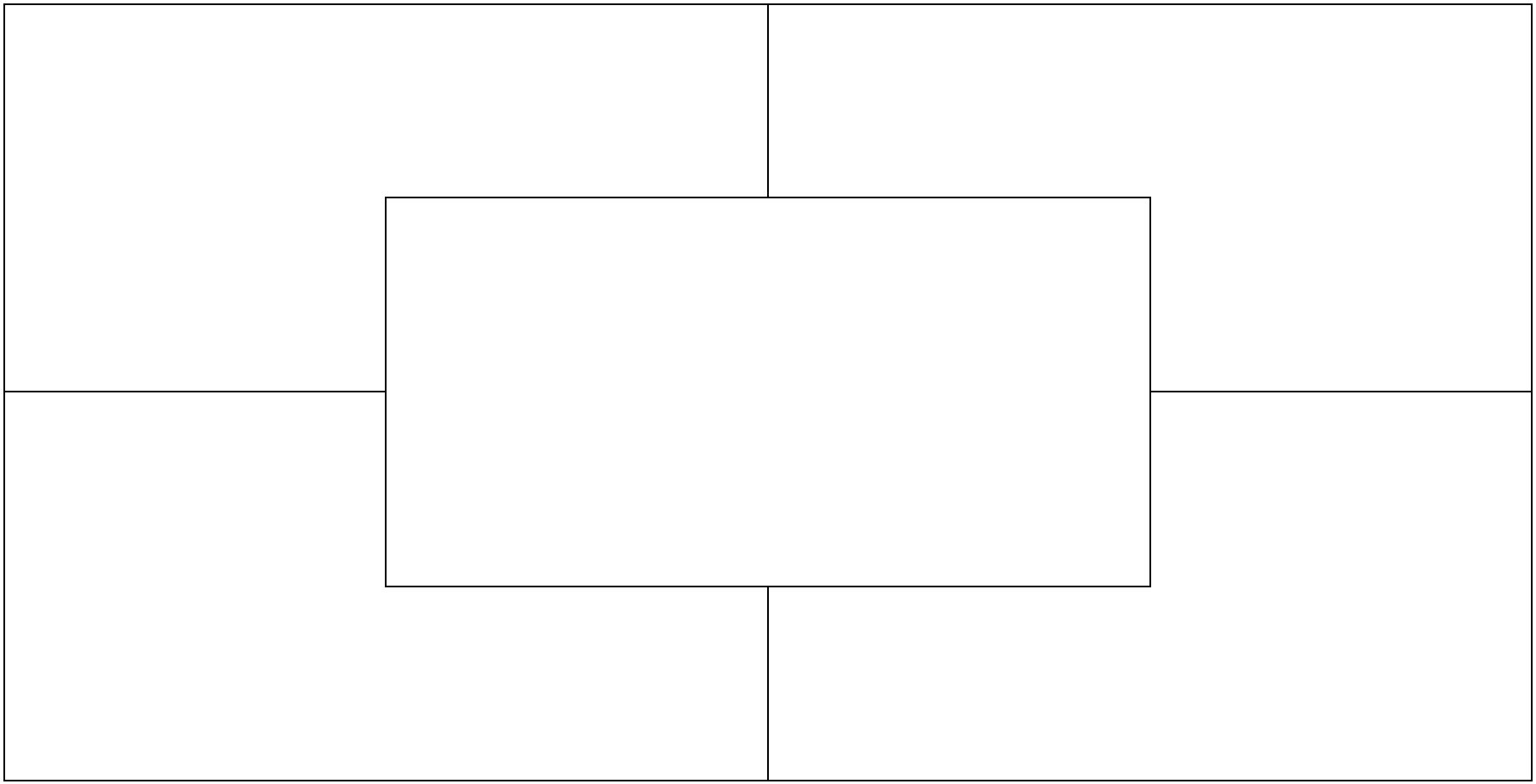
Unit 3: Day 2: Factoring Polynomial Expressions

MCT 4C

Minds On: 20 Action: 45 Consolidate:10 Total =75 min	**Learning Goals**: • Extend knowledge of factoring to factor cubic and quartic expressions that can be factored using common factoring, difference of squares, trinomial factoring and grouping.	**Materials** • BLM 3.2.1 – 3.2.5 • LCD projector or overhead projector (optional)

