Representing Vectors

Lesson Outline

BIG PICTURE

Students will:

- Introduce vectors in two-space and three-space
- Represent vectors geometrically and algebraically
- Determine vector operations and properties
- Solve problems involving vectors including those arising from real-world applications

Day	Lesson Title	Math Learning Goals	Expectations
1	*What's the Connection?*	• Explore connections between calculus and vectors	
2	*What's your Vector Victor?* *(Sample Lesson Included)*	• Represent vectors geometrically and algebraically in two-space. • Develop an understanding of equivalent vectors • Use geometric vectors to interpret information arising from real- world applications (Use applets described in Appendix A)	C1.1, 1.2
3	*Back and Forth with Vectors*	• Determine methods for changing from geometric (directed line segment) to algebraic (Cartesian) forms of a vector in two-space and vice versa.	C1.3
4	*Operating with Vectors*	• Add, subtract, and multiply vectors by a scalar in two-space, both geometrically and algebraically • Solve problems including problems arising from real-world applications involving vector operations in two-space	C2.1, 2.3
5	*The Dot Product*	• Determine the dot product of vectors in two-space geometrically and algebraically • Describe applications in two-space of the dot-product including projections	C2.4
6	*Jazz Day*	(Use applets described in Appendix A)	
7	*Summative Assessment*		
8	*Let's Go 3D*	• Represent both points and vectors algebraically in three-space • Determine the distance between points and the magnitude of vectors in three-space both geometrically and algebraically • Solve problems including problems arising from real-world applications involving vector operations in three-space	C1.4, 2.1, 2.3

9	*The Laws of Vectors*	• Investigate, with and without technology, the commutative, associative and distributive properties of the operations of addition, subtraction and multiplication by a scalar in two and three-space (Use Vector Laws applet described in Appendix A)	C2.2
10	*3D Dot Product*	• Determine the dot product of vectors in three-space geometrically and algebraically • Describe applications in three-space of the dot-product including projections	C2.4
11	*More on Dot Product*	• Determine through investigation the properties of dot product in two and three space	C2.5
12	*The Cross Product*	• Determine the cross product of vectors in three-space algebraically including magnitude and describe applications	C2.6
13	*More on Cross Product*	• Through investigation, determine properties of the cross product of vectors	C2.7
14	*Putting it All Together*	• Solve problems arising from real-world applications that involve the use of dot products, cross products, including projections	C2.8
15	*Jazz Day*		
16	*Unit Summative Assessment*		

Unit 5: Day 2: What's your Vector Victor?

MCV4U

Minds On: 10 Action: 45 Consolidate:20 Total=75 min	**Learning Goals**: • Represent vectors geometrically and algebraically in two-space. • Develop an understanding of equivalent vectors • Use geometric vectors to interpret information arising from real-world applications	**Materials** • Computer lab with GSP • Computer and data projector • BLM 5.2.1 • BLM 5.2.2

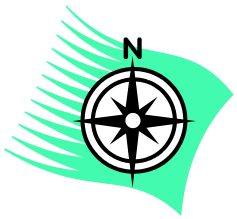
			Assessment Opportunities
	Minds On…	**Whole Class → Brainstorm** Brainstorm examples of real-world applications of vectors. **Whole Class → Discussion** Using GSP **Vector Basics.gsp** demonstrate vector basics and terminology.	Find additional vector applets and GSP files at the Ontario Educational Resource Bank ***http://www.elearningontario.ca/eng/Default.aspx*** See appendix A of the Course outlines for details
	Action!	**Whole Class –> Discussion** Draw examples of geometric vectors. Provide examples of vectors where direction is expressed in different ways (e.g. an object falls down, a plane flies on a heading of N20 ºW, the black car is 5 blocks east and 2 blocks north of the white car) **Pairs → Investigation** Using instructions on BLM 5.2.1 and GSP sketch **Gettothepoint.gsp**, students explore algebraic vectors. **Mathematical Process Focus:** Representing, Communicating	
	Consolidate Debrief	**Whole Class→ Discussion** Elicit responses from students in order to summarize properties of geometric and algebraic vectors. Highlight the differences in the way geometric and algebraic vectors are represented. **Curriculum Expectations/Presentation/Mental Note** Listen to student responses and assess their understanding of the learning goals of this lesson. **Pairs → Practice** Using GSP students complete exercises on BLM 5.2.2 to review properties of vectors.	
	Exploration Application	**Home Activity or Further Classroom Consolidation** Find other real-world examples of vectors. Complete extra practice questions as needed.	

