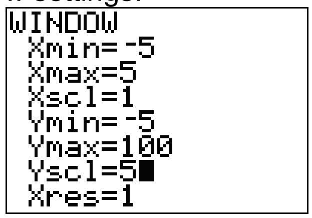
Unit 3: Day 1: Exploring Exponentials

Minds On: 15 Action: 40 Consolidate:20 Total= 75 min	**Learning Goal**: • Graph exponential functions to look at key features of the graph including rate of change • Compare exponential functions with linear and quadratic functions in real-world context • Explore rates of change using finite differences	**Materials** • Graphing calculators • BLM 3.1.1 • BLM 3.1.2 • BLM 3.1.3 • BLM 3.1.4

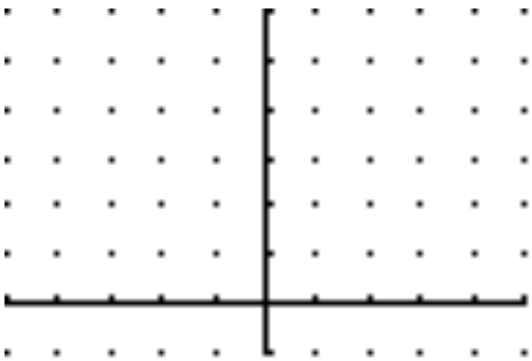
		Assessment Opportunities
Minds On…	**Pair/Share → Exploration** Distribute one of the BLM 3.1.1 activities to each pair of students along with a graphing calculator. Circulate as students complete the activity to ensure that appropriate connections are being drawn. Instruct students to compare their answers with others who completed a different handout. Have students summarize their findings using the Frayer Model - BLM 3.1.2 **Whole Class → Discussion** Complete class Frayer models on chart paper to be posted in the room summarizing the student's results. Instruct students to add to their models anything that they are missing. **Mathematical Process Focus:** Students **reflect** on the characteristics of exponential models.	Literacy strategy: Frayer Model
Action!	**Whole Class → Discussion** Distribute a copy of BLM 3.1.3 to each student. Ensure that students understand the task. **Small Groups → Investigate** Circulate to check student progress and assist as necessary. Students may require help remembering how to perform regression on a graphing calculator. **Learning Skills/Observation/Checkbric:** Assess teamwork skills as you circulate. **Mathematical Process Focus:** Students **represent** the given data and reason to select the most appropriate model.	
Consolidate Debrief	**Whole Class → Four Corners** Have students select the model which they feel best represents the data and move to that model (table of values, graph, equation, other). Have students share their thinking with the class. Develop appropriate terminology and language as student's dialogue about the models. **Individual → Practise** Students should begin BLM 3.1.4. Circulate as students work through these questions offering assistance where needed.	Literacy strategy: Four Corners
Exploration *Application*	**Home Activity or Further Classroom Consolidation** Complete BLM 3.1.4.	

3.1.1: Exploring Exponentials – Activating Prior Knowledge(1)

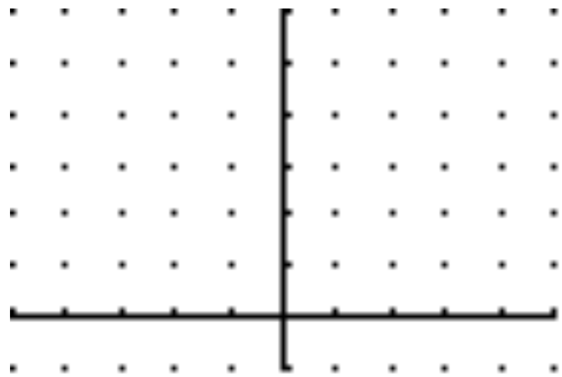
Use graphing technology to graph the following functions and sketch the graph in the spaces provided. Use these window settings:



$y = 3\times 2^x$



$y = 3\times 0.5^x$



For each of the functions, create a table of values (or view the table of values created by the graphing calculator) and determine the finite differences. These values also show the rate of change for the functions for each interval.

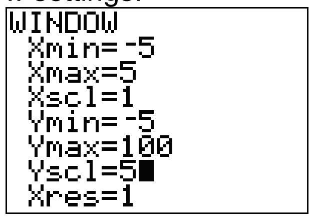
X	$y = 3\times 2^x$	Finite differences
-2		------------
-1		
0		
1		
2		
3		-------------

x	$y = 3\times 0.5^x$	Finite differences
-2		------------
-1		
0		
1		
2		
3		-------------

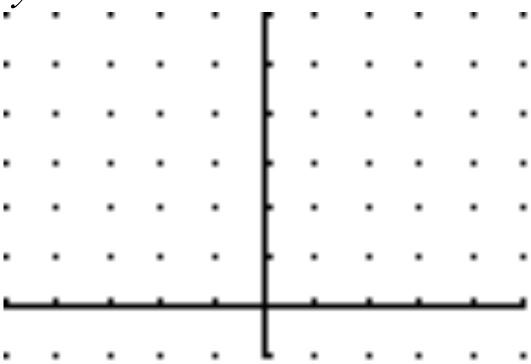
Finite differences show how quickly the y-values increase (or decrease) for each constant increase in x. Finite differences are also called "RATE OF CHANGE".

