

▲ TECHNOLOGY CORNER — EXPONENTIAL AND LOGARITHMIC MODELS

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Mary teaches mathematics at West Carleton Secondary School in Ottawa. She is a strong advocate for the appropriate use of technology in the classroom. She has presented workshops internationally, authored mathematics resources, is a past VP of OAME and a Past President of COMA.

An award-winning teacher, Mary continually strives to learn new and better ways of helping students learn and love mathematics.

Mathematical modelling is a great way of relating functions studied in class to real-world data. Graphing technology is very helpful with data modelling, as it allows us see how well the model fits the data. Although a graphing calculator can determine the equation of a line or curve of best fit, working through the process of developing the right model is a wonderful exercise for students. Finite differences can be used to determine polynomial models. Different strategies must be employed to model exponential and logarithmic data.

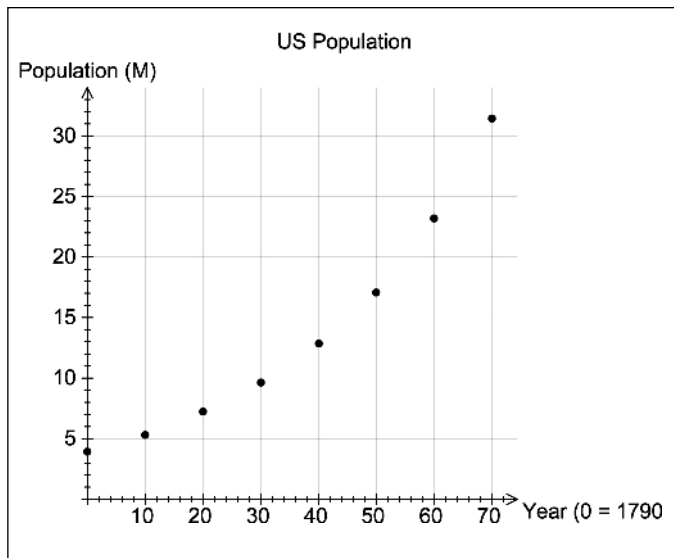
Here is an example of data that grows exponentially. In the years before the Civil War, the population of the U.S. grew rapidly, as shown in the following table. Determine a model for this growth.

Year	1790	1800	1810	1820	1830	1840	1850	1860
Population in millions	3.93	5.31	7.24	9.64	12.86	17.07	23.19	31.44

Using a graphing calculator to plot the data, let's make 1790 year 0 and enter this new year data and the population data. On a TI-84, enter the data in lists 1 and 2. On a TI-Nspire, enter the data in a Lists & Spreadsheet page. Set up a scatter plot and look at the shape of the plot.

L1	L2	L3	1
0	3.93	-----	
10	5.31		
20	7.24		
30	9.64		
40	12.86		
50	17.07		
60	23.19		

L1(1) = 0



The shape suggests either a polynomial graph of even degree or an exponential graph. Populations generally grow exponentially, so we should check for a common ratio to determine if this case is exponential.

year	populat...
0	3.93
10	5.31
20	7.24
30	9.64
40	12.86

$\frac{5.31}{3.93} = 1.35$	$\frac{7.24}{5.31} = 1.36$	$\frac{9.64}{7.24} = 1.33$	$\frac{12.86}{9.64} = 1.33$	$\frac{17.07}{12.86} = 1.33$	$\frac{23.19}{17.07} = 1.36$	$\frac{31.44}{23.19} = 1.36$
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Since the ratios are almost constant, we can use an exponential equation to model this data.

We will use the model $y = c \cdot a^x$, where c is the initial value and a is the growth factor.

The common ratio is somewhere between 1.327 and 1.363. Taking the average of the values suggests a common ratio of 1.35. To determine a , we must take into account the fact that the data points are 10 years apart. This means that to get from the first data point to the second data point, you would have to multiply by a 10 times:

$$3.93 \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a = 5.31$$

$$3.93 \cdot a^{10} = 5.31$$

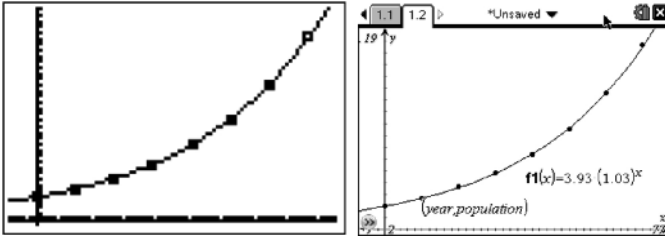
Therefore we need the 10th root of $\frac{5.31}{3.93}$, or 1.35.

