strategies/Observation/Mental Note: Observe and assess the expertise with which students are using the graphing calculators to compute binomial distributions. Use this to determine if you need to review any concepts with the class.
<u>Individual/Pairs → Investigation</u>
As a follow-up to the dice rolling in BLM 4.2.1, consider a “roll of 6” as a ‘success’, and any other roll a ‘failure’. Ask students to predict the outcome, and then complete a probability distribution table and histogram for five rolls of a die. They should also calculate the expected value. If time allows, actually using dice to compare empirical with theoretical results is helpful.
Ask students to then compare their results to the 5 ‘free throws’ in the BLM.
Mathematical Process Focus: Reasoning & Proving: Students will predict results and prove their prediction correct (or incorrect) using their probability distribution and histogram.</td><td></td></tr>
<tr><td>Consolidate Debrief</td><td><u>Whole Class → Discussion</u>
Engage student in a discussion of the investigation results. Instruct them to complete a journal entry describing, in their own words, “binomial distribution” based on their work today.</td><td></td></tr>
<tr><td>Application
Concept Practice
Differentiated</td><td><u>Home Activity or Further Classroom Consolidation</u>
On graph paper, create the probability distribution histogram and determine the expected value for the number of field goals or free throws assigned to you.</td><td></td><td>DI: Assign weaker students a # of free throws and stronger students a #of field goals</td></tr>
</table>

4.6.1: Binomial Distribution Investigation MDM4UI

John is a basketball star, with a field goal average of 45%, and free throw average of 70%. In any particular game, if John attempts 5 free throws, what is the probability of a certain number of successful throws?

A. Answer this question by hand first – the probability of a successful throw is 70% (i.e. p = ____ and q = (1 – p) = ____), and the number of trials (n) is 5.
Complete the table and a histogram (with labels, etc.):

x (# of successful free throws)	Probability $P(x)$
0	${}_5C_0(0.7)^0(0.3)^5 = .00243$
1	
2	
3	
4	
5	
Total =	

B. Now confirm the values you calculated in the table (*P(x)*) on a graphing calculator:

1. Use the *binompdf(* function (From the DISTR menu for the **TI83+,** or menu/7/5/D on the **TInSpire**). This function stands for 'binomial probability density function'.
2. On the **TI83+:** Clear All Lists (and make sure the columns for L1, L2, and L3 are present).
   Enter all possible values of *x* into L1, then enter the function binompdf(5,.7,L1) in L2.
   You should be able to compare your table values above to these ones now.
   On the **TInSpire**: After retrieving the Binomial Pdf function, enter the values n=5 and p=0.7 in the chart, and select X value column A. The calculator lists the P(x) values.
3. Once you are familiar with the operation, use the graphing calculator to determine the probability of John making at least 5 throws out of 7 (you'll have to add 3 values together): P(at least 5 out of 7) = ______ + ______ + ________ = _______.

C.1. Finally, we need to understand how to find the Expected Value (the number of free throws John would be *expected* to make). Using the original question, multiply *x* by *P(x)* for each row, then add the products together:

E(X) = 0(.00243) + 1(_____) + 2(______) + 3(______) + 4(______) + 5(______)
= ________

Or use the binomial expectation formula: *E(X) = np*
= (_____)(_____)
= ________

Or on the **TI83+**: In L3 enter the formula L1 × L2, then select *sum(* from the LIST OPS menu, and type *sum*(L3). Did all 3 calculations result in the same value? ___

2. The other day John played a game in which he attempted 7 free throws. What would his expected successful number of throws be? (Use any method.)
E(X) = ___________.

Unit 4 : Day 7 : Binomial Distributions III

MDM4UI

Time	Description/Learning Goals	Materials
Minds On: 15 Action: 35 Consolidate:25 Total=75 min	**Description/Learning Goals** • Calculate the probabilities associated with all values of a binomial distribution • Generalize the algebraic representation of the binomial distribution • Represent a binomial distribution numerically using a table and graphically using a probability histogram, and make connections to the algebraic representation • Calculate and interpret expected value, and make connections to the mean of the discrete random variable.	**Materials** • Marbles (or similar) • BLM 4.7.1

Assessment Opportunities

Minds On…

Whole Class → Discussion/Demonstration

Using a single box/bag containing 10 objects (ex. marbles or cube-a-links) of 2 colours in the ratio 7:3, ask different students to select 3, one at a time, with replacement. Have someone on the board record the results. Do this a few times to start a discussion of an hypothesis to predict the probability outcomes.

