Leading MATH Success

TIPS4RM Targeted Implementation and Planning Supports for Revised Mathematics

OAME/OMCA
GRADE 12 PROJECT
MCV 4U

SUMMER, 2007

Ontario Association for Mathematics Education

OMCA Ontario Mathematics Coordinators Association

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Grade 12 University Calculus and Vectors (MCV 4U)

Introduction

This package of materials has been created in response to the revised grade 12 mathematics curriculum to be implemented in September, 2007. The prepared lessons are not exhaustive, but rather were developed to give a flavour of the intended approach for this course. Attention was given to areas where there was a lack of resources, as well as to modelling how to bridge the understanding for students between the abstract and application. Teachers are encouraged to work together in school and board teams to develop lessons not included to extend their own learning as the writers in this project have done.

The following supports are included in this package.

- Course Overview based on the Ministry of Education version (order and timing of topics and summative assessments)
- Lessons and BLMs for 18 classes (identified as those which would most support teachers and complement the lessons developed under TIPS4RM)
- Unit summative performance tasks (available in the members area of OAME.on.ca and omca.ca
- Electronic Resources (GSP sketches, Fathom Data files, power point slide shows)

These resources are also posted on the OAME website: http://www.oame.on.ca:

Guiding Principles:

Writers and reviewers completed this resource package in order to:

- improve student success (model teaching considerations which support the profile of the learner)
- interweave and revisit the big idea of the course - conceptual understanding of derivatives, the use of vectors to introduce students to 3-D reasoning, the increase in time for development and consolidation of understanding
- emphasize problem solving and inquiry
- make the mathematical processes and literacy strategies explicit
- continue the use of TIPS4RM

Lesson Planning (Match Template)

The lessons and assessments have been created using the MATCH template from the TIPS4RM resource. The acronym MATCH is organized around a three part lesson, paying attention to:

Minds on getting students mentally engaged in the first few minutes of class
Action! the main portion of the lesson where students investigate new concepts
Consolidate/Debrief ideas for 'pulling out the math', and checking for understanding

Meaningful and appropriate follow-up to the lesson is provided in the Home Activity section.

The time allocation in the upper left corner suggests how much time should be devoted to each of the three parts of the lesson.

The materials section in the upper right corner identifies resources needed for the class.

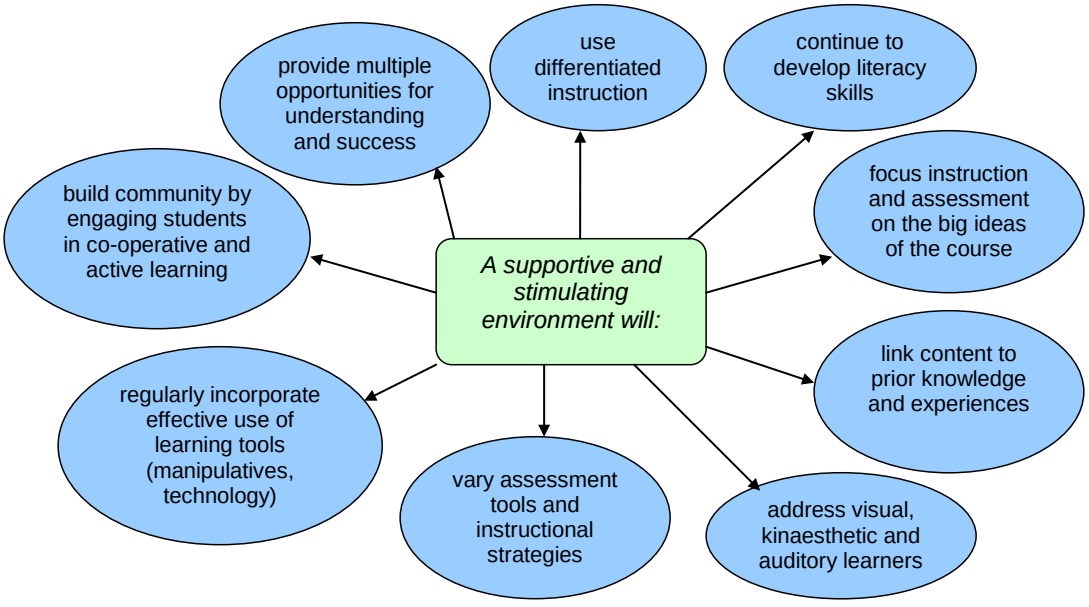
The right hand column offers Tips for teachers such as instructional strategies, references to resources, literacy strategies used, and explanations.

