# MDM4U
# Mathematics of Data Management
## University Preparation

# Mathematics of Data Management: Content and Reporting Targets

***Mathematical Processes across all strands:*** *Problem Solving, Reasoning and Proving, Reflecting, Selecting Tools and Computational Strategies, Connecting, Representing, and Communicating.*

| Unit 1 | Unit 2 | Unit 3 | Unit 4 | Unit 5 |
|---|---|---|---|---|
| **Counting and Probability**<br>• Introducing Probability vocabulary and notation in contexts involving simple counting<br>• Counting permutations and combinations<br>• Using mathematical notation to describe the number of permutations and combinations<br>• Solving counting problems<br>• Solving probability problems using counting principles | **Organization of Data for Analysis**<br>• Connecting questions and the data needed to answer them<br>• Understanding data concepts | **Statistics**<br>• Single-variable data<br>• Two-variable data<br>• Evaluating validity | **Probability Distributions**<br>• Discrete random variables | **Modeling Continuous Data**<br>• Continuous random variables |
| **Culminating Projects and Investigations**<br>**A: Permutations, Combinations, Probabilities; B: Statistics; C: Probability Distributions** | | | | |
| **Counting Stories Project**<br>• Choosing or creating a nursery rhyme or piece of literature that provides the potential for posing and answering connected problems that involve permutations, combinations, and probabilities<br>• Posing and solving the connected problems | **Culminating Investigation**<br>• Retrieving/gathering data from various sources (e.g., e-Stat)<br>• Dealing with/organizing various forms of data<br>• Fine-tuning their research question<br>• Learning to use data-analysis tools (e.g., Fathom) | **Culminating Investigation**<br>• Interpreting, analysing, and interpreting the data<br>• Drawing conclusions from the analysis of data<br>• Compiling report, presenting multimedia summary to audience, and answering questions<br>• Posing probing questions of presenters<br>• Critiquing work of others | **Games Fair Project**<br>• Creating a game of chance that has an expected value > 0 for the game provider, and that encourages people to play the game<br>• Calculating the probability distribution for the game and the expected values<br>• Hosting the game and gathering experimental data<br>• Comparing the theoretical probability distribution and the experimental data<br>• Critiquing work of others | **Culminating Investigation**<br>• Identifying area of interest and connecting it to probability<br>• Posing a problem of interest, finding sources of data, determining relevance of data<br>• Designing a plan to study the problem of interest, formulating a research question |

## Rationale

**Starting with Probability and Counting**

- Helps students understand that this course requires new ways of thinking mathematically that is less algebraic than earlier courses.
- Due to new pathways producing a more diverse group of students, starting with Probability and Counting will begin to train students to use more complex mathematical processes.
- Builds on prior experiences with and knowledge of probability to use a more formal mathematical approach, including its vocabulary and notation.
- Separates the establishment of this formal approach from the introduction to permutations and combinations.
- Offers an opportunity for success early in the course to students who may not have excelled in more algebraic approaches.
- Establishes this course as a university-preparation course
- Provides the opportunity to introduce concepts of variability.
- Prepares students for statistical analysis and probability distributions.

**Organization of Data for Analysis (in Unit 2)**

- Providing students with tools for working with data prepares them to begin work on Culminating Investigation B early in the course.

**Probability Distributions( later in the course)**

- Students benefit from seeing data distributions before applying those concepts to probability distributions

**Culminating Projects and Investigations (addressed in all units)**

- Provides time for students to develop quality projects and investigations.
- Provides time for students to incorporate feedback on Project A into Project and Investigation C
- Provides time for students to incorporate feedback on various stages of Investigation B into their presentation.
- Addresses all curriculum expectations through projects and investigations, thereby eliminating the need for an examination.
- Provides more opportunities for feedback and interactions.

