Unit 5: Day 4: Maximizing Area for a Given Perimeter

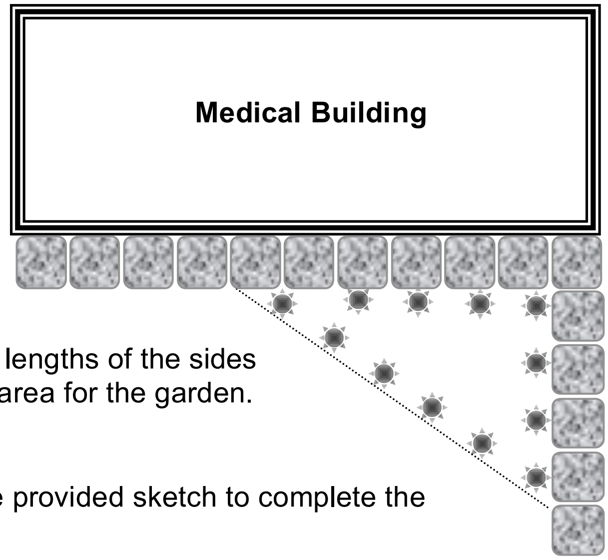
Minds On: 15 Action: 35 Consolidate:25 Total=75 min	**Learning Goal**: Students will • Explore optimization of two-dimensional figures • Solve problems to maximize area for a given perimeter	**Materials** • BLM 5.4.1 • BLM 5.4.2 • GSP Sketch • Chart Paper • Markers

Assessment Opportunities

Minds On…	**Placemat → Brainstorm and Discussion** Arrange students in groups of 4 and have them complete a placemat activity on different types of triangles and calculating the perimeter and area of triangles. This activity should be done on chart paper. The centre of the placemat should read **"Different Types of Triangles & Perimeter and Area"**. Give students 5 minutes to write individually and 5 minutes to summarize as a group in the centre of the placemat. Summarize the entire activity as a class on the board.Include Pythagorean Theorem in the class discussion. **Mathematical Process Focus: Reflecting –** Students will reflect upon their prior learning.		For more information on placemats, refer to the "Think Literacy – Mathematics, Grades 10-12" document, p. 66.
Action!	**Pairs → Exploration** Distribute BLM 5.4.1 and direct students to the GSP file on the computers (MAP_U5L4GSP1) to aid them in completing the investigation. Circulate to ensure students are maintaining a perimeter of 15 cm for their triangles. **Learning Skills(Teamwork/Initiative)/Observation/Anecdotal comments:**. Observe students and make anecdotal comments. **Mathematical Process Focus: Representing** - Students will **represent** the problem with diagrams to solve the problem.		If you do not have access to a computer lab, the activity could be done as a demonstration using a projector.
Consolidate Debrief	**Whole Group → Guided Instruction** Have students describe the triangle they created that had the maximum area (and conclude that it was a right-isosceles triangle). Ask students to help you determine the approx dimensions, using any method, of a garden with a 34' perimeter. Keep prompting students for 'another way' to solve this. Lead students through a general algebraic solution (refer to BLM 5.4.2) by asking about formulae that can be considered to make the 'guess and check' easier. Have students check to see if the general solution applies to the 34' perimeter as well as another given perimeter.		
Application	**Home Activity or Further Classroom Consolidation** For each of the given perimeters, find the lengths of the sides of the right triangle that will maximize the area. Be sure to show all of your work and provide a labelled sketch of each solution. Find the dimensions of 1 other triangle and show that your original triangle's area is maximized. 1. P = 32 inches 2. P = 151.3 cm 3. P = 5 yards **Content Expectations/Investigation/Rating Scale**: Collect the completed questions and assess for correctness.		

5.4.1: Maximizing a Triangle

A new medical center has just been built and it is ready for landscaping. The owners of the building want to put in a triangular planting area between the front sidewalk and the walk leading up to the building. They have enough solar lights to go around 15 m of garden. The lights will go around all three sides of the garden.



As the landscaper you need to determine the lengths of the sides of the triangle that will maximize the planting area for the garden.

Investigate

Open Geometer's SketchPad and use the provided sketch to complete the table on the back of the sheet as follows:

- Create at least 10 different triangles and record their information in the table. Remember, you are looking for the triangle with the greatest area.
- Include a sketch of each triangle in the last column. Your sketch does not have to be exact, but should be a good representation of the triangle (i.e. it should show which side is longer, a or b)

Make sure all of your triangles have a perimeter of 15 m.

Conclusions

When you have completed the chart, answer the following questions:

1. Which triangle had the greatest area?

2. Is there anything special about this triangle?

3. Give a hypothesis for how to maximize the area of any right triangle with a given perimeter.

5.4.1: Maximizing a Triangle (continued)

a	b	c	Perimeter	Area	Sketch
			15 m		
			15 m		
			15 m		
			15 m		
			15 m		
			15 m		
			15 m		
			15 m		
			15 m		
			15 m		

5.4.2 Teacher Notes for Guided Instruction

Using Pythagorean Theorem:

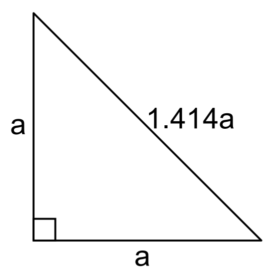
$$a^2 + b^2 = c^2$$

$$a^2 + a^2 = c^2$$

$$2a^2 = c^2$$

$$\sqrt{2}a = c$$

$$c = 1.414a$$



Therefore: Perimeter = a + a + 1.414a
P = 3.414a ← this formula will now work for all right isosceles triangles

So if P = 25m, then

25 = 3.414a
7.3m = a

c = 1.414 (7.3)
c = 10.32 m

Total area = 7.3 * 7.3/2
= 26.645 m^2

Unit 5: Day 9: Maximum Volume of a Triangular Prism

Minds On: 15	**Learning Goal**	**Materials**
Action: 30	• Investigate the relationship between dimensions of a triangular prism and surface area in order to maximize the volume of a triangular prism for a given surface area.	• BLM 5.9.1 - 3 • Tape, rulers, scissors, chart paper • Formula sheet for 2-D and 3-D measurements
Consolidate:30		
Total = 75 min		

Assessment Opportunities

Minds On…

Whole Group → Discussion

Review the ideas of maximizing area learned earlier in the unit and extend this to a discussion about optimal volume given a fixed surface area for a 3-D figure. Remind students of maximizing the volume for a rectangular prism (e.g., cube).

Students make a hypothesis about the optimal dimensions of a triangular prism. Students discuss the choice of equilateral triangular prism vs. right isosceles triangular prism. Which is more useful? Which has more optimal dimensions?

Provide a formula sheet which includes formulae for the volume and surface area for a triangular prism.

Mathematical Process Focus: Students will **connect** minimum perimeter to minimum surface area, and the purpose of this in a packaging context.

Action!

Groups → Six Corners Investigation

Show the students the nets of the six different triangular prisms. Put a stack of each net at six different stations in the room (a different net at each station). Have the students form groups at the station with the net that they think will produce the greatest volume.

In their groups, students should cut out the net, build the triangular prism, measure all necessary dimensions and then calculate the surface area and the volume of the prism. Once completed each student in the group should then go to one other station and complete the same steps for the new net in order to prove (or disprove) their conjecture. There must be at least one group member at each of the new stations. Students should then return to their original group and summarize their findings using chart paper and display their prisms.