3.1.1: Exploring Exponentials – Activating Prior Knowledge(2)

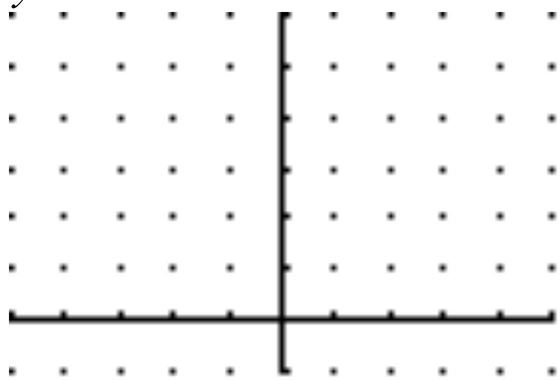
Use graphing technology to graph the following functions and sketch the graph in the spaces provided. Use these window settings:



$y = 2 \times 4^x$



$y = 2 \times 0.25^x$



For each of the functions, create a table of values (or view the table of values created by the graphing calculator) and determine the finite differences. These values also show the rate of change for the functions for each interval.

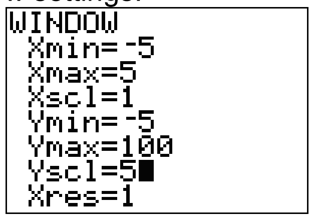
X	$y = 2 \times 4^x$	Finite differences
-2		------------
-1		
0		
1		
2		
3		-------------

x	$y = 2 \times 0.25^x$	Finite differences
-2		------------
-1		
0		
1		
2		
3		-------------

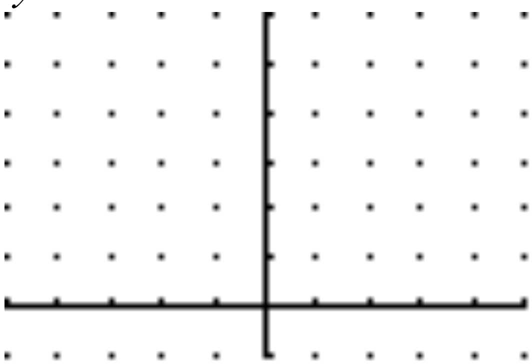
Finite differences show how quickly the y-values increase (or decrease) for each constant increase in x. Finite differences are also called "RATE OF CHANGE".

3.1.1: Exploring Exponentials – Activating Prior Knowledge(3)

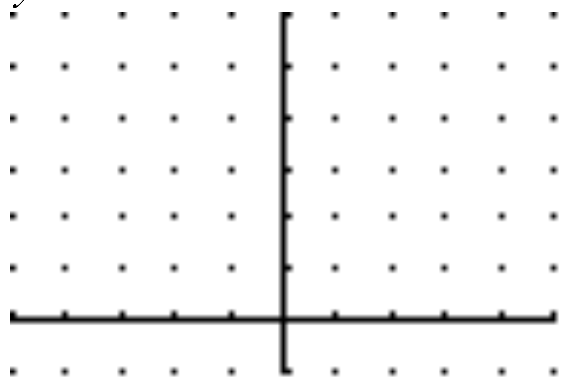
Use graphing technology to graph the following functions and sketch the graph in the spaces provided. Use these window settings:



$y = 4 \times 5^x$



$y = 4 \times 0.2^x$



For each of the functions, create a table of values (or view the table of values created by the graphing calculator) and determine the finite differences. These values also show the rate of change for the functions for each interval.

X	$y = 4 \times 5^x$	Finite differences
-2		------------
-1		
0		
1		
2		
3		-------------

x	$y = 4 \times 0.2^x$	Finite differences
-2		------------
-1		
0		
1		
2		
3		-------------

Finite differences show how quickly the y-values increase (or decrease) for each constant increase in x. Finite differences are also called "RATE OF CHANGE".

3.1.2 Exponential Functions

Complete the following Frayer models for the 2 types of exponential functions

Essential Characteristics	Non-Essential Characteristics
Examples	Non-examples

$$y = ab^x, b > 1$$

Essential Characteristics	Non-Essential Characteristics
Examples	Non-examples

$$y = ab^x, 0 < b < 1$$

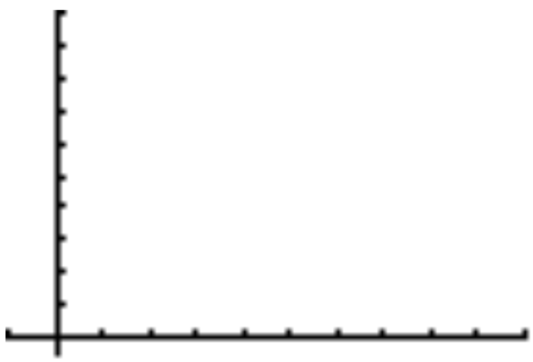
3.1.3: Exploring Exponentials – Leaky Tire

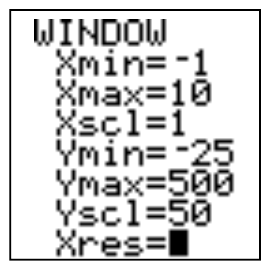
Investigating a Leaky Tire

Larry has a car tire with a slow leak. He measures the tire pressure every day for a week and records the following data:

