Unit 1 : Day 6 : Counting, Arrangements, and Selections

MDM4U

Minds On: 15	
Action: 40	
Consolidate:20	
Total=75 min	

Description/Learning Goals

- Solve problems using lists, tree diagram, role play that progress from small sets to more unwieldy sets to motivate the need for a more formal treatment.
- See examples where some of the distinct objects are used and where all the distinct objects are used.
- Discuss how counting when order is important is different than when order is not important.

Materials

- BLM 1.6.1
- BLM 1.6.2
- Coins
- Dice
- Chart paper

Assessment Opportunities

Minds On…

Small Groups → Exploration

Explore the flipping of a coin for 4 iterations and possible outcomes using a tree diagram. Students notice that the tree grows quickly and any patterns. Continue to explore tree diagrams by rolling of a six-sided dice for 2 iterations. Students predict the size of the next iteration. Discuss observations from this activity.

In groups of 4, students choose a president, vice-president, secretary and treasurer for their group. How many different ways can this be done? Students draw tree diagrams on large paper to represent this situation. How does this differ from the previous examples?

Action!

Whole Class → Investigation

Choose three students to come to the front of the room. Try to choose people who are wearing different types of outfits.

As a class, construct a tree diagram of all the possible combinations of outfits that can be made from the clothes the students are wearing. For example: (red shirt (person 1), blue jeans (person 2), running shoes (person 3).

Students discuss what changes when you add more choices. (4 people, include socks). Continue with investigating putting all students in the class in a line. Students attempt to make a tree diagram and discuss the problems with the construction. Start over again using only 5 people from the class to be put in a line. "How many choices do we have for the first, second, third, fourth, and fifth?" Students discuss and compare the total number of choices for each experiment.

Curriculum Expectations/Observation/Mental Note

Observe students as they work on BLM1.6.1 to assess understanding of repeated & no repeated elements.

Questions could also be answered as communication assignment or in journals

Pairs → Connecting

Let's look at a Postal Code. In Canada, we use the code LNL NLN. How many different possibilities for postal codes are there. How is this different from the previous example(numbers and letters can be repeated)

Pairs complete BLM 1.6.1.

Process Expectations: Connecting/Communicating: Students communicate with each other to hypothesize correct counting technique. Connect from their investigation to choose correct technique to apply to worksheet.

Consolidate Debrief

Whole Class → Discussion/Reflection

Engage students in a discussion as they respond to the following questions:

- When is a tree diagram appropriate to visually represent data and when isn't it?
- What is different from when all objects are chosen versus some chosen?
- When do you think order is important and when is it not important and give an example in each case.

Application

Home Activity or Further Classroom Consolidation

Complete BLM 1.6.2

1.6.1: Counting Techniques

For each of the following questions, decide whether or not the elements can be repeated or not. Use the appropriate counting technique to solve the problem.

1. In Ontario, our licence plates consist of 4 letters followed by 3 numbers. Determine the number of licence plates that can be issued.

Repeated Elements ☐Yes ☐No

2. How many seven-digit telephone numbers can be made if the first three digits must be different?

Repeated Elements ☐Yes ☐No

3. The Math Club has 15 members. In how many ways can President, Vice-President, and Secretary be chosen?

Repeated Elements ☐Yes ☐No

4. The Junior Boys Volleyball team has six members. In how many ways can a starting line-up be chosen?

Repeated Elements ☐Yes ☐No

5. A committee of three is to be formed from five Math teachers and four English teachers. In how many ways can the committee be formed if there:

a. are no restrictions

b. must be one math teacher

c. must be one English teacher

d. must be only math teachers

Repeated Elements ☐Yes ☐No

1.6.2: I Can Count

1. How many different combinations can be used for a combination lock with 60 numbers

   a. if it takes three numbers to unlock the lock?

   b. if the three numbers must be unique?

2. Draw a tree diagram to illustrate the number of possible paths Bill can take to get to London, England, if he has three choices of flights from Toronto to Montreal, 2 choices from Montreal to St. John's, and 4 choices from St. John's to London.

3. In how many ways can you choose three Aces from a deck of cards one after the other

   a. if the cards are not replaced between draws?

   b. if the cards are replaced between draws?

4. Subs to Go offers 5 choices for meat, 4 choices for vegetables, 6 choices for bread, and 3 choices for cheese, assuming a sandwich must have one from each choice. Would you be able to eat a different sub everyday of the year?

Unit 1 : Day 7 : Counting Permutations

MDM4U

Time	Description/Learning Goals	Materials
Minds On: 20 Action: 45 Consolidate:10 Total=75 min	• Develop, based on previous investigations, a method to count the number of permutations of all the objects in a set of distinct objects and some of the objects in a set of distinct objects. • Use mathematical notation (e.g. n!, P(n,r)) to count.	• BLM1.7.1 – 1.7.7 • Linking cubes • Jazz music CD

Assessment Opportunities

Minds On…

Whole Class/Pairs → Tap Your Toes

Students discuss what they know about jazz music and the idea of improvising music. Make the link of improvisation to music and play a piece of jazz music. Compare to making up stories on the spot and importance of the details in both stories and music. Consider the number of different rhythms that the jazz musician has to decide between when improvising. Use an acetate of BLM 1.7.1 to introduce the bar and beats.

Using BLM 1.7.1 and BLM 1.7.2, pairs of students find how many ways a musician can create a bar of music with four different ways of notating one beat. Students reflect on how a jazz musician must decide on rhythms in a split second when they are improvising.

Using the fractions note chart on BLM 1.7.2 teacher can help explain the value of one beat.

Action!

Pairs → Hang Ups

Students complete BLM 1.7.3 working in pairs and using the labelled cards. Students should understand the meaning of permutations, factorial notation and how to calculate total number of possible arrangements using P (n, r).

Pairs → Problem Solving

Use BLM 1.7.4 to help students recall prior learning on counting techniques and assist them in investigating the concept of factorial notation. After students have completed the page, discuss solutions with students.