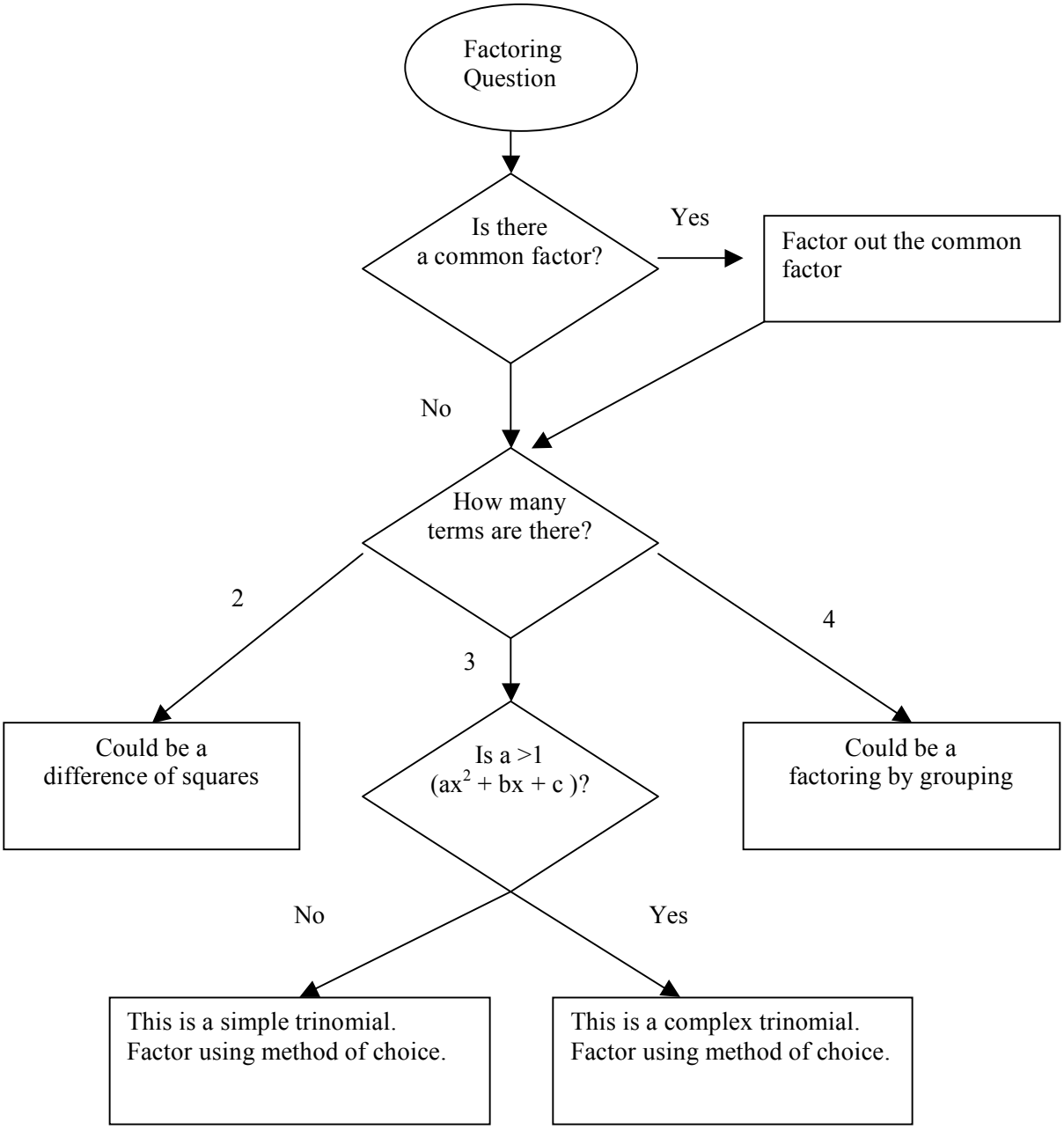
			Assessment Opportunities
Minds On…	**Groups of 4 → Placemat Activity** Students complete BLM 3.2.1 using their prior knowledge to write everything that they know about factoring. They should consider the following. 1. Defining the terms *factors* and *factoring*. 2. Providing examples of different methods of factoring and if possible, provide the names of those methods. 3. Indicating when factoring is useful. After students have completed their space on the placemat, groups will discuss their ideas and agree upon a response to be shared with the class, which is recorded in the centre box on the placemat. **Curriculum Expectations/Observation/Mental Note** Observe student responses and listen to student dialogue during the placemat activity. Assess student understanding of the curriculum expectations addressed to this point in the unit. **Whole Class → Discussion** One member of each group will be called upon to share their group's response with the whole class. Students should create a summary note from the discussion. If desired, the placemats can be posted in the classroom. **Mathematical Process Focus:** Reflecting - **Reflect** on prior knowledge of factoring Connecting - **Connect** prior factoring knowledge to factoring higher order polynomials		This activity will give a good overview of what the students remember about factoring, indicating the depth to which factoring needs to be reviewed as a class. More information about Placemats can be found in *Think Literacy*!
Action!	**Whole Class→ Discussion** Have students identify the thinking process involved in the factoring process. Students should identify the factoring strategies utilized and the characteristics of the polynomial that provide insight into the appropriate choice of strategies. BLM 3.2.2 provides an example of one way to organize this thinking process for your reference. Introduce factoring by grouping. Explain that sometimes questions involve more than one strategy in order to be fully factored. (There are examples of this in BLM 3.2.3, #6.) **Small Groups → Activity** Students practice these strategies as they complete BLM 3.2.3.		
Consolidate Debrief	**Partners→Activity** Using BLM 3.2.4, students match up edges of the small squares so that each polynomial is placed with its factored form in order to create a 4 by 4 square, using all of the small squares. The solution is given in BLM 3.2.4.		BLM 3.2.4 will have to be cut.
Concept Practice	**Home Activity or Further Classroom Consolidation** Complete BLM 3.2.5.		