Time, ***t*** (days)	Pressure, ***P*,** (kPa)
0	400
1	335
2	295
3	255
4	225
5	195
6	170
7	150

1. Graph the tire data using technology. Sketch the graph in the space below:





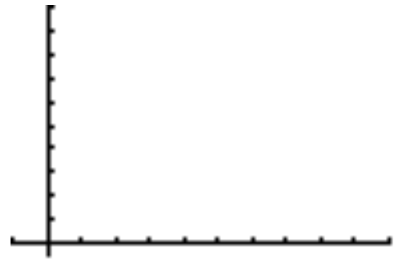
2. What kind of relationship seems to exist between time and pressure? Justify your answer.

3. Determine the rate of change of pressure in this data using finite differences. You may wish to add a column to the data table to record your results. What does this tell you about the data?

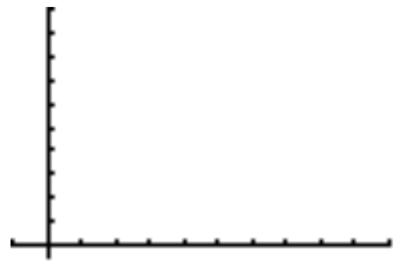
3.1.3: Exploring Exponentials – Leaky Tire (Continued)

4. Perform a regression analysis of the data on your graphing calculator using linear, quadratic, and exponential models. Record your results below giving the equation for each model. Sketch a graph of each model along with the data points.

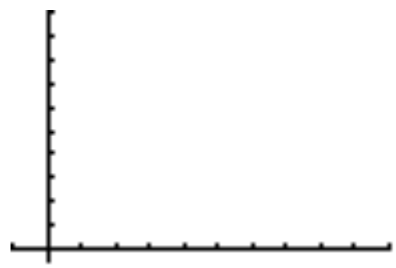
Linear Equation: ____________



Quadratic Equation: ____________



Exponential Equation: ____________



5. Which model best represents this data? Justify your answer.

6. Use your best model equation to answer the following questions. Show your work.

a) What will the pressure be after 10 days? 20 days?

b) How long will it take for the pressure to drop to 50 kPa?

c) When will the pressure drop to zero?

3.1.4: Exploring Exponentials – Practise

1) Complete the following analysis in your notes to determine whether a linear, quadratic or exponential model best represents the data.

This data gives the population growth of bacteria cells in a petri dish that was inoculated by a swab from an infected wound:

Time, t (hours)	Number of bacteria cells
0	250
1	525
2	1103
3	2315
4	4862
5	10210

a) Sketch the graph. Use graphing technology if available.

b) Determine the rates of change in bacteria population.

c) Determine the equation that best models this data. Use technology or algebraic methods.

d) Use your equation to answer the following questions:
   i) What will the bacteria population be after 12 h? 2 days?
   ii) When will the population reach 1 million?
   iii) In real life, will the bacterial population continue to grow like this?

e) Instead of growing as shown in the table above, the bacteria started with 250 cells and increased by a constant amount of 250 cells each hour. What type of equation would model this data?

f) Now suppose that the number of cells in the petri dish remained constant at 250 no matter how much time passed. What type of equation would model this data? Sketch a graph.

3.1.4: Exploring Exponentials – Practise (Continued)

2) **The compound interest formula is $A = P(1 + i)^n$ where A is the amount with interest, P is the principal (or starting amount), i is the interest rate as a decimal, and n is the number of compounding periods.**

Each of the following scenarios uses the compound interest formula. For each, complete the table of values and graph the function. Then, identify whether the function is linear, quadratic or exponential.

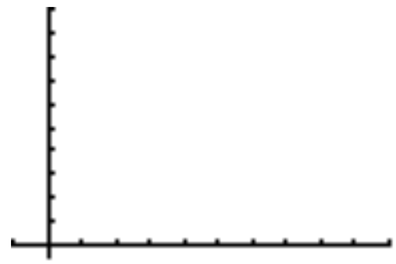
<u>*Scenario 1 – One Year*</u>: Bonita plans to deposit $250 in a savings account. She wonders what relationship exists between the interest rate of the savings account and the amount of money she will have at the end of <u>one year</u>.

The compound interest formula for one year is: $\mathbf{A = 250(1 + i)^1}$.

Complete this table of values.
Calculate the finite differences.

i	A	Finite differences
0.02		---------------
0.04		
0.06		
0.08		
0.10		
0.12		---------------

Sketch the graph. Label the axes!



Is *Scenario 1* linear, quadratic or exponential? How do you know?

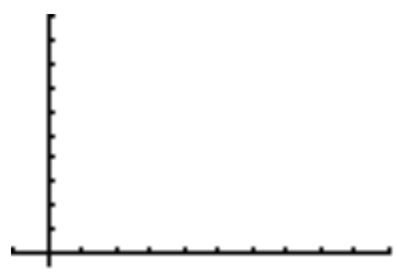
<u>*Scenario 2 – Two Years*</u>: Bonita still plans to deposit $250 in a savings account. She now wonders what relationship exists between the interest rate of the savings account and the amount of money she will have at the end of <u>two years</u>.