Process Expectation/ /Observation/Anecdotal
Selecting Tools andComputational Strategies
Observe students and make note of which strategies they use to solve problems and if they are appropriate.

Students can cut out cards or use coloured linking cubes to represent the pictures when carrying out the investigation.

Consolidate Debrief

Whole Class → Discussion

A variety of problems should be discussed on the board that involve choosing all or some of the distinct objects. (BLM 1.7.5)

Students can demonstrate their understanding of permutations by completing a Frayer Model for Permutations. See example BLM 1.7.7.

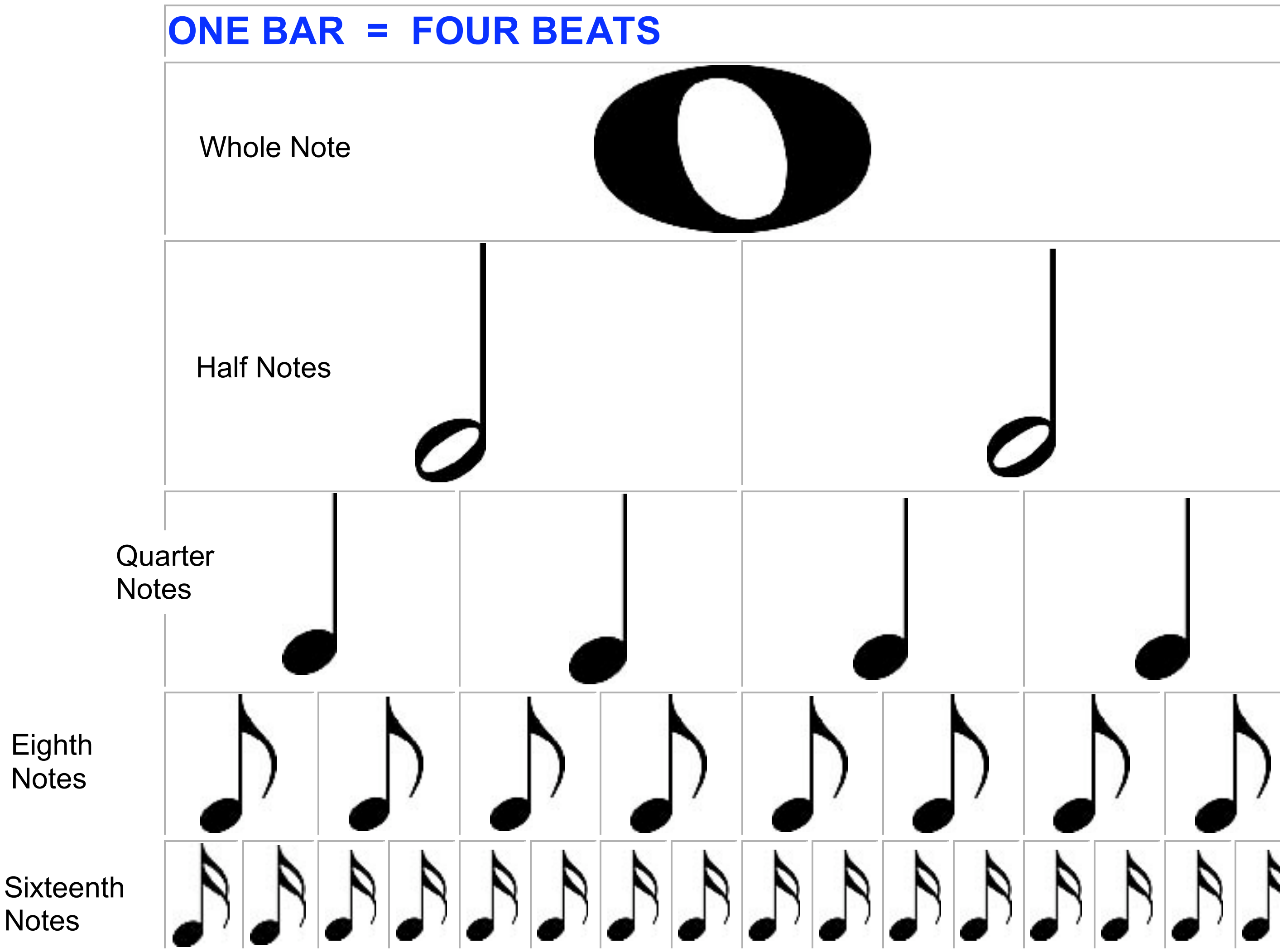
A Frayer Model is a visual organizer that helps students understand key concepts. Encourage students to use this organizer during assessments.

Application

Home Activity or Further Classroom Consolidation

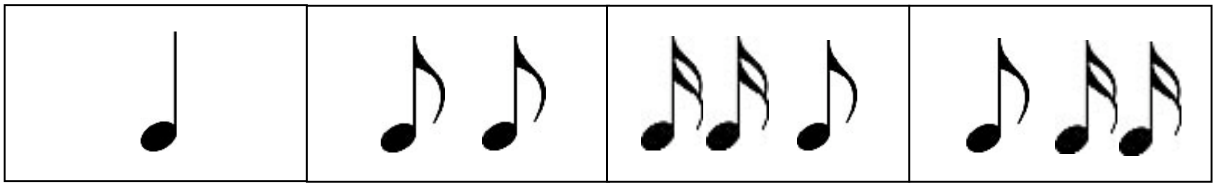
Students should demonstrate understanding of concepts through BLM 1.7.6 and explore the use of permutations to solve various problems.

1.7.1: Fractions of a Note



Using this chart, a drummer can choose different arrangements of notes for a bar of music as long as there are four beats. In music notation, this is called 4/4 time – four quarter notes in one bar.