The narrow column to the left of this suggests opportunities for assessment.

For further details about this organizer go to http://www.curriculum.org/lms/ Field Code Changed

Teaching Considerations

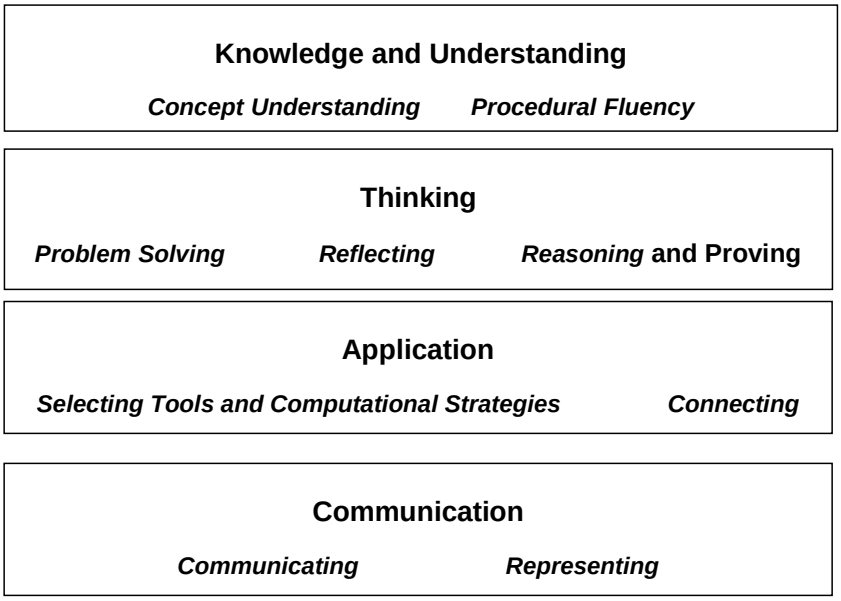
There are many considerations in the development of a positive learning environment which supports the grade 12 learner.



Processes

The seven mathematical processes can be referred to as the '***actions of math***.' In the revised curriculum, these process expectations have been highlighted in their importance since they support the acquisition and use of mathematical knowledge and skills. They can be mapped to three categories of the Achievement Chart – Thinking, Communication and Application. The fourth category, Knowledge and Understanding, connects to the overall and specific expectations of the course, which can be referred to as the '***mathematical concepts'***. Students apply the mathematical processes as they learn the content for the program.

The **combination** of the mathematical **processes** and **expectations** are **embedded** in the **achievement chart** as the following:



Students need multiple opportunities to engage in the processes. Many lessons included in this project highlight at least one process to be ~~developed .~~developed.

To assist students' development of these processes (instructional strategies, questions and feedback) see TIPS4RM Processes Package on the Leading Math Success website http://www.curriculum.org/lms/

Literacy Strategies

> Mathematics is the most difficult content area material to read because there are more concepts per word, per sentence, and per paragraph than in any other subject; the mixture of words, numerals, letters, symbols, and graphics requires the reader to shift from one type of vocabulary to another.
>
> Leading Math Success, Report of the Expert Panel for Mathematical Literacy Gr. 7 – 12

Improved student achievement demands an emphasis on developing literacy competencies linked to mathematics learning. To consolidate understanding, learners need opportunities to share their understanding both in oral as well as written form. Weakness in reading or writing skills provides barriers to success in problem solving. Many lessons utilize literacy strategies.