The compound interest formula for two years is: $\mathbf{A = 250(1 + i)^2}$.

Complete this table of values.
Calculate the finite differences.

i	A	Finite differences
0.02		---------------
0.04		
0.06		
0.08		
0.10		
0.12		---------------

Sketch the graph. Label the axes!



Is Scenario 2 linear, quadratic or exponential? How do you know?

3.1.4: Exploring Exponentials – Practise (Continued)

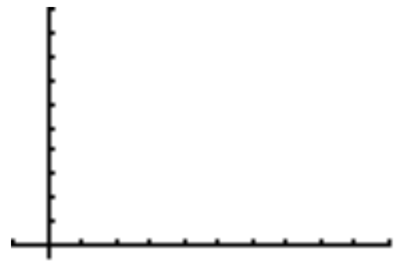
Scenario 3 – Unknown Time: Bonita deposits $250 in a savings account with an interest rate of 6%. She wonders what relationship exists between the number of years the money is compounded, and the interest rate.

The compound interest formula for 6% for a unknown time is: $\mathbf{A = 250\ (1 + 0.06)^n}$
$\mathbf{A = 250\ (1.06)^n}$

Complete this table of values.
Calculate the finite differences.

n	A	Finite differences
0		---------------
1		
2		
3		
4		
5		---------------

Sketch the graph. Label the axes!

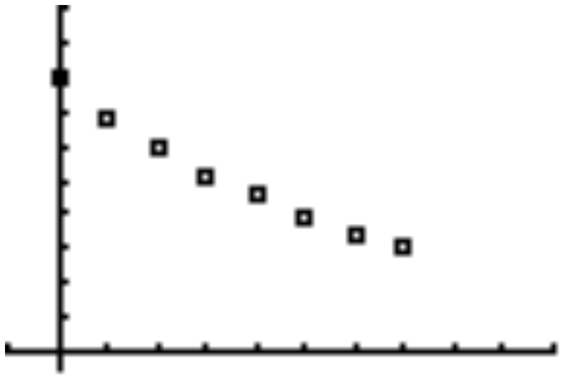


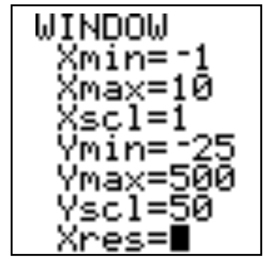
Is *Scenario 3* linear, quadratic or exponential? How do you know?

Which variable(s) in the formula $\mathbf{A = P\ (1 + i)^n}$ did you set to a constant to create a linear equation? a quadratic equation? an exponential equation?

3.1.3: Exploring Exponentials – Leaky Tire (SOLUTIONS)

1. Graph the tire data using technology. Sketch the graph in the space below:





2. What kind of relationship seems to exist between time and pressure? Justify your answer.

   There is a negative correlation between time and pressure. As the time increases, the tire pressure decreases.

3. Determine the rate of change of pressure in this data using finite differences. You may wish to add a column to the data table to record your results. What does this tell you about the data?

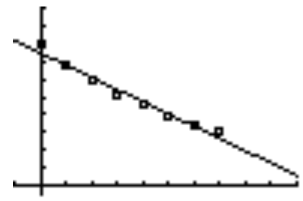
Time, ***t*** (days)	Pressure, ***P***, (kPa)	Finite Differences
0	400	--------------------
		-65
1	335	
		-40
2	295	
		-40
3	255	
		-30
4	225	
		-30
5	195	
		-25
6	170	
		-20
7	150	--------------------

The finite differences are not constant, so the data does not follow a linear relationship.

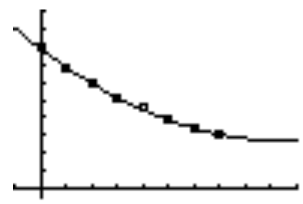
3.1.3: Exploring Exponentials – Leaky Tire (SOLUTIONS)

4. Perform a regression analysis of the data on your graphing calculator using linear, quadratic, and exponential models. Record your results below giving the equation for each model. Sketch a graph of each model along with the data points.

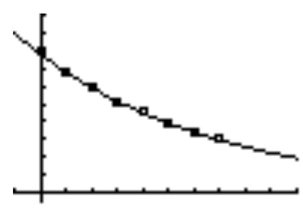
Linear Equation: $y = -34.58x + 374.17$



Quadratic Equation: $y = 2.89x^2 - 54.79x + 394.38$



Exponential Equation: $y = 390.73(0.87)^x$



5. Which model best represents this data? Justify your answer.

Both quadratic and exponential look very good. However, if you increase the domain of both models it becomes clear that the quadratic model is not correct for predicting future values. The quadratic model predicts the pressure will increase again which is not valid in the context.