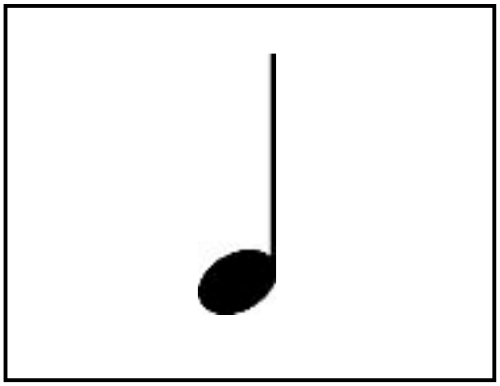
1.7.2: Tap Your Toes

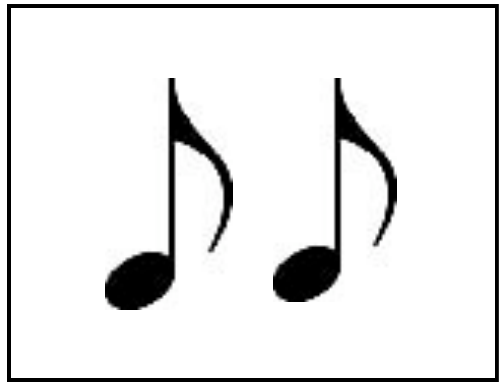
Rhythm is the drumbeat. When you tap your toes, you are hearing rhythm. These rhythms are grouped into recurring patterns that determine the piece of music. The basis of jazz music is called 4/4 time – four beats in a bar. For example, consider each space as one beat. This represents one bar of music that is made up of four beats.

There are different ways to notate one beat.
In your groups, experiment with permutations of the four different beats to create bars of music, without repeating a beat. Record them in the boxes below. Do not repeat a beat once it has been used in a bar.

1.7.2: Tap Your Toes (Continued)

How many different permutations of a bar can be made with the four different beats?

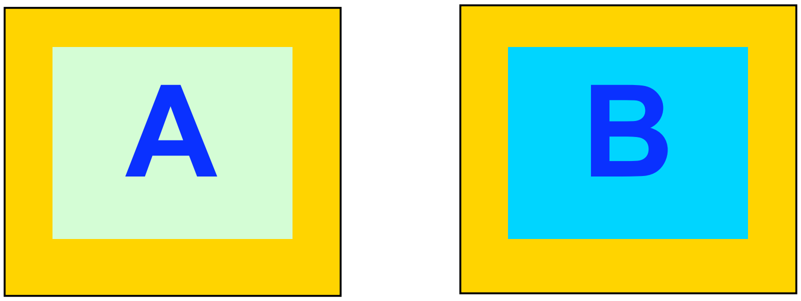
Cut out the following cards to help you arrange the beats in the bar.





1.7.3: Hang Ups

You have been given the job of hanging two pictures on the wall: A and B



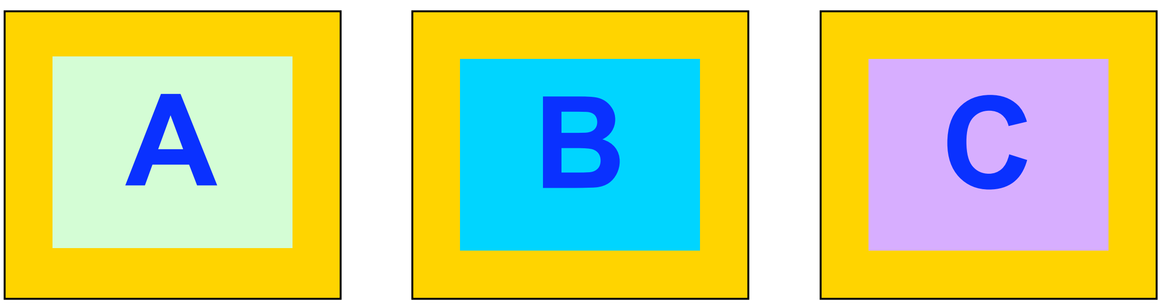
Using the cards, try it out.
You should have found two different ways.

Are both ways the same? __________

Permutation: the order of the events are important and it matters which picture is hung first.

Combination: The order of the events doesn't matter and it does not matter which picture is hung first.

This time you have three pictures to hang up: A, B and C.



Using the cards, determine how many ways you can hang three pictures on your wall. (in a row)

1.7.3: Hang Ups (continued)

First picture:

	or		or	
	or		or	

Third picture: There is no choice, only one picture is left.

What are the six possible permutations? 3 x 2 x 1 = 6

What about 5 pictures? A B C D E

Let's use spaces: ___ ___ ___ ___ ___

Fill in each space, one at a time.

How many pictures can we choose from for the first space? ___
Now how many do we have left to choose from for the second space? ___ ___
Third?___ ___ ___
Fourth?___ ___ ___ ____
Fifth? ___ ___ ___ ___ ___

5 x 4 x 3 x 2 x 1 can be written as 5! and is read as "Five factorial".
5! = ____

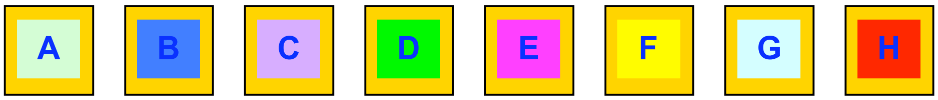
Aren't you glad you didn't have to draw them all out?

1.7.3: Hang Ups (continued)

Okay, you are now given 8 pictures, but only want 3 of them on the wall.

How many arrangements are possible?

Here are the eight pictures and the three spaces:



____ ____ ____

How many choices for each space?

How many total choices are there? _____

What if you had 10 pictures and 4 spaces on the wall to hang them?

There is a formula for this calculation.
The total number of possible arrangements of r objects out of a set of n:

$$_nP_r = \frac{n!}{(n-r)!}$$

1.7.4: Who Goes First?