Starting points for teachers:

- Use strategies to develop vocabulary and comprehension skills, including
  - word walls
  - Frayer model
  - concept circles
- Use strategies relating to the organization of information
  - "inking your thinking" – having students write down their thoughts
  - concept maps
  - anticipation guides
- Use strategies to help students understand features of textbooks and graphics
  - read problems aloud
  - highlight key words
  - think aloud

More details and strategies can be found in Think Literacy: Cross-Curricular Approaches, Mathematics, Grades 10-12, 2005, http://www.curriculum.org/thinkliteracy/library.html

Assessment

The primary purpose of assessment and evaluation is to improve student learning. Information gathered through assessment helps to provide feedback to students as well as guiding teachers' instruction.

Assessment must be based on the four categories of the achievement chart and include the mathematical processes.

Assessment should be varied in nature. The chart below provides suggestions for a variety of assessment tools and the categories that they could be connected to.

Category	Assessment Tools
Knowledge and Understanding	Quiz, Test, Exam, Checkbric, Demonstration, Short Answer , True/False , Multiple Choice, Observation
Thinking	Editorials, Observations, Portfolio/Digital Portfolio, Essays, Articles, Debates, Report, Investigations, Graphic Organizers, Open-ended Questions, Performance Assessment Tasks, Video Tapes, Plays, Student /Teacher Conferences
Communication	Concept Map, Journals, Plays , Multi media presentations , Oral presentations , Drawings , Discussions, Explanations , Performance Task Assessment, Student/Teacher Conferences, Portfolio
Application	Concept Map, Debates, Editorials, Portfolio, Observation, Tests, Quizzes, Open-ended Questions, Design of Products, Models/Concrete Representations, Discussion

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Note-: This is by no means an exclusive or exhaustive list. It is only a guide.

Manipulatives and Technology

Many expectations in the revised curriculum make reference to using a variety of tools, including manipulatives, calculators and computer software. All new learning should begin with exploration and using learning tools whenever possible to provide students with representations of abstract mathematical ideas in varied, concrete, tactile, and visually rich ways.

> Information and communication technologies provide a range of tools that can significantly extend and enrich teachers' instructional strategies and support students' learning. Technology can reduce the time spent on routine mathematical tasks thus allowing students to devote more of their efforts to thinking and concept development.
>
> *The Ontario Curriculum, Grade 12 Mathematics, Revised, 2007*

The lessons and assessment written for this support document identify these learning tools. Teachers need to make arrangements to have these materials available and for computer lab booking at the beginning of the course. The use of these learning tools

should not be considered an extra to the instructional component of the course, nor should they be considered as only beneficial to a select few.

Online Resources

Ontario Resources

TIPS4RM, Leading Math Success and TIPS resources	http://www.edu.gov.on.ca/eng/studentsuccess/
Think Literacy Mathematics Grades 7 – 10	www.oame.on.ca/main/index1.php?lang=en&code=ThinkLit
Ontario Association for Mathematics Education	www.oame.on.ca
Statistics Canada	http://estat.statcan.ca/
Ontario Mathematics Coordinators Association	www.omca.ca

Learning Resources

Learning Math Series	www.learner.org
Math Forum	www.mathforum.org
NCTM	www.nctm.org
Regina University - Rich Math Tasks	http://mathcentral.uregina.ca
Rich Math Tasks - UK	www.nrich.maths.org.uk/

Virtual Manipulatives

National Library of Virtual Manipulatives	http://nlvm.usu.edu/en/nav/vlibrary.html

Lesson Outline

Day	Lesson Title	Math Learning Goals	Expectations
1	*Rates of Change Revisited (TIPS4RM Lesson)*	• Describe real-world applications of rate of change (e.g., flow) problems using verbal and graphical representations (e.g., business, heating, cooling, motion, currents, water pressure, population, environment, transportations) • Describe connections between average rate of change and slope of secant, and instantaneous rate of change and slope of tangent in context	A1.1, A1.2
2	*Determine Instantaneous Rate of Change using Technology (TIPS4RM Lesson)*	• With or without technology, determine approximations of and make connections between instantaneous rates of change as secant lines tends to the tangent line in context	A1.3
3	*Exploring the Concept of a Limit (TIPS4RM Lesson)*	• Explore the concept of a limit by investigating numerical and graphical examples and explain the reasoning involved • Explore the ratio of successive terms of sequences and series (use both divergent and convergent examples) (e.g., Explore the nature of a function that approaches an asymptote (horizontal and vertical)	A1.4
4,5	*Calculating an instantaneous rate of change using a numerical approach (TIPS4RM Lesson)*	• Connect average rate of change to $\frac{f(a+h)-f(a)}{h}$ and instantaneous rate of change to $\lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$	A1.5, A1.6
6,7	*Jazz / Summative Assessment*		