Leave the results on the board for later.

Note that this will be an excellent bridge to hypergeometric distributions – same apparatus, but with no replacement.

Action!

Individual → Investigation

Handout and instruct students to complete the investigation on BLM 4.7.1.

Along with the BLM, boxes with 7 blue and 3 green (or any colour combination) marbles or comparable shapes could be given out to small groups in order to continue to test and compare empirical values with the theoretical values computed on the BLM.

Process expectations/Observation/anecdotal comments: Continue to monitor their application of the binomial theorem and the binomial distribution function to this activity.

Mathematical Process Focus: Problem Solving – students will use problem solving to complete the BLM.

The BLM (question B4) assumes they have learned the term 'Binomial Theorem' – if not, this is a good time to review it!

The final questions (B7) will likely require discussion.

Consolidate Debrief

Whole Class → Investigation

Now that the theoretical probability distribution has been worked out, students who finish early should continue the experimental results on the board. When time is up, inquire as to whether the experimental results are close to theoretical, and then ask if increasing the number of independent trials would make the values align even better.

ProbSim could again be used as an experimental 'check'.

Pairs → Discussion

Have students discuss their results and then work through the theoretical probability expectations of selecting 4 from the 10 shapes. Check to ensure that they recognize the relationship to the binomial theorem (i.e. for $(g+b)^4$) and the binomial probability distribution.

Application
Exploration
Reflection

Home Activity or Further Classroom Consolidation

1. On BLM 4.1.1, flipping coins produced a frequency and a probability distribution (both empirical and theoretical). Since this was, in fact, a binomial distribution, you must now review and calculate the expected value of the # of heads (answer in #3a), and compare this to the mean experimental value.
2. Write a description of what you found on the back of BLM 4.1.1.
3. Calculate the theoretical probabilities with the binomial distribution formula instead of drawing a tree diagram.
4. Complete another experiment with 3 or 4 coins at home, answering the same set of questions.

4.7.1: Binomial Distributions Investigation MDM4U1

A1. Define 'binomial distribution': ____

____

A2. Answer True or False:

a) A binomial distribution (b. d.) is a type of probability distribution. ____________
b) Our coin-flip experiment from the beginning of this Unit was a b. d. ____________
c) Our dice-roll investigation from second day of this Unit was a b. d. ____________

B. A box contains 7 blue and 3 green marbles. Three marbles are chosen at random, one at a time <u>with</u> replacement (and let's say 'blue' = a 'success'). Answer the following questions:

B1. Determine each of the following probabilities (as fraction and decimal):

P(blue marble) = ________________ P(green marble) = _________________

B2. Are the events "choosing a blue marble" and "choosing a green marble" independent events? Why or why not?

B3. Complete the following table (for a draw of 3 marbles from the box):

# of blue marbles	**0**	**1**	**2**	**3**
List <u>all</u> possibilities		**ggb** **gbg** **bgg**		

B4. Express each list as indicated in the example.

↓	↓	↓	↓
+	**$3g^2b$** +	+	

Once done, what do you notice? What **theorem** does the list resemble? ___________
Now expand: $(g + b)^3$ = ______________________________

B5. Write a **general term** for the above example: _______________________________
(where *r* represents __________________ and (3-*r*) represents _________________)

B6. Let the **discrete random variable** be **B**, the number of blue marbles selected. Complete the following probability distribution table & graph (with title, scale, & labels):

B	Probability [P(B)=0.7]
0	$\binom{3}{0}(0.3)^3(0.7)^0 = 0.027$
1	
2	
3	

B7. Determine the following:

a) P(B=0+1+2+3) b) P(**exactly** 2 blue marbles) c) P(**at least** 1 blue marble)