
▲ COMMUNICATION IN A BLENDED MATH-TALK COMMUNITY: EXTENDING THE BOUNDARIES OF CLASSROOM COLLABORATION

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In September 2012, Geoff retired from a 39-year career in education as a secondary school mathematics teacher and department head (Timmins); board curriculum consultant (Timmins); Ministry of Education, Education Officer (Sudbury); and professor at the Faculty of Education, Queen's University. He continues to be involved in mathematics education; leading workshops, exploring environments for online mathematics collaboration (<http://collabmath.pbworks.com>) and developing the websites www.math-towers.ca and www.kids-math.ca.

Like the National Council of Teachers of Mathematics (2000), the Ontario Ministry of Education (OME, 2005) emphasizes that communication is an essential process in learning mathematics. The Ontario Mathematics Curriculum does not expand on what communication looks like in an online environment, perhaps because online learning was less prevalent in 2005 and 2007, but the Ontario Ministry of Education does identify: (i) *what* it means to communicate in mathematics, and (ii) *how* communication should be supported. These two components are fundamental in any mathematics classroom. In this article, we focus on what mathematics communication may look like and how it can be supported in a blended collaborative mathematics

learning environment. In particular, we describe how a wiki, dynamic mathematics software, and screencast videos were used to support communication in the classroom and online in the Principles of Mathematics (MPM2D), Grade 10, Academic course.

Communication in a Blended Collaborative Learning Environment

Communication, according to the Ontario Mathematics Curriculum guidelines, is “the process of expressing mathematical ideas and understanding orally, visually, and in writing, using numbers, symbols, pictures, graphs, diagrams, and words” (OME, 2005, p. 16). With the technological tools that we employed, students were able to express and record their thinking in all of these forms. Furthermore, dynamic mathematics software allowed students to interact with graphical, algebraic, and numeric representations in mathematics (see Roulet & Lazarus, 2013).

In addition to changing the way students express their thinking in mathematics, we considered the impact of blended learning in a classroom environment, where students are actively contributing to mathematics learning. Instead of using the wiki and video screencast technologies to communicate information to students, students were involved in online collaboration. This approach reflects thinking in the Ontario Mathematics Curriculum about how communication should be supported in a mathematics learning environment. In particular, according to the Ontario Ministry of Education, “effective classroom communication requires a supportive and respectful environment that makes all members of the class comfortable when they speak and when they question, react to, and elaborate on the statements of their classmates and the teacher” (OME, 2005, p. 16).

The learning environment envisioned in the curriculum guidelines reflects components of a math-talk learning community. The math-talk framework, which was supported throughout the implementation of this project, is referenced in teacher resource documents that are provided by the Ministry of Education (OME, 2008a, 2008b). According to Hufferd-Ackles et al. (2004), in a math-talk learning community, there is a shift from the teacher asking questions to *students* and the *teacher* posing questions. Students are expected to *explain* their thinking and to elaborate on their peers' thinking. There is a shift away from the teacher as a *source of all mathematical ideas* to student ideas also influencing the direction of lessons. Students also take more

responsibility for learning for themselves and for each other (Hufferd-Ackles, Fuson, & Sherin, 2004). As we demonstrate below, blending technologies and bringing the collaboration to an online environment provided students with alternative ways to communicate their thinking. The blended approach allowed for the math-talk community to be extended across time and space.

Facilitating the collaborative learning that is emphasized in a math-talk community requires careful consideration. Delich, Kelly, and McIntosh, for example, acknowledge that online collaboration requires “both structure and flexibility” (2008, p. 18). Structure can be provided, for example, by giving clear instructions. Flexibility may be promoted “by being willing to move in new directions that emerge during the collaborative exchanges” (Delich, Kelly, & McIntosh, 2008, p. 18). These authors also emphasize that “teaching and learning still relies on people—expert learners and beginning learners—more than technology” (2008, p. 18). Simply creating a wiki and telling students to interact is unlikely to lead to meaningful collaboration (Lavin, Beaufait, & Tomei, 2008).

Combining Tools to Facilitate Blended Collaboration

Integrating technological tools provides a range of possibilities for facilitating blended collaboration in mathematics. In the approach presented here, “blended” means offline classroom activities along with asynchronous online sharing. Online collaboration was supported with a wiki, dynamic mathematics software, and screencast software. The three free online tools and our approach to integrating them to facilitate a blended math-talk learning community are outlined below.