Note: *TIPS4RM Lesson* refers to a lesson developed by writing teams funded by the Ministry of Education. These lessons are not included with this package. They will be available at a later date. Details will be posted on the OAME web site. (www.oame.on.ca)

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Unit 2 Calculus and Vectors

Exploring Derivatives

Lesson Outline

Day	Lesson Title	Math Learning Goals	Expectations
1	*Key characteristics of instantaneous rates of change (TIPS4RM Lesson)*	• Determine intervals in order to identify increasing, decreasing, and zero rates of change using graphical and numerical representations of polynomial functions • Describe the behaviour of the instantaneous rate of change at and between local maxima and minima	A2.1
2	*Patterns in the Derivative of Polynomial Functions (TIPS4RM Lesson)*	• Use numerical and graphical representations to define and explore the derivative function of a polynomial function with technology, • Make connections between the graphs of the derivative function and the function	A2.2
3	*Derivatives of Polynomial Functions (Sample Lesson Included)*	• Determine, using limits, the algebraic representation of the derivative of polynomial functions at any point	A2.3
4	*Patterns in the Derivative of Sinusoidal Functions (Sample Lesson Included)*	• Use patterning and reasoning to investigate connections graphically and numerically between the graphs of *f(x) = sin(x)*, *f(x) = cos(x)*, and their derivatives using technology	A2.4
5	*Patterns in the Derivative of Exponential Functions (Sample Lesson Included)*	• Determine the graph of the derivative of f(x) = a^x using technology • Investigate connections between the graph of *f(x)* = a^x and its derivative using technology	A2.5
6	*Identify "e" (Sample Lesson Included)*	• Investigate connections between an exponential function whose graph is the same as its derivative using technology and recognize the significance of this result	A2.6
7	*Relating f(x)= ln(x) and* $f(x) = e^x$ *(Sample Lesson Included)*	• Make connections between the natural logarithm function and the function $f(x) = e^x$ • Make connections between the inverse relation of *f(x) = ln(x)* and $f(x) = e^x$	A2.7

Day	Lesson Title	Math Learning Goals	Expectations
8	*Verify derivatives of exponential functions (Sample Lesson Included)*	• Verify the derivative of the exponential function $f(x)=a^x$ is $f'(x)=a^x \ln a$ for various values of a, using technology	A2.8
9	Jazz Day / Summative Assessment *(Sample Assessment Included)*		
10, 11	*Power Rule*	• Verify the power rule for functions of the form $f(x) = x^n$ (where n is a natural number) • Verify the power rule applies to functions with rational exponents • Verify numerically and graphically, and read and interpret proofs involving limits, of the constant, constant multiple, sums, and difference rules	A3.1, A3.2 A3.4
12	*Solve Problems Involving The Power Rule*	• Determine the derivatives of polynomial functions algebraically, and use these to solve problems involving rates of change	A3.3
13, 14, 15	*Explore and Apply the Product Rule and the Chain Rule*	• Verify the chain rule and product rule • Solve problems involving the Product Rule and Chain Rule and develop algebraic facility where appropriate	A3.4 A3.5
16, 17	*Connections to Rational and Radical Functions (Sample Lessons Included)*	• Use the Product Rule and Chain Rule to determine derivatives of rational and radical functions • Solve problems involving rates of change for rational and radical functions and develop algebraic facility where appropriate	A3.4 A3.5
18, 19	Applications of Derivatives	• Pose and solve problems in context involving instantaneous rates of change	A3.5
20	*Jazz Day*		
21	*Summative Assessment*		

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Unit 3 **Calculus and Vectors**