3.2.1: Factoring: You must remember this!



3.2.2: Factoring Strategies (Teacher Notes)

Factoring is the process where a polynomial expression is written as a **product** of other algebraic expressions.



Factoring is used to solve polynomial equations and to graph polynomial functions. Factoring will be used in this unit to accomplish these tasks.

3.2.3: Let's Practice Factoring

1. Factor by common factoring.

a) $a^3b^2 + ab^3$

b) $25x^3 - 30x^5 + 35x$

c) $7x(x+2) - 5(x+2)$

2. Factor as a simple trinomial.

a) $t^2 + 3t - 10$

b) $x^2 - 10x - 24$

c) $x^2 - 8x + 16$

d) $x^4 + 6x^2 + 8$

3. Factor.

a) $3m^2 - m - 30$

b) $8m^2 - 5m - 3$

c) $7x^2 + x - 8$

3.2.3: Let's Practice Factoring (continued)

4. Factor as a difference of squares.

a) $y^2 - 81$

b) $9m^2 - 1$

c) $169x^2 - 144z^2$

5. Factor by grouping.

a) $a^2 - 2a + ad - 2d$

b) $x^4 - 3x^3 + 2x - 6$

c) $y^3 + y^2 + 2y + 2$

6. Factor fully. It might be necessary to use more than one factoring strategy in order to fully factor these polynomial expressions.

a) $x^3 - 3x^2 + 2x$

b) $2x^4 - 18x^2$

c) $x^3 - x^2 - 4x + 4$

3.2.4: Factoring Game

$2x^2 + 13x + 15$ $16x^2 - 36$	$(2x + 3)(x + 5)$ $5xy - 2x$	$x^2 + 12x - 45$ $(x + 15)(x - 3)$	$x^2 + 3x - 28$ $(x + 7)(x - 4)$
$4(2x + 3)(2x - 3)$ $15x^2 + 3x - 12$	$x(5y - 2)$ $3(5x - 4)(x + 1)$	$x^3 + 5x^2 + 6x$ $(x + 8)(x - 5)$	$2x^2 - 6x + 2$ $(3x - 1)^2$
$10x^2 + x - 3$ $(5x + 3)(2x - 1)$	$5xy - 2y$ $y(5x - 2)$	$x(x + 3)(x + 2)$ $(x - 8)(x + 5)$	$x^2 - 3x - 40$ $2(x^2 - 3x + 1)$
$3 + 4x - 15x^2$ $10x^2 - x - 3$	$(3 - 5x)(1 + 3x)$ $50x^2 - 72$	$15x^2 + 3x - 12$ $3(5x - 4)(x + 1)$	$15x^3 - 30x^2 + 10x$ $(3x + 1)^2$
$(5x - 3)(2x + 1)$ $5x^2 - 10x$	$2(5x - 6)(5x + 6)$ $5x(x - 2)$	$x^4 - 81$ $(x + 8)(x + 5)$	$3x(5x - 4)(x + 1)$ $9x^2 - 1$
		$(x^2 + 9)(x + 3)(x - 3)$	$x^4 + 13x^3 + 40x^2$ $(3x - 1)(3x + 1)$
			$x^2(x + 8)(x + 5)$