Content Expectations/Observations/Comments: Check for understanding and give students verbal feedback on their work.

Mathematical Process Focus: Reasoning & Proving – Students will prove/disprove their conjecture by testing other nets and comparing results.

When copying BLM 5.9.1 make sure that you have enough of each in case many students select the same net.

Ask students to think of examples with right triangular prisms used in real life.

Consolidate Debrief

Whole Group → Discussion

Discuss the general dimensions of the prisms that created large volumes vs. the dimensions of the prisms that created small volumes. Was one of those prisms necessarily the 'best' possible? Could we have done better?Discuss how using a right angled triangular prism might be more useful.

Show students how to calculate the height of a prism if the surface area and length of a leg of the triangle are known This will prepare students for completing the assignment on BLM 5.9.3. See BLM 5.9.2 for the derivation.

Mathematical Process Focus: Students **represent** the different dimensions using a table and by using a graph.

You can either show students the derivation in general or for a specific numerical example.

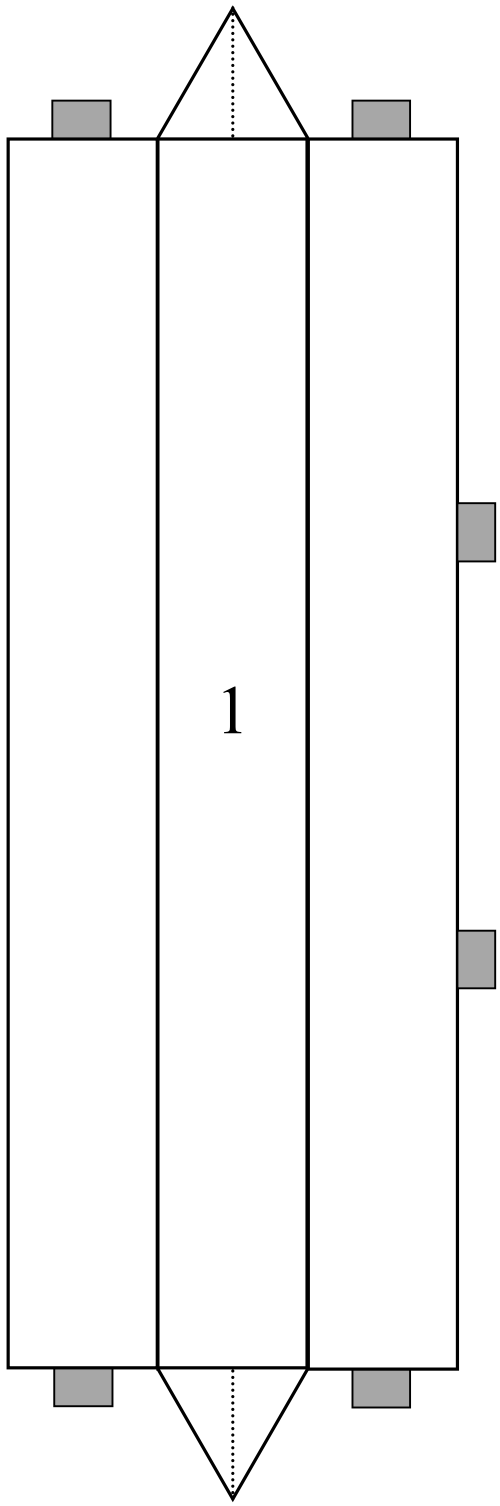
Exploration
Application

Home Activity or Further Classroom Consolidation

Complete BLM 5.9.3.

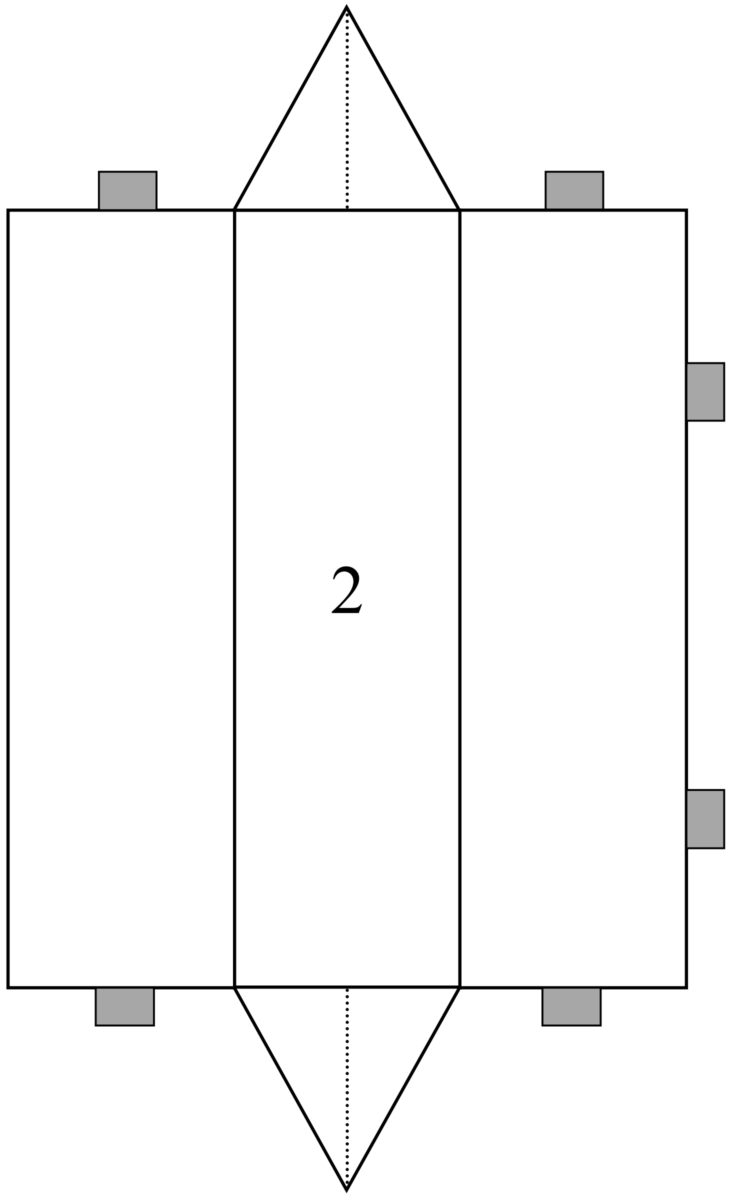
5.9.1: Triangular Prism Pattern MAP4C

Use the following pattern to create an equilateral triangle based prism. Measure the necessary dimensions (in cm, to one decimal place) to calculate the surface area and the volume of the prism.