Applying Properties of Derivatives

Lesson Outline

Day	Lesson Title	Math Learning Goals	Expectations
1, 2, 3,	*The Second Derivative (Sample Lessons Included)*	• Define the second derivative • Investigate using technology to connect the key properties of the second derivative to the first derivative and the original polynomial or rational function (increasing and decreasing intervals, local maximum and minimum, concavity and point of inflection) • Determine algebraically the equation of the second derivative $f''(x)$ of a polynomial or simple rational function $f(x)$, and make connections, through investigation using technology, between the key features of the graph of the function and those of the first and second derivatives	B1.1, B1.2, B1.3
4	*Curve Sketching from information*	• Describe key features of a polynomial function and sketch two or more possible graphs of a polynomial function given information from first and second derivatives – explain why multiple graphs are possible.	B1.4
5, 6	*Curve Sketching from an Equation*	• Extract information about a polynomial function from its equation, and from the first and second derivative to determine the key features of its graph • Organize the information about the key features to sketch the graph and use technology to verify.	B1.5
7	*Jazz Day*		
8	*Unit Summative (Sample Assessment Included)*		

Note: The assessment on day 8, and an assessment for a jazz day, is available from the member area of the OAME website and from the OMCA website (www.omca.ca).

Unit 4
Rate of Change Problems Calculus and Vectors

Lesson Outline:

Day	Lesson Title	Math Goals	Expectations
1, 2, 3	*Rate of Change Problems (Sample Lessons Included)*	• Make connections between the concept of motion and the concept of the derivative in a variety of ways. • Make connections between graphical and algebraic representations and real-world applications • Solve problems in wide variety of contexts and interpret the results	B2.1 B2.2, B2.3
4, 5, 6, 7, 8	*Optimization Problems*	• Solve a variety of optimization problems given an algebraic model • Solve a variety of optimization problems requiring the creation of an algebraic model	B2.4
9	*Solve problems from data*	• Solve problems arising from real-world applications by applying a mathematical model and the concepts and procedures associated with the derivative to determine mathematical results, and interpret and communicate results. • Revisit some of the rate of change and rate of flow problems from Unit 1	B2.5
10	*Jazz Day*		
11, 12, 13	*Summative Assessment for Units 3 – 4 (Sample Assessment Included)*		

Note: The assessment on day 11 is available from the member area of the OAME website and from the OMCA website (www.omca.ca).

Unit 5 Calculus and Vectors

Representing Vectors

Lesson Outline

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Students will:

- Introduce vectors in two-space and three-space
- Represent vectors geometrically and algebraically
- Determine vector operations and properties
- Solve problems involving vectors including those arising from real-world applications

Day	Lesson Title	Math Learning Goals	Expectations
1	*What's the Connection?*	• Explore connections between calculus and vectors	
2	*What's your Vector Victor? (Sample Lesson Included)*	• Represent vectors geometrically and algebraically in two-space. • Develop an understanding of equivalent vectors • Use geometric vectors to interpret information arising from real- world applications (Use applets described in Appendix A)	C1.1, 1.2
3	*Back and Forth with Vectors*	• Determine methods for changing from geometric (directed line segment) to algebraic (Cartesian) forms of a vector in two-space and vice versa.	C1.3
4	*Operating with Vectors*	• Add, subtract, and multiply vectors by a scalar in two-space, both geometrically and algebraically • Solve problems including problems arising from real-world applications involving vector operations in two-space	C2.1, 2.3
5	*The Dot Product*	• Determine the dot product of vectors in two-space geometrically and algebraically • Describe applications in two-space of the dot-product including projections	C2.4
6	*Jazz Day*	(Use applets described in Appendix A)	
7	*Summative Assessment*		
8	*Let's Go 3D*	• Represent both points and vectors algebraically in three-space • Determine the distance between points and the magnitude of vectors in three-space both geometrically and algebraically • Solve problems including problems arising from real-world applications involving vector operations in three-space	C1.4, 2.1, 2.3

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9	*The Laws of Vectors*	• Investigate, with and without technology, the commutative, associative and distributive properties of the operations of addition, subtraction and multiplication by a scalar in two and three-space (Use Vector Laws applet described in Appendix A)	C2.2	**Formatted:** Bullets and Numbering
10	*3D Dot Product*	• Determine the dot product of vectors in three-space geometrically and algebraically • Describe applications in three-space of the dot-product including projections	C2.4	**Formatted:** Bullets and Numbering
11	*More on Dot Product*	• Determine through investigation the properties of dot product in two and three space	C2.5	**Formatted:** Bullets and Numbering
12	*The Cross Product*	• Determine the cross product of vectors in three-space algebraically including magnitude and describe applications	C2.6	**Formatted:** Bullets and Numbering
13	*More on Cross Product*	• Through investigation, determine properties of the cross product of vectors	C2.7	**Formatted:** Bullets and Numbering
14	*Putting it All Together*	• Solve problems arising from real-world applications that involve the use of dot products, cross products, including projections	C2.8	**Formatted:** Bullets and Numbering
15	*Jazz Day*			
16	*Unit Summative Assessment*			

Unit 6 Calculus and Vectors

Representing Lines and Planes

Lesson Outline

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Students will:

- Represent lines and planes in a variety of forms and solve problems involving distances and intersections
- Determine different geometric configurations of lines and planes in three-space
- Investigate intersections of and distances between lines and/or planes

Day	Lesson Title	Math Learning Goals	Expectations
1	*Lines in Two-Space*	• Recognize that a linear equation in two-space forms a line and represent it geometrically and algebraically • Represent a line in two-space in a variety of forms (scalar, vector, parametric) and make connections between the forms	C3.1 C4.1
2	*Lines in Three-Space*	• Recognize that a line in three-space cannot be represented in scalar form • Represent a line in two-space in a variety of forms (vector and parametric) and make connections between the forms	C4.2
3	*Planes in Three-Space*	• Recognize that a linear equation in three-space forms a plane and represent it geometrically and algebraically • Determine through investigation geometric properties of planes including a normal to a plane • Determine using the properties of the plane the scalar, vector and parametric equations of a plane	C3.2 C4.3 C4.5
4	*Lines and Planes in Three-Space*	• Determine the equation of a plane in its scalar, vector or parametric form given another of these forms. • Represent a line in three-space by using the scalar equations of two intersecting planes	C4.6 C4.2
Refer to Smart Ideas file **Overview.ipr** for a flowchart of the concepts covered in lessons 5 through 10.			
5	*Lots of Lines (TIPS4RM Lesson)*	• Recognize that a linear equation in two-space forms a line represent it geometrically and algebraically • Recognize that the solution to a system of two linear equations in two-space determines a point in two space if the lines are not coincident or parallel • Solve and classify solutions to systems of equations in two-space in vector and parametric forms and understand the connections between the graphical and algebraic representations	C3.1 C4.1

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6	*Concrete Critters (TIPS4RM Lesson)*	• Determine through investigation different geometric configurations of combinations of up to three lines and/or planes in three space • Classify sets of lines and planes in three space that result in a common point, common line, common plane or no intersection	C3.3
7	*Interesting Intersections I (TIPS4RM Lesson)*	• Determine the intersections of two lines, and a line and a plane in three space given equations in various forms and understand the connections between the geometric and algebraic representations	C3.3 C4.7
8	*Interesting Intersections II (TIPS4RM Lesson)*	• Determine the intersections in three-space of 2 planes and 3 planes intersecting in a unique point given equations in various forms and understand the connections between the graphical and algebraic representations of the intersection	C3.3 C4.3 C4.4 C4.7
9	*Interesting Intersections III (TIPS4RM Lesson)*	• Determine the intersections of 3 planes in three space given equations in various forms and understand the connections between the graphical and algebraic representation of the intersection • Recognize that if $\vec{a} \bullet \vec{b} \times \vec{c} \neq 0$ is true then the three planes intersect at a point • Solve problems involving the intersection of lines and planes in three-space represented in a variety of ways	C4.4 C4.7
10	*How Far Can it Be? (TIPS4RM Lesson)*	• Calculate the distance in three-space between lines and planes with no intersection • Solve problems related to lines and planes in three-space that are represented in a variety of ways involving intersections	C3.3 C4.3 C4.7
11	*Jazz Day*		
12, 13, 14	*Summative Assessment for Units 5 and 6*		