3.2.5: Factoring Fun

Determine which type of factoring is needed in each problem and then factor each expression. Remember that an expression might require more than one factoring strategy in order to be fully factored!

1. $x^3 - 2x^2$

2. $x^3 + 3x^2 - x - 3$

3. $3x^3 - 9x$

4. $3x^3 + x$

5. $-2x^3 + x^2 + 2x - 1$

6. $-2x^3 - 6x$

7. $x^4 - 3x^3$

8. $x^4 - 5x^2 + 4$

9. $x^4 - x^3 - 3x^2 + 3x$

10. $-x^4 + x^3 + 4x^2 - 4x$

11. $-x^4 + 4x^2$

12. $-x^4 + 5x^2 - 4$

Unit 3: Day 3: Cubic and Quartic Equations

MCT 4C

Minds On: 20 Action: 25 Consolidate:30 Total=75 min	**Learning Goal**: • Solve equations, of degree no higher than four, and verify the solutions using technology. • Solve cubic and quartic equations in standard form that can only be factored using common factoring, difference of squares, trinomial factoring and/or the quadratic formula.	**Materials** • PPT 3.3.1 • PPT 3.3.2 • BLM 3.3.1 • LCD projector • Graphing calculators

			Assessment Opportunities
	Minds On…	**Pairs → Discussion** Compare answers from the homework activity 3.2.5 from the previous lesson. **Whole Class → Discussion** Discuss any questions that pairs were not able to answer. **Whole Class → Presentation/Investigation** Present PPT 3.3.1 and work through the investigation together. Stress the point that to find when the height is zero, we would want to make $h = 0$. Discuss strategies employed in the past to solve for zeros (solving directly for linear equations; factoring; quadratic formula). Highlight the goal of this lesson which is to develop a strategy to solve equations of degree greater than two.	The teacher may want to a have a copy of BLM 3.2.5 on the overhead projector. Note: The connection between the roots or solutions to an equation and the zeros of a function should have been introduced in the first lesson of Unit 3.
	Action!	**Whole Class → Presentation** Observe, using PPT 3.3.2, the connection between *factoring expressions* and *solving equations*. By using graphing calculators during the presentation, the connection between *solving for the roots* and *what they represent will be established*. **Mathematical Process Focus:** Connecting – Students **connect** prior knowledge of factoring to solving for the roots of cubic and quartic functions in standard form. Students **connect** with the understanding that roots are x-intercepts.	Students will still need a graphing calculator for this portion of the lesson.
	Consolidate Debrief	**Individual → Activity** Complete BLM 3.3.1 **Curriculum Expectations/Performance Task/Checklist** Collect BLM 3.3.1 and assess student understanding. This information can inform planning for Day 4.	Portions of BLM 3.3.1 can be assessed for student learning.
	Exploration *Application*	**Home Activity or Further Classroom Consolidation** Decide how to solve the following equations: $x^3 - 2 = 0$ $x^4 - 16 = 0$ $x^4 + 1 = 0$ Try your strategy and solve for the roots, if possible.	

3.3.1: Solve & Graph it!

1. Solve.

a) $x^3 – x^2 – 2x = 0$

b) $0 = x^4 – 2x^3 – 5x^2 + 6x$

c) $0 = x^4 – 1$

d) $0 = 3x^2 + 12x + 12$

e) $0 = -x^3 + 2x^2 + 4x – 8$

f) $-x^4 – 4x^3 – 4x^2 = 0$

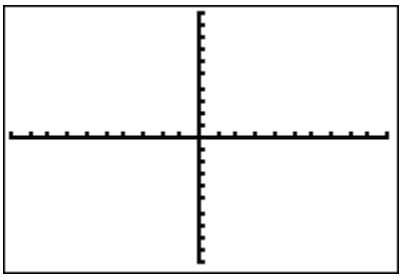
g) $x^3 – 9x^2 + 9x – 27 = 0$

h) $0 = x^4 – 5x^3 + 6x^2 + 4x – 8$

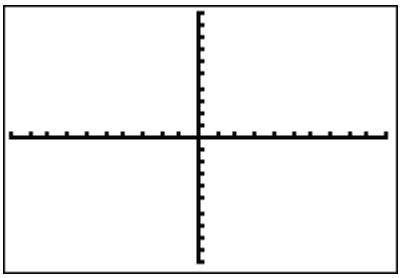
3.3.1: Solve & Graph it! (Continued)

2. Use your graphing calculator to sketch a graph of the functions below. Compare the roots from #1 with the sketches to determine if these roots and x-intercepts match.

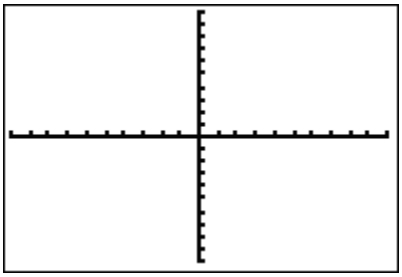
a) $y = x^3 - x^2 - 2x$



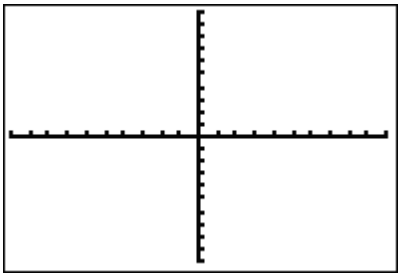
b) $y = x^4 - 2x^3 - 5x^2 + 6x$



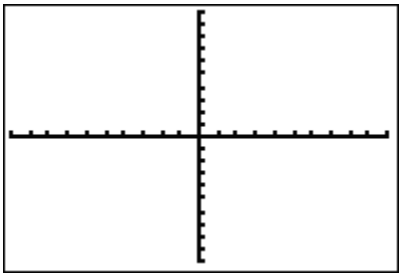
c) $y = x^4 - 1$



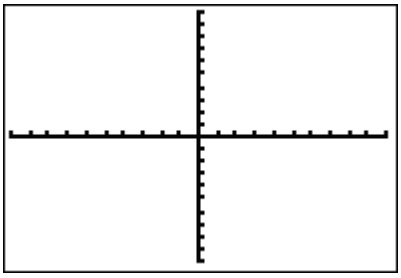
d) $y = 3x^2 + 12x + 12$



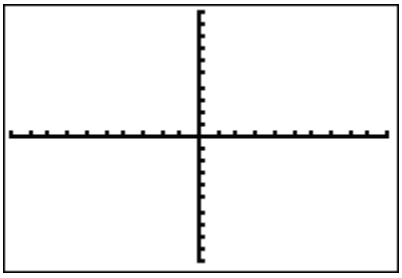
e) $y = -x^3 + 2x^2 + 4x - 8$



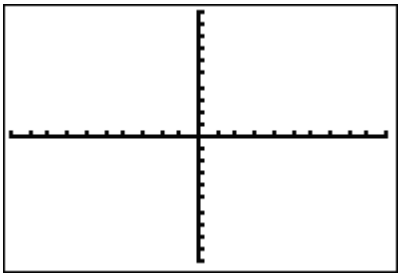
f) $y = -x^4 - 4x^3 - 4x^2$



g) $y = x^3 - 9x^2 + 9x - 27$



h) $y = x^4 - 5x^3 + 6x^2 + 4x - 8$



3.3.1: Solve & Graph it! (Continued)

3. Solve. Factor and use the quadratic formula where needed.

a) $x^4 + 4x^3 + 5x^2 = 0$

b) $0 = x^4 + x^2$

c) $-4x^3 + 10x^2 - 2x = 0$