Exponential with domain of 20 days:

Quadratic with domain of 20 days:

6. Use your best model equation to answer the following questions. Show your work.

a) 10 days? ≈ *97.07 kPa*
20 days? ≈ *24.11 kPa*

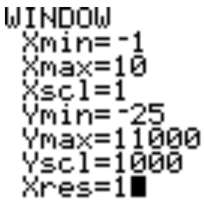
b) 50 kPa? ≈ *14.76 days*

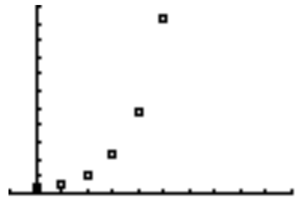
c) zero kPa? *Never! This model does not reach 0 kPa (horizontal asymptote)*

3.1.4: Exploring Exponentials – Practise (SOLUTIONS)

1) Bacteria:

a) Sketch the graph. Use graphing technology if available.





b) Determine the rates of change in bacteria population.

Time, *t* (hours)	Number of bacteria cells	Finite differences
0	250	---------------------
1	525	275
2	1103	578
3	2315	1790
4	4862	2547
5	10210	5348

The finite differences are not constant, so the data does not follow a linear relationship.

c) Determine the equation that best models this data. Use technology or algebraic methods.

$y = 250.03(2.10)^x$

d) Use your equation to answer the following questions:

i) 12 h? *1 839 178.2 cells* 2 days? ≈ *732 010 000 000 000 000 cells*

ii) When will the population reach 1 million? ≈ *11.18 hours*

iii) In real life, will the bacterial population continue to grow like this? *No! It will be confined by variables including the size of the petri.*

3.1.4: Exploring Exponentials – Practise (SOLUTIONS)

e) Instead of growing as shown in the table above, the bacteria started with 250 cells and increased by a constant amount of 250 cells each hour. What type of equation would model this data?

This is a linear model since the rate of change is constant (250 cells/hour).

f) Now suppose that the number of cells in the petri dish remained constant at 250 no matter how much time passed. What type of equation would model this data? Sketch a graph.

This is a linear model with a constant rate of change of 0 cells/hour. The graph is a straight horizontal line with all y-values at 250 cells.

2) Compound Interest:

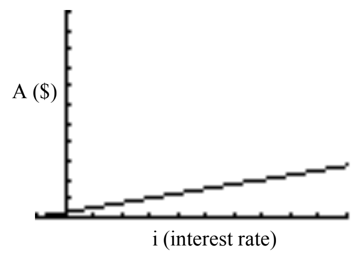
Graphs need a larger domain to clearly show the curve!

<u>Scenario 1 – One Year</u>: **A = 250 (1 + i)^{1}**.

Complete this table of values.
Calculate the finite differences.

i	A	Finite differences
0.02	255	---------------
		5
0.04	260	
		5
0.06	265	
		5
0.08	270	
		5
0.10	275	
		5
0.12	280	---------------

Sketch the graph. Label the axes!



Is *Scenario 1* linear, quadratic or exponential? How do you know?

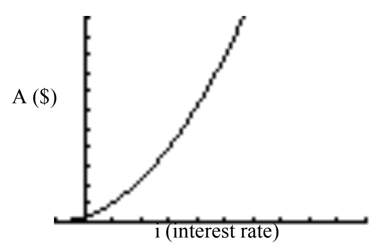
It is linear. The finite differences are constant, the graph is a straight line, and the equation is linear.

<u>Scenario 2 – Two Years</u>: **A = 250 (1 + i)^{2}**.

Complete this table of values.
Calculate the finite differences.

i	A	Finite differences
0.02	260.1	---------------
		10.3
0.04	270.4	
		10.5
0.06	280.9	
		10.7
0.08	291.6	
		10.9
0.10	302.5	
		11.1
0.12	313.6	---------------

Sketch the graph. Label the axes!



Is Scenario 2 linear, quadratic or exponential? How do you know?

It is quadratic. The finite differences are increasing by a constant amount, and the equation is quadratic (power of 2).

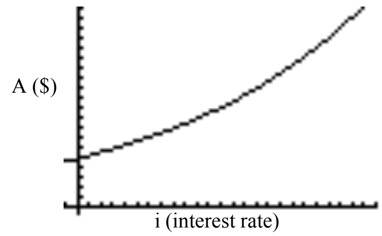
3.1.4: Exploring Exponentials – Practise (SOLUTIONS)

Scenario 3 – Unknown Time: **A = 250 (1.06)^n**

Complete this table of values.
Calculate the finite differences.

n	A	Finite differences
0	250	---------------
		15
1	265	
		15.9
2	280.9	
		16.855
3	297.755	
		17.865
4	315.62	
		18.9375
5	334.5575	---------------

Sketch the graph. Label the axes!



Is *Scenario 3* linear, quadratic or exponential? How do you know?

It is exponential. The finite differences are not constant and they are not increasing by a constant amount. Also, the equation is exponential of the form y=ab^x.

Which variable(s) in the formula **A = P (1 + i)^n** did you set to a constant to create a linear equation? a quadratic equation? an exponential equation?