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Appendix A: Electronic Learning Objects to Support MCV4U

Texas Instruments Nspire Activities

The Activities Exchange web site features three activities that could be used to enhance some of the lessons in units 1, 2 and 4. These activities are described below. The activities may be downloaded from http://education.ti.com/educationportal/activityexchange/activity_list.do?cid=us

Activity	ID	Associated Lesson	Handheld
The Tangent Line Problem	8315	Unit 1 Lesson 2	Nspire or Nspire CAS
Derivative Trace	8110	Unit 2 Lesson 2	Nspire or Nspire CAS
Falling Ladder	8318	Unit 4 Lesson 9	Nspire CAS

E-Learning Ontario Web Site: MGA4U Unit 3 Vectors
http://www.elearningontario.ca/eng/Default.aspx

Activity 2: Vector Laws

The last applet on Vector Laws allows the user to investigate the commutative, associative, distributive properties of 2–space vectors in geometric form.
At the bottom of activity 2 is a link to the University of Guelph's Physics department where a tutorial for vectors is provided.

Activity 3: Applications of Geometric Vectors

The second applet in the Velocity Java Applets allows the user to investigate the resultant vector for a boat crossing a river. The user controls 2-space vectors in geometric form for the boat's velocity and the current.

Activity 5: Algebraic Vectors

The first applet allows users to interactively explore the connections between geometric and algebraic forms of vectors in 2-space.

At the end of this activity is a link to a 3-space Graphing Tool that allows students to graph points, lines and planes in various forms.

Activity 6: Operations with Algebraic Vectors

There are four applets on addition of vectors, scalar multiplication, unit vectors and position vectors. They allow the user to interactively manipulate 2-space vectors.

E-Learning Ontario Web Site: MGA4U Unit 5 Vector Methods with Planes and Lines

Activity 1: Equations of Lines in 2-space

There are five guided and three interactive applets on forms of vector equations, how to convert between forms, distance from a point to a line.

Activity 3: Intersection of Lines

There are two guided applets on intersection of lines in 2-space and 3-space.

Activity 5: Equations of Planes

There are four guided applets on the forms of equations of planes and how to convert between forms.

Activity 6: Intersection of a Line and a Plane

There is one guided applet.

Activity 7: Intersection of Planes

There is one guided applet on solving systems of planes algebraically.

Activity 8: Task: X, Y & Z Factor

An open ended task using the 3-space Graphing Tool allows students to consolidate vector concepts.

Vector Applets on the Web

NCTM
http://standards.nctm.org/document/eexamples/chap7/7.1/index.htm

This site has two applets. The first illustrates the components of a vector to control a car. The user interactively controls the speed and direction. The second illustrates vector addition for an aircraft flying that is acted upon by wind. The user controls the speed and direction of both the aircraft and wind.

Syracuse University
http://physics.syr.edu/courses/java-suite/crosspro.html

This applet demonstrates cross product of two vectors in 3-space. It allows users to interactively change the vectors and see the resulting cross-product. The two vectors are limited to one plane but the plane can be moved to different viewing angles.

International Education Software
http://www.ies.co.jp/math/products/vector/menu.html

This Japanese site has a collection of applets that cover a wide variety of 2-space and 3-space vector topics. The controls are not very user-friendly but there are topics covered here like vector forms of lines in 2-space and 3-space that are not covered on other sites.

Professor Bob's Physics Lab (Rob Scott)
http://www.after4.ca/SchoolStuff/PhysicsLab/roomnojpgtest.html

This interactive site has flash applets on various Physics topics. Some topics such as Milliken and Momentum labs allow students to apply vector concepts.

B.Surendranath Reddy **(Physics Teacher in India)**
http://surendranath.org/Applets.html

This site has several applets that can be used in MCV4U. For vectors there are applets for addition, cross product of vectors, converting between Cartesian and directed line segment forms and several kinematics applets. For calculus there are applets for instantaneous speed and velocity.