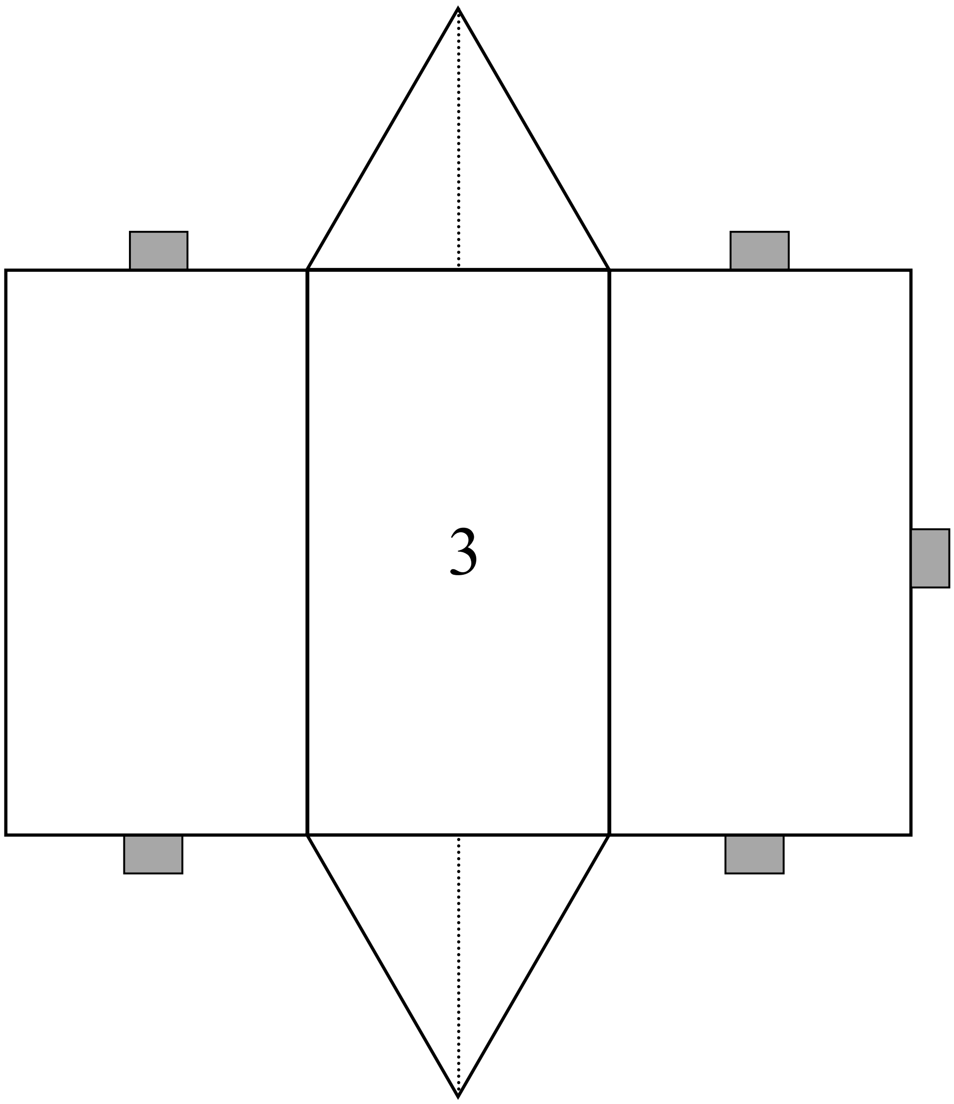
5.9.1: Triangular Prism Pattern (continued) MAP4C

Use the following pattern to create an equilateral triangle based prism. Measure the necessary dimensions (in cm, to one decimal place) to calculate the surface area and the volume of the prism.



5.9.1: Triangular Prism Pattern (continued) MAP4C

Use the following pattern to create an equilateral triangle based prism. Measure the necessary dimensions (in cm, to one decimal place) to calculate the surface area and the volume of the prism.



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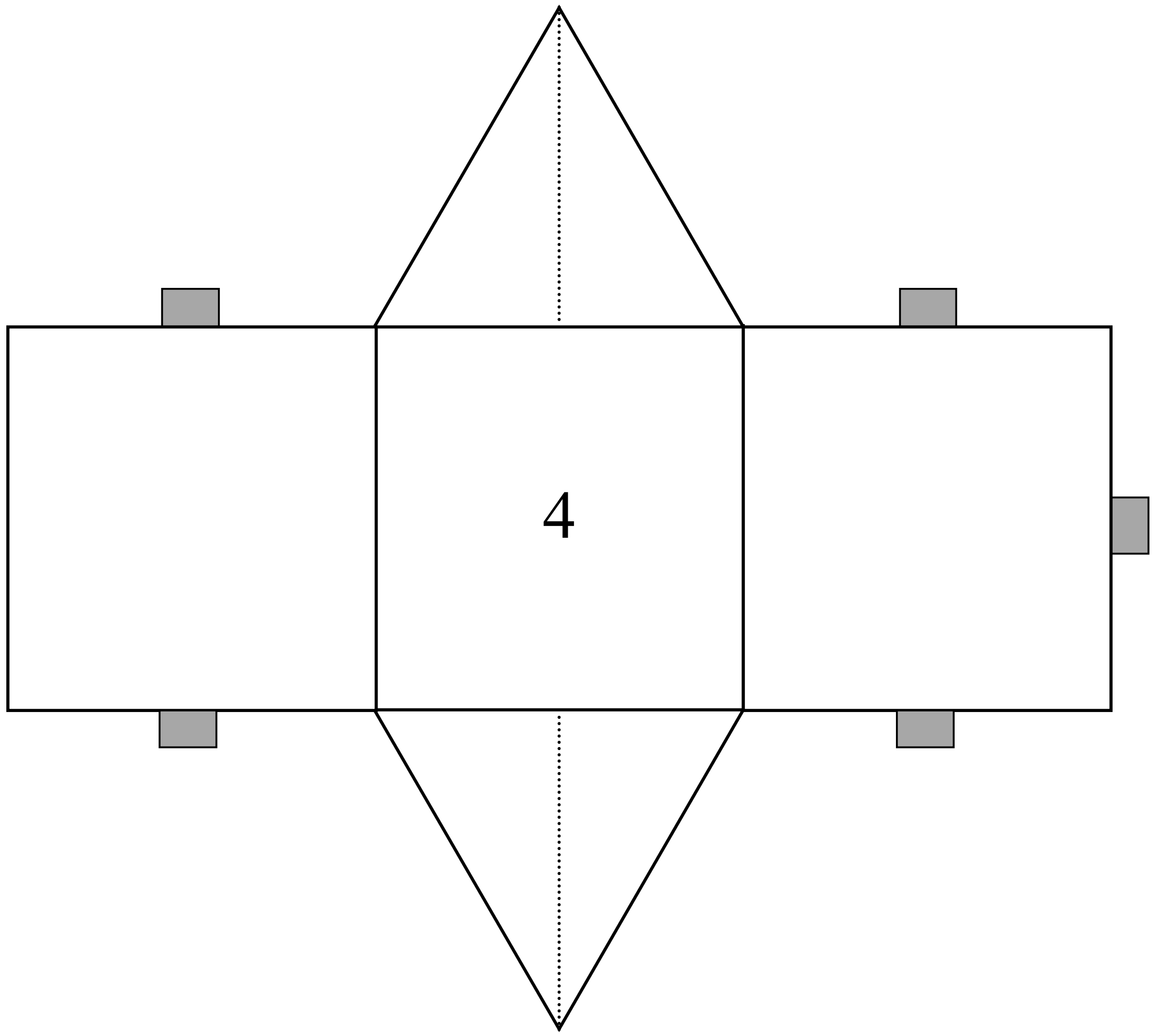
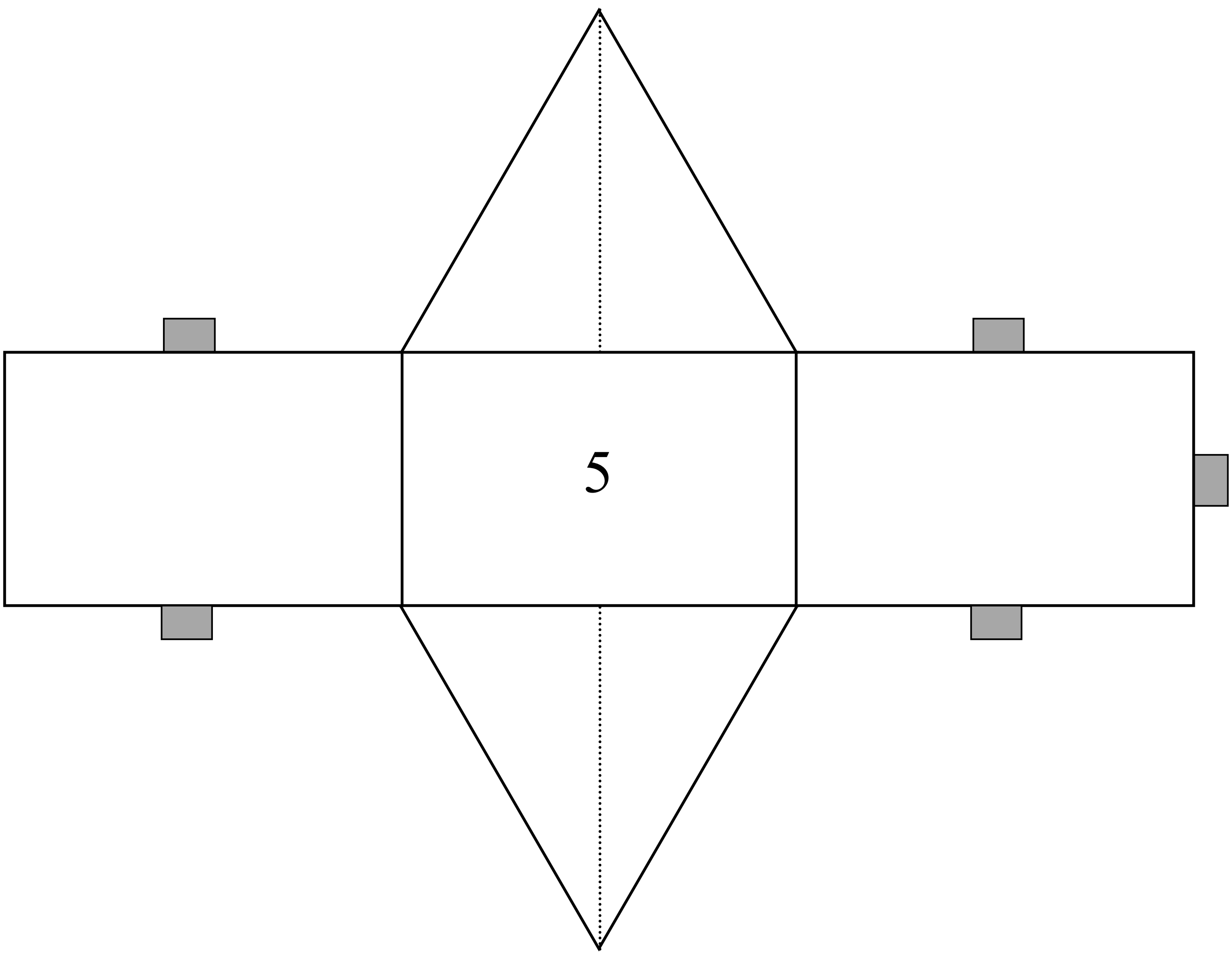