$${}^{10}\sqrt{1.35} \approx 1.030465312$$

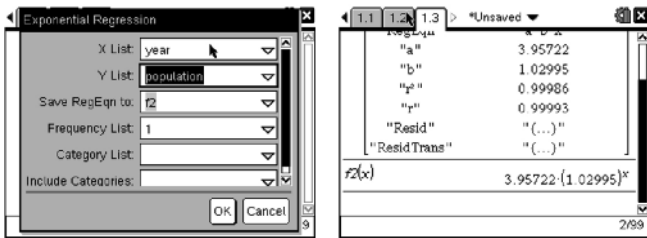
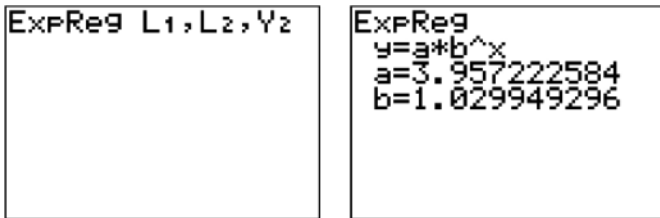
Store this value in your calculator.

The model is now $y = c \cdot 1.03^x$. Our initial value is 3.93, so we can use that for c . If we did not have the y -intercept, then we could use one data point for x and y

and calculate the value of c . For example, using (60, 23.19), $c = 3.94$, our model is now: $y = 3.93(1.03)^x$. Using a graphing calculator, we can see how well our model fits the data:



The graphing calculator can also provide an equation for us by doing an exponential regression:

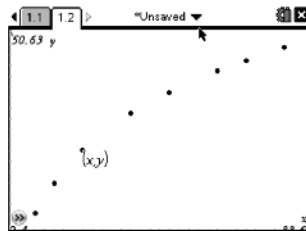


Let's now look at logarithmic data. Determine a logarithmic model, if appropriate, for the following data:

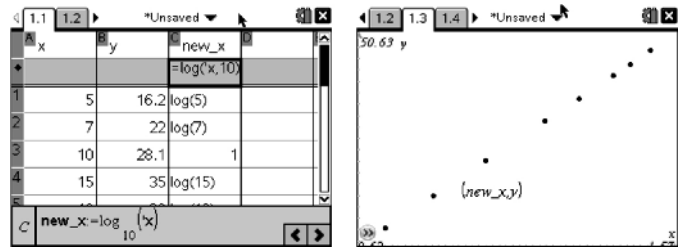
x	5	7	10	15	19	24	27	31
y	16.2	22	28.1	35	39	43	45	47.5

Enter the data and plot them.

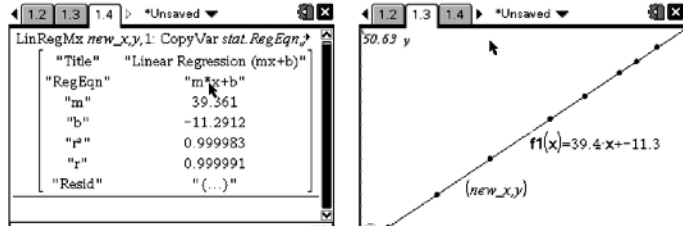
To determine if a logarithmic model is appropriate, you can linearize the data. If y represents a logarithmic function, taking the logarithm of the data in x will create a linear relationship. If the graph of the new plot appears to be linear, then a logarithmic model is a good choice for this data.



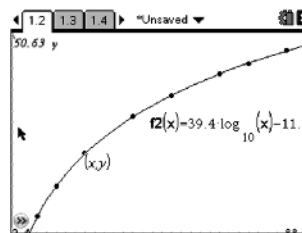
You need to plot $\log(x)$ and y . In your lists, make the 3rd column equal to $\log(1st\ column)$; then set up your scatter plot.



Does the graph look linear? If so, perform a linear regression on it and look at the correlation coefficient to see how well it fits the data. The correlation coefficient is high in this case.



The linear equation is $y = 39.4x - 11.3$. Since we replaced x with $\log x$, we need to replace x with $\log x$ in our linear equation. The result is the logarithmic equation that fits the original data.



Logarithmic equation:
 $y = 39.4 \log x - 11.3$. ▲