5.2.1 Get to the Point – Investigating Algebraic Vectors

Instructions for Geometer's Sketchpad Vector Exploration

1. Open the GSP file *Gettothepoint.gsp*
2. Click on the Custom Tool, Arrowhead (closed) and construct a vector from the point A(-6, 1) to the point B(1, 5). Label the points.
3. Construct a horizontal line through A (select point A and the x-axis, **Construct, Parallel Line**) and a vertical line through B (select point B and the y-axis, **Construct, Parallel Line)**. Construct the point at the intersection of these two lines (select the two constructed lines, **Construct, Point at Intersection**). Label it C.
4. Determine the distances between points A and C and between points B and C (From the **Measure** menu choose **Coordinate Distance**). These distances are the magnitudes of the horizontal and vertical components respectively of vector $\overrightarrow{AB}$. Hide the constructed lines at this time (select each line, Display, Hide Lines).
5. What translation would be required to move point A to the origin? Apply the translation to the entire vector AB (Select all of the parts of the vector then, **Transform, Translate, By Rectangular Vector,** enter the horizontal and vertical components).
6. What are the new coordinates of the head, B' ?
7. Compare the coordinates of B' to the magnitudes of $\overrightarrow{AC}$ and $\overrightarrow{CB}$.
8. How do the directions of the vectors $\overrightarrow{AC}$ and $\overrightarrow{CB}$ compare to the signs of the coordinates of point B' ?
9. How do the horizontal and vertical components of the translated vector $\overrightarrow{A'C'}$ compare to the coordinates of point B' ? Comment on direction as well as magnitude. The vector A'B' is referred to as a **position vector**.
10. Test your conjectures by clicking on the head of the position vector and dragging it to locations in all four quadrants.
11. A vector that has been translated so that its tail is at the origin of the Cartesian plane is called a position vector. Position vectors have interesting characteristics that make them convenient to work with. For example, given the coordinates of any position vector, how would you find the magnitude of that vector?
12. Summarize the results of your investigation of these vectors.

5.2.2 Vector Practice



Compass bearings rotate clockwise from N. (0°)

1. Applications of Vectors:
Draw vectors to represent

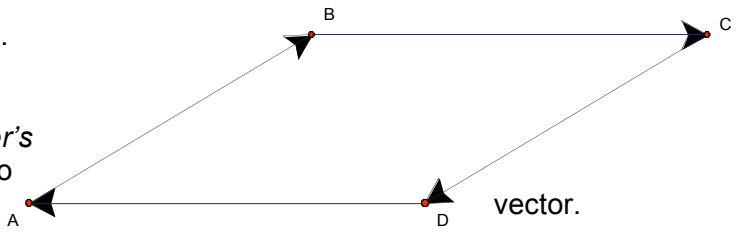
- A displacement of 60km Southeast.

- A weight of 35N acting vertically downward.

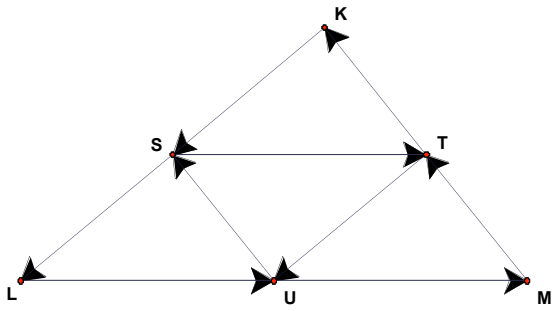
- A velocity of 150 m/s on a bearing of 188°

2. Applying Properties of Vectors:

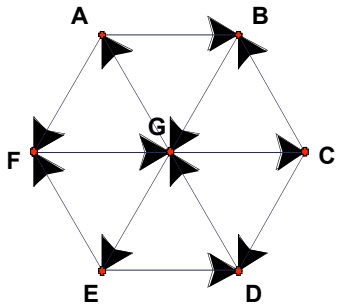
For quadrilateral ABCD:
a) Verify that ABCD is a parallelogram.
b) State 2 pairs of equal vectors.
c) State 2 pairs of opposite vectors.
d) If you have access to *The Geometer's Sketchpad*, move individual points to investigate the effect on each vector.



3. Use the geometric properties of vectors to list all pairs of equal vectors.



S, T, and U are midpoints of segments KL, KM, and LM.



ABCDEF is a regular hexagon.