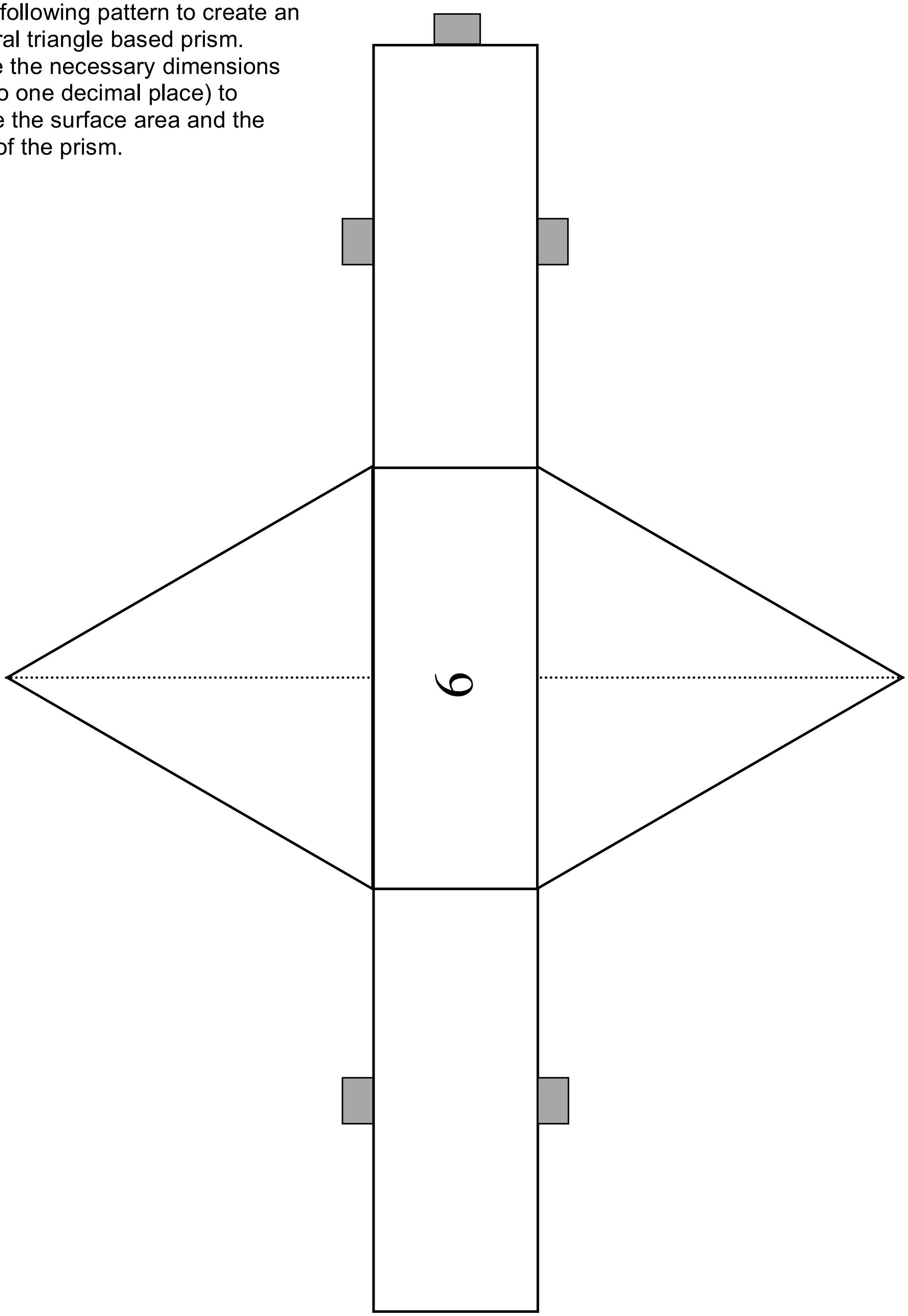
5.9.1: Triangular Prism Pattern (continued) MAP4C

Use the following pattern to create an equilateral triangle based prism. Measure the necessary dimensions (in cm, to one decimal place) to calculate the surface area and the volume of the prism.