Suppose there are eight students that are running for class president (Adam, Bob, Christine, Darlene, Emmett, Francis, Greg and Helen). They each have the opportunity to give a brief speech. Consider how you could determine the number of different orders in which they speak.

a. If there are only two that are going to speak, list the possible orders in which they could speak.

b. If there are now three who wish to speak, list the possible orders in which they could speak.

c. If there are now four who wish to speak, list the possible orders in which they could speak.

d. Is there an easier method to organize the list, so that you include all the possibilities? Explain why or why not.

e. Could you use your method to predict the number of different orders in which all eight students could give speeches? Determine the number of different orders.

1.7.5: Factorials and Permutations

Example: The senior choir has rehearsed five songs for an upcoming assembly. In how many different orders can the choir perform the songs?

Solution: Listing 5 X 4 X 3 X 2 X 1 = ____

Factorial 5! = ____

Permutation: P(5,5) = _____ = ______

Complete the following three questions by the three above methods.

1. How many ways can you arrange the letters in the word 'Factor'?

2. How many ways can Joe order four different textbooks on the shelf of his locker?

3. Seven children line up for a photograph. How many arrangements are possible?

1.7.6: Permutations

1. Find the number of arrangements of the word

   a. PENCIL

   b. BEETS

   c. DINOSAUR

2. Find the number of 4 letter words that can be created from the word GRAPHITE.

3. A twelve-volume library of different books numbered from 1 to 12 is to be placed on a shelf. How many out-of-order arrangements of these books are there?

4. Mei is trying to choose a new phone number and needs to choose the last four digits of the number. Her favourite digits are 2, 5, 6, 8, 9. Each digit can be used at most once.

   a. How many permutations are there that would include four of her favourite digits?

   b. How many of these would be odd?

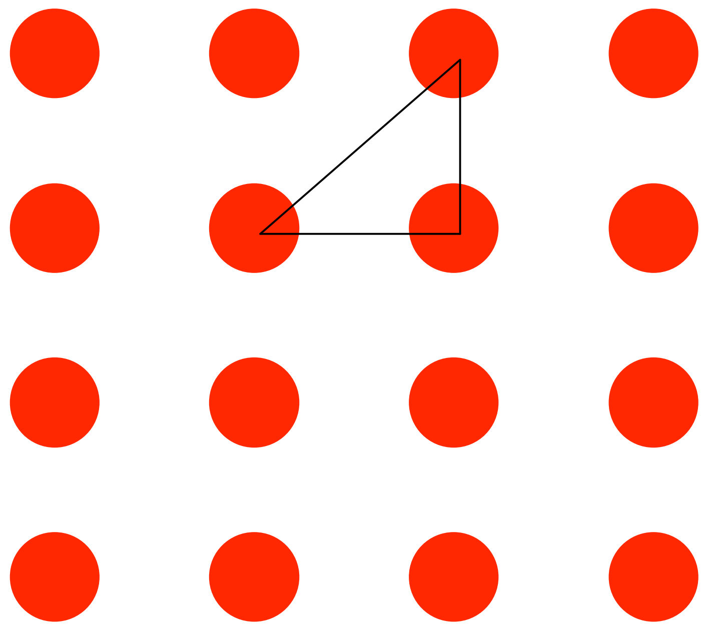
   c. How many of these would end with the digit 2?

5. In a particular business, everyone has a three-letter designation after their name. What is the smallest number of people employed by the business if there must be at least two people with the same three-letter designation?

1.8.1: Triangle Tally

On a square peg board there are sixteen pegs, four pegs to a side.
If you connect any three pegs, how many triangles can you form?

You can use a geoboard to help you solve this problem.



1.8.2: Co-Chairs

Suppose the students at your school elect a council of eight members - two from each grade. This council then chooses two of its members to be co-chairpersons. How could you calculate the number of different pairs of members who could be chosen as the co-chairs?

Number of students to choose from	Number of possible ways to choose
2	
3	
4	
5	
6	
7	
8	

1. What is the pattern emerging?

2. Use this pattern to predict the number of ways two co-chairs can be chosen from 10 students.

3. How does this differ from permutations?

1.8.3: A Novel Idea

The Bargain Book Bin is having a sale on their paperback novels. They are charging $1.00 for its Mix ‘n Match selection, which allows you to choose three novels from the following genres: Romance, Science Fiction, Fantasy, Mystery, Biographies, and Humour.

How many different Mix ‘n Match selections are possible?

Brainstorm with your group how you will solve this problem. Do not forget to include “repeat” combinations such as three romance novels.

On the chart paper provided, show your group’s solution, clearly showing your steps. Include lists, tables, diagrams, pictures or calculations you have used to arrive at your answer.

Be prepared to share your work with the whole class.

1.8.4: Three Corners

1. How many groups of three toys can a child choose to take on vacation if the toy box contains 10 toys?

2. In how many ways can we choose a Prime Minister, Deputy Prime Minister and Secretary from a class of 20?

3. In how many ways can Kimberly choose to invite her seven friends over for a sleepover assuming that she has to invite at least one friend over?

4. In how many ways can the eight nominees for Prime Minister give their speeches at a rally?

5. In how many ways can a teacher select five students from the class of 30 to have a detention?

1.8.5: Combination Conundrums

1. In how many ways can a committee of 7 be chosen from 16 males and 10 females if

   a. there are no restrictions?

   b. they must be all females?

   c. they must be all males?

2. From a class of 25 students, in how many ways can five be chosen to get a free ice cream cone?

3. In how many ways can six players be chosen from fifteen players for the starting line- up

   a. if there are no restrictions

   b. if Jordan must be on the starting line.

   c. if Tanvir has been benched and can't play.

1.8.6: Example of Frayer Model

Combination

Definition:

A selection from a group of items without regard to order is called a combination

$$C(n,r) = \frac{n!}{(n-r)!r!}$$

Facts/Characteristics:

Order does not matter in the arrangements of the objects.

Examples:

In how many ways could someone choose 20 songs to play at a dance from a selection of 30?

C(30,20) = 30,045,015

Any example where it is a selection from a group of items and order does not matter would be appropriate in this space.