Linear → P = 250 (any constant will work); n = 1
Quadratic → P = 250 (any constant will work); n = 2
Exponential → P = 250 (any constant will work); i = 0.06 (any constant will work)

Unit 3: Day 5: Problem Solving with Exponentials

Minds On: 15 Action: 35 Consolidate:25 Total=75 min	**Learning Goal**: Students will • Solve equations of the form $x^n = a$ having rational exponents using inverse operations • Using a real world formula, determine the value of a variable of degree no higher than three by substituting known values and then solving for the unknown variable • Solve problems involving exponential equations	**Materials** • Construction paper, tape (optional) • BLM 3.5.1 • BLM 3.5.2 • Manipulatives • Formula sheet for 2-D and 3-D measurements
		Assessment Opportunities
Minds On…	**Pairs → Pair/Share/Guided Instruction** Write the following questions on the blackboard one at a time. Give students enough time to think about the problem & discuss it with their partner, then summarize student answers on the board. 1. How would you solve for x if $x^3 = 27$? List as many ways as you can think of to do this. 2. Solve the following without using a calculator: a) $x^3 = 64$ b) $x^3 = 125$ c) $x^3 = 343$ 3. Solve using a calculator. Why is it necessary to use a calculator for these questions? a) $x^3 = 26$ b) $x^3 = 65$ c) $x^3 = 100$ 4. Solve $4x^3 = 108$. What additional step is necessary? 5. Solve the following: a) $3x^3 = 192$ b) $5x^3 = 1080$ 6. A box is in the shape of a cube and has a volume of 1000 cm^3. What are the dimensions of the box? **Mathematical Process Focus: Selecting Tools and Computational Strategies** by selecting an appropriate method for solving the sample questions. **Learning Skill (Teamwork)/Observation/Checkbric:** Observe and record students' collaboration skills.	
Action!	**Groups of 3 → Problem Solving Carousel** Set up carousel activity with enough copies for each student of each part of BLM 3.5.1 at each station. Encourage the use of manipulatives during the activities. Introduce the carousel activities and write these instructions on the blackboard: "Join your group of 3 at an assigned station. Work on the station's problem. There is an instruction sheet at each station. When directed, rotate to the next station. At the THIRD station visited by your group, prepare a complete written solution on chart paper." **Mathematical Process Focus: Problem solving** by developing, selecting and applying a strategy for each problem.	Set up four different stations for the carousel. Provide a formula sheet at each station for students to use. Literacy strategies: Carousel
Consolidate Debrief	**Whole Class → Group Presentation** Have students present their complete solution to the whole class. As a class, work through Problem #5 on BLM 3.5.1. **Expectations/Performance Task/Oral Feedback:** Give feedback to groups on their presentations and clarify any misunderstandings.	
Exploration *Application*	**Home Activity or Further Classroom Consolidation** Complete BLM 3.5.2	

3.5.1: The 10-Cubed Problems

PROBLEM 1: THE SHOE BOX PROBLEM

A shoebox has a volume of 1 000 cm^3. The width of the shoebox is double the height and the length is triple the height. What is the height of the box?

Steps:

a) Draw a representative diagram of the shoebox.

b) Choose an appropriate volume formula to solve this problem.

c) Write equations that relate the length of the box to the height, h, and the width of the box to h.

d) Substitute all known values and expressions into the volume formula. You will now have an equation with only one variable, h.

e) Solve the equation for the height, h.

f) Build the shoebox from materials provided.

PROBLEM 2: THE SPORTS BALL PROBLEM

A sports ball has a volume of 1 000 cm^3. What is the diameter of the ball?

Steps:

a) Draw a representative diagram of the ball.

b) Choose an appropriate volume formula to solve this problem.

c) Substitute all known values into the volume formula. You will now have an equation with only one variable, r.

d) Solve the equation for the radius.

e) Calculate the diameter of the ball. What sport would use a ball that was about this size?

3.5.1: The 10-Cubed Problems

PROBLEM 3: THE JUICE CAN PROBLEM

A juice can has a volume of 1 000 cm^3. The height of the can is equal to the diameter. What is the radius of the can?

Steps:

a) Draw a representative diagram of the juice can.

b) Choose an appropriate volume formula to solve this problem.

c) Write an equation that relates the height to the radius, r.

d) Substitute all known values and expressions into the volume formula. You will now have an equation with only one variable, r.

e) Solve the equation for the radius.

f) Build the juice can from materials provided. What kind of juice is sold in this size of container?

PROBLEM 4: THE WAFFLE CONE PROBLEM

A waffle cone has a volume of 1 000 cm^3. The radius of the cone is one quarter of the height. What is the height of the cone?

Steps:

a) Draw a representative diagram of the waffle cone.

b) Choose an appropriate volume formula to solve this problem.

c) Write an equation that relates the radius to the height, h.

d) Substitute all known values and expressions into the volume formula. You will now have an equation with only one variable, h.

e) Solve the equation for the height.

f) Build the cone from materials provided. Is this a reasonable size for a waffle cone?

3.5.1: The 10-Cubed Problems

PROBLEM 5: THE TRAY PROBLEM

You may use graphing technology to solve this problem.

You must make an open-topped box (tray) that has a volume of 1 000 cm^3 using a flat piece of cardboard that is 25 cm by 25 cm. To make the box, you cut a square out of the corners of the cardboard and fold up the sides. What should the length of the cut-out be so the box has the correct volume?

Steps:

a) Draw a representative diagram (3D) of the tray. Also draw a net of the tray (2D) showing the corner cut-outs and fold lines.

b) Choose an appropriate volume formula to solve this problem.

c) Let the length of the cut-out be x units. Label the dimensions of your 3D and 2D diagrams. Write equations that relate the length and width of the given piece of cardboard and the cut-out size, x.

d) Substitute all known values and expressions into the volume formula. You will now have an equation with only one variable, x.

e) Solve the equation for the height of the tray, which is the variable x. You may use graphing technology to solve this equation.

f) How many solutions did you find? Do they both result in a “reasonable” tray?

g) Build the tray from materials provided.

3.5.1: The 10-Cubed Problems (Teacher Notes)

Suggested possible manipulatives:

For all stations –have the following available in the classroom if possible

- 3D geometric shapes
- Cube-a-links
- Paper, scissors and tape

Specific extras that would be nice to have at the given station:

Station 1: The Shoe Box Problem

- An Actual shoe box

Station 2: The Sports Ball Problem

- A sports ball cut in half
- Or some other sphere shaped object cut in half (orange?)

Station 3: The Juice Can Problem

- A 1L can of juice (like apple juice)
- A 255mL can of juice would do as well OR
- A soup can would help if you don't have one of the above

Station 4: The Waffle Cone Problem

- Paper to fold into a waffle cone would work
- Actual waffle cones – then they can be eaten!

Station 5: The Tray Problem

- A clothing gift box that fits this pattern that can be taken apart.

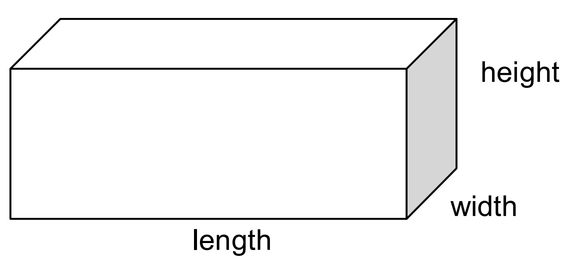
Suggested solutions to the five problems:

SOLUTION 1: THE SHOE BOX PROBLEM

A shoebox has a volume of 1 000 cm^3. The width of the shoebox is double the height and the length is triple the height. What is the height of the box?

Steps:

a) Draw a representative diagram of the shoebox. (not to scale)



b) Choose an appropriate volume formula to solve this problem.

$$V = l \times w \times h$$

c) Write equations that relate the length and width to the height, h.

$$l = 3h \qquad w = 2h$$

d) Substitute all known values and expressions into the volume formula.

$$V = l \times w \times h$$
$$1000 = (3h)(2h)(h)$$

e) Solve the equation for the height, h.

$$1000 = 6h^3$$
$$\frac{1000}{6} = h^3$$
$$\left(\frac{1000}{6}\right)^{\left(\frac{1}{3}\right)} = h$$

```
(1000/6)^(1/3)
      5.503212081
```

$$\therefore h \approx 5.50\,cm$$

f) Build the shoebox from materials provided.

This is a very small box that could be used for baby shoes.

SOLUTION 2: THE SPORTS BALL PROBLEM

A sports ball has a volume of 1 000 cm^3. What is the diameter of the ball?

Steps:

a) Draw a representative diagram of the ball.

b) Choose an appropriate volume formula to solve this problem.

$$V = \frac{4}{3}(\pi)(r)^3$$

c) Substitute all known values into the volume formula. You will now have an equation with only one variable, r.

$$1000 = \frac{4}{3}(\pi)(r)^3$$

3.5.1: The 10-Cubed Problems (Teacher Notes)(continued)

d) Solve the equation for the radius.

$$1000 = \frac{4\pi}{3}(r)^3$$

$$\frac{3}{4\pi}(1000) = r^3$$

$$\frac{3000}{4\pi} = r^3$$

$$\left(\frac{3000}{4\pi}\right)^{\frac{1}{3}} = r$$

$$\therefore r \approx 6.2cm$$

e) Calculate the diameter of the ball. What sport would use a ball that was about this size?

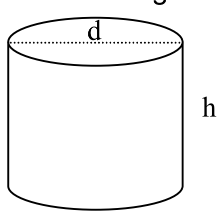
The diameter of the ball would be 12.4 cm, which is a bit bigger than a softball.

SOLUTION 3: THE JUICE CAN PROBLEM

A juice can has a volume of 1 000 cm^3. The height of the can is equal to the diameter. What is the radius of the can?

Steps:

a) Draw a representative diagram of the juice can.



b) Choose an appropriate volume formula to solve this problem.

$$V = (\pi)r^2 h$$

c) Write an equation that relates the height to the radius, r.

$$h = d = 2r$$

d) Substitute all known values and expressions into the volume formula. You will now have an equation with only one variable, r.

$$1000 = (\pi)r^2(2r)$$

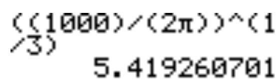
e) Solve the equation for the radius.

$$1000 = (\pi)r^2(2r)$$

$$1000 = (\pi)(2)(r^3)$$

$$\frac{1000}{2\pi} = r^3$$

$$\left(\frac{1000}{2\pi}\right)^{\frac{1}{3}} = r$$



$$\therefore r \approx 5.42\,cm$$

f) Build the juice can from materials provided. What kind of juice is sold in this size of container?

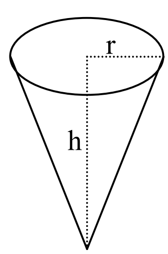
The height of the can is 10.84 cm. Stewed tomatoes and chilli sauce are sold in a can about this size.

SOLUTION 4: THE WAFFLE CONE PROBLEM

A waffle cone has a volume of 1 000 cm^3. The radius of the cone is one quarter of the height. What is the height of the cone?

Steps:

a) Draw a representative diagram of the waffle cone. (not to scale)



b) Choose an appropriate volume formula to solve this problem.

$$V = \frac{1}{3}\pi r^2 h$$

3.5.1: The 10-Cubed Problems (Teacher Notes)(continued)

c) Write an equation that relates the radius to the height, h.

$$r = \frac{h}{4}$$

d) Substitute all known values and expressions into the volume formula. You will now have an equation with only one variable, h.

$$1000 = \frac{1}{3}\pi\left(\frac{h}{4}\right)^2 h$$

e) Solve the equation for the height.

$$1000 = \frac{\pi h^3}{48}$$

$$\pi h^3 = 48000$$

$$h^3 = \frac{48000}{\pi}$$

$$h = \left(\frac{48000}{\pi}\right)^{\frac{1}{3}}$$

$$\therefore h \approx 24.8cm$$

f) Build the cone from materials provided. Is this a reasonable size for a waffle cone?

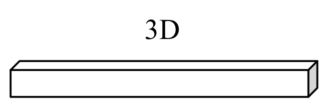
This is quite a large waffle cone, but it certainly is possible!

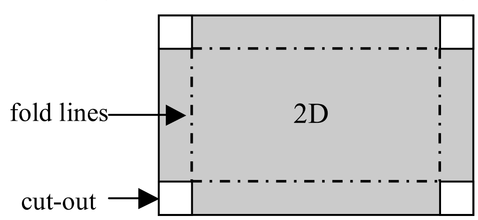
PROBLEM 5: THE TRAY PROBLEM

You must make an open-topped box (tray) that has a volume of 1 000 cm^3 using a flat piece of cardboard that is 25 cm by 25 cm. To make the box, you cut a square out of the corners of the cardboard and fold up the sides. What should the length of the cut-out be so the box has the correct volume?

Steps:

a) Draw a representative diagram (3D) of the tray. Also draw a net of the tray (2D) showing the corner cut-outs and fold lines.





b) Choose an appropriate volume formula to solve this problem.

$$V = l \times w \times h$$

3.5.1: The 10-Cubed Problems (Teacher Notes)(continued)

c) Let the length of the cut-out be x units. Label the dimensions of your 3D and 2D diagrams. Write equations that relate the length and width of the given piece of cardboard and the cut-out size, x.

$l = 25 - 2x \qquad w = 25 - 2x$

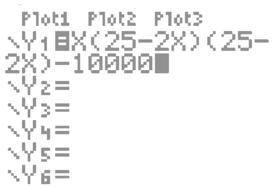
d) Substitute all known values and expressions into the volume formula. You will now have an equation with only one variable, x.

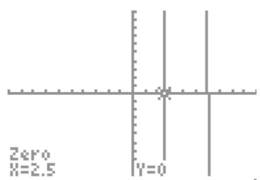
$$1000 = x(25 - 2x)(25 - 2x)$$

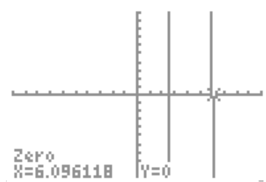
e) Solve the equation for the height of the tray, which is the variable x. You may use graphing technology to solve this equation.

$$0 = x(25 - 2x)(25 - 2x) - 1000$$

Use the graphing calculator to graph the function and solve for x (find the zeros, or trace).







Therefore $x = 2.5$ cm or $x \approx 6.1$ cm

f) How many solutions did you find? Do they both result in a “reasonable” tray?

The two solutions are a height of 2.5cm with width and length of 20cm, and height of 6.1cm with width and length of 12.8cm. The first solution makes sense for a tray, while the second solution is more of a box!

3.5.2 Problem Solving with Exponentials: Home Activity

The following problems are designed to consolidate and reinforce today's learning.

Prepare complete solutions on notepaper.

1) \$1500 was invested for 2 years in an account that pays interest compounded annually. What was the interest rate if the investment was worth \$1800 after two years?
2) \$25000 was invested for 3 years in an account that pays interest compounded annually. What was the interest rate if the investment was worth \$28500 after three years?
3) The fuel consumption of a particular make of small car is related to the car's speed by the equation: $F = 6.0 + 0.001(v - 90)^3$ where F is the fuel consumption in L/100 km and v is the average speed in km/h. The formula is only valid for speed in excess of 90 km/h. If this car is consuming 7.2 L/100km what is its average speed?
4) A propane storage tank consists of two hemispheres attached to the ends of a cylinder. The length of the cylindrical part is equal to the diameter of the hemispherical ends. If the tank holds 10 000 m^3 of propane, what is the diameter of the tank?

Answers:

1) $i \approx$ 0.095 or 9.5%
2) $i \approx$ 0.045 or 4.5%
3) $v \approx$ 100.6 km/h
4) $d \approx$ 19.7m