5.9.1: Triangular Prism Pattern (continued) MAP4C

Use the following pattern to create an equilateral triangle based prism. Measure the necessary dimensions (in cm, to one decimal place) to calculate the surface area and the volume of the prism.



5.9.1: Triangular Prism Pattern (Solutions) MAP4C

These are the dimensions that should have been measured for each triangular prism.

1. S.A. = 100
   s = 2
   h = 16.09
   V = 27.87

2. S.A. = 100
   s = 3
   h = 10.24
   V = 39.93

3. S.A. = 100
   s = 4
   h = 7.18
   V = 49.74

4. S.A. = 100
   s = 5
   h = 5.22
   V = 56.54

5. S.A. = 100
   s = 6
   h = 3.82
   V = 59.60

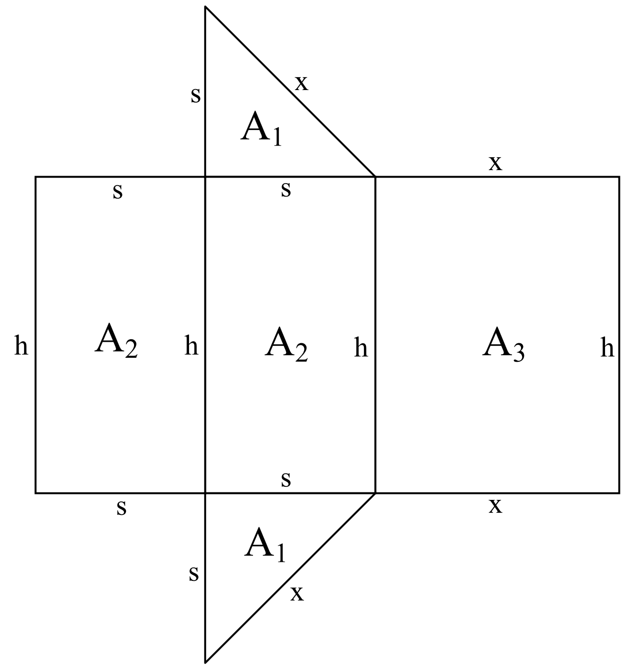
6. S.A. = 100
   s = 7
   h = 2.74
   V = 58.16

Optimal Dimensions:

S.A. = 100
s = 6.20
h = 2.90
V = 59.70

5.9.2: Height Derivation MAP4C

The following shows how to determine the height of a right angled, isosceles triangle based prism. This is how the hint in BLM 5.9.3 was determined. It may be worthwhile to show the students this derivation or perhaps show it for a particular numerical example.



Area 1: $A_1 = \dfrac{s^2}{2}$

Area 2: $A_2 = sh$

Area 3:

$$x^2 = s^2 + s^2$$
$$x^2 = 2s^2$$
$$x = \sqrt{2s^2}$$
$$x = 1.414s$$

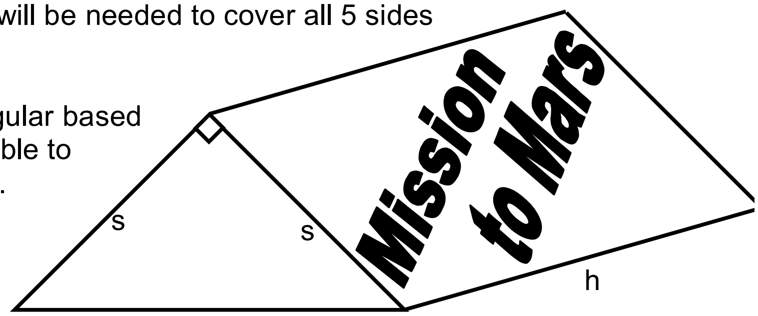
$$A_3 = xh$$
$$A_3 = 1.414sh$$

Total Surface Area

$$S.A. = 2A_1 + 2A_2 + A_3$$
$$S.A. = 2\left(\frac{s^2}{2}\right) + 2(sh) + (1.414sh)$$
$$S.A. = s^2 + 2sh + 1.414sh$$
$$S.A. = s^2 + 3.414sh$$
$$200 = s^2 + 3.414sh \qquad \leftarrow \text{Total Surface Area} = 200 \text{ yd}^2$$
$$200 - s^2 = 3.414sh$$
$$\frac{200 - s^2}{3.414s} = h$$

5.9.3: Mission to Mars MAP4C

NASA is designing a tent structure that can be used as a habitat for a possible settlement on Mars. The tent structure will allow for winds to pass easily over the habitat but should also allow for a suitable amount of space inside the habitat. The material required for the tent is expensive, so NASA has determined that the tent will be made out of 200 square yards of material, which will be needed to cover all 5 sides of the structure.



The tent is going to be designed as a triangular based prism as shown. Complete the following table to determine the maximum volume of the tent.

Graph the results (side length vs. volume) on the grid on the back of this page.

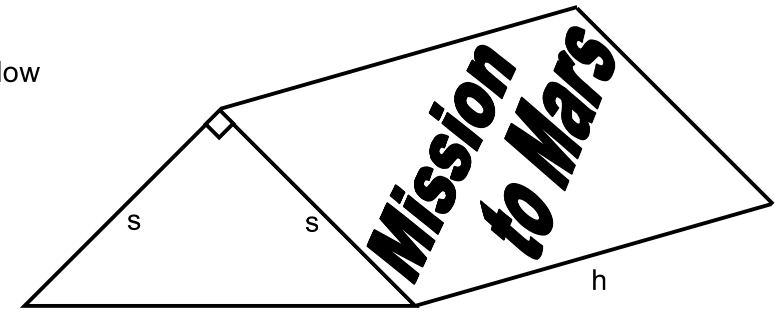
(Round your answers to two decimal places)