# Mathematics of Data Management
# Year Outline – Planning Tool

**P** Number of pre-planned lessons (including instruction, diagnostic and formative assessments, summative assessments other than summative performance tasks)

**J** Number of jazz days of time (instructional or assessment)

**T** Total number of days

**SP** Summative performance task (see Assessment – Grade 9 Applied)

| Unit | Cluster of Curriculum Expectations | Overall and Specific Expectations | P | J | T | SP |
|---|---|---|---|---|---|---|
| 1 | Counting and Probability<br>Introducing Probability vocabulary (e.g. sample space, outcomes, events, trials, discrete, continuous, theoretical probability, experimental probability, mutually exclusive, random number generator, Venn diagram, independent and dependent events, conditional probability, complement, simulation) and notation (e.g., P(~A), P(A and B), P(A or B), P(A \| B)) in contexts involving simple counting (e.g. where the sample space is given)<br>Distinguish between and make connections between situations involving the use of permutations and combinations<br>Develop, through investigation the number of permutations and combinations<br>Using mathematical notation (e.g., $n!, P(n,r), \binom{n}{r}$) to count<br>Solving counting problems using counting principles – additive, multiplicative<br>Solving probability problems using counting principles<br>Culminating Investigation – identifying area of interest and connecting it to probability | **A1** solve problems involving the probability of an event or a combination of events for discrete sample spaces<br>**A2** solve problems involving the application of permutations and combinations to determine the probability of an event<br>**E1** design culminating investigation that requires the integration and application of the knowledge and skills related to the expectations of this course<br>**E2** communicate the findings of a culminating investigation and provide constructive critiques of the investigations of others | 16 | 2 | 19 | 2 |

| Unit | Cluster of Curriculum Expectations | Overall and Specific Expectations | P | J | T | SP |
|---|---|---|---|---|---|---|
| 2 | Organization of Data for Analysis<br>Demonstrate an understanding of the role of data in statistical studies<br>Describe the characteristics of a good sample and compare sampling techniques<br>Design an effective survey and collect data<br>Understand how data is organized<br>Find sources of data, refine topic of interest and design a plan in preparation for the Culminating Investigation | **C1** demonstrate an understanding of the role of data in statistical studies and the variability inherent in data, and distinguish different types of data<br><br>**C2** describe the characteristics of a good sample, some sampling techniques, and principles of primary data collection, and collect and organize data to solve a problem<br><br>**E1** design culminating investigation that requires the integration and application of the knowledge and skills related to the expectations of this course | 10 | 1 | 11 | |
| 3 | Statistics<br>Explore, analyse, interpret, and draw conclusions from one-variable data<br>Explore, analyse, interpret, and draw conclusions from two-variable data<br>Investigate and evaluate validity of statistical summaries<br>Culminating Investigation<br>Analyse, interpret, draw conclusions and write a report of their research<br>Present summary of findings<br>Critique presentations of their peers | **D1** analyse, interpret, and draw conclusions from one-variable data using numerical and graphical summaries<br><br>**D2** analyse, interpret, and draw conclusions from two-variable data using numerical, graphical, and algebraic summaries<br><br>**D3** demonstrate an understanding of the applications of data management used by the media and the advertising industry and used in various occupations<br><br>**E1** design and carry out a culminating investigation that requires the integration and application of the knowledge and skills related to the expectations of this course<br><br>**E2** communicate the findings of a culminating investigation and provide constructive critiques of the investigations of others | 18 | 2 | 20 | 10 |

| Unit | Cluster of Curriculum Expectations | Overall and Specific Expectations | P | J | T | SP |
|---|---|---|---|---|---|---|
| 4 | Probability Distributions<br>Understand probability distributions for discrete random variables<br>Explore and connect Binomial and Hypergeometric distributions<br>Recognize that the differences between a probability histogram and a frequency histogram may be the result of variability<br>Complete a Games Fair culminating project | **B3** demonstrate an understanding of discrete probability distributions, represent them numerically, graphically, and algebraically, determine expected values, and solve related problems from a variety of applications;<br><br>**E1** design and carry out a culminating investigation that requires the integration and application of the knowledge and skills related to the expectations of this course<br><br>**E2** communicate the findings of a culminating investigation and provide constructive critiques of the investigations of others | 10 | 1 | 11 | 2 |
| 5 | Modelling Continuous Data<br>Describe the shapes of distributions of continuous data<br>Extend the concept of a discrete probability distribution to a continuous probability distribution<br>Understand the features of the normal distribution<br>Apply normal distributions to real-world situations recognizing the role of variability | **D1** analyse, interpret, and draw conclusions from one-variable data using numerical and graphical summaries;<br><br>**B2** demonstrate an understanding of continuous probability distributions, make connections to discrete probability distributions, determine standard deviations, describe key features of the normal distribution, and solve related problems from a variety of applications. | 10 | 1 | 11 | |
| | Summative Performance Tasks | | | | | 14 |
| | Total Days | | 64 | 7 | 72 | 85 |

The number of prepared lessons represents the lessons that could be planned ahead based on the range of student readiness, interests, and learning profiles that can be expected in a class. The extra time available for "instructional jazz" can be taken a few minutes at a time within a pre-planned lesson or taken a whole class at a time, as informed by teachers' observations of student needs.

The reference numbers are intended to indicate which lessons are planned to precede and follow each other. Actual day numbers for particular lessons and separations between terms will need to be adjusted by teachers.

# Unit 1: Counting and Probability

**Grade 12**

## Lesson Outline

### Big Picture

Students will:

- solve problems involving probability of distinct events;
- solve problems using counting techniques of distinct items;
- apply counting principles to calculating probabilities;
- explore variability in experiments;
- demonstrate understanding of counting and probability problems and solutions by adapting/creating a children's story/nursery rhyme in a Counting Stories project;
- explore a significant problem of interest in preparation for the Culminating Investigation.

| Day | Lesson Title | Math Learning Goals | Expectations |
|---|---|---|---|
| 1–2 | Mathematical Probability | • Investigate probabilities generated from experiments (e.g., spinners, sampling, numbered cubes, coins, cards) and use mathematical vocabulary (e.g., sample space, outcomes, events, trials, theoretical probability, experimental probability, mutually exclusive, non-mutually exclusive, independent and dependent events, complement) and notation (e.g., *P(A), P(~A), P(A* and *B), P(A* or *B), P(A \| B))* in contexts involving simple counting (e.g. where the sample space is given) and tools (e.g. tree diagrams, organized lists, Venn diagram).<br>• Determine whether 2 events are independent or dependent and whether one event is conditional on another.<br>• Recognize that the sum of the probabilities of all possible outcomes in the sample space is 1. | A1.1, A1.2, A1.3, A1.5, A1.6 |
| 3 | Counting Stories Project | • Introduce and understand one culminating project, Counting Stories Project (e.g. student select children's story/nursery rhyme to rewrite using counting and probability problems and solutions as per Strand A).<br>• Create a class critique to be used during the culminating presentation. | E2.3<br>E2.4 |
| 4–5 | Using Simulations to Show Variability | • Determine, through investigation using class-generated data and technology-based simulation models, the tendency of experimental probability to approach theoretical probability as the number of trials in an experiment increases.<br>• Graph the experimental probability versus the number of trials, and describe any trend. | A1.4 |
| 6 | Counting Arrangements and Selections | • Solve problems that progress from small sets to more complex sets using lists, tree diagram, role play to establish the need for a more formal strategy.<br>• See examples where some of the *distinct* objects are used and where all the *distinct* objects are used.<br>• Discuss how counting when order is important is different from when order is not important to distinguish between situations that involve, the use of permutations and those that involve the use of combinations. | A2.1 |

| Day | Lesson Title | Math Learning Goals | Expectations |
|---|---|---|---|
| 7 | Counting Permutations | • Develop, based on previous investigations, a method to calculate the number of permutations of all the objects in a set of *distinct* objects and some of the objects in a set of *distinct* objects.<br>• Use mathematical notation (e.g., *n!, P(n, r)*) to count. | A2.1, A2.2 |
| 8 | Counting Combinations | • Develop, based on previous investigations, a method to calculate the number of combinations of some of the objects in a set of *distinct* objects.<br>• Make connection between the number of combinations and the number of permutations.<br>• Use mathematical notation (e.g., $\binom{n}{r}$) to count<br>• Ascribe meaning to $\binom{n}{n}, \binom{n}{1}, \binom{n}{0}$.<br>• Solve simple problems using techniques for counting permutations and combinations, where all objects are distinct. | A2.1, A2.2 |
| 9 | Counting Stories Project | • Use counting and probability problems and solutions to create first draft of Counting Stories Project. | A1.1, A1.3, A1.5, A1.6, A2.1, A2.2, A2.3 |

| Day | Lesson Title | Math Learning Goals | Expectations |
|---|---|---|---|
| 10–11 | Pascal's Triangle | • Investigate patterns in Pascal's triangle and the relationship to combinations, establish counting principles and use them to solve simple problems involving numerical values for $n$ and $r$.<br>• There is only one way to choose all of the elements (i.e., $\binom{n}{n} = 1$).<br>• There is only way to choose none of the elements (i.e., $\binom{n}{0} = 1$).<br>• There are n ways to choose one element from n elements (i.e., $\binom{n}{1} = 1$).<br>• Choosing $r$ elements from $n$ elements is the same as choosing $n–r$ elements from $n$ elements (i.e., $\binom{n}{n} = \binom{n}{n-r}$) (e.g., Choosing 3 girls from 8 girls for a committee is the same as choosing 5 girls not to be on the committee).<br>• The number of collections of any size from $n$ elements is $2^n$. (i.e., $\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \ldots + \binom{n}{n-1} + \binom{n}{n} = 2^n$) (e.g., the number of different playlists selected from 10 tunes is $2^{10}$).<br>• The total number of selections of r elements from n elements is made up of selections that either include a particular element or not (i.e., $\binom{n}{r} = \binom{n-1}{r-1} + \binom{n-1}{r}$) (e.g., the number of unordered playlists with 5 tunes chosen from 10 tunes either includes a specific tune or not. If it includes it, there are $\binom{9}{4}$ ways of choosing the remaining tunes. If it doesn't include it, there are $\binom{9}{5}$ ways of choosing the five tunes. So $\binom{9}{4} + \binom{9}{5}$ is the number of ways of choosing 5 tunes from 10, which is $\binom{10}{5}$.).<br>• Investigate pathway problems. | A2.4 |
| 12 | Mixed Counting Problems | • Distinguish between and make connections between situations involving the use of permutations and combinations of distinct items.<br>• Solve counting problems using counting principles – additive, multiplicative. | A2.3 |

| Day | Lesson Title | Math Learning Goals | Expectations |
|---|---|---|---|
| 13 | Probability | • Solve probability problems using counting principles involving equally likely outcomes, e.g., two cards are drawn randomly from a standard 52-card deck. What is the probability that the two cards are both aces if the first card is replaced? If the first card is not replaced? | A2.5 |
| 14 | Counting Stories Project | • Complete final version of Counting Stories Project. | A1.1, A1.3, A1.5, A1.6, A2.1, A2.2, A2.3, A2.4, A2.5, F2.4 |
| 15 | Culminating Investigation | • Identify a significant problem of interest for Culminating Investigation.<br>• Brainstorm ideas, e.g., mind mapping, for organization and analysis of data related to a related significant problem. | E1.1 |
| 16–17 | Jazz/Summative | | |

# Unit 2: Organization of Data for Analysis Grade 12

## Lesson Outline

### Big Picture

Students will:

- demonstrate an understanding of the role of data in statistical studies;
- describe the characteristics of a good sample and compare sampling techniques;
- design an effective survey and collect data;
- understand how data is organized;
- find sources of data, refine topic of interest, and design a plan in preparation for the Culminating Investigation.

| Day | Lesson Title | Math Learning Goals | Expectations |
|---|---|---|---|
| 1 | Brainstorming | • Use prepared data to:<br>– recognize and describe the role of data in statistical studies;<br>– describe examples of applications of statistical studies;<br>– recognize that conclusions drawn from statistical studies of the same relationship may disagree, and explain why. | C1.1, C1.2 |
| 2 | Distinguishing Types of Data | • Use prepared data to distinguish different types of statistical data that is discrete from continuous; qualitative from quantitative; categorical from numerical; nominal from ordinal; primary from secondary; experimental from observational; micro data from aggregate data. | C1.3 |
| 3 | Sampling Jigsaw | • Describe and compare sampling techniques i.e., simple random; systematic, stratified, convenience, voluntary.<br>• Describe principles of primary data collection.<br>• Demonstrate an understanding of the difference between population and sample. | C2.2 |
| 4 | Data Validity | • Describe the characteristics of a good sample, i.e., bias free, random, representative.<br>• Distinguish between population and sample, and understand why sampling is necessary.<br>• Understand how using random samples with a bias or non random samples can affect the results of a study. | C2.1, C2.2, C2.3 |
| 5 | Surveys | • Describe the characteristics of an effective survey.<br>• Collect data from primary sources, through experimentation, organize data with one or more attributes. | C2.4, C2.5 |
| 6 | Census of school, collecting data | • Design questionnaires. | C 2.4 |
| 7 | Culminating Investigation Searching for data<br><br>*(lesson not included)* | • Collect data from secondary sources, e.g., by using the internet to access reliable data from a well-organized database such as e-stat; by using print sources such as newspapers and magazines. | C2.5, |

| Day | Lesson Title | Math Learning Goals | Expectations |
|---|---|---|---|
| 8 | Demographics and Beverage Consumptions | • Collect data from secondary sources (e.g., by using the internet to access reliable data from a well-organized database such as e-stat; by using print sources such as newspapers and magazines).<br>• (Beer demographics activity available at http://www.statcan.ca/english/edu/mathmodel.htm). | C2.5 |
| 9 | Project Day | • Find sources of data in preparation for the Culminating Investigation.<br>• Refine topic of interest for Culminating Investigation.<br>• Design a plan to investigate topic. | C2.5 E1.1, E1.2, E1.3 |
| 10–11 | Jazz/Summative | | |

# Unit 3: Statistics

Grade 12

## Lesson Outline

### Big Picture

Students will:

- explore, analyse, interpret, and draw conclusions from one-variable data;.
- explore, analyse, interpret, and draw conclusions from two-variable data;
- investigate and evaluate validity of statistical summaries;
- culminating Investigation;
- analyse, interpret, draw conclusions, and write a report of their research;
- present summary of finding;
- critique presentations of their peers.

| Day | Lesson Title | Math Learning Goals | Expectations |
|---|---|---|---|
| 1–2 | Summary Statistics | • Recognize that the analysis of one-variable data involves the frequency of one attribute, and determine, using technology, the relevant numerical summaries (mean, median, mode, range, variance, and standard deviation).<br>• Determine the positions of individual data points within a one-variable data set using quartiles, percentiles, and *z*-scores; use the normal distribution to model one-variable data sets, and recognize these processes as strategies for one-variable data analysis. | D1.1, D1.2 |
| 3–4 | Graphical displays of data | • Generate, using technology, the relevant graphical displays of one-variable data based on the type of data provided.<br>• Explore types of data, e.g., categorical, ordinal, and quantitative. | D1.3 |
| 5 | Culminating Investigation | • Interpret, analyse, and summarize data related to the study of the problem.<br>• Draw conclusions from the analysis of the data, evaluate the strengths of the evidence, specify limitations, suggest follow-up problems or investigations.<br>• Focus on one-variable analysis. | E1.4, E1.5 |
| 6–7 | Interpreting Data Using Summary Statistics and Graphs | • Interpret and compare two related one-variable prepared data sets.<br>• Formulate and critique conclusions.<br>• Communicate conclusions orally and in writing make connections to the culminating investigation. | D1.5, E 1.4, E1.5 |
| 8–9 | Analysing Two Variable Data | • Graph two numerical variables on a scatter plot.<br>• Determine the appropriateness of a linear model to describe the relationship between two numerical attributes.<br>• Recognize the meaning of the correlation coefficient, using a prepared investigation.<br>• Compare a quantitative and a categorical variable (e.g., gender vs. Income) using appropriate displays, e.g., stacked box plots.<br>• Compare two categorical variables, e.g., gender vs. Colour-blindness, using a contingency or summary table and computing proportions. | D2.1, D2.3 |

| Day | Lesson Title | Math Learning Goals | Expectations |
|---|---|---|---|
| 10–11 | Understanding Correlation | • Explore different types of relationships between two variables, e.g., The cause-and-effect relationship between the age of a tree and its diameter; the common-cause relationship between ice cream sales and forest fires over the course of a year; the accidental relationship between your age and the number of known planets in the universe.<br>• Interpret statistical summaries to describe and compare the characteristics of two variable statistics. | D2.2, D2.5, E1.4, E1.5 |
| 12–13 | Interpreting and Making Inferences | • Perform linear regression using technology to determine information about the correlation between variables.<br>• Determine the effectiveness of a linear model on two variable statistics.<br>• Investigate how statistical summaries can be used to misrepresent data.<br>• Make inferences and justify conclusions from statistical summaries or case studies.<br>• Communicate orally and in writing, using convincing arguments. | D2.2, D2.4, D2.5, E1.4, E1.5 |
| 14 | Culminating Investigation | • Interpret, analyse, and summarize data related to the study of the problem.<br>• Draw conclusions from the analysis of the data, evaluate the strengths of the evidence, specify limitations, suggest follow-up problems or investigations.<br>• Focus on two-variable analysis. | E1.4, E1.5 |
| 15 | Assess Validity | • Interpret and assess statistics presented in the media, e.g., Promote a certain point of view, advertising, including how they are used or misused to present a certain point of view.<br>• Investigate interpretation by the media based on lack of knowledge of statistics, e.g., drug testing, false positives.<br>• Examine data collection techniques and analysis in the media, e.g., sample size, bias, law of large numbers.<br>• Scrapbook of statistical observations from the media. | D3.1, D3.2, E1.5 |
| 16–17 | Culminating Investigation related to occupations | • Use journalism as an example to demonstrate applications of data management in an occupation.<br>• Gather, interpret, and describe how the information collected in their project relates to an occupation, e.g., insurance, sports statistician, business analyst, medical researcher.<br>• From their projects identify university programs that explore the applications. | D3.3, E1.3 |
| 18 | Culminating Investigation | • Edit and compile a report that interpret, analyses, and summarizes data related to the study of the problem.<br>• Draw conclusions from the analysis of the data, evaluate the strengths of the evidence, specify limitations, suggest follow-up problems or investigations. | E1.4, E1.5, E2.1 |
| 19–20 | Jazz/Summative | • | |
| Reserve time 10 days | Culminating Investigation | • Present a summary of the culminating investigation to an audience of their peers.<br>• Answer questions about the culminating investigation and respond to critiques.<br>• Critique the mathematical work of others in a constructive manner. | E2.2, E2.3, E2.4 |

# Unit 4: Probability Distributions of Discrete Random Variables

Grade 12

## Lesson Outline

**Big Picture**

Students will:

- Understand probability distributions for discrete random variables.
- Explore and connect binomial and hypergeometric distributions.
- Recognize that the differences between a probability histogram and a frequency histogram may be the result of variability.
- Complete a Games Fair culminating project.

| Day | Lesson Title | Math Learning Goals | Expectations |
|---|---|---|---|
| 1–2 | Formalizing Discrete Random Variables<br><br>Relating Probability Histograms and Frequency Histograms | • Recognize and identify a discrete random variable.<br>• Calculate the probabilities associated with all values of a random variable, with and without technology.<br>• Generate, tabulate, and graph a probability distribution.<br>• Compare a probability histogram to the frequency histogram of a related experiment conducted previously in the course.<br>• Make connections between the frequency histogram and the probability histogram. | B1.1, B1.3 |
| 3 | Spin and Win | • Calculate, interpret, and apply expected value.<br>• Construct and analyse simple fair and unfair games, e.g., using familiar examples from Unit 1. | B1.2 |
| 4 | Are You Game? | • Calculate, interpret, and apply expected value.<br>• Make connections between the expected value and the weighted mean of the values of the discrete random variable.<br>• Introduce the Games Fair project and its connection to experimental and theoretical probability.<br>• Develop a critique for peer assessment for the Games Fair project. | E1, E2, B1.2 |

| Day | Lesson Title | Math Learning Goals | Expectations |
|---|---|---|---|
| 5–7 | The Binomial Distribution | • Recognize and identify situations resulting in a binomial discrete random variable.<br>• Calculate the probabilities associated with all values of a binomial random variable.<br>• Compare binomial probability histograms to frequency histograms of related experiments conducted previously in the course.<br>• Recognize that the differences between a probability histogram and a frequency histogram may be the result of variability and/or other factors (e.g., when tossing a coin 10 times and recording the number of heads, the frequency distribution should have some of the features of the probability distribution but will not match it exactly due to variability. As well, the model may not be correct due to bias such as an unfair coin or poor experimental technique or that independence is not a good assumption).<br>• Investigate how increasing the number of independent trials makes the frequency histogram better match the probability histogram.<br>• Generalize the algebraic representation of the binomial probability distribution.<br>• Calculate, interpret, and apply expected value.<br>• Compare the expected value of a binomial random variable to the mean of an experimental data set. | B1.4, B1.6, B1.1, B1.2, B1.3 |
| 8–9 | The Hypergeometric Distribution | • Recognize and identify situations resulting in a hypergeometric random variable.<br>• Calculate the probabilities associated with all values of a random variable.<br>• Compare hypergeometric probability histograms to frequency histograms of related experiments conducted previously in the course.<br>• Calculate, interpret, and apply expected value.<br>• Compare the expected value of a hypergeometric random variable to the mean of an experimental data set.<br>• Investigate how changing the probability of "success" affects the shape of the probability histogram. | B1.5, B1.6, B1.1, B1.2, B1.3 |
| 9–10 | Solve Problems | • Solve problems involving uniform, binomial, and hypergeometric distributions.<br>• Apply probability distributions to real-world situations.<br>• Solve problems related to the design of a game for the Games Fair project, (ensure that the expected value of the winnings falls within a given range). | B1.7, E1, E2 |
| 11-12 | Games Fair Project data collection and presentation | • Present games and collect data.<br>• Organize the data.<br>• Compare the probability model to the data gathered at the *Games Fair*.<br>• Interpret, analyse, and summarize the data.<br>• Communicate findings. | E1, E2 |
| 13 | Jazz | | |

# Unit 5: Modelling Continuous Data

Grade 12

## Lesson Outline

### Big Picture

Students will:

- describe the shapes of distributions of continuous data;
- extend the concept of a discrete probability distribution to a continuous probability distribution;
- understand the features of the normal distribution;
- apply normal distributions to real-world situations recognizing the role of variability.

| Day | Lesson Title | Math Learning Goals | Expectations |
|---|---|---|---|
| 1–2 | Look At Continuous Data | • Identify a continuous random variable.<br>• Distinguish between situations that result in discreet vs. continuous frequency distribution.<br>• Recognize standard deviation as a measure of the spread of a distribution.<br>• Determine the mean and standard deviation of a sample of values, with and without technology.<br>• Recognize the need for mathematical models to represent continuous frequency distributions.<br>• Use intervals to represent a sample of values of continuous random variables numerically (frequency table) and graphically (frequency histogram and polygon).<br>• Use technology to compare the effectiveness of the frequency polygon as an approximation of the frequency distribution.<br>• Recognize that the probability of a continuous random variable taking any *specific* value is zero. | B2.1, B2.2, B2.3, B2.4, B2.5 |
| 3–5 | Normal Distributions | • Recognize important features of a normally distributed data, e.g., bell-shaped, the percentages of data values within one, two, and three standard deviations of the mean.<br>• Recognize and describe situations that might be normally distributed.<br>• Investigate the conditions under which the shape of a binomial distribution approaches a normal distribution, i.e., as the number of trials increases and/or the probability of "success" gets closer to one-half.<br>• Investigate the conditions under which the shape of a hypergeometric distribution approaches a normal distribution, i.e., as the number of dependent trials increases and/or the probability of "success" gets closer to one-half.<br>• Use a discrete probability distribution to approximate the probability that a normal random variable takes on a specific range of values.<br>• Recognize that a continuous probability distribution is used to calculate the probability that a random variable takes on a range of values. | B2.5, B2.6, B2.7 |

| Day | Lesson Title | Math Learning Goals | Expectations |
|---|---|---|---|
| 6–7 | Probabilities In A Normal Distribution | • Define and calculate *z*-scores. | D1.2, B2.8 |
| 8–9 | Solving Problems Using The Normal Distribution | • Use the normal distribution to model one-variable data sets after determining that such a model might be suitable.<br>• Interpret, for a normally distributed population, the meaning of a statistic qualified by a statement describing the margin of error. Recognize that this is one way to account for variability.<br>• Solve probability problems involving normal distribution using a variety of tools and strategies.<br>• Apply normal distributions to real-world situations. | D1.4, B2.8 |
| 10–11 | Summative, Jazz | | |