Non examples:

In how many arrangements could 5 people give speeches at a student assembly? This can't be done with combinations since order matters in this example.

Unit 1 : Day 10 : Pascal's Triangle		MDM4U
Minds On: 20 Action: 45 Consolidate:10 Total=75 min	**Description/Learning Goals** • Investigate patterns in Pascal's triangle and the relationship to combinations, establish counting principles and use them to solve simple problems involving numerical values for n and r. • Investigate pathway problems	**Materials** • BLM 1.10.1 – 1.10.5

		Assessment Opportunities
Minds On…	**Small Groups → Experiment** Students are introduced to Pascal's Triangle by conducting coin probability experiments. Students are given blank Pascal's Triangle worksheets, a coin experiment recording sheet and five coins. To begin, give only one number on Pascal's Triangle – the top 1. The rest of the number will be discovered as student flip coins. (BLM 1.10.1) Students engage in a discussion on the numerical patterns seen with Pascal's Triangle.	More on Pascal's Triangle found at http://mathforum.org/workshops/usi/pascal/hs.color_pascal.html Students cut out "slices" with toppings to help with the activity. .
Action!	**Pairs → Pascal's Pizza Party** Students investigate combinatoric patterns using BLM 1.10.2 and BLM 1.10.3. **Curriculum Expectations/Observation/Checklist** Assess students' understanding of combinatoric patterns by observing and questioning them as they work. **Whole Class → Case of the Stolen Jewels** Students extend their knowledge of Pascal's Triangle by solving the "Case of the Stolen Jewels" (BLM 1.10.4). They predict the number of paths from Canard's house to the thief's location and problem solve to find the number of paths in a grid, supporting their paths by listing the moves. Using BLM 1.10.5 students practice using Pascal's Triangle and combinatorics to solve pathway problems. **Mathematical Process/Problem Solving/Connecting**: Students problem solve to find patterns within Pascal's Triangle. Students make a connection between Pascal's Triangle and combinations.	Answers could be placed in a journal or collected for assessment.
Consolidate Debrief	**Whole Class → Discussion** Questions to consider: What is the pattern that produces Pascal's Triangle? t(n,r)=t(n-1,r-1)+t(n-1,r) List three patterns found within Pascal's Triangle. What do combinations and Pascal's Triangle have in common?t(n,r)=C(n,r)	
Application	**Home Activity or Further Classroom Consolidation** Read the book "Oh, the Places You'll Go" by Dr. Seuss, and create your own map on a grid using the places mentioned in the book. Create a pathways problem (with solution) using this map.	

										1										
									1		1									
								1		2		1								
							1		3		3		1							
						1		4		6		4		1						
					1		5		10		10		5		1					
				1		6		15		20		15		6		1				
			1		7		21		35		35		21		7		1			
		1		8		28		56		70		56		28		8		1		
	1		9		36		84		126		126		84		36		9		1	
1		10		45		120		210		252		210		120		45		10		1

1. What is the pattern used to create each row?

2. What is the pattern in the second diagonal within Pascal's triangle?

3. What is the pattern in the third diagonal?

4. Add the terms is the first row (row 0) ______
   Add the terms is the second row (row 1) ______
   Add the terms in the third row (row 2) ______
   Add the terms in the fourth row (row 3) ______
   Add the terms in the fifth row (row 4) ______

5. What conclusion could you make about the sum of the terms in the row and the row number?

6. Find another pattern within Pascal's triangle.

1.10.2: Pascal's Pizza Party

Pascal and his pals have returned home from their soccer finals and want to order a pizza. They are looking at the brochure from Pizza Pizzaz, but they cannot agree on what topping or toppings to choose for their pizza.
Pascal reminds them that there are only 8 different toppings to choose from. How many different pizzas can there be?
Descarte suggested a plain pizza with no toppings, while Poisson wanted a pizza with all eight toppings.
Fermat says, "What about a pizza with extra cheese, mushrooms and pepperoni?"
Pascal decides they are getting nowhere.
Here are the toppings they can choose from:
Pepperoni, extra cheese, sausage, mushrooms, green peppers, onions, tomatoes and pineapple.

Using the cut-out pizza slices, look for patterns and answer the following questions:

1. How many pizzas can you order with no toppings?
2. How many pizzas can you order with all eight toppings?
3. How many pizzas can you order with only one topping?
4. How many pizzas can you order with seven toppings?
5. How many pizzas can you order with two toppings?
6. How many pizzas can you order with six toppings?
7. Can you find these numbers in Pascal's triangle?
8. Can you use Pascal's triangle to help you find the number of pizzas that can be ordered if you wanted three, four, or five toppings on your pizza?
9. How many different pizzas can be ordered at Pizza Pizazz in total?

1.10.2: Pascal's Pizza Party (continued)

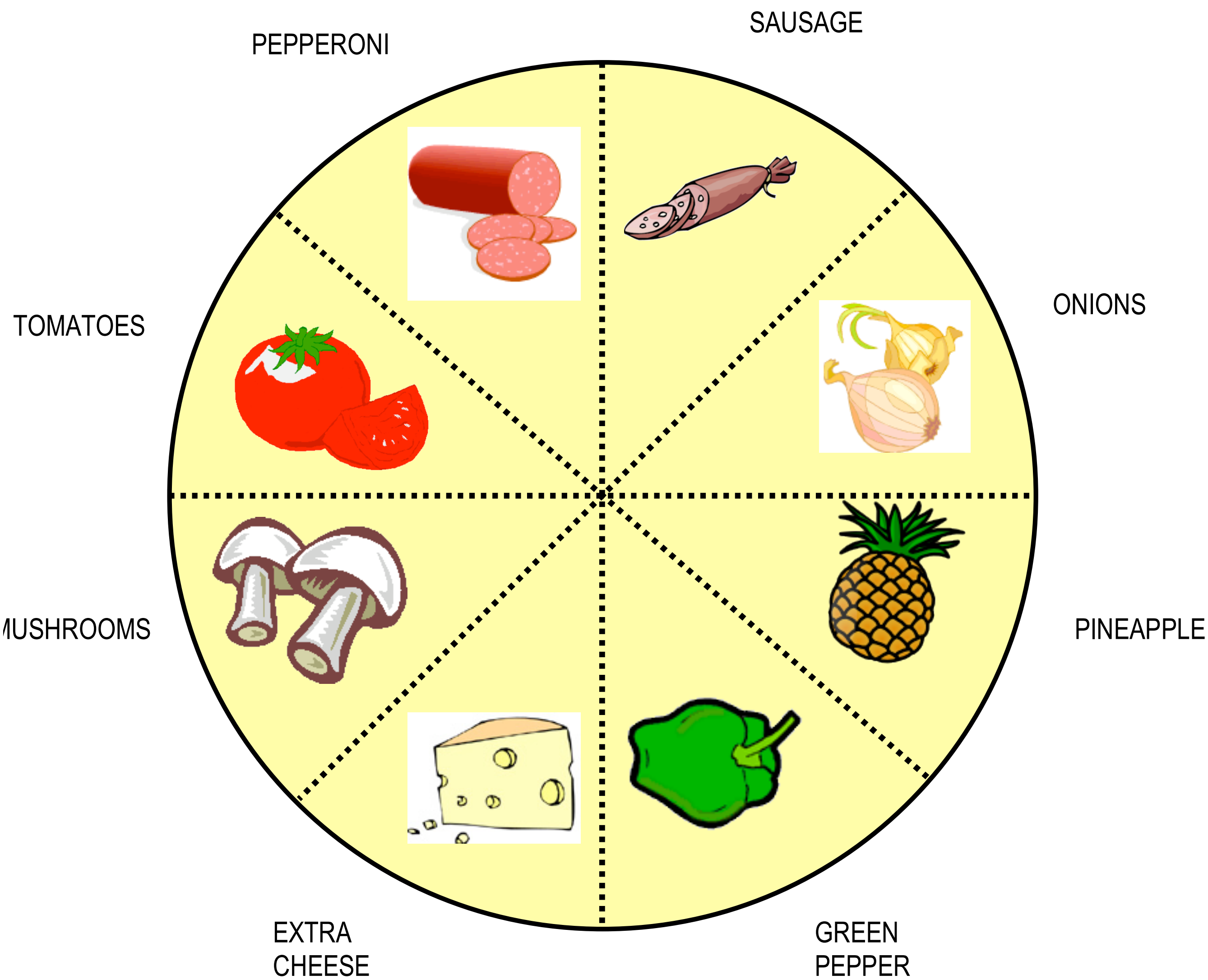
Pascal could have asked the following questions to help the group decide on their order:

1. Do you want pepperoni?
2. Do you want extra cheese?
3. Do you want sausage?
4. Do you want mushrooms?
5. Do you want green peppers?
6. Do you want onions?
7. Do you want tomatoes?
8. Do you want pineapples?

How would you use the answers to these questions to find the total number of different pizzas that can be ordered?

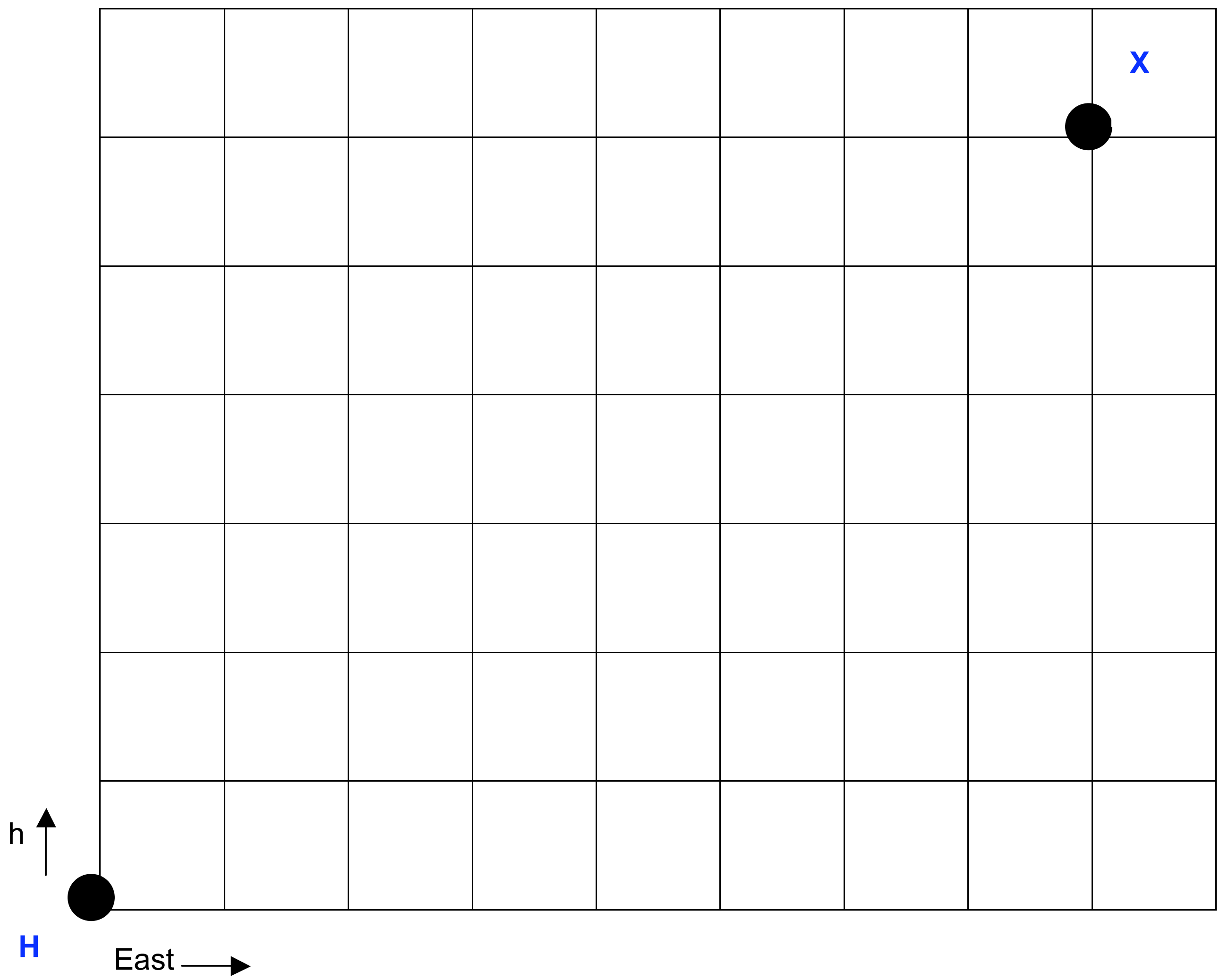
1.10.3: Pizza Pizzaz Toppings

Cut out the different toppings and use the “slices” to help you with the activity.



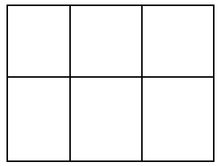
1.10.4: The Case of the Stolen Jewels

Here is a street map of part of the city of London. Inspector Canard's next case involved a million dollars worth of jewellery stolen from a hotel suite in the city. This map shows the hotel marked with the letter H. Inspector Canard is certain that the thieves and the jewels are located at the spot marked by the letter X. In order to catch the thieves, Canard must determine all the possible routes from H to X. The inspector is driving and all the streets are one-way going north or east. How many different routes do you think Inspector Canard has to check out?



1.10.5: Pathfinders

1. A

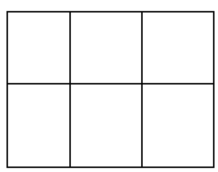


B

a. Count and draw the number of paths from A to B by only going south or east.

b. Starting at corner A begin placing Pascal's Triangle. At each successive corner continue with Pascal's Triangle pattern until corner B. How does the number at corner B relate to the number of paths you found in part a?

A



B

c. If n = the number of rows plus the number of columns (in grid AB) and r = the number of rows or columns. Find C(n,r). What do you notice?

1.10.5: Pathfinders **(continued)**

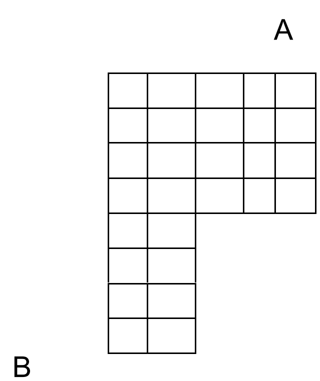
2. Solve the following problems using both Pascal's Triangle and/or Combinations.

a. A school is 5 blocks west and 3 blocks south of a student's home. How many different routes could the student take from home to school by going west or south at each corner. Draw a diagram.

b. In the following arrangements of letters start at the top and then procede to the next row by moving diagonally left or right. Determine the number of different paths that would spell the word PERMUTATION.

P
E E
R R R
M M M M
U U U U U
T T T T T T
A A A A A
T T T T
I I I
O O
N

c.



Find the number of paths from point A to Point B by only going south or west.

Unit 1 : Day 13 : Probability

MDM4U

Minds On: 15 Action: 50 Consolidate:10 Total=75 min	**Description/Learning Goals** • Solve probability problems using counting techniques involving equally likely outcomes	**Materials** • BLM 1.13.1 – 1.13.5 • Linking Cubes • Counters • dice • chart paper

		Assessment Opportunities
Minds On…	**Whole Class → Feeling Lucky** Students read the BLM 1.13.1 and discuss the outcome of the Powerball lottery and use of the fortune cookies for the selection of numbers and the probability of winning a lottery. **Pairs → Lewis Carroll's Pillow Problem** Using BLM 1.13.2, students try and solve the pillow problem in pairs. Solutions provided by Lewis Carroll are presented and students analyze them.	Manipulatives can be used to help solve this problem.
Action!	**Small Groups → Marble Mystery** Students work through BLM 1.13.3 in groups. All work and solutions should be recorded on chart paper. Students will share their strategies and solutions with the whole class. Linking cubes could be used with BLM 1.13.4 to determine experimental probability before theoretical probability is calculated. **Learning Skills/Observation/Rubric** Through observations during the investigation, assess students' teamwork skills. **Mathematical Process/Connecting/Selecting Tools/Problem Solving: students reflect on past learning and problem solve to incorporate the use of counting techniques.**	Students should use their prior knowledge on counting techniques to work through the solution to the Marble Mystery.
Consolidate Debrief	**Whole Class → Gallery Walk** All solutions to the Marble Mystery should be sorted and posted in groupings according to strategies used for different solutions. Students go on a Gallery Walk to reflect on alternate approaches to the final answer, different solutions, and other observations on probabilities. Students discuss the connections made to counting techniques, understanding of probabilities and application to real-world events such as sports, weather, game designs, lotteries, etc.	
Application *Concept Practice* *Differentiated* *Exploration* *Reflection* *Skill Drill*	**Home Activity or Further Classroom Consolidation** Play the game on BLM 1.13.5 with a partner. Record results on the table provided. Were you surprised with the results when you were playing the game? Can you explain the results of the game using probabilities? Cross-Curricular Activity Rosencrantz and Guildenstern are Dead	

1.13.1 Feeling Lucky

May 12, 2005 , New York Times

BY Jennifer Lee

Who Needs Giacomo? Bet on the Fortune Cookie

James Estrin/The New York Times

Many different brands of fortune cookies come from Wonton Food's Long Island City factory.

Powerball lottery officials suspected fraud: how could 110 players in the March 30 drawing get five of the six numbers right? That made them all second-prize winners, and considering the number of tickets sold in the 29 states where the game is played, there should have been only four or five. But from state after state they kept coming in, the one-in-three-million combination of 22, 28, 32, 33, 39.

It took some time before they had their answer: the players got their numbers inside fortune cookies, and all the cookies came from the same factory in Long Island City, Queens.

Chuck Strutt, executive director of the Multi-State Lottery Association, which runs Powerball, said on Monday that the panic began at 11:30 p.m. March 30 when he got a call from a worried staff member.

The second-place winners were due $100,000 to $500,000 each, depending on how much they had bet, so paying all 110 meant almost $19 million in unexpected payouts, Mr. Strutt said. (The lottery keeps a $25 million reserve for odd situations.)

Of course, it could have been worse. The 110 had picked the wrong sixth number - 40, not 42 - and would have been first-place winners if they did.

"We didn't sleep a lot that night," Mr. Strutt said. "Is there someone trying to cheat the system?"

He added: "We had to look at everything to do with humans: television shows, pattern plays, lottery columns."

Earlier that month, an ABC television show, "Lost," included a sequence of winning lottery numbers. The combination didn't match the Powerball numbers, though hundreds of people had played it: 4, 8, 15, 16, 23 and 42. Numbers on a Powerball ticket in a recent episode of a soap opera, "The Young and the Restless," didn't match, either. Nor did the winning numbers form a pattern on the lottery grid, like a cross or a diagonal. Then the winners started arriving at lottery offices.

"Our first winner came in and said it was a fortune cookie," said Rebecca Paul, chief executive of the Tennessee Lottery. "The second winner came in and said it was a fortune cookie. The third winner came in and said it was a fortune cookie."

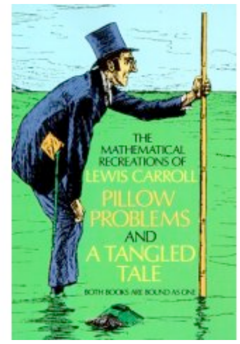
Investigators visited dozens of Chinese restaurants, takeouts and buffets. Then they called fortune cookie distributors and learned that many different brands of fortune cookies come from the same Long Island City factory, which is owned by Wonton Food and churns out four million a day.

"That's ours," said Derrick Wong, of Wonton Food, when shown a picture of a winner's cookie slip. "That's very nice, 110 people won the lottery from the numbers."

The same number combinations go out in thousands of cookies a day. The workers put numbers in a bowl and pick them. "We are not going to do the bowl anymore; we are going to have a computer," Mr. Wong said. "It's more efficient."

1.13.2: Lewis Carroll’s Pillow Problem

Author Lewis Carroll had insomnia and used the time to create “pillow problems”.
Here is an example of one of these problems:



> *A bag contains a counter, known to be either white or black. A white counter is put in, the bag is shaken, and a counter is drawn out, whi proves to be white. What is now the chance of drawing a white coun*

1. Solve this problem with your partner. Justify your solution.

1.13.2: Lewis Carroll's Pillow Problem (continued)

Lewis Carroll provided two solutions to this problem:

Solution #1

As the state of the bag, *after* the operation, is necessarily identical with its state *before* it, the chance is just what it was, viz. 1/2.

Solution #2

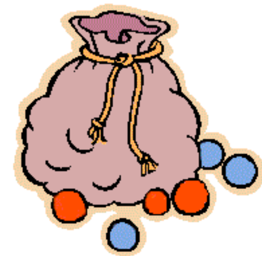
Let B and W1 stand for the black or white counter that may be in the bag at the start and W2 for the added white counter. After removing white counter there are three equally likely states:

Inside bag	Outside bag
W1	W2
W2	W1
B	W2

In two of these states a white counter remains in the bag, and so the chance of drawing a white counter the second time is 2/3.

2. Which one is correct? Explain.

1.13.3: Marble Mystery



A bag contains two red marbles, three blue marbles, and four green marbles. Yusra draws one marble from the jar, and then Chang draws a marble from those remaining. What is the probability that Yusra draws a green marble and Chang draws a blue marble? Express your answer as a common fraction.

Remember that to find a basic probability, with all outcomes equally likely, we make a fraction that looks like this:

$$\frac{\text{number of favourable events}}{\text{number of total events}}$$

1.13.4: More on Probability

1. Find the probability of drawing two red cubes simultaneously from a box containing 3 red, 5 blue, and 3 white cubes.

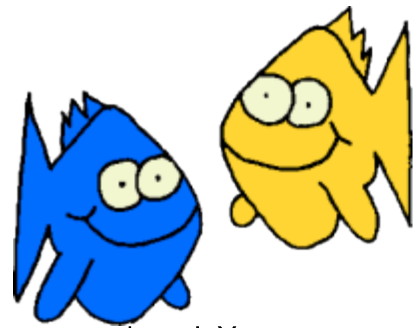
2. Find the probability of drawing two red cubes from the same box. This time you draw one cube, note its colour, set it aside, shake the bag and draw another cube. (Hint: there are two events in this problem.)

3. Find the probability of choosing first a red cube, then a blue, then a white if:

   a. each cube is replaced between choices.

   b. each cube is not replaced between choices.

1.13.5: Something's Fishy Game



Equipment Needed
- Game board for each player
- 6 counters (fish) for each player
- 2 dice per pair of players

Rules

1. Each player can place their fishes into any aquarium on their own game board. You can place one in each aquarium, or two in some aquariums and none in others, or even all six in one aquarium.

2. Take turns to roll the two dice. Calculate the difference between the two numbers. You can release one fish from the aquarium with that number. For example, if the difference is 2, you can release one fish from aquarium #2.

3. The winner is the first to release all their fish.

4. Keep a record of where you place your fishes for each game, then record the ones that are winners.

	Aquariums						Winners
	0	1	2	3	4	5	
Fishes							

1.13.5: Something's Fishy Game Board

How do you place your fishes in these aquariums to release them quickly?