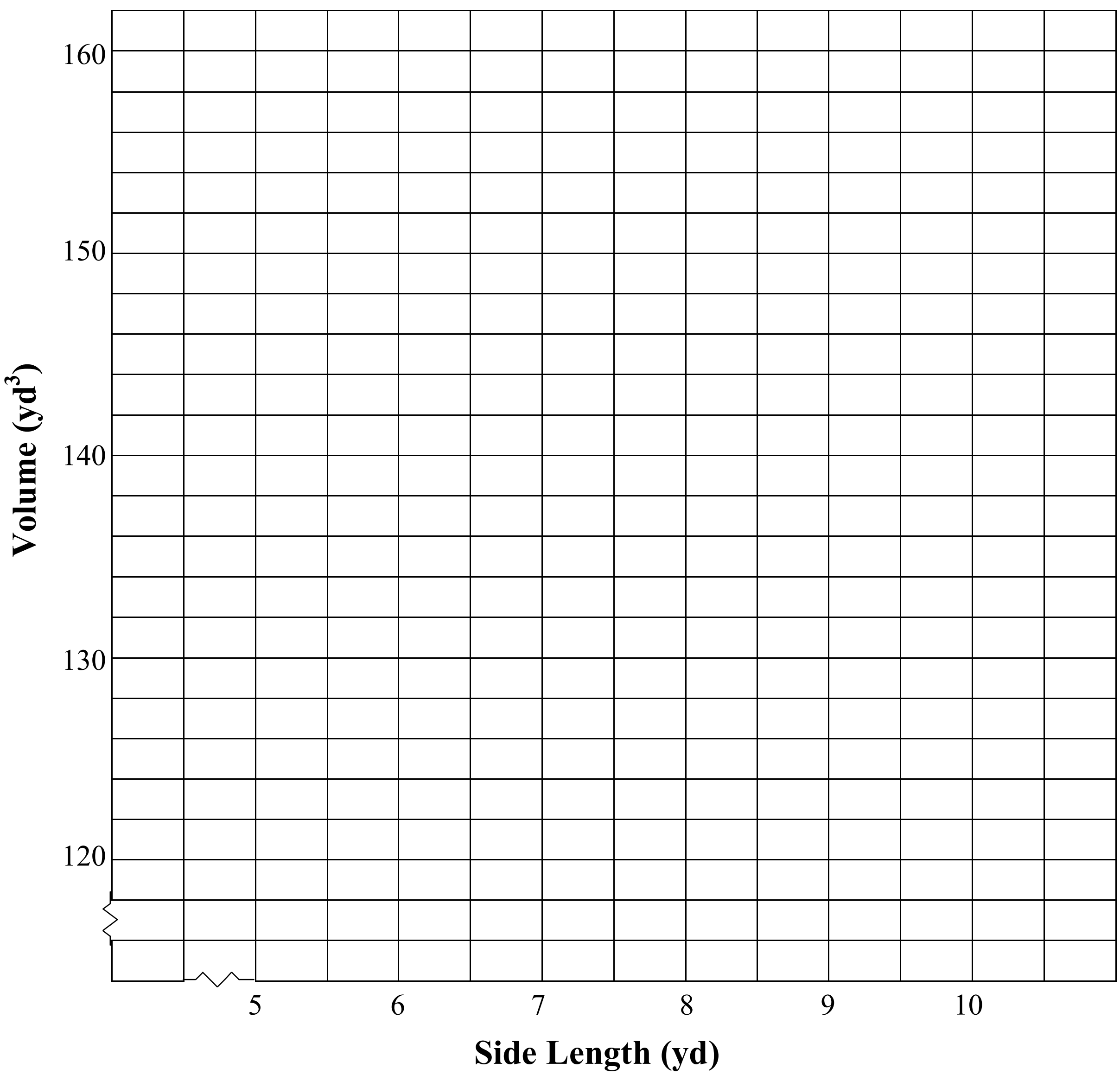
Hint:
Calculate h, using the formula: $h = \dfrac{200 - s^2}{3.414s}$

s (in yards)	**h** (in _______)	**Volume** (in ______)
5		
6		
7		
8		
9		
10		

5.9.3: Mission to Mars (continued) MAP4C

Graph your results on the grid below and draw a curve of best fit.





Optimal Dimensions: ________________________________

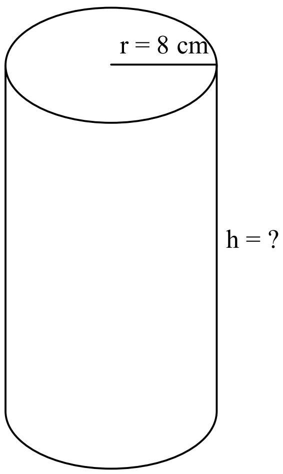
Unit 5: Day 10: Minimum Surface Area of a Cylinder

Minds On: 15 Action: 30 Consolidate:30 Total = 75 min	**Learning Goal**: • Investigate the relationship of dimensions in order to minimize the surface area of a cylinder for a given volume.	**Materials** • BLM 5.10.1 • BLM 5.10.2 • A variety of cans • Rulers or tape measures

Assessment Opportunities

Minds On…	**Pairs → Think, Pair, Share** Show the class a variety of different containers that are cylinders (eg. cans of soup, stew, tuna, fruit, apple juice → use a variety of sizes). Students should think about what the company was considering when they decided on the dimensions of the can (eg. easy size to hold, large for advertising, short so they are easy to stack, minimize cost of production etc.). Students should share their ideas with their partner and then discuss as a class some of the ideas. **Whole Group → Discussion** Review ideas of minimizing perimeter, learned earlier in the unit. Link this to trying to minimize the surface area of a cylinder in order to minimize the cost of production. It may be necessary to review how to solve for height in a cylinder given the volume and radius. Eg. Have students complete the following question using Think/Pair/Share. "Determine the height of a cylinder that has a radius of 5cm and a volume of 500mL." **Mathematical Process Focus: -** Students will **connect** the minimum perimeter to minimum surface area, and the purpose of this in a packaging context.	Literacy Strategy used: Think, Pair, Share Copy lesson cards onto cardstock to give them a more official look – and allow for reuse.
Action!	**Groups of 4 → Investigation** Students will work in groups of 4 to complete the problem cards found on BLM 5.10.1 to investigate the relationship between height and radius in a cylinder. Each group should be given a different set of problem cards, and after calculating the surface area for each card, they should order them from smallest to largest surface area. Students discuss and determine the relationship between height and radius for their cylinder with the smallest surface area. **Learning Skill (Teamwork)/Observation/Checkbric:** Observe and record students' collaboration skills. **Mathematical Process Focus: Reasoning –** Students discover how adjusting the radius and height in a cylinder will affect the surface area.	Use π = 3.14 in order to get dimensions that are whole numbers. Differentiate instruction by giving problems 3 and 4 to stronger groups, since they must solve for r, instead of h. A chart on the black board or chart paper would be helpful to organize the information.
Consolidate Debrief	**Whole Group → Discussion** Have each group write the volume, radius, height and surface area of their cylinder with minimum surface area on the board. As a class, discuss the similarities in all of the cylinders. Focus should be on h = 2r. **Groups of 4 → Investigation** Give each group one of the cylinders that you originally discussed in the Minds On portion of the lesson. Have the groups measure the height and radius. Determine if any of the cylinders have the optimal dimensions or if any have close to optimal dimensions. Engage students in a discussion as to why some companies choose not to use optimal dimensions for their products.	
Exploration *Application*	**Home Activity or Further Classroom Consolidation** 1. Complete BLM 5.10.2 2. Reflect on the activities of the last 2 days. Is there a common connection between the 3 shapes and their optimal dimensions? Write a journal entry to present your conclusions	Students may need to be reminded of the 3 shapes – rectangular prism (in Minds On), triangular prism, and cylinder.