A Wiki for Collaboration

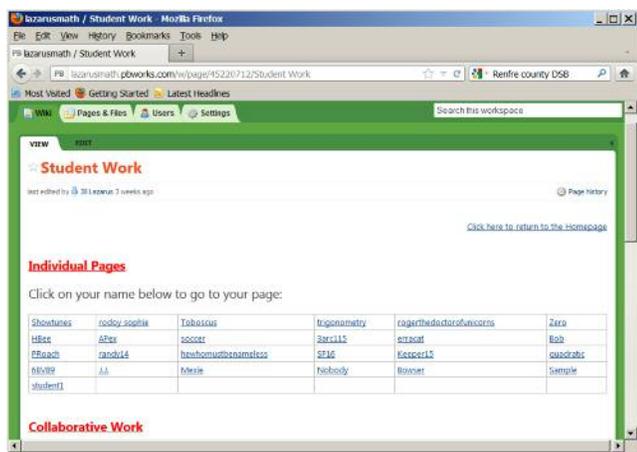


Figure 1: Wiki links to student pages

A wiki, in this case PBworks (<http://pbworks.com>), provides a medium for supporting student interactions. Wikis are “collaboratively editable websites that can be used for various purposes” (Lavin, Beaufait, & Tomei, 2008, p. 391). The collaborative editing feature allows for the aspects of communication identified in mathematics curriculum documents: questioning, reacting, and responding to each other’s statements. As Lavin, Beaufait, and Tomei acknowledge, typically, “where a team of people is working on creating a knowledge base of some kind, a wiki implies engagement with information and ideas more than people, though of course it was ultimately people who produced those ideas” (2008, p. 295). Thus, by blending our math-talk community, students worked together in the classroom in addition to interacting with information and ideas online. Although PBworks is a free tool, and in the past we used the free version, for this project we chose to pay to upgrade, which allowed us to keep some pages public and other pages closed to the teacher and students in the class (see Figure 1). Our approach to controlling page access is explained in more detail below.

Representing and Interacting with Representations, Using GeoGebra

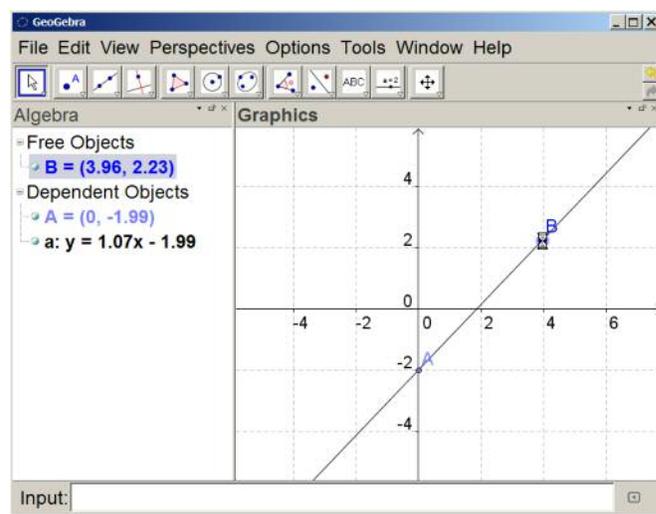


Figure 2: GeoGebra construction

Dynamic mathematics software, GeoGebra (International GeoGebra Institute, 2012), allows students to represent their thinking graphically, algebraically, and numerically (see Figure 2). In GeoGebra, which is free and may be used online or offline, the representations are linked so that in addition to communicating in these forms, students can explore and describe the interactions between representations (see Roulet & Lazarus, 2013). In terms of online sharing, GeoGebra

constructions may be embedded into wiki pages as applets so that students can post a dynamic sketch; someone else can access and work with it, and then she or he can re-post. Thus, since the wiki alone did not allow for easy mathematics representations, embedding GeoGebra applets was important for supporting communication using mathematics notation.

Screencast Videos Using Jing

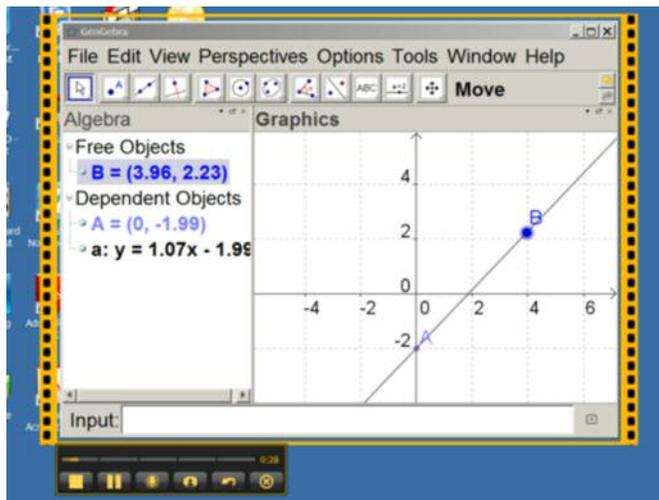


Figure 3: Screencasting a GeoGebra construction with Jing

With Jing (www.techsmith.com/jing.html), free screenshot and screencast software, students may record their screens and voices as they explain their thinking (see Figure 3). This reduces the need for text and provides a YouTube-like experience for students. To reflect that students are an important source of mathematical ideas in the math-talk community, videos posted to the wiki were primarily student generated rather than teacher generated or online videos that presented information to students. For more information on the software and technical details involved in online mathematical collaboration, see the Collabmath wiki at <http://collabmath.pbworks.com>.

Implementing a Blended Math-Talk Learning Community

To provide structure and flexibility, we carried out this project in a series of stages. This was essential because students had limited or no prior experience with the technologies. According to the Ontario Mathematics Curriculum, when students lack prior knowledge of the technologies that are being employed, “it is important that teachers introduce their use in ways that build students’ confidence and contribute to their understanding of the concepts being investigated” (OME,

2005, p. 14). To maintain this position, we carried out the integration of technologies in six stages: (i) Setting Up the Environment, (ii) Posting Mathematics Text, (iii) Embedding GeoGebra Applets, (iv) Embedding Jing Videos, (v) Commenting on Each Others’ Work, and (vi) Combining Skills.

Stage 1: Setting Up the Environment

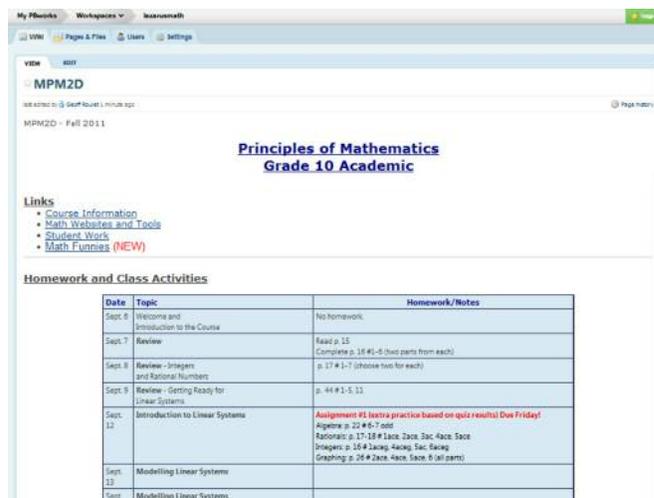


Figure 4: Wiki home page

An important first step in this approach to blended learning was to introduce students gradually to the wiki, the dynamic mathematics software, and to collaborating in a math-talk learning community. In preparing students to participate in a blended collaborative learning culture, a math-talk learning community was introduced in the classroom during the first week of school. The classroom was set up so that students sat in groups of four. They also contributed to co-constructing criteria for communicating in group and whole-class activities. Some of the criteria they identified in this activity included listening, everyone participating, and “sharing is caring.” Math-talk activities were supported throughout the entire course.

As the community developed, students were introduced to the wiki and to GeoGebra. Initially, the wiki was a place where students or their parents could access course information, including details about daily work and homework (See Figure 4). This allowed for a gradual introduction to the wiki, which is important because initially, the various features of a wiki may be confusing to students (Lavin, Beaufait, & Tomei, 2008). In the same way, GeoGebra was also introduced in stages. This tool was used by the teacher to facilitate whole-class discussions and by students as they explored mathematical ideas in class.

Online safety was an important consideration in this classroom. On the wiki, students were identified by self-created pseudonym names. Access to wiki pages was also controlled by the teacher. We used a wiki that allowed control of access to each page. We had control over which pages were open for individual, group, class, or public access for viewing/editing. Pages that contained course information that would be of interest to parents, were open to the public for viewing, but not for editing. Pages that students contributed to were password protected. Each student had a username and password and had access to view and/or edit the pages that we approved. For example, a page for individual student work could be open only to that individual or to the entire class. A page for group work could be closed to the group or open to the entire class for viewing and/or editing. Page access depended on the purpose of the task.

Stage 2: Posting Mathematics Text

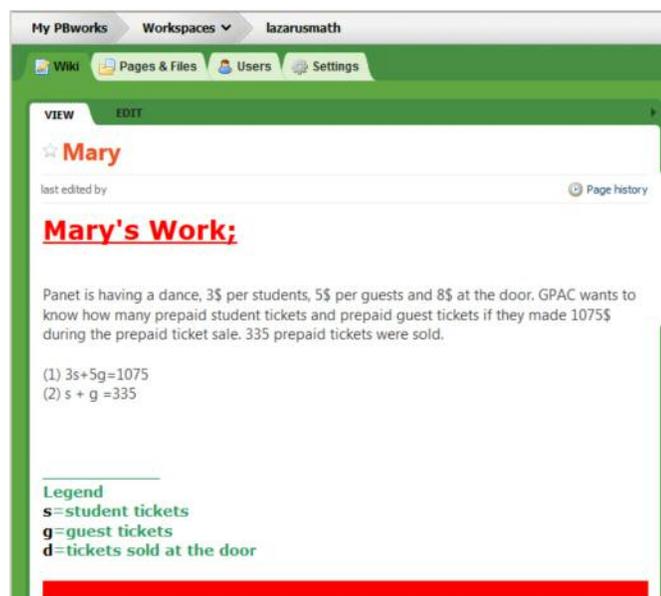


Figure 5: Individual student wiki page

For their first online experience in this course, students posted text to the wiki. This was done in class so that any issues could be addressed. Given the potential for delays, such as computer loading time, we found that it is important to have a task for students to work on during wait time.

Students worked individually for this activity, posting to their own wiki pages (see Figure 5). Each student created and posted two unique linear systems word problems along with the corresponding algebraic representations. One problem was required to have a

unique solution and the second either infinite or no solutions. Prior to this posting, students had opportunities in class to create different types of problems and to get feedback from the teacher and peers. Their wiki postings can be used in a variety of ways. For example, students can visit each other's pages and comment on the constructions or contribute other representations. We found that such interaction requires classroom initiation and modelling, and for this, a few examples were shared and discussed in class.

Stage 3: Embedding GeoGebra Applets

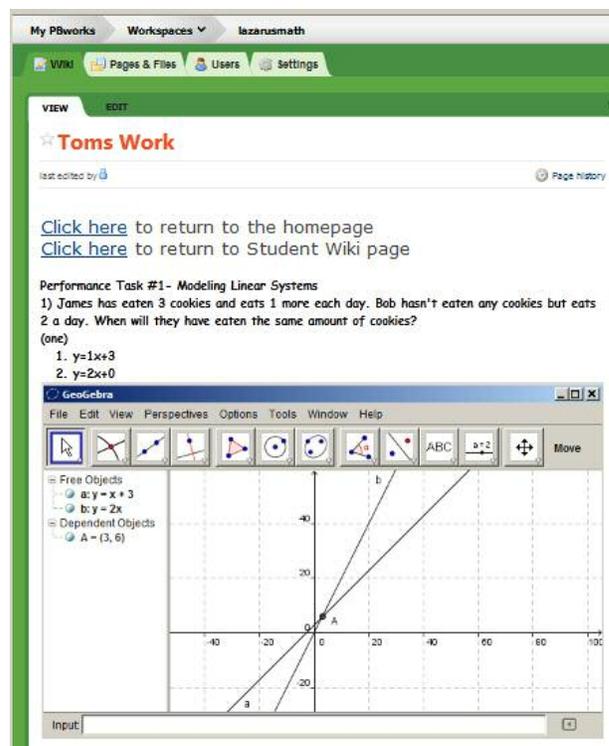


Figure 6: Embedding GeoGebra applets

Once students were comfortable with accessing and posting text to the wiki, they were introduced to embedding GeoGebra applets to wiki pages. This activity was also done in class to allow for troubleshooting any problems. Students embedded graphical models of the problems that they had created (see Figure 6).

Stage 4: Embedding Jing Videos



Figure 7: Season's Greeting Jing video

Jing, screencast software, was introduced next. Jing allowed students to record their screens and voices. This software was introduced in a fun way, just before the holiday season, when students worked in groups to create and share a season's greeting that they developed using GeoGebra (see Figure 7). For more information on how to embed GeoGebra applets and Jing videos on wiki pages in PBworks, visit the Collabmath wiki (<http://collabmath.pbworks.com>).

Stage 5: Commenting on Each Other's Work

Once they gained experience with participating in a math-talk learning community and with using the technologies, students started to comment on each other's work (see Figure 8). For this online collaboration, they worked together to develop a strategy for using similar triangles to determine the height of an inaccessible object. This collaboration was part of the "MPM2D Performance Task" (see below) that was initiated in the classroom, but students also contributed to the wiki between classes.

Students were encouraged to post text, GeoGebra models, and videos to explain their approaches. Prior to participating in this activity, the class contributed to a second conversation about communication in mathematics, this time with respect to what it means to contribute in an online collaboration. Similar to the

Method 2: Mirror

last edited by [user] Page history

VIEW EDIT

Tree Object
 - @ B = (0, 4)
 - @ D = (4, 3)
 - @ F = (4, 0)
 - @ A = (0, 0)
 - @ C = (2, 0)
 - @ E = (2, 3)
 - @ G = (2, 4)
 - @ H = (5, 4)

Dependent Object
 - @ I = (2, 0)
 - @ J = (2, 3)
 - @ K = (2, 4)

13m 2m 2m 5m
 2m 1m 54cm
 2m 1m 54cm
 1m 54cm
 1m 54cm
 1m 54cm

Height

1m 54cm
 2m
 2m
 5m

1. Subtract the points and the equation
 $-4(1) + 4(2) = 0$

2. Take the measurements of the known lengths and substitute them into the equation
 $3x + 5 = 7(2)$

3. Check multiply
 $4x = 33 \text{ cm}$

4. Solve

Using the mirror method, there are a few easy steps...

- 1) find an inaccessible object to measure the height of, our group chose the light in the class room
- 2) Put a metre stick at the base of the object, and measure a distance of choice away from the object to place the mirror (our group chose 2 metres)
- 3) Then we placed the mirror 2 metres away from the base of the object
- 4) A member from our group (Leslie) then backed away from the mirror away from the object until she could see the object in the mirror
- 5) Then we measured the distance from Leslie to the mirror which was (1 metre and 5 cm)
- 6) Then we had to measure the height of Leslie (1 metre and 54 cm)
- 7) We then used the triangle method to calculate the height of the object
- 8) This is showed on our geogebra diagram

WRITTEN BY Margaret

Comments (7) Delete all comments

nobody said
 at 11:54 am on Jan 10, 2012
 Reply Delete

For step 4 (above): use something to place where you are in terms of the mirror and measure after because it would be easier when you aren't the object

jj said
 at 11:54 am on Jan 10, 2012
 Reply Delete

for step 2 how do you know how far back to place the mirror? (M,M)

hewhomustbenamless said
 at 11:59 am on Jan 10, 2012
 Reply Delete

Make sure the "triangles" are similar or this won't work

Figure 8: Collaborating on the wiki

approach to co-constructing criteria for participating in a math-talk community, we developed criteria for collaborating on a wiki. These criteria, which emphasized expanding on their peers contributions rather than merely agreeing or disagreeing, were included in a communication rubric for evaluating participation in the online discussions.

Stage 6: Combining Skills

Students combined their technological and mathematical skills in an individual task. Each student selected a different inaccessible object and applied the group-generated strategy to determine the height of that object. For this individual component of the performance task, students were encouraged to submit their work on a wiki page (Figure 9) or in a form that they felt was most appropriate (e.g., paper and pencil).

Performance Task**Solving Problems Related to Similarity****Curriculum Expectation**

Use your knowledge of ratio and proportion to investigate similar triangles and solve problems related to similarity.

Task:

- i. Choose an inaccessible object, the height of which you will measure.
- ii. Select an appropriate method for measuring the height of this object. Choose one from the back of this page or develop a new one, if you wish.
- iii. Think of how you can use the tools and a diagram similar to the one shown to measure the height of the object.

Part I: Online Collaboration and Jing Video for Developing a General Strategy

- iv. Visit lazarusmath.pbworks.com and go to the *Student Work* page. Click on *Height of an Inaccessible Object*. Select a link to the method that you chose and post your strategy.
- v. You must respond to the strategies posted by other members of this group. You can use text, a GeoGebra model, and/or a Jing video to share your thinking.
- vi. Then, with a partner or in a group of up to four people, you must create a Jing video that summarizes the strategy. Use GeoGebra to model your work for this video. Refer to the ideas that are already posted to the web page.
- vii. You will post this video to a new page on the website. Miss Lazarus will help with this. Your contributions will be graded using the rubric generated by our class for *Thinking and Communication*.

Part I is due by _____.

Part II: Measuring the Height of Your Object

- viii. Apply the strategy that your group came up with to measure the height of the object that you selected in (i).
- ix. Your solution may be submitted in writing or posted to your web page, using a Jing video. This is your choice. For this final solution, you must summarize all steps and calculations for measuring the height of your object. Refer to the strategy that your group generated. You should include at least one diagram in your own solution. Explain how accurate your diagram is and consider any possible sources of error. This individual report will also be graded using a rubric for *Thinking and Communication*.

Part II is due by _____.

Findings and Considerations

Blended collaboration provides new ways for students to interact with each other and with mathematical ideas. Combining a wiki, dynamic mathematics software, and screencast videos influenced how students questioned, reacted to, and elaborated on ideas. When a GeoGebra applet was shared on the wiki, for example, students could elaborate on the construction or explore other possibilities by manipulating the object and then re-posting to the wiki page. Screencast videos were also valuable, as they allowed students to post more detailed explanations as they recorded their screens and voices while interacting with objects.

Since the wiki and GeoGebra exist on the Web, access was available beyond class time, and students were encouraged to use these tools on their home computers or on the computers available in the school library. Taking into account that in some cases computer access was limited, when students collaborated online, they were generally given class time, and the activities were extended for longer periods so that they had sufficient time to visit the library, if they needed to. Initially, student commitment to using the online tools varied. There was a wide range of experience, skills, and motivation. As students gained computer access, experience, and comfort, however, they also tended to gain enthusiasm. They demonstrated this on the wiki, where in addition to sharing mathematical thinking, they were also posting jokes, season's greetings messages during the holiday, and thank-you messages.

Introducing tools in stages helped students develop the required skills and reduced the potential for frustration, but still some had considerable difficulty mastering all of the online activities. This range in skills raised issues related to assessment and evaluation. In terms of formative assessment, the online collaborative component provided a lens into student thinking that was valuable for supporting student learning. The range in technological skills, however, presented a problem for summative assessment that we addressed by giving more in-class support and providing students with choice in how they submitted their work. In adapting to meeting the technological needs of students in this class, we question the implications for assessment and evaluation in a technology-rich learning environment.

Our experience with this project showed that a wiki, GeoGebra, and Jing can be combined to provide an online collaborative environment that complements in-class activity. Looking back over the project, we see

My PBworks Workspaces lazarusmath

Wiki Pages & Files Users Settings

VIEW EDIT

proach similar triangles task

last edited by Page history

E-G is the total Height of Object
 First, measure the length from your eyes to the ground (in our example, 1.5m)
 Then measure the length of your arm (L) and hold the ruler so that the same distance is protruding vertically from your hand.
 Walk backwards until you see just the very top of your inaccessible objects over the metre stick.
 Mark the spot you are standing at, and measure from that spot on the floor to the object.
 You now have 2 similar triangles.
 Since $L \propto 3.5$ gives us the length from the person to the object, $L \propto H$ will give us 'H'.
 To get the total height of the object, add the eye level measurement (1.5m) to 'H', that is the height of your inaccessible object.

MathTalk.org - A Math Talk Resource
 PBworks Special Triangle

Figure 9: Combining the wiki, GeoGebra, and Jing

many ways in which student opportunities to share online could be increased or enhanced. Even though we may not have explored all of the possibilities, we found that students did benefit from sharing their work and exploring contributions on the wiki.

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