Problem 1 - Cards

<table>
<tr><td colspan="2">Problem 1 – Card 1</td></tr>
<tr><td rowspan="4">Volume = 10 851.84 cm^3

</td><td>Volume = _________</td></tr>
<tr><td>Radius = _________</td></tr>
<tr><td>Height = _________
Calculations:</td></tr>
<tr><td>Surface Area = _________
Calculations:</td></tr>
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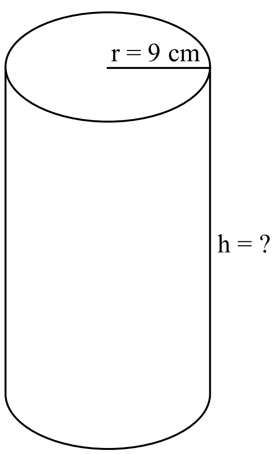
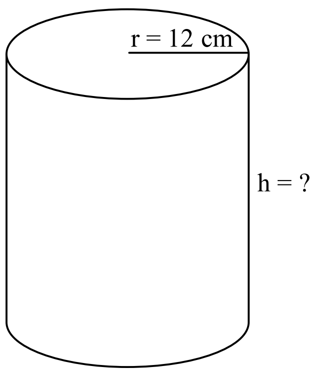
<table>
<tr><td colspan="2">Problem 1 – Card 2</td></tr>
<tr><td rowspan="4">Volume = 10 851.84 cm^3

</td><td>Volume = _________</td></tr>
<tr><td>Radius = _________</td></tr>
<tr><td>Height = _________
Calculations:</td></tr>
<tr><td>Surface Area = _________
Calculations:</td></tr>
</table>

Problem 1 – Card 3

Volume = 10 851.84 cm^3



Volume = _________

Radius = _________

Height = _________

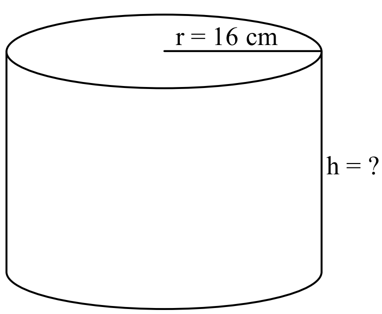
Calculations:

Surface Area = _________

Calculations:

Problem 1 – Card 4

Volume = 10 851.84 cm^3



Volume = _________

Radius = _________

Height = _________

Calculations:

Surface Area = _________

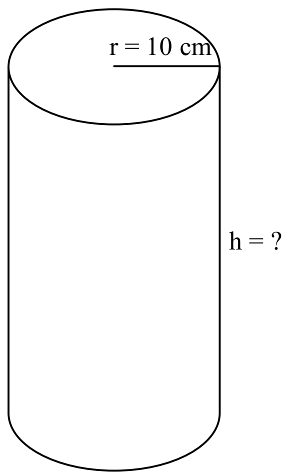
Calculations:

5.10.1: Problem Cards (continued) MAP4C

Problem 2 - Cards

Problem 2 – Card 1

Volume = 21 195 cm^3



Volume = ________

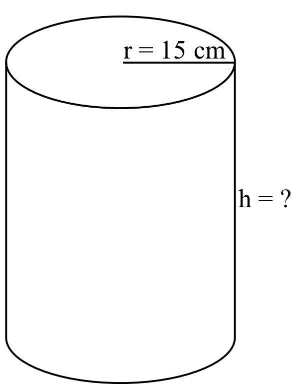
Radius = ________

Height = ________
Calculations:

Surface Area = ________
Calculations:

Problem 2 – Card 2

Volume = 21 195 cm^3



Volume = ________

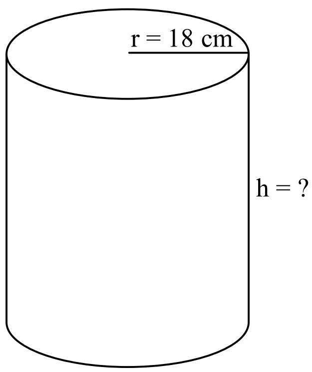
Radius = ________

Height = ________
Calculations:

Surface Area = ________
Calculations:

Problem 2 – Card 3

Volume = 21 195 cm^3



Volume = _________

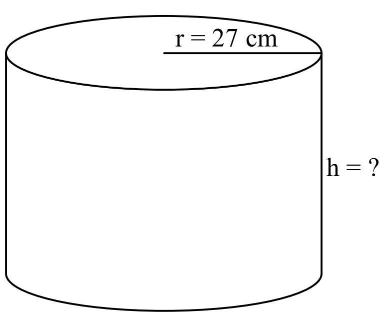
Radius = _________

Height = _________
Calculations:

Surface Area = _________
Calculations:

Problem 2 – Card 4

Volume = 21 195 cm^3



Volume = _________

Radius = _________

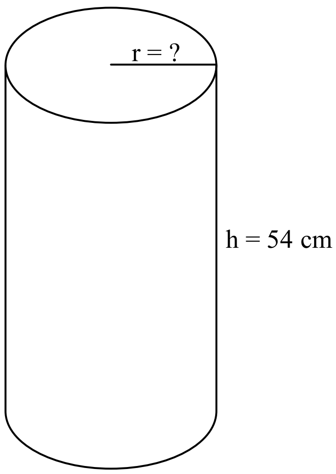
Height = _________
Calculations:

Surface Area = _________
Calculations:

Problem 3 - Cards

Problem 3 – Card 1

Volume = 36 624.96 cm^3



Volume = _________

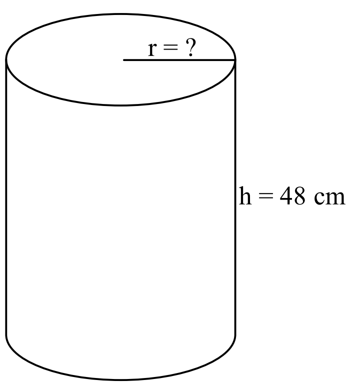
Height = _________

Radius = _________
Calculations:

Surface Area = _________
Calculations:

Problem 3 – Card 2

Volume = 36 624.96 cm^3



Volume = _________

Height = _________

Radius = _________
Calculations:

Surface Area = _________
Calculations:

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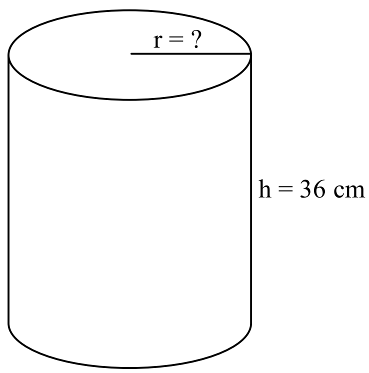</td><td>Volume = _________</td></tr>
<tr><td>Height = _________</td></tr>
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Calculations:</td></tr>
<tr><td>Surface Area = _________
Calculations:</td></tr>
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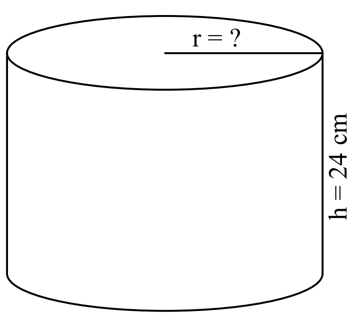
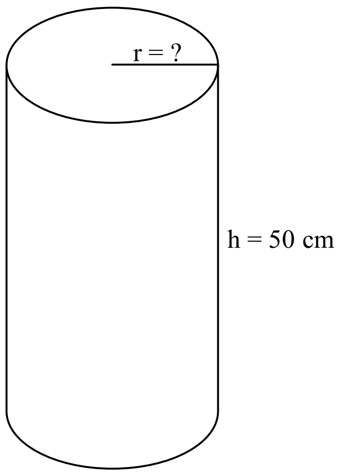
<table>
<tr><td colspan="2">Problem 3 – Card 4</td></tr>
<tr><td rowspan="4">Volume = 36 624.96 cm^3
</td><td>Volume = _________</td></tr>
<tr><td>Height = _________</td></tr>
<tr><td>Radius = _________
Calculations:</td></tr>
<tr><td>Surface Area = _________
Calculations:</td></tr>
</table>

Problem 4 - Cards

Problem 4 – Card 1

Volume = 50 240 cm^3



Volume = ________

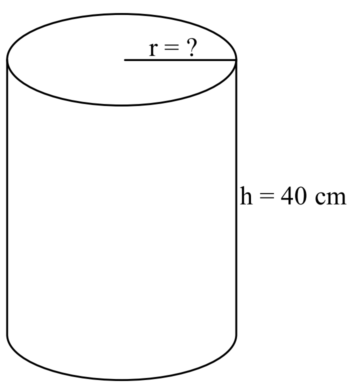
Height = ________

Radius = ________
Calculations:

Surface Area = ________
Calculations:

Problem 4 – Card 2

Volume = 50 240 cm^3

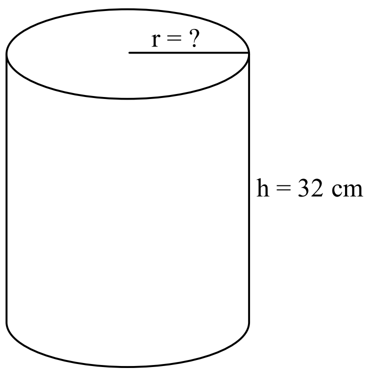


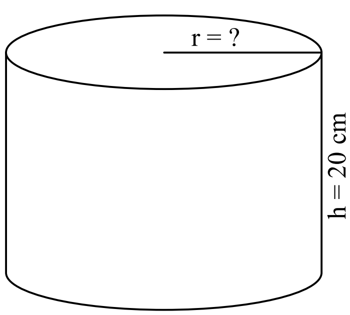
Volume = ________

Height = ________

Radius = ________
Calculations:

Surface Area = ________
Calculations:

Problem 4 – Card 3	
Volume = 50 240 cm^3 	Volume = ________
	Height = ________
	Radius = ________ Calculations:
	Surface Area = ________ Calculations:

Problem 4 – Card 4	
Volume = 50 240 cm^3 	Volume = ________
	Height = ________
	Radius = ________ Calculations:
	Surface Area = ________ Calculations:

5.10.2: Dairy King MAP4C

Dairy King wishes to sell 1350mL tubs of ice cream at all of their restaurant locations . The base and cylindrical part of the container will be made from specially coated cardboard.However, the lid is made from plastic. To ensure the lowest price for their product, Dairy King would like to minimize the cost of the cardboard needed to make each tub.

Complete the following table to determine what dimensions would minimize the amount of cardboard for one tub of ice cream.

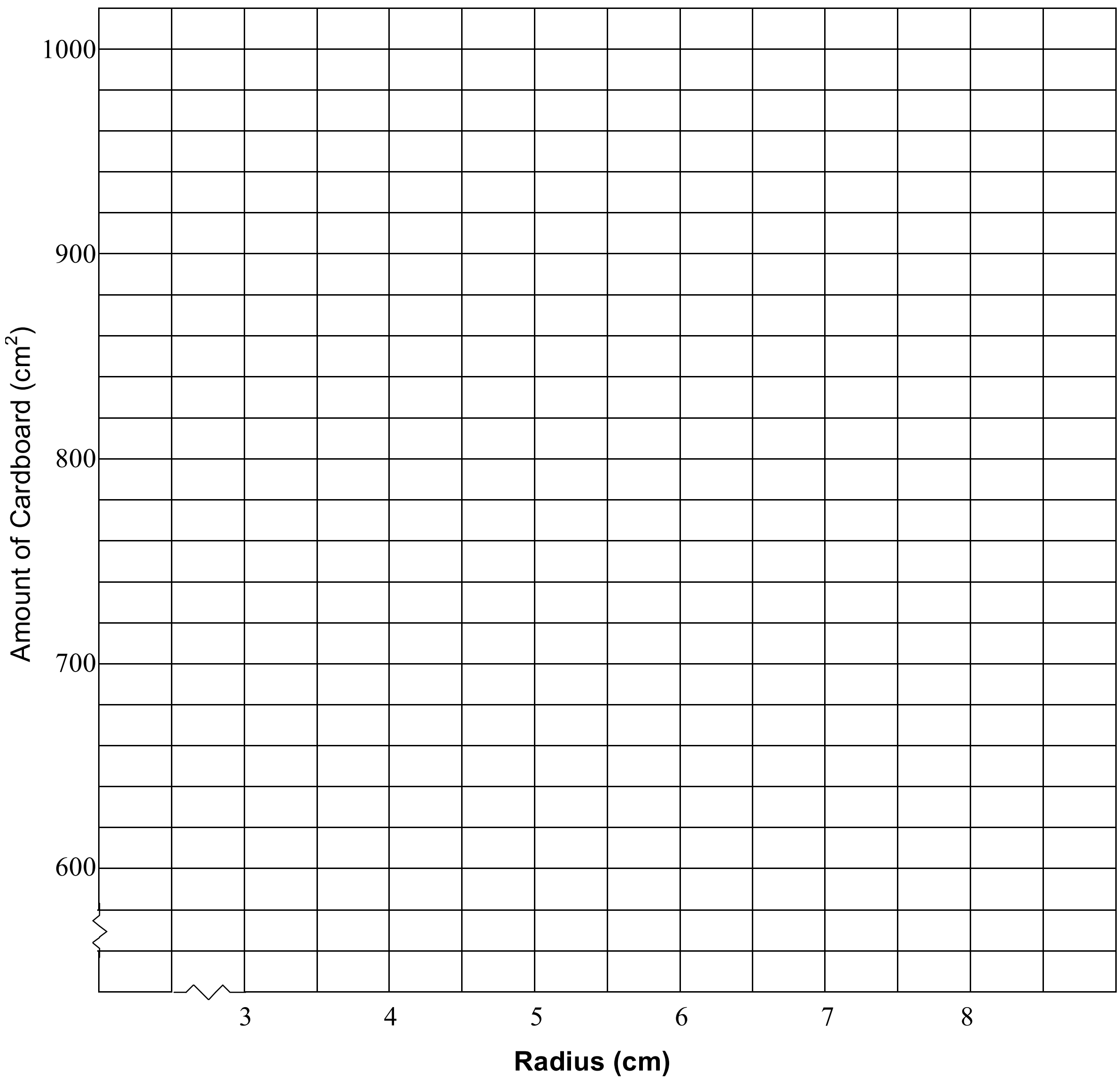
Graph the results (Radius of Tub Base vs. Amount of Cardboard) on the grid on the back of this page.

(Recall: 1 mL = 1 cm^3)
(Round your answers to two decimal places)

r	h	Amount of Cardboard (Area of Base and Tube)
3 cm		
4 cm		
5 cm		
6 cm		
7 cm		
8 cm		

5.10.2: Dairy King (continued)

Graph your results on the grid below.
Draw a curve of best fit.



Recommendation to Dairy King: