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Ontario Mathematics Gazette

OAME – ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION

AOEM – ASSOCIATION ONTARIENNE POUR L'ENSEIGNEMENT DES MATHÉMATIQUES

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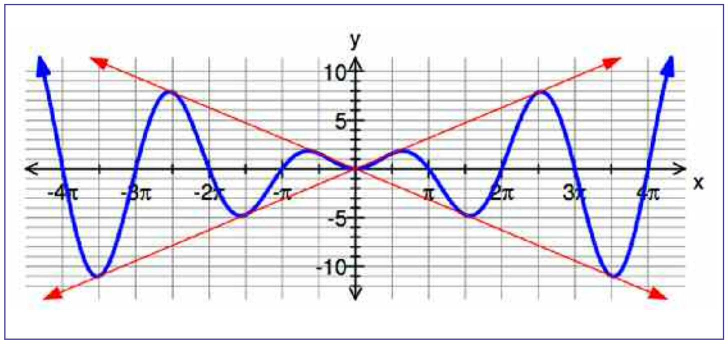

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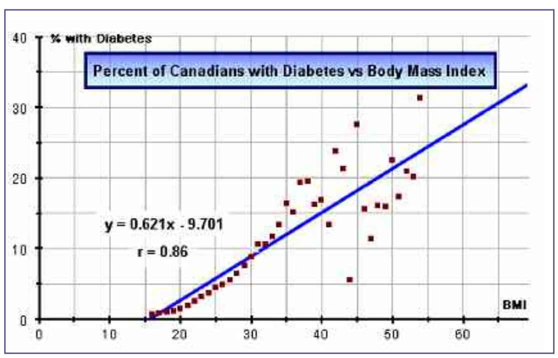

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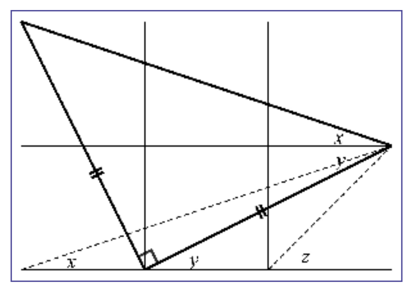

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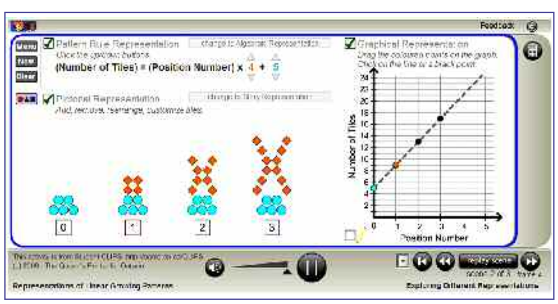

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Submission of Articles

The *Ontario Mathematics Gazette* (*OMG*) is looking for news items, articles, and good ideas that are useful to mathematics teachers and mathematics teacher education. We are seeking submissions, preferably from mathematics teachers K–12 and other mathematics education professionals, that describe innovative and creative approaches to mathematics teaching.

Please keep in mind the following criteria when making submissions to the *OMG*:

- The ideas/activities must be of interest to the readership.
- The ideas/activities must be fresh and innovative.
- The mathematics content must be appropriate for the readership.
- The mathematics content must be accurate.
- The article must be well written and easily understood.
- The article and its ideas must be free of sexual, ethnic, racial, or other bias.
- The article must not have been previously published, nor should it be out for review by other publications.
- The article must be original.

Articles must be word-processed in MS Word, double-spaced with wide margins, not exceeding 10 numbered pages of text, and prepared according to the *Publication Manual of the American Psychological Association, Sixth Edition*. Figures and diagrams should be drawn by computer, if possible, or drawn in black ink in camera-ready form. Embedded images must also be submitted separately in jpeg or tif format. Proof of the photographer's permission is required, and for **photos of students** under the age of 18, the written permission of a **parent or guardian is required**.

You must submit **one complete copy** of your article, embedded with any tables, figures, and captioned photographs or graphics, to the Editor, Marian Small, along with **separate files for each of the text, graphics, and/or photographs**. Please e-mail all files to Marian Small at **marian.small@gmail.com**.

Your name should not appear anywhere in your article, including websites, so that your article can be sent out for blind review. Your name, full mailing address, and e-mail address must be included on a separate sheet. Upon review, you will be notified as to whether your article has been accepted for publication (as is, or pending minor or major revisions) or rejected.

The Editor reserves the right to edit manuscripts prior to publication. Once an article is published, it becomes the property of OAME.

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▲ About the Ontario Mathematics Gazette

The *Ontario Mathematics Gazette*—ISSN 0030–3211—is indexed in the Canadian Education Index and is published four times per year. Its Canadian Publication Mail Product Sales Agreement Number is 40051074.

▲ Editor's Message

MARIAN SMALL

I was travelling on a plane recently and got into a conversation with my seatmate. It turned out that she was a Grade 4 teacher. We had a great discussion about her approaches to teaching math. One of the things that she said struck me enough that I thought it was worth mentioning in this editor's message; it's about her textbook.

This teacher told me that she values the textbook she uses, yet she feels that she is being pressured not to use textbooks anymore. It was even suggested that she had better enjoy it while she could, since soon it would not be allowed. Her concern is that she is not knowledgeable enough, herself, to create the rich problems or the follow-up questions that the textbook provides. It is her opinion that the people who wrote the books know a lot more about math than she does, so she did not understand why this "ban" on textbooks was coming. I have been to a number of boards where this has come up. It is often consultants or coaches who are, perhaps inadvertently or perhaps not, leading teachers to believe that they are doing something wrong if they ever have students open a textbook.

Although I am a textbook author and am disclosing this potential "conflict," speaking as a professional, I think that telling teachers they are harming their students by using texts is the wrong message. What I believe that we want is for teachers to be thoughtful about whatever they use, whether a textbook, problems downloaded from the Internet, the TIPS materials developed by Ontario teachers and posted by the Ministry, activities shared by colleagues, or some combination thereof. We want teachers to consider the students in front of them, and how the materials they use need to be adapted to be appropriate for those students, and perhaps adapted differently for different students. We want teachers to ensure that they use whatever resources they do in a way that supports students to become more independent thinkers, which might mean denying them access to scaffolds or direct instruction provided in those resources until or unless it is deemed appropriate.

The curriculum for which teachers are responsible is provided in Ministry documents, and teachers should be aware that there could be material in texts they choose to use, particularly older ones, that is no longer relevant, or there could be material missing. The curriculum document is the launching-off point; teachers should be and must be free to use their professional judgment to employ whatever resources are accessible to them to do the best job they can for their students. In the U.S., if you ask teachers what curriculum they are following in math, they normally cite the name of their textbook. Canadian teachers seem to be more aware that the textbooks to which they have access are resources and not the curriculum. In fact, to successfully complete the curriculum, they must pick and choose what parts of that resource or any others they use make sense.

Some argue that the evidence that texts are the wrong way to go is that students don't like them or that students would prefer not to use them. Because texts are dense, static print documents, it is no wonder students would find it difficult to have to navigate them on their own. It is not surprising at all that no matter how hard editors work to simplify text, it will never be simple enough for some students. Sometimes, teachers see the alternative as work on a single activity or single problem that the teacher provides orally; that could be a good thing, but often, perhaps not always, there needs to be follow-up, where the mathematics underlying that activity is explored from a number of different angles. When students tell teachers they don't like their texts, it is possible that it is not really about the text at all, but the appreciation of having only one question to which to respond and not many questions.

Some might worry that if the message isn't unequivocably "Don't use the textbook," teachers will not be able to be judicious in their use of the texts. I think we should give teachers more credit than that.

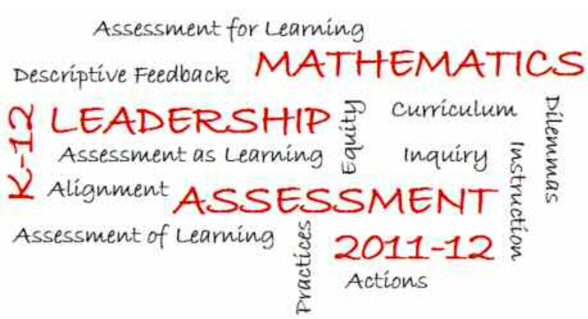
Perhaps rather than denying teachers access to these resources, it would be useful for numeracy leaders to help teachers learn to personalize these materials for their own students in the interest of developing strong mathematical thinkers. It would also be helpful if the system could find ways to provide time and support for teachers to work with colleagues to consider the questions or activities provided in resources they use and really analyze their purpose and whether that purpose suits their own instructional goals. I think my acquaintance on the plane was correct—it is not reasonable to expect a teacher, particularly an elementary teacher without much math training and lessons in many subjects to prepare each day, to always start from scratch. ▲

▲ President's Message

CONNIE QUADRINI, PRESIDENT

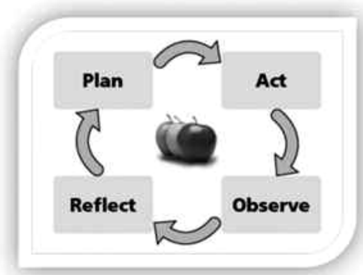
Connie Quadrini is currently the Grades 7–12 Mathematics Program Consultant for the York Catholic District School Board. Her current work focuses on facilitating networked, job-embedded professional learning involving Grades 4–12 teachers within families of schools, as well as supporting school teams in the collaborative development and implementation of school improvement plans. Connie has served as an OAME director for 6 years and Y4MA executive member for 13 years. She was the co-chair of the OAME 2008 Annual Conference, and was the recipient of the OAME Union Gas Award for Outstanding Leadership in Mathematics Education in 2003. Connie has taken a leadership role in numerous Math GAINS initiatives, including Steering Team Lead for the 2008–2009 Coaching for Math GAINS; co-organizer of Math CAMPPP 2008, 2009, 2010, 2011; and co-facilitator of several professional learning series, including the Mathematics Coaching Cycle, PRIME Leadership Framework Adobe Connect Book Study, and Leadership in K–12 Mathematics System Planning. This year, she presents mathematics leaders with an opportunity to explore assessment within a new OAME Leadership Conference format.

OAME 2011–2012 Leadership Conference



This year's OAME Leadership Conference, entitled "Creating Communities of Assessment Inquiry and Practice: A Vision for Leadership in Mathematics," presented a new conference format for our 243 delegates.

The virtual Pre-Adobe Connect session that took place in October offered delegates the opportunity to examine dilemmas involving assessment through two different lenses; the first focused on alignment among curriculum, instruction and assessment, formative assessment, and summative assessment as reflected in NCSM's PRIME Leadership Framework for the Assessment Domain indicators (2008); and the second focused on Windschitl's Framework (2002), which presents four types of dilemmas when implementing "constructivist philosophies of teaching," specifically, conceptual, pedagogical, cultural, and political dilemmas. Delegates were introduced to the Mathematics Assessment Leadership Professional Learning Cycle, which would provide a process for delegates to plan, act, observe, and reflect upon their inquiry.



www.edugains.ca/resourcesDI/ProfessionalLearningCycle/ProfessionalLearningCycleBlankNov2010.doc

During the face-to-face sessions on October 27, 28, and 29, delegates had an opportunity to work collaboratively with colleagues within grade-band and by role breakout sessions. A K–12 continuum of learning mural involving patterning and algebra was created by delegates, with a focus on alignment of curriculum, instruction, and assessment, as well as formative assessment, specifically assessment for learning strategies, and specific feedback based on student thinking. Delegates would then make connections among the categories of achievement, assessment strategies, and triangulation of data (observations, conversations, and products) during the summative assessment segment. Delegates gathered together by role to examine dilemmas involving assessment and use the lenses presented during the Pre-Adobe Connect session to make sense of them, and consider actions that would support themselves and other colleagues in their pursuit to address them.

Before the end of the face-to-face sessions, delegates formed discussion groups based on their inquiry area of interest, using an Open Space Technology approach. Topics included assessment in mathematical inquiry, alignment, balanced assessment planning, using assessment to differentiate learning, success criteria, descriptive feedback, summative assessment, using a variety of assessments, assessing mathematical processes and/or achievement chart categories, assessment within a system (culture), formative assessment, and recording/tracking assessment information. Feedback from the conference indicated that delegates valued the Open Space Technology group discussions and in response, additional opportunities for synchronist conversations focused on these topics were made available in January and February 2012 via Adobe Connect.

Asynchronous conversations continue on the OAME Leadership Conference wiki. You are invited to visit our wiki at **www.oame-leadership-conference-2011.wikispaces.com/** to "e-lurk" into these conversations and explore resources that have been posted to continue to support learning, reflection, and action related to these topics.

In March 2012, delegates will engage in a Post-Adobe Connect session to share the journey of their Assessment Leadership Inquiry, solicit feedback and suggestions from colleagues, and explore potential collaborations based on common interests of inquiry. Delegates will have the opportunity to reconvene at the OAME 2012 Annual Conference in Kingston during a Leadership Conference sharing session, as well as attend other sessions that will continue to delve deeper into assessment concepts that were presented during the October 2011 face-to-face sessions.

I have valued conference feedback from delegates. Thank you for your willingness to share your "aha!" moments, challenges, successes, and needs. A special thank you to the OAME Leadership Team, Dr. Christine Suurtamm, plenary speaker, and facilitators, Shirley Dalrymple, Anna Jupp, Mary Lou Kestell, Caroline Rosenbloom, Liisa Suurtamm, and Shelley Yearley for sharing their expertise and for their dedication to the conference, the OAME Executive Directors, Fred and Lynda Ferneyhough, for all their support and help in organizing the conference, and to delegates for their willingness to embrace a new OAME Leadership Conference format. I am grateful for the opportunity to have learned with you, and look forward to hearing about the blooms that are produced in classrooms, schools, school boards, and OAME local chapters as a result of the assessment seeds that were sown this year.

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National Council of Supervisors of Mathematics (NCSM). (2008). *The PRIME leadership framework: Principles and indicators for mathematics education leaders.* Denver, CO: Author.

Windschitl, M. (2002). Framing constructivism as the negotiation of dilemmas: An analysis of the conceptual, pedagogical, cultural, and political challenges facing teachers. *Review of Educational Research, 72*(2), 131–175. ▲

▲ Technology Corner – Investigating Combinations of Functions

MARY BOURASSA

Mary teaches mathematics at West Carleton Secondary School in Ottawa. She is a strong advocate for the appropriate use of technology in the classroom. She has presented workshops internationally, authored mathematics resources, is a past VP of OAME and a Past President of COMA. An award-winning teacher, Mary continually strives to learn new and better ways of helping students learn and love mathematics.

Combinations of functions are the culmination of the Advanced Functions course, yet despite having a solid understanding of the individual functions, combining them can be quite challenging for students. Although not seen as such, many simple combinations are investigated throughout the secondary academic curriculum. These are generally presented in terms of transformations, such as adding a constant function to a polynomial function, creating a vertical translation, or multiplying a function by a constant to create a vertical stretch or compression. However, more complicated combinations are made accessible to students with graphing technology.

At my school, we start by having students explore what the graphs of the combinations of simple functions look like. They are provided with two sets of cards, which they must match up. Some have two individual functions and others have one combined function, which could be created by adding, subtracting, multiplying, or dividing the original functions. This exploration allows students to start making connections between the different types of functions they know and the properties they find in the combined function. They use graphing technology to answer the "what if" questions that arise amid their discussions.

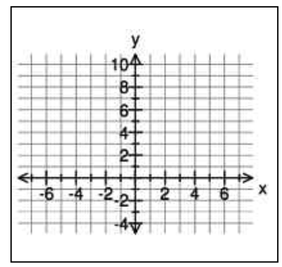
This activity is followed by a more formal investigation. Students are given sets of functions and particular combinations in order to understand how combining them works numerically, graphically, and algebraically. Here is the beginning of the investigation, each of the sets of functions, and the resulting graphs (students are not given these!).

Given each of the following sets of functions, complete the graphs/sketches and tables below. Use a graphing calculator as needed.

Set 1

x	*f(x)=*	*g(x)=*	*h(x)=*	*k(x)=*	*m(x)=*
-2					
-1					
0					
1					
2					

$f(x) = x^2$
$g(x) = 3$
$h(x) = f(x) + g(x)$
$k(x) = f(x) - g(x)$
$m(x) = g(x) - f(x)$

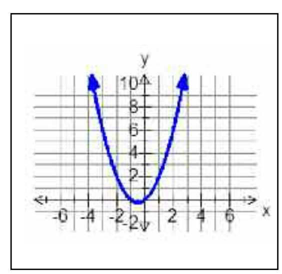


	Degree	Domain	Range	Even/Odd/ Neither	Interval(s) of Increase	Interval(s) of Decrease	Max./Min. Points	# of Zeros
f(x)=								
g(x)=								
h(x)=								
k(x)=								
m(x)								

The same type of table as the second one shown above is included with each set of functions. The degree column is only included for polynomial functions. Students are also asked to draw conclusions as they work through the investigation.

Set 2

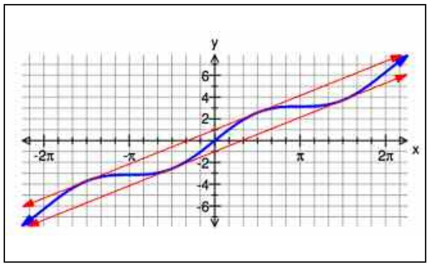
$f(x) = x^2$
$g(x) = x$
$h(x) = f(x) + g(x)$



Students should notice that if they factor the combined function $h(x) = x^2 + x$, they get $h(x) = x(x+1)$, which is a parabola with zeros at -1 and 0.

Set 3

$f(x) = x$
$g(x) = \sin x$
$h(x) = f(x) + g(x)$

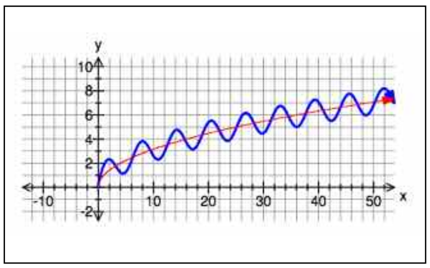


* Add the lines $y = x + 1$ and $y = x - 1$ to your graph.

The extra lines provide the envelope of the graph and make the amplitude of the graph more obvious. The line defined by $f(x) = x$ is the axis of the curve.

Set 4

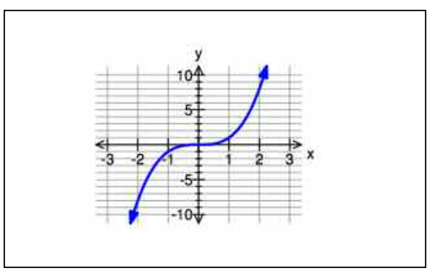
$f(x) = \sqrt{x}$
$g(x) = \sin x$
$h(x) = f(x) + g(x)$



When students see the graph of $f = \sqrt{x}$ on the same grid as the combined function, it becomes clear that this is the axis of the curve. This may be the first time they see that the axis of the curve can itself be a curve. Cool!

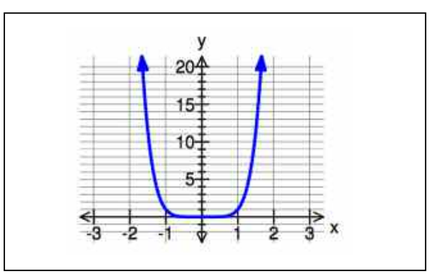
Set 5

$f(x) = x^2$
$g(x) = x$
$h(x) = f(x) \cdot g(x)$



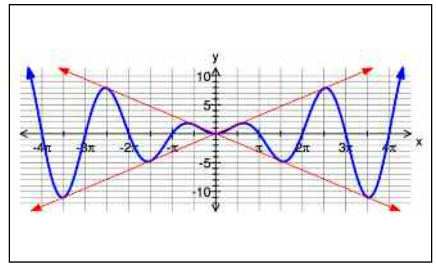
Set 6

$f(x) = x^2$
$g(x) = x^4$
$h(x) = f(x) \cdot g(x)$



Set 7

$f(x) = x$
$g(x) = \sin x$
$h(x) = f(x) \cdot g(x)$

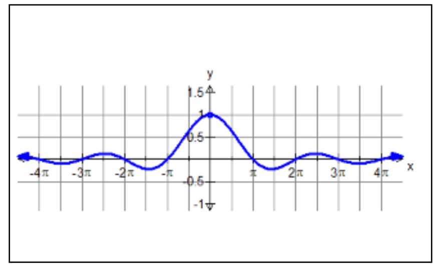


* Add the lines $y = x$ and $y = -x$ to your graph.

Another interesting envelope is shown here.

Set 8

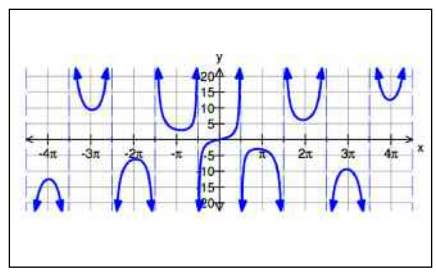
$f(x) = \sin x$
$g(x) = x$
$h(x) = \dfrac{f(x)}{g(x)}$



Although it may not be visible on the graph, there is a hole at $x = 0$. Since the value of the numerator varies between -1 and 1, the value of the combined function will approach zero as the value of the denominator becomes a larger positive or negative number.

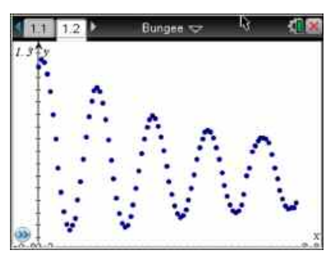
Set 9

$f(x) = x$
$g(x) = \cos x$
$h(x) = \dfrac{f(x)}{g(x)}$



This produces another interesting graph with vertical asymptotes corresponding to the zeros of the cosine function.

Students are now well positioned to model real-world data such as CO_2 emissions or damped harmonic motion. An easy way to collect data for the latter is using a CBR, on the floor, and a weight suspended from rubber bands. The trick to get the weight moving up and down before the data collection begins is to hold still for the duration of the data collection. An example of the result is shown here.

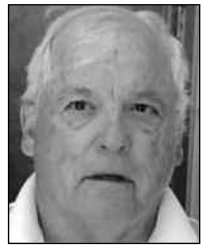


This data can be modelled with an exponential decay function and a sinusoidal function. This activity is rich, as it requires students to correctly model the exponential decay and correctly model the sinusoidal function, and then combine them in a logical manner. The exponential decay represents the changing amplitude, so it must be multiplied by the sinusoidal function.

Some teachers may think that they don't have time to allow students to investigate concepts, but with a well-designed investigation and the appropriate use of technology, students can learn more, have important discussions with their peers, and retain the mathematics longer. They are also taking ownership of their learning and having fun along the way. ▲

▲ OAME/NCTM Report March 2012

TODD ROMIENS



Todd recently retired from teaching K–12 methodology in the Faculty of Education at the University of Windsor. He is a past president of OAME and a life member, as well as a previous editor of the Gazette..

Pi day is this month and I hope you will be able to celebrate it with your students. As you probably know, the most special part of that day, March 14, is 1:59 a.m. (03 14159). March 14 is also Albert Einstein's birthdate, as he was born on that day in 1879. Pi is an irrational number, continuing infinitely without repeating. It is usually estimated to the hundredths place, but with the use of computers, it has been calculated to over a trillion decimal places. I have a friend who has memorized over 6000 decimal places of pi. Although the ratio has been around for about 4000 years, the symbol (π) just turned 300 years old in 2006. The symbol for pi was first used in 1706 by a Welsh man, William Jones. The symbol was made popular after the Swiss mathematician, Leonhard Euler, adopted its use in 1737.

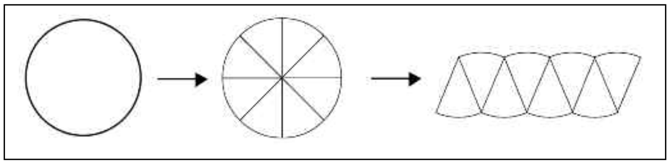
NCTM has produced a number of lessons/activities that you can access through the "illuminations" website. Some I found to be fairly routine and well known, while others were more interesting and perhaps lesser known. A few are outlined here.

Activity 1.

Using a variety of circular objects, a piece of string, and a ruler, students investigate the ratio of the circumference of the circle to the diameter. It is important that they measure accurately so they come up with an accurate approximation to π, i.e., $\frac{C}{d} = \pi$ or $C = \pi d$ or $C = 2\pi r$.

Activity 2.

Students can find the formula for the area of a circle by dividing the circle into sectors:



then rearranging the sectors in the shape shown

As the number of sectors becomes larger, the shape becomes closer to a rectangle with length πr and width r. Hence the area of the circle becomes πr^2.

Activity 3.

In this activity, students measure the diameter and circumference of a variety of circular objects (data from activity 1 can be used), plot the measurements on a graph, and relate the slope of the line to π. There are detailed notes showing how to carry out this activity, as well as activity sheets to help students with the graph. This is a nice activity for Grade 9, as it deals with lines, slopes of lines, intercepts, as well as the formula

$$\frac{C}{d} = \pi \; .$$

Activity 4.

Archimedes was a Greek mathematician who was born in 287 BC and was killed in 212 BC by a Roman soldier. He is credited with advancing the fields of physics, engineering, and mathematics. Among his mathematical accomplishments was a method to approximate the value of π. His method was to look at the area of inscribed and circumscribed polygons around a circle with unit radius.

Students need to be familiar with the trigonometric functions, as well as the formula for the area of any regular polygon, i.e., $A = \frac{1}{2}(ap)$, where p is the perimeter of the polygon and a is the length of the apothem (the perpendicular distance from the centre of the polygon to the midpoint of a side). Students could verify that this formula works for the area of an equilateral triangle as well as a square. NCTM provides two detailed lessons with activity sheets that lead students through the method used by Archimedes to approximate the value of π. Some nice mathematics ensues.

On another note, I mentioned in the last *Gazette* issue that NCTM is moving into the realm of YouTube presentations. You can find the first one at **www.youtube.com/watch?v=SkxJ3XbcB38**.

It outlines a "magic number trick" that I have used many times with many grade levels, with great success. You start with a number grid of any square dimension; I like a 5 x 5, although the video uses a 4 x 4, which is fine. Show the grid to your students without saying how it was composed. Ask a student to pick any number on the grid. Circle this number and stroke out all the other numbers in the same row and column of the number picked. Continue this process until all numbers are either circled or crossed out. Add up the circled numbers chosen at random by your students. Of course, you already know what the sum will be, based on how the grid was made. Now amaze your students by having this sum placed in a sealed envelope or, as in the movie, concealed on your arm and revealed by rubbing eye shadow over it. To see how all of this can be done and why it works, visit the website. ▲

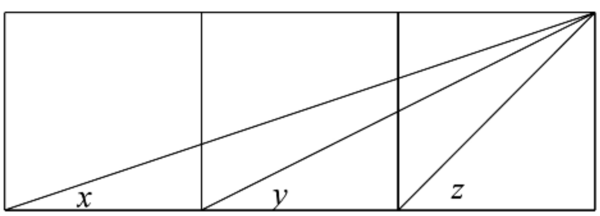
▲ What's the Problem? Making it Right

SHAWN GODIN

Shawn is the head of Mathematics, Technology, and Family Studies at Cairine Wilson Secondary School in Orleans, Ontario. He is a past and present member of the OAME Board of Directors and is currently Editor-in-Chief of Crux Mathematicorum with Mathematical Mayhem.

Welcome back problem solvers. Last time, I left you with the following problem:

The diagram shows three squares and angles x, y, and z. Find the sum of the angles x, y, and z.



This problem is question #4 from the *2009 Maritime Mathematics Competition*. It appeared in the column *Skoliad Corner* from the November 2009 issue of *Crux Mathematicorum with Mathematical Mayhem*. The solution appeared in the September 2010 issue and an additional solution and comment appeared in the April 2011 issue. You can check out more problems at the *Crux Mathematicorum with Mathematical Mayhem* website at **www.journals.cms.math.ca/CRUX/**.

We can attack this problem in a number of ways. At the most basic level, we can measure the angles either by hand or with the aid of software like the Geometer's Sketchpad. Upon measuring, we see that $x + y + z = 90^\circ$. This leads us to think that there might be a more attractive solution to the problem.

Readers Robert Keightley and Peter Harrison sent in several trigonometric solutions. Their solutions ranged from using the primary trigonometric ratios to determine the angles, to using the addition formulas for the sine and tangent functions (separately) to determine $x + y = 45^\circ = z$. A similar solution is shown below using the cotangent function and the identity

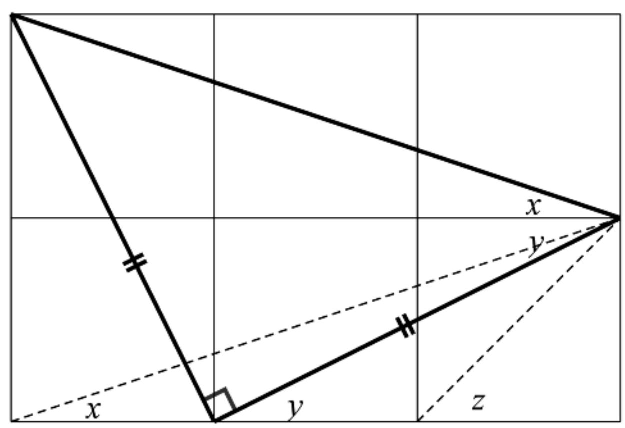
$$\cot(A+B) = \frac{\cot A \cot B - 1}{\cot A + \cot B}.$$

Using the identity twice, we get

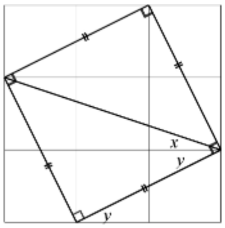
$$\begin{aligned}\cot(x+y+z) &= \frac{\cot(x+y)\cot z - 1}{\cot(x+y) + \cot z} \\ &= \frac{\dfrac{\cot x \cot y - 1}{\cot x + \cot y}\cot z - 1}{\dfrac{\cot x \cot y - 1}{\cot x + \cot y} + \cot z} \\ &= \frac{\dfrac{3\times 2 - 1}{3+2}\times 1 - 1}{\dfrac{3\times 2-1}{3+2} + 1} \\ &= 0\end{aligned}$$

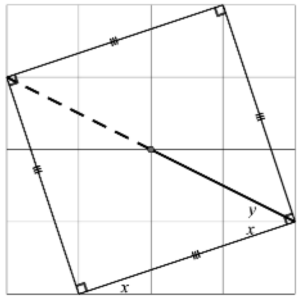
Since the three angles are acute, we must have $0^\circ < x + y + z < 270^\circ$. Thus, the only possibility in this range is $x + y + z = 90^\circ$. You may want to see for yourself if you can come to the same conclusion, using the addition formulas for other trigonometric functions.

David Gilbert sends a nice geometric solution. If some of the segments are repositioned, you get the diagram below.



Notice the bold triangle is right isosceles. Thus, $x + y = 45^\circ = z$. The construction could be extended so that the hypotenuse becomes a diagonal of a square. Interestingly, if four copies of the longer segment are used as sides of a square, then the diagonal will be two copies of the middle segment and the same result follows.





Finally, Peter Harrison sent in one more solution that uses similar triangles. I will leave it to the readers to discover the similar triangles in the original diagram.

Now for your homework:

Two cans, X and Y, both contain some water. From X, Tim pours as much water into Y as Y already contains. Then, from Y, he pours as much water into X as X already contains. Finally, he pours from X into Y as much water as Y already contains. Each can now contains 24 units of water. Determine the number of units of water in each can at the beginning.

Until next time, happy problem solving. ▲

▲ Hey, It's Elementary – It's Time to Advocate for Who We Are and What We Do

LYNDA COLGAN

Lynda Colgan's career has included roles as a classroom teacher, a university professor, and newspaper columnist. Her contributions to mathematics and its teaching have been recognized through awards such as the Marshall McLuhan Foundation Distinguished Teacher Award. Lynda always exhibits a passion for mathematics and views her professional mission as dispelling the myth that math is the bad guy.

Have you heard what's being said about you (the elementary math teacher) and me (the Faculty of Education professor) in the popular press lately? If not… read on.

On Thursday December 15, 2011, Margaret Wente, *Globe and Mail* columnist, wrote an article called *Why Alex can't add (or subtract, multiply, or divide)*. The article begins:

> A parent I know went to an information session about math at his kid's school. After listening to the visiting curriculum expert explain how important it was for students to "understand" the concepts, he asked: "So, how important is it for them to learn the times tables?" The expert hemmed and hawed and wouldn't give an answer.
>
> Parents across Canada might be surprised to learn that the times tables are out. So are adding, subtracting, and dividing. Remember when you learned to add a column of numbers by carrying a number over to the next column, or learned to subtract by borrowing, then practised your skills until you could add and subtract automatically? Forget it. Today, that's known as "drill and kill," or, even worse, "rote learning." And we can't have that.

In her column, Wente goes on to criticize the mathematics curriculum, saying that the focus on *understanding*, rather than *mechanics* at the elementary school level is to blame for low student achievement in calculus, poor pupil attitudes toward mathematics, the booming business in Kumon math franchises, and a two-tiered education system (disenfranchising students whose parents cannot teach them math at the kitchen table or afford personal tutoring).

The December column was her second attack on the topic; the first (*Too many teachers can't do math, let alone teach it*) appeared on Thursday, September 29, 2011.

That column opens with:

> Is your kid struggling with math? Is she flustered by fractions and laid low by long division? Here's a secret: Her teacher may be struggling, too.
>
> An alarming number of elementary-school teachers are so uncomfortable with math, they can't teach it properly.... Across the country, university math professors report that the math skills of students who are studying to become teachers are generally abysmal. Basic skills such as adding fractions or calculating percentages are frequently beyond them.

Wente goes on to argue that both prospective elementary and secondary teachers are woefully underqualified in terms of content knowledge upon completion of undergraduate degrees, and that faculties of education are wrong to admit candidates with only high school level mathematics (for elementary teacher education programs) or one university course (for secondary B.Ed. qualifications).

She continues:

> You might think that the nation's faculties of education—the institutions that teach the teachers—would be concerned about this problem. After all, their job is to ensure that teachers know their stuff by the time they're unleashed on the classroom.
>
> But this concept of teacher training is pathetically behind the times. Today's faculties of education have much loftier goals in mind. According to them, their main job is to sensitize our future teachers to issues of social justice and global inequality.
>
> "Classes in elementary schools have complex human interactions that involve political, racial, economic and gender issues," writes Cecilia Reynolds, the dean of education at the University of Saskatchewan. Her faculty is now considering whether to make the math course an elective—meaning that future teachers wouldn't have to demonstrate any proficiency at all. She thinks math training should be more child-focused, "taking into consideration if that child is aboriginal, if that child has autism, whether that child ate a breakfast that morning." Her own professional interests are in gender relations, equity and social justice.
>
> Dr. Reynolds is a product of the OISE school of pedagogy, by far the most influential in Canada. And improving student achievement through effective teaching methods is not a priority for the Ontario Institute for Studies in Education. It has a research and advocacy arm, the Centre for Urban Schooling, that's designed to connect it with schools in inner cities. As part of its commitment to "social justice and equity for all students," the centre "works collaboratively on education projects that challenge power relations based on class, race, gender language, sexuality, religion, ethnicity and ability in all aspects of education both formal and informal." If only it were interested in math.
>
> Unfortunately, the people who educate the educators sound like the wacky wing of the NDP. Here's Fern Snart, the dean of education at the University of Alberta: "To educate students beyond the superficial," she writes, "we must engage them in transformational processes and deep thinking such that they understand the Western position of privilege that is often reflected in issues of diversity, power, and justice, and that they move to an internalization of responsibility related to this privilege."
>
> No wonder little Emma doesn't know her times tables. She's way too busy learning how her Western position of privilege entrenches gender relations. Or something like that.

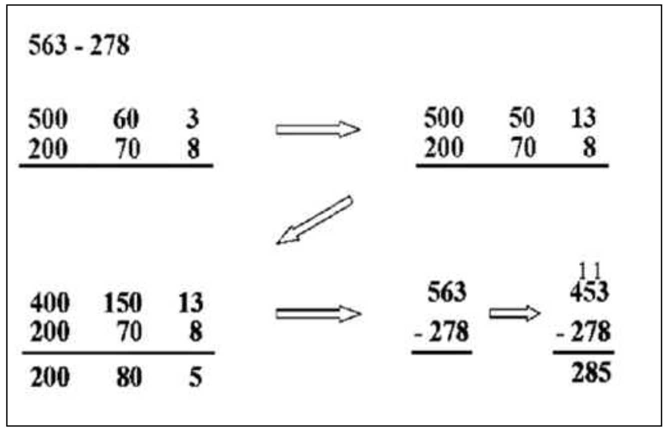
These provocative, verbatim excerpts will give you the flavour of what some 1.3 million readers across the nation read about teachers and teacher educators in just two of Wente's recent education-targeted columns. The first column generated about 1000 written reactions from readers from coast to coast, and the second, about 700. To my dismay, the authors of the responses (members of the general public, including secondary school mathematics teachers, parents, employers, college instructors, and university professors) were almost unanimous in sharing Wente's opinions, praising her for her courage to speak up on these "hot button" issues and congratulating her for "hitting the nail squarely on the head."

Where were the responses from hard-working, front-line elementary teachers who must know that Wente's confrontational assessment of the current curriculum is dead wrong? The Ontario mathematics curriculum policy document emphasizes computational proficiency and procedural fluency: knowing how and knowing when to use specific operations and algorithms. There are explicit expectations that address estimation and the application of mental math strategies. There are sensible progressions that build solid skill sets based on facility with number. One example, for multiplication, is shown below.

When Wente wrote:

> The common methods used to add and subtract are known as standard algorithms. They are efficient and foolproof. But, instead of being taught these methods, students are encouraged to find "strategies," such as breaking numbers into units of thousands, hundreds, tens and ones and working horizontally. It works, but it's not efficient. And every time a student sees a new problem, he has to start from scratch—and pick his "strategy." It's like playing the piano without ever learning scales, or hockey without basic drills.

where were the articulate rejoinders from mathematics curriculum leaders, showing her the error of her thinking? Did she know that she was describing a milestone, not a destination? Did she know that her broad generalizations about "common methods" used to add and subtract are based on her limited familiarity with regional, not universal, algorithms?



Multiplication						
Method	Understand multiplication as arrays Describe an array as repeated addition	Repeated addition on a number line	Grid method	Grid method	Multiply without a grid	Other methods
Recording	3 x 4 = Record mental calculations using x and = signs	7 x 4 = 28 +7 +7 +7 +7 0 7 14 21 28	18 x 5 x | 10 | 8 5 | 50 | 40 50 + 40 = 90	23 x 35 x | 20 | 3 30 | 600 | 90 5 | 100 | 15 600 + 100 + 90 + 15 = 805	17 x 32 10 x 30 = 300 10 x 2 = 20 7 x 30 = 210 7 x 2 = 14 = 544	Other methods may be introduced for more able children to investigate
Points to note	Begin to see links between tables e.g. 7x = 5x + 2x	Make links to division by linking to repeated subtraction	Ensure children understand and use the place value of a number to multiply 2 numbers together Use factorising to help understand multiplication of multiples of 10 e.g. 40 x 30 = (4 x 10) x (3 x 10) = 4 x 3 x 10 x 10 = 1200 Ensure children have necessary addition skills to add up the grid total		Have a method of keeping track of which digits have been multiplied e.g. 17 x 32	
Mental strategies needed	Know that multiplication can be done in any order Begin to know some multiples of 2, 5 and 10	Able to count on in steps of a constant size (know multiples) Use known facts where appropriate e.g. doubles	Know multiplication facts up to 10 x 10 Able to multiply a single digit by a multiple of 10 e.g. 50 x 6 = 300	Know multiplication facts up to 10 x 10 Able to multiply a single digit by a multiple of 10 e.g. 50 x 6 = 300 using known facts Able to multiply 2 multiples of 10 e.g. 60 x 30 = 1800 using known facts		

In reply to her vitriolic diatribe about the pathetic state of teacher education and its focus on social issues such as bullying, where were the voices of mathematics education professors (including me!) to counter that bullying is a mathematics issue and it is addressed, not at the expense of mathematics content, but as a supplement, complement, and extension to discussions about the complexities of teaching mathematics in the complex arena of today's classroom?

No doubt, you are familiar with the expression perception is reality. What the expression means is that what people see in the world around them is not the world as it is... it is the world as they believe it is. What this expression tells us is that how others see a person, event, or situation is their reality, no matter what might have been intended.

The lesson behind the expression is to see ourselves (and in this case, our curriculum and our classrooms) as others do, so that we can understand our blind spots (tricky because we don't realize they exist) and know our weaknesses (so that we are able to compensate for them).

If, as mathematics educators, our collective weaknesses and blind spots have gotten us into trouble in terms of the pan-Canadian public perception that ***is*** reality, then it is time for ***all*** of us in the mathematics education community to stand up and be counted—to let people know our opinions and speak the truth about what is happening in math class and why. Mathematicians and the people who support teaching and learning of mathematics (from teachers to teacher educators, curriculum leaders, Ministry officials, university faculty, and policy makers), must work collectively and with one single purpose to change perceptions and encourage more students to study mathematics; share mathematics' ubiquity and worth; and champion the subject and its best teaching practices. We must actively work to dispel myths by celebrating student mathematics achievements as loudly and proudly as we acknowledge sports triumphs. We need to shout from the rooftops: everyone can learn and, more importantly, enjoy mathematics. We must have convincing stories that shine light on the fact that math is an essential life skill, and like reading, must be an essential component of every child's basic education. We are obligated to shine the light on mathematics as a human activity carried out by creative, hard-working everyday people, rather than some sort of magic pursued by and done by an elite few. We need to find the content that will inspire and fire students' imagination and enthusiasm. We need to challenge the media… from the popular CBC host who regularly and proudly declares his hatred of all things "mathy" to the outrageous rantings of biased and uninformed provocateurs.

If we, as a united community of mathematicians and mathematics educators, do not confront the enormous conflict between what mathematics is perceived to be — boring and irrelevant, poorly and inexpertly taught, and what it actually is—highly applicable, highly valued by industry, and at the cutting edge of science; taught conceptually, with depth and to high standards—then who will? ▲

▲ Data Rocks – New Health Data for Teaching MDM4U

JOEL YAN

Joel Yan, a volunteer education consultant for Statistics Canada, is a mathematician who is keen to help educators use Canadian data in mathematics teaching and learning.

Why use the updated health data for teaching MDM4U?

Students are intrinsically interested in health-related topics. To explore these topics, they quite often go hunting for data to analyze and quickly discover that health data are not readily available. Statistics Canada is the primary source of social and economic data in Canada, and one of its surveys, the Canadian Community Health Survey (CCHS) contains data that align with students' natural interests. A previous edition of the CCHS microdata was provincially licensed through the Ontario Ministry of Education in 2008. This meant that the 2005 data were available to all publicly funded school boards and faculties of education through the Ontario Software Acquisition Program Advisory Committee (OSAPAC) and their contacts in each board across Ontario. For further details, see our article (by Yan, Temmer and Steinke), published in the September 2008 edition of the OAME *Gazette*.

The updated Canadian Community Health Survey (CCHS) Public Use Microdata File (PUMF) for 2009/2010 and 2010 is now available for free from Statistics Canada (please consult the "Getting started" section below for directions to obtain the PUMF). As with previous releases, detailed data are provided for a representative sample of over 130 000 Canadians. This PUMF actually contains two separate microdata files, each with over 1250 variables, one for 2009/2010 and the second for 2010. The *Beyond 20/20* software, which is included, provides fast extraction from this large data set and automatic aggregation of the data to selected geographic levels. Detailed documentation including the complete questionnaire, user guide, and more are provided with the data and software on the CD-ROM. Additional teacher support documentation is available online at **www.teacherweb.com/ON/Statistics/Math/photo7.stm** (website maintained by Joel Yan, independent of Statistics Canada). This site provides:

- links to ten online instructional videos funded by OSAPAC
- step-by-step user guides
- teaching strategies developed by teachers across the province
- exemplary student culminating projects based on these data

What is provided on this CCHS CD-ROM?

The CD-ROM contains data from Statistics Canada, as well as a browser (*Beyond 20/20*) to help access and extract data. The data sets are the 2009/2010 and 2010 Canadian Community Health Survey (CCHS) Public Use Microdata Files (PUMF). The data are about the health of the Canadian population, aged 12 and older, and related socio-economic data. The survey is designed to provide reliable cross-sectional estimates at provincial, territorial, and health region levels.

All common content modules, for which questions were asked of all survey respondents, can be used to produce estimates at all three geographic levels. On the other hand, optional content modules vary among provinces. For 2009/2010 and 2010, certain modules were only selected by certain provinces. Based on the Ontario selection, the 2009/2010 and 2010 PUMF provides provincial and health region estimates on questionnaire modules such as:

- access to healthcare services
- weekly alcohol usage
- dental visits
- driving and safety
- patient satisfaction with health care services
- and much, much more

Getting started

To obtain a copy of the Canadian Community Health Survey: Public Use Microdata File, 2009/2010 and 2010 (82M0013X, free), or to ask questions about the CCHS data, contact Statistics Canada's National Contact Centre (613-951-8116; toll-free 1-800-263-1136; **infostats@statcan.gc.ca**), Communications Division.

The *Beyond 20/20* browser must be installed to access the CCHS data. Installation instructions are provided with the CD-ROM. Once you have installed the *Beyond 20/20* browser, you can begin exploring all that CCHS data have to offer and perform cross-tabulations on multiple variables. For example, as shown in graphs below, you can examine the rate of diabetes by age, gender, BMI, and physical activity level.

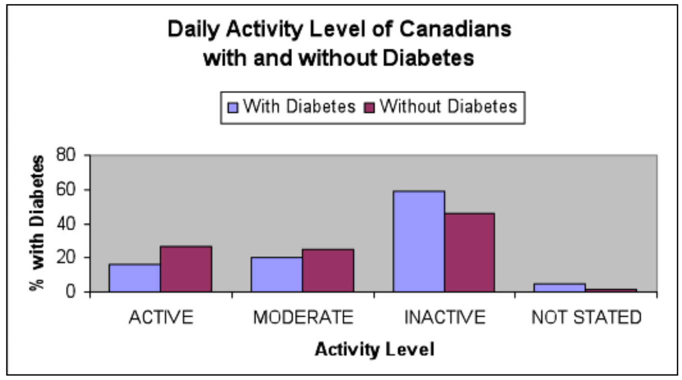
Note: The *Beyond 20/20* browser does not work on Mac computers. However, data can be downloaded on a Windows computer and then analyzed on a Mac, using *Fathom*, Excel, or other software.

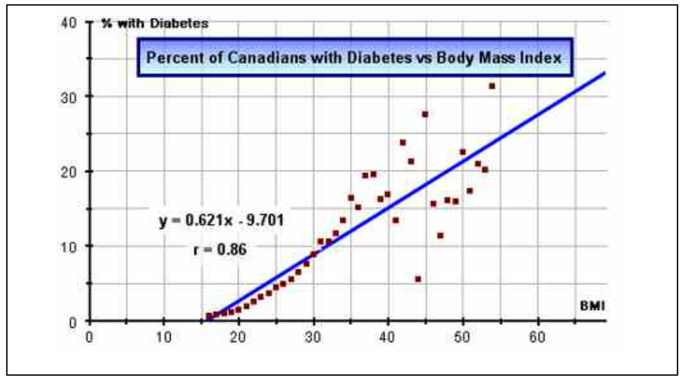
What teachers are saying about the CCHS microdata

The CCHS data set is tremendously useful for my MDM4U data students. The data sets are relevant, current, and available for a variety of time periods for a time-series analysis. I really like that the students can choose a wide variety of attributes to examine for a variety of age groups, socio-economic groups, by geographic location, etc. It allows the students the flexibility to explore issues that are particularly interesting to them. The data are easily examined using the provided Beyond 20/20 *software or exported into Excel. The vast micro health data set allows the students to investigate correlations like an actual statistician would!*

Kelly Blair, Rideau District High School, Upper Canada District School Board

*We have been using the CCHS microdata in our Data Management classes for at least five years. We were first introduced to the data (Version 1.1) at an OAME provincial conference in 2004 and were elated to have finally found a data source that was interesting to students, large enough to do reliable statistical analysis, and accessible to all students in the class. Previously, students struggled to find good data with large enough samples to do histograms. They are no longer frustrated, and the investigations that they are able to produce are much more meaningful and rich than those prior to the discovery of the CCHS microdata. With much collaboration between ourselves and a close colleague, Betty Temmer, we have been able to use this data with our students and also share our teaching practices with other colleagues in our board and in the province. Sample graphs we have produced using the data (*Beyond 20/20 *and* Autograph, *respectively) are shown below.*





Source for these three graphs: Statistics Canada, Canadian Community Health Survey Public Use Microdata file on CD-ROM , Catalogue number 82M0013XCB, cycle 3.1, 2005.

We hope to be leading a workshop at the 2012 OAME provincial conference in May and look forward to sharing with others how this data set can be effectively used for the statistical investigation in the Grade 12 Data Management course. Please come to this workshop or feel free to email us if you have questions about using CCHS with your classes. Shelah Pickett and Patricia Misner, Grand Erie District School **Boardshelah.pickett@granderie.ca patricia.misner @granderie.ca**

Other related data resources

There are several other health-related resources available through the Statistics Canada website that may be of further interest to students seeking more health data.

- CANSIM tables – CANSIM provides fast and easy access to a large range of the latest statistics available in Canada. CANSIM contains over 3000 tables, including over 500 tables dealing with health. CANSIM tables can be accessed at **www5.statcan.gc.ca/cansim/home-accueil?lang=eng**.
- Health Profile – An application on the Statistics

Canada website that provides quick access to the latest health-related data available for a selected health region. The Health Profile application can be accessed at **www12.statcan.gc.ca/health-sante/82-228/index .cfm?Lang=E** .

- Health Trends – An application that provides quick access to recent trends that can be customized by health indicator or province. The Health Trends application can be accessed at **www12.statcan.gc.ca/health-sante/82-213/index.cfm?Lang=ENG** .

For a more complete view of available health-related resources, please visit the Health in Canada portal on the Statistics Canada website at **www.statcan.gc.ca/health-sante**.

Acknowledgement:

The author would like to thank Patricia Misner, Shelah Pickett, and Kelly Blair for sharing their experiences on their successful classroom use of these health microdata, and Magda Ferguson for her contribution to the content of this article. Magda is a web product coordinator with the Health Statistics Division at Statistics Canada. ▲

Note: *While Joel retired in 2009 from the Statistics Canada team responsible for the education outreach work, he maintains, independently from Statistics Canada, the site* **www.teacherweb.com/ON/Statistics/Math/***, to provide support based on Canadian data for mathematics teachers.*

▲ Call for Gazette Editor

The *Ontario Mathematics Gazette* is taking applications for a two-year term as editor commencing with the September 2012 issue.

Job shadowing available now.

For more details,
please contact:
Fred and Lynda Ferneyhough

▲ In the Middle – Infusing Technology: Using Social Media and Other Technology Tools for Teaching

CAROLINE ROSENBLOOM

Caroline Rosenbloom is currently on secondment as an Elementary Coordinator in the J/I Midtown Technology Option and Mathematics Instructor in the pre-service program at OISE/UT. She has enjoyed working with intermediate students and teachers as a middle school teacher, Mathematics Instructional Leader for the Toronto District School Board, and Education Officer at EQAO. Caroline is co-author of "Investigating Mathematics Using Polydrons."

As teachers, we constantly seek exciting lessons and new ideas. Resources are available to us and are just a Google search away. However, in a blink of an eye, something attracts us, making last year's technologies *passé*. The questions we have to ask ourselves (as it was with calculators years ago) are:

- What questions are we asking students when using technology?
- How does this help students? How does this help teachers?
- How much time do we allocate to using these tools?
- Is it equitable for all students?
- What do I have to learn as a teacher to prepare my students for the twenty-first century?

A quick web/Twitter or other social media search leads to a myriad of ways technology is being infused into the classroom. Even a non-technical teacher can find and learn how to use different tools for instruction and assessment. A quick search will also yield tools that enable student expression and enhance creativity.

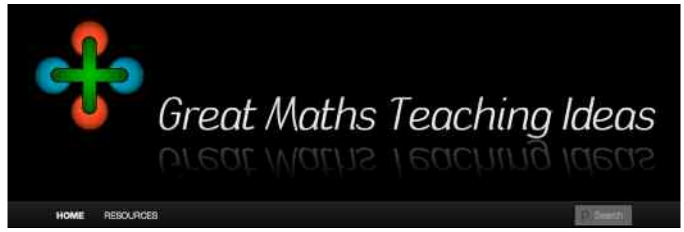
Social media tools, such as Twitter, provide another platform for sharing expertise and great ideas. I have recently been following "experts" on Twitter who share their ideas on various subject areas. Their tweets point me to interesting articles, ideas, websites, and tools, leaving me eager to try them with my own students. Here are just a few tools that can help you infuse technology into your own program.

- **VoiceThread** is a great collaborative tool that allows students to post different types of comments—text, audio, or video—to a community discussion. As an example, I found a few photos on **Flickr** and used them as topics for student-generated mathematical questions. Students would look at the pictures I posted and then contribute their own mathematical questions to the **VoiceThread** website. For example, Caley added the question "If you know the diameter of a pool ball, can you determine the area of the triangle?" to the billiards photo . Justin added the question "What is the ratio of bananas to total fruit candies?" to the list of questions for the candies photo below.

In addition, VoiceThread allows users to also animate their comments through doodling. Caley could outline the diameter of the billiard ball and outline the height and base of the triangle as she explains how to find the area of the triangle.

- **Edublogs** – There are many great education and technology blogs. Two that I have found particularly useful are the following:

- **www.greatmathsteachingideas.com/** (Twitter: **@Maths_Master)**

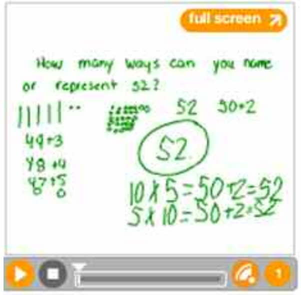


William Emeny has some great ideas on mathematics teaching. His blog includes great videos on how to do math, video links to great speakers, and ideas on integrating technology into the classroom. His posts provide great ideas for particular math concepts. He created many YouTube videos, available for free download on iTunes (MathsMaster.org).

- **www.edudemic.com/category/in-the-classroom/** (Twitter: @**Edudemic**)

Edudemic is a great blog with lots of articles on how to integrate technology into the classroom. You can find many lists such as: 20 Ways Teachers Are Using Legos in the Classroom, The 350 Best Education Resources Chosen by You, 100 iPad Apps Perfect for Middle School. There are lots of great ideas to connect to mathematics teaching.

- Digital Pens – When we think of technology, we tend to think of how technology can enhance instruction and learning. But what about assessment? Digital pens help us see and understand student thinking throughout the problem-solving process. It gives us access to their thought process, even when we aren't able to observe the thinking process in real time. These tools can create an audio recording simultaneously with a video recording of what the student is writing when solving a math problem. If students are working in pairs, we can hear the collaboration and the decisions students are making. What a great way to hear and see their thinking!



You can view an example from Livescribe. Although this is an elementary example, it's a great one, demonstrating how you can see the different ways a student has approached a problem. There are many other examples created by both students and teachers of a variety of grades that are searchable on Google.

When thinking about infusing technology into our programs, we have to think about what it is that we need to learn as teachers. It doesn't mean tossing out what we are already good at, but adding to our tools for teaching to enhance learning. Teaching is not about what tools we use, but the questions we ask.

We also need to think about all our learners. Not all students will have access to technology outside the classroom. We need to collaborate with other teachers in our schools to infuse these tools into our programs to make learning equitable for all.

If we know how to implement the appropriate resources and still provide stimulating questions to promote creativity and critical thinking, then why not use these tools? As Jim Shimabukuro says in his online article, *Online Learning 2012: Six Issues That Refuse to Die*,

> Those born or growing up in the last twenty to thirty years are different from previous generations. The mobile web is their primary medium for information and communication, and they expect to participate at anytime from anywhere. If it's not and they can't, they'll wonder why.

For more great links on infusing technology into the classroom, follow me @MathPlay_net. I'd love to hear your ideas too!

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Shimabukuro, J. (2011, December 29). Online learning 2012: Six issues that refuse to die. *Educational Technology & Change (ETC)*. Retrieved from **www.etcjournal.com/2011/12/29/online-learning-2012-six-issues-that-refuse-to-die/**

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▲ Directors' Dialogue

ANN MICHELE STENNING

Ann Michele Stenning is a mathematics teacher at South Collegiate Institute with the Thames Valley District School Board. She enjoys mentoring new teachers and learning new ways of how to address the various learning styles of her students.

OAME Board of Directors Meeting

Director's Diaolgue is written to summarize the minutes from OAME's Board of Directors meetings. These meetings occur three times a year.

Friday, October 14

- OAME's president, Connie Quadrini, welcomed everyone to the meeting and announced that we would be doing some work around the Prime Frameworks later in the meeting.
- Lynda Ferneyhough, OMAE's executive director, welcomed new chapter representatives and noted that OAME's membership has gone from 1000 members to a current membership of 1125 educators.
- Todd Romiens, the NCTM representative, encouraged board members to think about joining NCTM, attending their conferences, and speaking at their events. Board members were also told of videos that can be seen of lessons being taught.
- Lynda Ferneyhough thanked co-chairs David Petro and Dragana Martinovic for their time and effort in organizing OAME's 2011 annual conference. Earlier in the day, David and Dragana also received plaques as a result of their contributions. Henry Santos announced that OAME's 2012 annual conference would be held in Kingston.
- Wayne Erdman reported on the recent Ontario Mathematics Olympics competition held at York University last June. Wayne was given a plaque in honour of his work to organize the event.
- Connie Quadrini announced that the upcoming Leadership conference would feature Adobe Connect sessions. Participants would also have an opportunity to interact, using a wiki, in advance of the Leadership conference.
- Demetra Saldaris and Judith Taylor from the Ministry of Education made a presentation on a new document: *Paying Attention to Mathematics Education*. The document represented the Ministry's intention to focus on EQAO results. Board members had an opportunity to discuss the document and ask questions.

Board members also worked on solving a mathematical problem that was featured at last summer's CAMPPP. Led by CAMPPP presenters, board members were asked to collate the solutions so that different approaches could be identified and discussed. Then board members received copies of T*he Prime Leadership Framework* from Connie Quadrini.

Saturday, October 15, 2011

- Lynda Ferneyhough presented board members with details of the Financial Report for 2010–2011. She highlighted OAME's deficit of $18 000. A proposed budget for 2011–2012 was also presented by Lynda. This budget was accepted unanimously.
- Various motions were addressed and passed. The board members agreed on the Executive Directors' honoraria and that Chapter rebates be 30% of the membership fee for current and retired educators, and a 0% rebate for teacher candidate memberships. After some discussion, 26 out of 30 board members supported the plan to have each chapter in OAME appoint one Chapter Representative for a three-year term on a rotating basis, with one director at large for K–6 and one director at large for Grades 7–12+, each for a three- year term. The board also approved having the OAME's executive include the past president, president, president-elect, plus two vice-presidents elected for a two-year term. All but two board members voted to set OAME's membership fee to $55 in order to receive printed copies of the *Gazette*, while a $45 membership fee would include digital copies of the *Gazette*. Board members unanimously voted to have organizers of the OAME Leadership Conference and the OAME Annual Conference determine a registration fee based upon their financial analysis, then add an additional fee for a standard membership. Since the OAME membership would be included with every registration, board members agreed to make the membership transferable, subject to an expiry date.

Chairs of committees, provided the following summaries of their committees' work.

- The Communications committee, led by Robert Sherk, discussed the possibility of changing the number of Gazette publications from four per year to three per year, partially to save costs; no decision was taken. Greg Clarke, OAME's webmaster, presented board members with new updates available from OAME's website. Greg also noted that Shirley Dalrymple, a past OAME president, was the recipient of the Prime Minister's Award for Teaching Excellence. OAME has acknowledged her accomplishment on the website.
- The Outreach committee, led by Cheryl Geoghan, considered the usefulness of selecting additional central conference sites that could be available to smaller chapters. The committee also discussed the advantages of hiring a conference planner, who could help new organizers of the annual conference from year to year.
- Connie Quadrini announced that OAME members have been providing curriculum input to OMCA and The Field's Institute in the event of changes to the curriculum; however, this input was coming to an end. Susan Pitre, who spoke on behalf of the curriculum committee, had board members discuss ways to encourage the use of the seven math processes in assessment.
- The SPARC committee, led by Pat Kehoe, asked that board members address equity in their discussions and that terms for elected members match the dates of OAME's fiscal year. The committee also suggested that the executive and the board have one less meeting a year. To address this suggestion, Fred Ferneyhough, OAME's executive director, will devise a way to implement these changes if these changes are accepted. The SPARC members will also provide suggestions to OAME's terms of reference and the constitution at the next board meeting in February 2012.

Board members voted 25 to 6 that OAME's conference planner allow conference organizers to determine an exhibitor's fee for all groups at the annual conference who are selling merchandise. The board also voted that the number of issues published by the *Gazette* would not be specified in OAME's bylaws and terms of reference.

- Mary Lou Kestell introduced board members to the Financial Literacy Project, which began last summer in 2011. These lessons are now being piloted and the final product should be available in the summer of 2012 from the EduGAINS website.

Written reports were submitted by the following chapters.

CHAMP: Our Fall conference was held on Tuesday, October 18 at St. Marcellinus Catholic Secondary School. After the sessions, dinner was accompanied by a keynote address from George Gadanidis (Stretching Math Ideas Across Grades 1–12). CHAMP is also excited that the chapter's proposal to host the provincial conference in 2014 has been accepted by the OAME executive. We are looking forward to playing host to math educators from around the province.

COMA: COMA hit the ground running in September! We kicked off the year with a hugely successful Secondary Forum featuring renowned Mathematics Educational Consultant and Tech wiz, Tom Reardon, from Austintown, Ohio, who wowed a packed crowd with all kinds of tips for effectively integrating SMART™ Boards in the Math classroom. Two weeks later, we hosted our much-loved annual Fall Social at the beautiful Centurion Centre, offering great food, networking, and a wonderfully interactive presentation by Laura Gini-Newman, a facilitator with the Critical Thinking Consortium and a Mathematical Literacy Resource Teacher. She shared her valuable insight on engaging students in critical thinking in the math classroom. We also launched the latest instalment of our pay-it-forward campaign, drawing the names of five lucky members who each win up to $750 in expenses to attend the annual conference this year in Kingston. An additional 15 members will be drawn over the course of this year, and plans are already under way to fill the COMA coach on its way to Kingston. Our Ronald C. Bender Memorial Mini-Conference at the University of Ottawa took place in November, and this provided a wonderful opportunity to gather and learn as a community of mathematics educators.

ISOMA: ISOMA hosted its annual Fall conference at Appleby College on Tuesday, November 1, 2011. The theme of the evening was "Teaching Mathematics in the 21st Century." The conference consisted of two sessions, dinner, and a keynote speaker, Dr. Miroslav Lovrics. Sessions covered a wide variety of topics and was suitable for teachers of Grades 7–12. Thank you to Elizabeth Johnson and her colleagues at Greenwood College for hosting a great 2011 ISOMA Math Olympics. It was a day filled with teamwork, problem solving, and fun. Greenwood College will again be hosting the ISOMA Math Olympics this year, and we are sure it will be another success.

MAC²: MAC² had great responses to our mini-conferences this year. We celebrated Technology Tuesday for our Spring mini-conference. Teachers across all divisions were provided with professional development in various technological mathematical programs. Many teachers recently came out to our Fall mini-conference, which addressed the theme "Responding in the Moment to Student Math Thinking." Our webmaster, Greg Clarke, received a Lifetime achievement award from OAME. Our future plans include creating a newsletter to be sent out to members and conference participants in order to increase chapter membership.

NOMA: The keynote address for our Fall conference, "Building Our Content Knowledge Together," focused on the continuum of Patterning to Algebra learning K–12. It was presented live in Sudbury, with links through satellite locations. After the keynote, each satellite site hosted sessions designed to meet local learning needs. NOMA has a tremendously dedicated team committed to building connections across our chapter!

NWOAME: In addition to plans for our very successful Pi-Day Celebrations and our Regional Math Olympics, we are planning a mini-conference with a focus on math PD for teachers as well as on increased chapter membership.

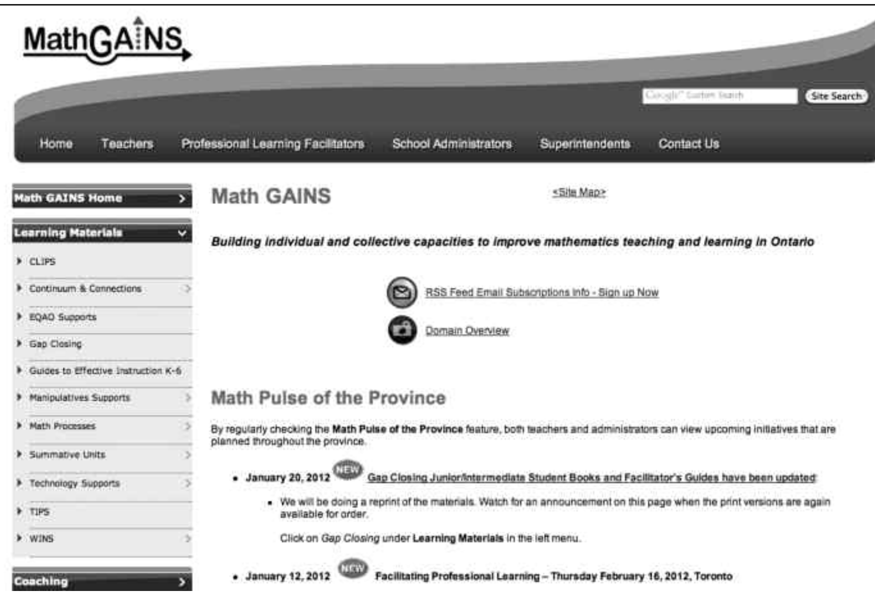
PRMA: PRMA hosted a mini-conference on November 8, 2011 at Holy Cross Catholic Secondary School in Peterborough. The theme of the conference was "Responding to Student Thinking: Assessment for Learning in the Mathematics Classroom." There was a breakout session, a catered dinner, and a keynote address by Dr. Marian Small. There was also an exhibitors display. We plan to host our meetings in a variety of locations throughout the Pine Ridge area, and we may even experiment with video-conferencing. Please check our website at **www.chapters.oame.on.ca/prma/** for the latest information regarding our upcoming meetings and events.

SWOAME: SWOAME is grateful for the support provided by all chapters with our 2011 "Put Math on the Map" conference, held in Windsor. We are actively planning our next big event, the 2012 Ontario Math Olympics. This will be held June 8 and 9, 2012 at St. Clair College in Windsor. We are pleased to see many new faces taking on leadership roles for this event. We are also finalizing our own Windsor playdowns for OMO and focusing on getting the message to as many schools as possible. We look forward to seeing everyone and enjoying the competition.

WOMA: WOMA hosted a Fall mini-conference, attracting a record of 155 participants, thanks to our keynote speaker, Dr. Marian Small. Dr. Small spoke on "Developing Creativity in Mathematics" and showed how to make math concepts more interesting to students. Using student examples, she demonstrated how open-ended questions can assess students' prior knowledge. Marian's creative use of questioning techniques was both entertaining and inspiring. The following day, Dr. Small spoke to educators at the Thames Valley District School Board office. WOMA would like to thank the Thames Valley District School Board for partnering with us in order to bring Dr. Small to London. ▲

▲ Are you Looking for Some Great Classroom Resources?

Go to the Ministry of Education website: **http://www.edugains.ca/newsite/math2/index.html** for the latest and greatest.



▲ Fields Institute – MathEd Forum Notes

STEWART CRAVEN

Stewart is a senior mathematics teacher recently retired from the Toronto District School Board. He continues to teach at York University and actively participates in the Fields Institute Mathematics Education Forum. He was the previous editor of the Gazette*.*

Our meeting on November 26 attracted a great deal of attention—our attendance was at least double our usual numbers. The theme was "The Heart of Mathematics," and our keynote speaker, William Byers (professor emeritus of mathematics and statistics at Concordia University in Montreal), attracted an audience that was eager to engage in discussion. Professor Byers opened the meeting with a presentation that was followed by two discussants, John Mighton and Judy Mendaglio, who were asked to identify important ideas, provide a comment or two based on their interpretation of the issues, and pose follow-up questions to get the discussion rolling. It didn't take long for the audience to become involved in a conversation that easily carried the meeting to its conclusion at 2:00 p.m.

The following is an abstract of Professor Byers' talk:

> What motivates people to get into mathematics? Why do we love math? What is the ingredient that brings it to life? In an attempt to get at this magical ingredient, let us distinguish as philosophers do between mathematical process and content. Content is what many of us think of as "real" math. I will take the position that mathematics is process; in other words, that the creative acts of doing and understanding mathematics are what is real and what we call content is only a snapshot of our understanding at a given moment in time.
>
> Everyone has a philosophy of math, but for most of us it is implicit, not explicit. How does questioning these assumptions that we bring to the table change us as teachers and communicators of math? I believe that the shift to process that I propose is radical, and once made, we will come to see mathematics in a new light. For example, some think that the essence of math is its logical structure, but I claim that ambiguity is present everywhere in math, that ambiguity is, in fact, a key to getting a deeper appreciation for math. Second, if math (and/or science) is something that you do, then it is dynamic, not static, as is our understanding. It follows that you can never say, "I understand continuity or randomness (definitively)." There are inevitable blind spots in our knowledge and our understanding. The problematic and the uncertain are essential aspects of mathematics that need to be acknowledged and addressed.
>
> This talk will draw on my experience as a university teacher of math—the things I tried to do, my successes, and my frustrations. My eventual conclusion was that something is wrong; something basic is missing. In an attempt to isolate this missing element, I had to go back to the beginning. I feel that such a reexamination is important, not only for teachers and students of math, but for all people, from the Prime Minister and CEOs to university administrators, all of whom use mathematics to help them define and solve problems that they face in their personal and professional lives.

To gain a deeper understanding of this issue, we recommend that you read one or both of Professor Byers' recent books: How Mathematicians Think: Using Ambiguity, Contradiction, and Paradox to Create Mathematics (2007), and The Blind Spot: Science and the Crisis of Uncertainty (May 2011). ▲

▲ Call for Manuscripts

The *Ontario Mathematics Gazette* is inviting manuscripts for all grade levels.

Instructions for submission of manuscripts are found on page 1 of any *OMG*.

Contact the Editor for further details.

▲ Formula Discovery

EMILY RODFORD
AND GRAHAM HILDEBRANDT

Emily and Graham are students in the Halton District School Board. Emily has just begun high school this year and dreams of becoming a dietician and an author. She also plays piano, writes poetry, and enjoys swimming, skiing, and reading. Graham, in the same grade, enjoys math, playing video games, and playing guitar.

Not too long ago, I (Emily) was comparing the difference between 12^2 and 13^2: it is 25. I realized that 25 is the same as 12 + 13, the sum of the square roots. Later, at a school dance, I voiced this idea to my friend Graham. He asked me if maybe there was a formula for that idea. We left that dance very excited that we had invented a theorem. Immediately we told a math teacher in our school and she became interested in it. Our second formula, for cube numbers, was invented at another dance, this one on our school-end trip to Quebec. It is important to note that when using these formulas $a < b$. To use these formulas, you must know the values of a, b, a^2, and, for the cube formula, a^3.

For squares...

$b^2 - a^2 = (a + b)(b - a)$

In other words, the difference between squares is the sum of the numbers being squared multiplied by their difference.

For example...

$a = 20,\ \ b = 23$

We know that $a^2 = 400$ and might want to predict b^2. If our formula is right, we would just add

$(20 + 23)(23 - 20) = 43 \times 3 = 129$.

$23^2 = 529$, so we were right.

This fit with our discovery, since

$13^2 - 12^2 = (13 + 12)(13 - 12)$, as we had noticed.

Graham says he uses this formula frequently. It is useful if you have a simpler small number and a more complex larger number (like above, or sometimes if b is a decimal). Graham also says that this formula is good for mental math, if you don't have a calculator on hand.

For cubes...

$b^3 - a^3 = b(a + b)(b - a) + a^2(b - a)$

For example...

if $a = 10$, $b = 11$

To get 11^3, you would add

$11(10 + 11)(11 - 10) + 10^2(11 - 10)$ to 10^3, which is 1000.

$= (11)(21)(1) + 100(1)$

$= 231 + 100$

And $331 + 1000 = 1331$, which is 11^3.

Obviously this formula, having so many more steps, since cubes are generally larger numbers, isn't as practical as the first when it comes to mental math. This theorem is more just for knowledge than for actual use.

Our formulas aren't precisely saying what b^2 or b^3 are, but rather identifying the relationship between a^2 and b^2 or a^3 and b^3. They do this by representing that a^2 + the difference = b^2 or a^3 + the difference = b^3. ▲

Note from Editor: *Secondary students might work at proving why these formulas are correct.*

▲ Are You Looking for Some Fun Open Questions to Try Out?

Consider these:

Primary

- Which number does not belong? Why?

  40 120 123 240

- Fill in the blanks:

  Another number that I think is a LOT like 33 is _____ because____.

Junior

- A fraction is SLIGHTLY more than ¾. What could it be?

Intermediate

- If you know that 2n + 3 = 18, what do you know about:

  - $2n + 4$? - $4n + 6$? - $2n - 5$?

  What else do you know based on the original information?

Senior

- A spinner has four numbered sections. The expected value when you spin is -2. Draw a possible spinner with unequal sections.

▲ Using Clips

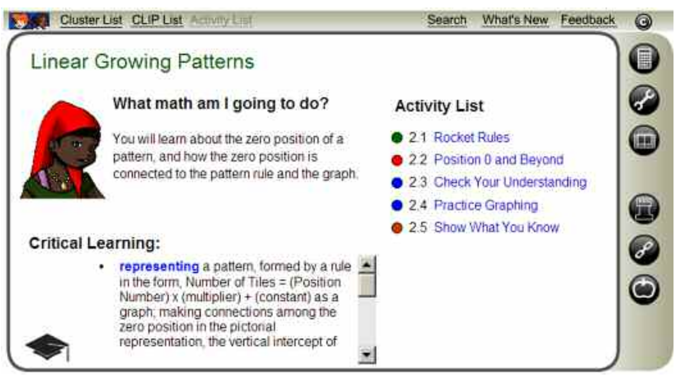
JAKE JOUPPI

Jake has been teaching for ten years. He is currently teaching Grades 7 and 8 in the Rainbow District School Board. He has past experience in teaching primary and junior grades, as a special education teacher and holds an honours specialist degree in Contemporary Studies.

This article will describe some of the benefits of broadening the use of technology in the math classroom. It is based on my use of CLIPS in my intermediate classes.

Using CLIPS in the Classroom

I have used CLIPS for about three years in my Grades 7 and 8 math classes, with students gaining the most experience from using the Linear Growing Patterns materials. I have used CLIPS with the whole class, in computer labs, and as a home support for students and parents.

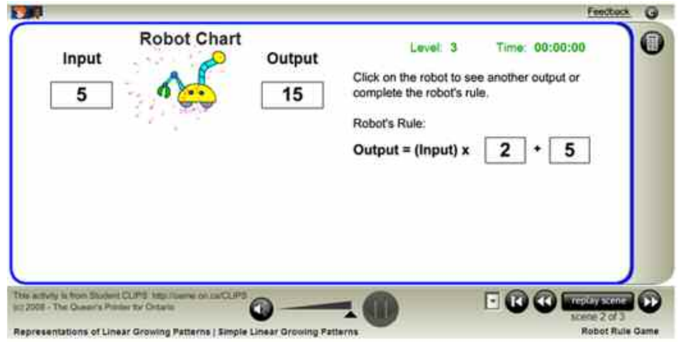


Why I Decided to Use CLIPS

I was first made aware of CLIPS through a series of after-school workshops, conducted by our school board's math coordinator. I loved their interactive format, and the feedback users received was meaningful and helpful in developing their conceptual understanding.

How Students Reacted to CLIPS

Students were immediately engaged! They loved the games, and the animations helped them make the connections they were lacking. Students at all levels were interested in the activities—top students were challenged and not bored, while students who had struggled were comfortable and could follow along. There were many "Aahha!" and "I never realized that before" moments.



Varied Uses of CLIPS

I integrated various CLIPS into different parts of my lessons. For example, on the first day of the unit, we used the Robot Transformer activity, in which students had to figure out the rule, given an input and an output. We did this as a *Minds On* activity, and then explored more concepts as a class. We consolidated our understanding by playing the *Robot Rule Game*. In later lessons, I integrated CLIPS into different parts of the lesson, mainly as the *Action* piece. At times, students worked in the computer lab in pairs or individually, and at other times, students worked together in the classroom. I found it worked well to use CLIPS in a variety of ways.

Increasing Retention

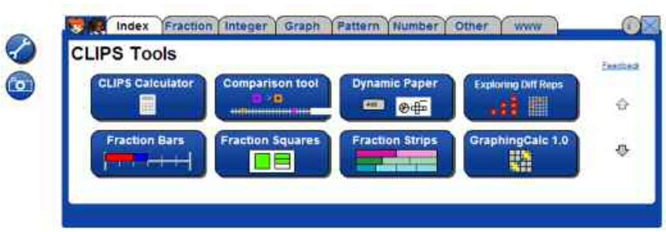
I was strongly encouraged by the initial reaction students had and that they seemed genuinely engaged from the beginning. What I found really exciting were the results in students' ability to demonstrate strong conceptual understanding as they progressed through the activities. When it came time for students to demonstrate their knowledge, they showed more success than with anything I had ever tried before. I was lucky enough this past year to have much of the same group for Grade 8 as I had taught in the Grade 7 class the previous year. When we revisited topics covered using CLIPS, not only did the students remember the activities and get excited about them, but they remembered the concepts. I was able to push this group even further because of the amazing retention of concepts.

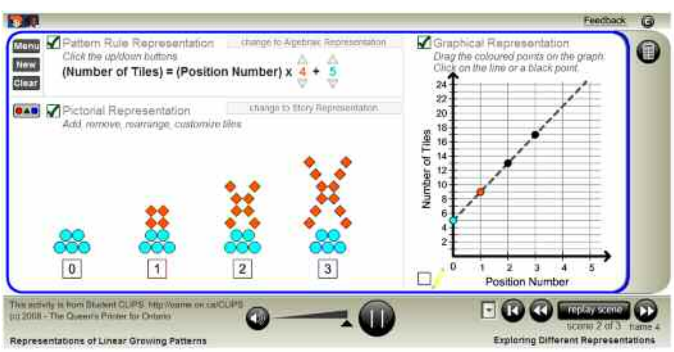
The Greatest Benefit

The obvious benefit to me is the amazing growth and retention of conceptual knowledge by each student who has used the materials. A side benefit has been my own growth in understanding of the concepts and developing sound teaching methods that I can trust have been grounded in thorough research.

Particularly Power Activities

My personal favourites are the activity "Exploring Different Representations," found in the Linear Growing Patterns CLIP, and the Tools Pod. "Exploring Different Representations" is an adaptable way to make connections between and among the different representations of linear growing patterns. I have never seen a more powerful single math tool. The Tools Pod is another excellent resource. It is full of interactive tools that can be used in any number of contexts.





Barriers to Using CLIPS

When I first started using CLIPS, it took a while to familiarize myself with all the materials available to me (because there is so much there!), and to gather the resources to make it a success. I had no interactive whiteboard, and very limited access to a computer lab. I also had limited Internet access in my classroom. I downloaded the flash files for off-line use, and my first CLIPS-based lessons were projected onto chart paper on the chalkboard, with students taking turns interacting and the class observing, discussing, and predicting. I learned early on that there are many ways CLIPS can be accessed, and almost instantly how valuable they were.

First-Time Use Suggestions

I would advise a new user to take some time to explore the resources available at **www.mathclips.ca**. You will find CLIPS to be the one resource that will make the greatest difference in your teaching, and your students' understanding—and it's free!

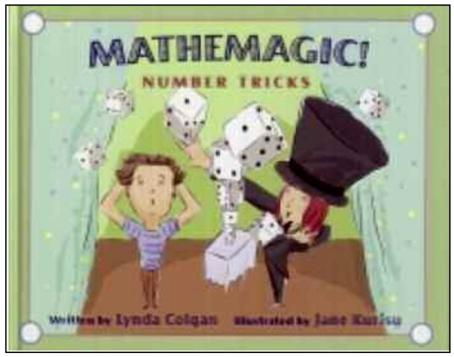
Stay Tuned…

Look forward to the last CLIPS article in this series, which focuses on secondary perspectives and experiences with CLIPS. ▲

▲ Book Review: *Mathemagic! Number Tricks*

Reviewed by Thomas Falkenberg, a teacher educator specializing in mathematics education, in the Faculty of Education at the University of Manitoba.

This article, a review of *Mathemagic*! *Number Trick*s by Lynda Colgan and illustrated by Jane Kurisu, published by Kids Can Press, in 2011, is reprinted from CM Volume XVII, April 22, 2011, with permission of the Manitoba Library Association, the journal's publisher.



Mathemagic! Number Tricks is a book that gives the reader exactly what its title promises, namely a collection of tricks with numbers that, if executed properly, must seem to the general audience like magic—whether it is finding the sum of the hidden numbers on a die, identifying a secret number chosen by the audience, or other forms of magical surprises. The book contains 12 major number tricks and very well-designed, age-appropriate explanations of how each of the tricks works. In addition, the book contains some variations for some of the number tricks, as well as very readable expansions on the mathematical ideas underlying some of the tricks. A one-page glossary of mathematical terminology used in the explanations completes the book.

Mathemagic! Number Tricks is very well-suited for different educational purposes in a school context, be it as a book in the school library or a resource book for teachers to engage their students in the magical sides of mathematical ideas.

Most of the number tricks presented in the book are based on quite sophisticated mathematical ideas involving numbers. This has two desired effects. First, it makes it difficult for the audience to see the magician "into the cards," so to speak, which keeps the tricks "magic." Second, for those students (and teachers) who want to get into understanding the tricks and their underlying mathematical ideas, the explanations of the tricks will most likely develop a deeper number sense in them than they had before. The excellent explanations of the tricks and their underlying mathematical ideas have a very high educational value. To be accurate, the explanations use the relevant mathematical terminology, like "the first digit in the product" and "multiples of 9," which are, however, explicated in the glossary at the end of the book.

But even for want-to-be magicians (students or teachers) who are not interested in how the tricks work, or who find the underlying mathematics of some of the tricks too difficult to (yet) understand, the book should be of great value because the steps the magician needs to take to execute each of the tricks are very clearly outlined in a step-by-step fashion. A magician can be magical even if he or she does not know why a trick works! And for aspiring mathemagicians, the book provides a number of helpful hints on how to perform the magic tricks well.

Fortunately, it is only the magician's outfit that is stereotypical in the illustrations that accompany each of the tricks: the middle-school-aged mathemagician wears a tuxedo and top hat, but is female, has long red hair, and performs in front of her peers, who represent a range of ethnic backgrounds. Overall, I find the book to be an outstanding example of a book on "mathemagic," useful to middle school students to study on their own and to teachers to use with their students as part of their regular or not-so-regular programs of mathematics teaching.

Highly Recommended. ▲

▲ Book Preview: *Sally Strange And How She Learned to Stop Worrying and Love Grade 7 Math:* A Novel

NICO ROWINKSY

This piece is a preview of a novel written by Nico Rowinsky, a Toronto District School Board math teacher for over ten years.

The book is represented by Ali McDonald at The Rights Factory: **alison@therightsfactory.com** +1.647.226.7328.

Chapter 1 – I've Got Problems

tuesday september 15th

If I was given the choice between going to math class or going to the orthodontist for a tightening, I'd probably choose the orthodontist. But I'm only 11 and I don't get to make those choices.

Yesterday, I had the painful tightening. Today, I'm here. Math class.

I move through the room towards my seat and say hi to Chin as I squeeze by his chair. Before I get a chance to sit, the bell goes, and the familiar voice of Niles comes on the PA, "Please stand for the National Anthem." I plop my bag down in the little area between my desk and Arial's. The noise of everyone getting up from their chairs carries on into the first few bars.

O Canada!
Our home and native land!

While some decide to stand quietly, others are still kinda moving and continue their morning chat in whispers. I look over to Lindsay across the room and we make weird faces for a moment until Evan interrupts our friendly game by walking in late.

With glowing hearts we see thee rise,

I watch Lindsay's weird face turn into a cute smile, followed by a tiny, flirty wave pointed towards Evan. He smiles back but continues his march. He passes my desk and gives me a nod. My heart skips a beat. Or does it beat twice as fast? I'm not sure. I can't think for a moment. It's not even 9:05 and I'm already needing some help.

God keep our land glorious and free!

Oh, I'll be fine. Evan and I are close friends. We've known each other since grade one. We like to joke around and tell people we're cousins, even though, I don't know, this year—something's different.

O Canada, we stand on guard for thee.

I try not to stare at him too obviously,

Ooooo Caaaa-na-da,

as he turns into his aisle,

we stand on guaaaard

and timed perfectly,

foooor

reaches for his chair,

theeeeeeeeeeee

and sits.

Seemingly on *his* cue, we all sit down.

"Good morning Winona! Today is Tuesday, September 15th—a Day 6 on our cycle—and these are your morning announcements." Niles. Where does he get the energy to be so cheery in the morning?

The announcements continue as I look down and read the graffiti in my desk to see if there is anything new since Friday. I read over Gavin's name for the millionth time. He left me a few messages last week. One of them was simply, *Good Game*! No *Hello*. No *Sally*. Just, *Good game*. He'd watched me play and wanted me to know. Both Evan and Gavin are trying out for the volleyball team, just like we are, and they're showing their support for the girls.

I pick at my teeth—the elastics are annoying and everything feels tight in the morning—before I reach for my pencil case. Today, we're starting something new because yesterday was the last of the so-called "review."

After the announcements end, Mr. Rowe slowly walks from his desk to the front of the room, faces the class, and with way more drama than needed, holds up two pieces of paper. He tries to make math fun. He tries.

I'm half-listening, not ready to fully commit my attention to my overly excited math teacher. It's too early, my mouth feels too tight, and two pieces of paper aren't going to do it for me.

Our room door is open and I see Niles in the hall, walking past. He pauses for a split second and looks to see if I'm okay before he continues to his Grade 8 homeroom class. He's like that. After the announcements, he checks in on me, every day. I'm not sure why, but maybe it's because it's still September. Maybe it's because I'm in Grade 7 and he's in Grade 8. Or, maybe it's because this is his second year at Winona Drive Senior School, and it's my first. But most likely, it's just what big brothers do—check on their little sisters.

Back to the action. I missed something. I turn to Chin. "What do we have to do?"

Chin is this tall, friendly giant in our class. I would say fat, but that seems rude. He's just big, I guess. He's not only friendly, he seems to pay attention just a little more than I do, so he's always there when I have one of my 'zone-out' moments.

"Pay attention," he tries to sound upset. "We have to make a cylinder out of this piece of paper."

I grab the sheet from Chin and wrap one side onto the other, making a tube. "Ta daaaa!" I throw my hands up and announce to my group, "I'm a math genius!"

"Sally, do you want to share with the class?"

Shoot. My hands went up just for show, now I'm booked. I'm totally not a math genius.

"Umm, ya."

I feel like I'm getting smaller. I hate being on the spot. Reason number 24 to hate math.

"I..." shrinking

"folded it like this…" shrinking

"to make the thingy..." shrinking

"like you said..." Mr. Rowe looks at me in silence. Shrunk!

And then says, "Good. Perfect." And rolls up one of his sheets, just like mine, tapes it together and places on the front ledge.

What? I think to myself.

"What?" Arial says half laughing at me.

"Anyone come up with a different solution?" he asks.

I'm in shock. My short tube sits proudly on the ledge, looking a little fat (not to be rude). Lindsay shoots up her hand and responds with her own solution. Her butt almost leaves her seat as she shows off her answer. Her tube is the same as mine, just the longer ends coming together.

"Very good Lindsay," he says as he turns to showcase Lindsay's solution beside mine.

"So, the question is..."

Here we go, I knew it couldn't have been that easy. Here comes the question only the math teacher actually cares about.

Mr. Rowe raises his hand slowly as he asks, "Which cylinder would hold the most water?" His hand clearly indicates we're not supposed to yell this one out.

The usuals raise their hands with confidence (how do they know this already?), followed by a few stragglers. Then Evan calls out, "They're the same!"

Our math teacher looks directly at him with no sign of emotion. Keeping his hand up, he slowly walks over to Evan.

"Someone with their hand up, please," and he calls on Gloria while whispering something to Evan.

"The taller one holds more," comes a shy answer from Gloria, sounding more like a question.

"Why?" the math teacher's favourite response to any unsuspecting student.

"Because," but she is not the type to just say because, "because, it's bigger, taller, so it holds more."

"Good." He leaves Evan and now moves to the back of the class. Most of us turn to follow him, except for Evan who now might be regretting walking in late AND blurting out his answer.

"Anyone agree with Gloria?" More than half the hands go up.

"Anyone disagree?" No hands.

Wait. One hand. It's Evan, back from his momentary mental detention.

"Evan," he calls on him as if to say 'thank you for putting up your hand this time.'

"Uhh, I think they're both the same."

Mr. Rowe nods his head, satisfied that he has our attention. "Good." He walks back to the front of the class.

Good? What kind of answer is 'good'? That doesn't answer anything. Which tube holds more? The taller tube must hold more, right? Gloria agrees. More than half the hands in the class agree. I wait a sec to see what Mr. Rowe is about to say.

Standing in front of the board he begins again, "Good. Now here's your challenge for today."

Challenge? What the...? What happened to the tubes?

Before he can continue, it's Arial who asks (on behalf of most of the class), "So, which cylinder holds more, Mr. Rowe?"

"Oh. Right. Ummm, I don't know yet. We'll have to figure that out. Should we have a quick discussion before our challenge?"

So there's a discussion alright, but it doesn't give us the answer, and neither does our teacher, just some more questions.

Oh, Mr. Rowe. I guess he sets it up this way. It's a week into school and although I haven't figured out any of the math yet, I think I'm beginning to figure him out a little.

This time it's a "challenge," but it's always a different word with teachers. Challenge, task, questions, problems. Problems, really? *I have a problem for you. Work on these problems. Did you finish your math problems?* It all sounds so negative. I clearly have a problem with the word "problem."

HOMEWORK:

Write an explanation as to why one tube holds more than the other tube.

Info (as discussed in class):

To find how much it holds, multiply the size of the circle by the height of the cylinder.

Short tube Circle: 62cm^2

Tall tube Circle: 37cm^2

I did most of the math myself (with a calculator), but Niles helped me figure out what to do. He also said that I haven't explained *why*. I hate "explain why" in math. I would like Mr. Rowe to explain why we need to explain in math. I wrote because the number is bigger, which is right. Don't ask me to explain why!

Chapter 2 – It's a Trap

wednesday september 16th

Where Sally figures out that most math lessons are really a trap. ▲

▲ A Computer Tool to Link Arithmetic and Algebra

DON WATSON

Don has an MA in Mathematics and a Postgraduate Diploma in Computer Science. He is a former mathematics teacher in Royal Air Force School, College, and University Institutions, and was former Director of Computing Services at Lakehead Universtiy. He has also been a volunteer tutor for a gifted Grade 5 student, which motivated his development of this tool.

Introduction

In general, students have always struggled with algebra. Schifter, Monk, Russell, and Bastable (2008) note the consequences for those unable to win that struggle:

> Among those barriers… is school Algebra, which continues to baffle large numbers of middle and high school students, including many who have been generally successful in their first 6 or 7 years of mathematics study… children who do not succeed in algebra are largely shut out of such mathematics-related careers as medicine, engineering and business. (p. 415)

Some of the difficulty originates from the nature of algebraic expressions, which contain two fundamental entities—functions and variables. Algebra uses variables as its basis. Functions are defined through variables (e.g., x^2, $bx + c$, $F(x,y)$), and expressions are built with variables. Students need to develop a secure understanding of variables before functions and expressions can effectively be introduced.

As students study arithmetic, functions are not introduced; little is learned about the structure of expressions involving literal variables, at least in most of the elementary years, because literal variables have not yet been addressed.

Some would suggest that large conceptual leaps such as the one from operating on numbers in arithmetic to operating on expressions in algebra need to be broken into small conceptual steps. Each of these steps would likely require an understanding only of the conceptual steps that precede it. However, if students learn little about functions or expressions until they meet variables and begin the manipulation of algebraic expressions, there is not a lot to build on. Some students face the introduction of literal variables, functions, and expressions with little previous experience to build on. Many students probably never overcome this lack of background, and even those who do are often left with an inadequate concept of function.

Teachers face a daunting challenge in the early years to provide conceptual experiences upon which a sound understanding of algebra can later be developed. This paper describes a distinctly different, but simple, tool to support teachers in this task and complement the successful approaches already in use. It makes use of the physical movement possible on computer screens and interactive whiteboards. This allows abstract ideas from the curriculum to be turned into concrete two-dimensional action.

The tool is designed to provide teachers with a range of options. It is simple and easy to learn, allowing teachers to choose how it might be useful within their individual classroom programs and grade levels.

Foundation

In order to assist teachers in providing conceptual experiences, it seems reasonable to consider a larger role for computers. Foundation is a graphic computer language that turns a computer into a "mathematics machine." It is a tool that enables:

1. functions, variables, and expressions to be presented to students as processes
2. functions to be presented as entities that accept input and produce output
3. arithmetic operations to be presented as simple functions
4. expressions to be built from these functions, using simple, graphic movement.

   (At all times, students are conscious that their building blocks are functions.)

In order to achieve this, Foundation uses functions instead of variables as its basis. These functions can be described to students as "number machines." A number machine processes by accepting number inputs, calculating, and producing number outputs.

There are several basic functions in Foundation. For example, "□ × □" is the multiplication function, $\sqrt{\square}$ is the square root function, $\frac{\square}{\square}$ is the division function, and $\square^2$ is the squaring function. The "□" symbols are input boxes through which numbers are fed to the function.

A satisfactory demonstration of motion is difficult to convey in a static paper. Nevertheless, the example below does its best to show how a basic Foundation function (or number machine) works. A solid arrow will indicate dragging on the computer screen, and a dashed arrow will indicate pointing:

A copy of the function can be dragged down from a toolbar:	□ x □
Pointing to one input box can expand the box and ready it for input:	□ x □
A number can be keyed into the expanded box:	$\boxed{3.42179}$ x □
The first box can be closed by pressing "enter."	
Pointing to the other box can open it for keying:	$\boxed{3.42179}$ x $\boxed{2.53819}$
The second box can be closed:	3.42179 × 2.53819
The result can be calculated:	8.68515

Each function in Foundation can be viewed as a set of three computer instructions. For the multiplication function, these three instructions are as follows:

Function	Instructions
□ × □	Step 1: Accept two numbers as input. Step 2: Multiply them. Step 3: Produce the result as output.

- In Step 1, the input boxes, "□", are responsible for the "accepting of input" part of the process.
- Step 2 is the normal exercise of arithmetic calculation, familiar to students.
- Step 1, Step 2, and Step 3 together form the complete function, so that it is seen as a processing entity that "accepts input and produces output."

Introducing Functions and Variables

Sfard (1991) identifies two separate, but complementary, conceptions of the same mathematical entity:

- structural (where the entity is an abstract "**object**" for manipulation)
- operational (where an entity is a "**process**")

The fundamental algebraic entities (variables and functions) are met and used in algebraic expressions as abstract structural “objects” for manipulation. However, Sfard (1989) states:

> It would be of little or no avail to introduce a new mathematical… (entity) by means of its structural description… The structural is much more abstract than the operational… In the classroom, therefore, the operational approach should precede the structural. (p. 152)

Thus Sfard’s premise implies that the introduction of variables and functions as objects to be manipulated in algebraic expressions is ineffective because it is too abstract.

An advantage of Foundation is the fact that its focus is entirely on process. It offers students the opportunity to meet variables, functions, and expressions first as processes—“...the operational approach… precede(s) the structural.” This gives students several years of operational experience with input boxes and number machines before they begin structural exercises in algebra. Sometime before this:

- the input box, “$\square$”, will have been introduced to students as Foundation’s ***variable***
- number machines will have been introduced as ***functions***

Expressions in Foundation

When students are ready, Foundation expressions can be built by dragging copies of entities into input boxes. For example, an expression for calculating the hypotenuse of a right-angled triangle could be created by using addition, “$\square + \square$,” squaring, “$\square^2$” and square root “ $\sqrt{\square}$”:

Drag down an addition function:	$\square + \square$
Drag down a copy of the squaring function, $\square^2$	
• into the right variable of the addition function,	$\square + \square^2$
• and into the left variable of the addition function giving the Foundation expression:	$\square^2 + \square^2$
Drag this expression into a square root function, giving:	$\sqrt{\square^2 + \square^2}$

The expression $\sqrt{\square^2 + \square^2}$ is simply another process containing a more complex set of instructions that can complete the whole Pythagorean calculation:

Drag down a copy of the function $\sqrt{\square^2 + \square^2}$:	$\sqrt{\square^2 + \square^2}$
Point to its left variable and key “324.6”:	$\sqrt{\boxed{324.6}^2 + \square^2}$
Point to its right variable and key “427.2”:	$\sqrt{324.6^2 + \boxed{427.2}^2}$ $\sqrt{324.6^2 + 427.2^2}$
Close the variable, giving:	$\sqrt{105365.16 + 427.2^2}$

Using a “Stage” icon, the teacher is able to show how a calculation works by executing it one stage at a time. The next stage of the calculation is executed each time the “Stage” icon is touched.

(Stage 1) - Square the first number:	$\sqrt{105365.16 + 427.2^2}$
(Stage 2) - Square the second number:	$\sqrt{105365.16 + 182499.84}$
(Stage 3) - Add the numbers:	$\sqrt{287865}$
(Stage 4) - Find the square root:	536.53

Once the process has been understood, Stages 1, 2, and 3 can be eliminated by pointing to a “Calculate” icon. However, having a “Stage” icon causes the calculation to become transparent. It allows for explicit teaching of how and why each calculation stage occurs.

It is interesting to note that nowhere has the word “structure” been used. Functional entities are simply being entered into input boxes in the same way as numbers. There is no need to use the word before algebraic expressions are introduced, but students might more easily be able to build algebraic structures when they have unknowingly been building Foundation structures for years.

Bridging the Algebra Gap

Students face a large and difficult conceptual transition from arithmetic to algebra. Kieran (1992) shows the magnitude of that challenge by defining the two sides of the chasm to be crossed:

> Procedural refers to arithmetic operations carried out on numbers to yield numbers… The term structural, on the other hand, refers to a different set of operations that are carried out, not on numbers, but on algebraic expressions. (p. 392)

However, the task of bridging the gap between operating on numbers and operating on expressions that is so major in algebraic notation is fairly simple in

Foundation. In the Pythagorean example above, the squaring function, "$\square^2$", was first dragged into input boxes. Then an expression, $\square^2 + \square^2$," was dragged into another input box. Thus, "...a set of operations… not on numbers, but on… expressions…." is already being carried out. Such easy conceptual experiences in Foundation can develop understanding that will help students later to cross the algebra gap.

Three Representations

Sfard (1991) refers to "…processes and objects as different sides of the same coin…." The analogy is a good one. On one side, Foundation sees functions, variables, and expressions as processes. On the other side of the coin, algebraic notation sees them as objects. The separation between the two is simple and elegant.

$$\sqrt{\square^2 + \square^2} \longleftrightarrow \sqrt{x^2 + y^2}$$

The only change required to move from Foundation to algebraic notation is to replace all variable boxes with one-letter variable names.

The difference between arithmetic and algebra is equally simple, a question only of what is entered into the Foundation variables (input boxes). Entering nothing but numbers produces arithmetic. Entering at least one letter produces algebra. Thus, Foundation is a common foundation upon which either algebra or arithmetic can be built:

$$\sqrt{3^2 + 4^2} \longleftrightarrow \sqrt{\square^2 + \square^2} \longleftrightarrow \sqrt{x^2 + y^2}$$

More than one author uses the analogy of theoretical physics, in which entities can be viewed as either a wave or a particle, according to which view is most helpful for the particular application. Similarly, functions, variables, and expressions can be viewed as processes in Foundation, objects for manipulation in algebraic notation, and calculation algorithms in arithmetic. Each offers a different, but linked, perspective to enrich student conceptual understanding.

Experimentation

Relating to the real world is important to students. Balacheff (2001) states:

> But all authors insist on the tendency of students to come back to a reference world outside of Algebra in order to check either the relevance of the actions they performed or the validity of their solution. (p. 253)

This directly compares to the way that students relate theory to the real world in science laboratories. They might, for example, vary voltages and resistances and measure currents. They never prove that Ohm's Law is true, but if they cannot find an example where it does not work, they accept that it is a hypothesis upon which they can depend. Even though equivalent mathematical experimentation is logically unnecessary, for some students, it might help confidence and understanding if they find that mathematical laws or rules are also dependable.

For example, consider the distributive law. Students are shown that "$a \times (b + c)$" and "$a{\times}b + a{\times}c$" are equivalent. In Foundation, the two expressions can be joined together with an equal sign: $a{\times}(b + c) = a{\times}b + a{\times}c$. This allows the same thing to be done to both expressions at the same time (expressions can also be joined by "<" and ">" signs for teaching inequalities).

Pointing at an algebraic variable causes it to revert to a box, opening up as an input process:

Point to a variable "a" in the expression:	$a \times (b + c) = a \times b + a \times$ c
"a" opens up and a number is keyed:	$\boxed{24} \times (b + c) = a \times b + a \times c$.
Closing the box gives the value 24 to all instances of "a":	$24 \times (b + c) = 24 \times b + 24 \times c$
Give values to "b" and "c" in the same way:	$24(3 + 7) = 24 \times 3 + 24 \times 7$
Calculate:	$240 = 240$

Another copy of "$a{\times}(b + c) = a{\times}b + a{\times}c$" can be dragged down, and different values, including large ones and decimals, can be given to "*a*," "*b*," and "*c*." Students find that whatever values "*a*," "*b*," and "*c*" have, both expressions always produce the same answer. This does not remove the need for all of the other conceptual justifications that are given students to convince them to accept the law. It is just another valuable justification that Balacheff suggests students need.

Beyond Elementary School

At any stage in a student's education, Foundation can provide algebraic insights, since the challenge of algebra also causes problems in the upper years of mathematics education. Carraher and Schliemann (2007) emphasize the importance of a strong understanding of function in later student education:

The concept of a function has… rather recently… become recognized as having a pivotal role to play in mathematics education at the secondary level and beyond. Schwartz, for example, has proposed functions… be the fundamental object of algebra instruction… We have proposed that arithmetic operations themselves be conceived as functions (p. 687).

Foundation provides a vehicle to achieve their goal because it is entirely based upon functions. Even the strongest students could benefit, since studies show that they also struggle with the concept of a function. The magnitude and significance of this problem are described by Marilyn Carlson and Michael Oehrtman (2005):

> …Studies have revealed that learning the function concept is complex, with many high performing undergraduates (e.g., students receiving course grades of A in calculus) possessing weak function understandings... Even understanding functions in terms of input and output can be a major challenge for most students….

Thus it could prove beneficial to experiment with Foundation in high school, teacher education, or even university mathematics courses.

Conclusion

Foundation is a tool. It is not intended to replace any of the current teaching approaches, nor is it intended to introduce formalism at an earlier stage. No suggestion is being made concerning where it fits in the curriculum. Judgment on the conceptual readiness of the student for different applications of the tool is intended to be left up to the individual teacher.

Its purpose is to enable the computer to help in providing conceptual experience of functions, variables, and expressions as processes. This would provide another method of scaffolding conceptual understanding of algebra. This paper is about what Foundation is, why it is useful, and how it works.

While the tool is intended for the pattern and algebra strand in Grades 1 to 8, it may also be useful for the number sense strand. Its use could begin with the simplest applications early in a student's education ($\square$ x $\square$ or $\square$ + $\square$). The option of colour-coded boxes is also available.

Research and teacher practice are needed to test Foundation and these hypotheses. To assist both research and practice, a software implementation of Foundation has been made available for SMART Board™, Macintosh, PC, and iPad platforms. Foundation and its documentation can be found at **www. foundationnotation.com**. Improvements will be guided by input from its users. There is a fee for use, beyond a trial period

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▲ Mathematical Images

FRANK MARINGOLA

Frank is currently working for the Toronto Catholic District School Board. He is interested in the potential of Communications Technology to evoke interest in math.

This feature shares some images around us that are connected to mathematical patterns and concepts.

This is an example of radial symmetry found in nature. Not only is it photogenic, but it is useful in engaging students who are disengaged by the standard curriculum.

Communications Technology can interest certain students in math, even if it is a subject they may be struggling with, through the use of photography. Photographing mathematical patterns in nature can introduce math from a different and fresh angle. This way, students can be motivated to take a closer look at math.

This is an example of Fibonacci Phyllotaxis. Focus into this fruit from Asia and you will see patterns.

Students can photograph this and then photo-shop it to get a close view of the patterns.

Communications Technology is a language, much like French or mathematics. Students can communicate not only through the use of technology in the classroom, but also through the study of nature. This can stimulate dendrites to open up new worlds to them—biology, science, math, technology.

This is also an example of Fibonacci Phyllotaxis, or a spiral pattern, found in nature. A close-up view of this fruit will display patterns that students can explore through independent learning. By encouraging thought and critical-thinking skills, students can become active math learners instead of looking for the formula or answer in the back of the textbook.

Communications Technology can accommodate thinking and exploring through photography, animation, and video editing. Students can begin by photographing interesting mathematical patterns in nature and then photo-shopping the images for a portfolio. ▲

▲ Minus Times Minus Equals Plus

VERA SARINA

Vera teaches mathematics at Pleasant View Junior High School in the Toronto District School Board.

There are a few concepts in the school curriculum that seemingly do not have an intuitive appeal. One of them is the famous rule for multiplication/division of integers: "*Minus times minus equals plus*." One can prove this rule mathematically by, for example, using the distributive property or using patterning. Nevertheless, it is often beneficial for students to answer the question "Why?" outside the limits of pure mathematics. To understand abstract concepts within mathematics, metaphors play an important role. "Metaphor is not a mere embellishment; it is the basic means by which abstract thought is made possible" (Lakoff & Núñez, 2000, p. 39). To unravel the metaphor behind mathematical concepts, it is often helpful to look into the history of these concepts.

Included in this article will be a brief look into the history of negative numbers. A teaching strategy based on the metaphor for integers and historical approaches to integers will be presented.

Negative numbers – what do they mean anyway?

Although negative numbers have been present in mathematics around the world for thousands of years, mathematicians fully accepted them only by the seventeenth century.

Negative numbers are counterintuitive. How can one compare or envision quantities that are less than nothing? History of mathematics shows that outside the realm of algebra, negative numbers were most exclusively confined to financial dealings. For instance, in *Nine chapters on the mathematical art* (one of the most famous and influential mathematical treatises in China, written around 2000 years ago), "the amount sold is treated as positive (because of receiving money), the amount spent in purchasing as negative (because of paying out). Similarly, a money balance is positive, what is taken out is negative" (Yan & Shiran, 1987, p. 49).

Indian mathematician of seventh century, Brahmagupta, "gave rules for zero and negative numbers in terms of 'fortunes,' which represent positive numbers, and 'debts,' indicating negative numbers" (Joseph, 1991, p. 370). Leonardo Fibonacci, who in his Liber Abaci (1202) introduced the Hindu-Arabic numerals to Europeans, freely used negative numbers, but almost exclusively in the problems that involved finances (Sigler, 2002, p. 9). Even today, if you ask somebody to explain what negative numbers really mean, often you would hear the "assets versus debts" analogy. But assets and debts can be recorded and calculated without negative numbers. For example, here's a quotation from an official U.S. government form: *If line 61 is more than line 54, subtract line 54 from line 61. This is the [positive] amount you OVERPAID.*

So, where do negative numbers come from? They come from algebra. Algebra made negative numbers a part of the number family (Cooke, 2008). "Negative numbers came up all the time in solving equations" (Seife, 2000, p. 133). Algebraists of early history, though seeking solutions to equations only in the domain of positive numbers, used negative numbers in intermediate computations. Greek mathematician Diophantus (third century AD), one of the fathers of algebra, never accepted negative solutions of equations, but had no choice but to use negative numbers in intermediate computations, calling positive numbers "availability" and negative "deficiency." For instance, "while solving problem III8, he needs to subtract $2x + 7$ from $x^2 + 4x + 1$. The result is $x^2 + 2x - 6$, i.e., he carries out the operation $1 - 7 = -6$" (Bashmakova, 2002, p. 49). In medieval India (seventh to twelfth centuries), negative numbers were used to produce the general form of the quadratic formula (the very one we use today!), and negative roots of quadratic equations were calculated. However, negative solutions were not approved and not accepted as "real" solutions.

The same attitude toward negative numbers persisted in Europe up to the seventeenth century. Negative numbers were called "absurd," "fictitious," and "false," but were used in algebraic calculations (Cajori, 2007, p. 233). The reluctance to accept negative numbers ceased when mathematicians started describing numbers not only by their magnitude, but also by their directionality. The understanding of integers came when positive and negative numbers became seen as opposition of directions on a line (Cajori, 2007). The visual or graphical representation of integers produced a reliable metaphor for them. French mathematician René Descartes (1596–1650) described positive and negative numbers as segments with opposite directions (Bashmakova, 2002, p. 96), e.g., 3 versus – 3 as →versus←. The visual representation of numbers was solidified by English mathematician John Wallis (1616–1703) by his idea of the number line (Sultan, 2011, p. 231). On this line, the numbers are represented geometrically, with absolute values of the positive numbers increasing to the right and absolute values of the negative numbers to the left.

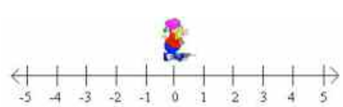
With geometrical representation, integers could be seen as answers not only to a "How many?" or "How much?" question, but also "Which way?" They were defined by two factors—magnitude and directionality: "Integers are the whole numbers and their opposites on the number line, where zero is its own opposite" (Randall, 2005, p. 13).

Walking along the number line in Egypt

Geometrical representation of integers is a metaphor that says: "Opposite integers are line segments that go in opposite directions." The following teaching strategy is based on this metaphor and inspired by some mathematical symbols of ancient Egypt; this strategy has proven to be successful in my classroom (Grades 8 and 9).

The ancient Egyptians used a symbol representing a pair of legs walking backwards for plus and walking forwards for minus . (Note that the direction for Egyptian writing is opposite to ours.) (Eves, 1990, p. 55). I tell my students to imagine that there is a tiny gnome who lives on a number line. The number line is his only possible habitat; he cannot step out from it and he can only move along it. His name is Computing Gnome because his only entertainment is to make calculations. And he will do all your calculations for you!

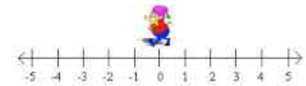
His **initial position** is at 0, looking into the "positive" direction:



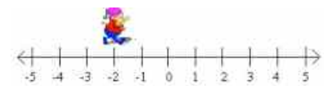
He interprets each arithmetical question as a series of commands: direction commands: "+" means *stay put* and "–" means *turn around*; and **distance commands**: numbers mean *number of steps to take*. After each move, he turns to the "positive" direction.

Let see how our Computing Gnome would perform –2 + (–3) – (–4):

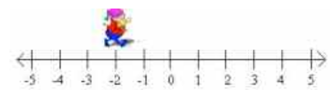
– (The gnome turns around.)



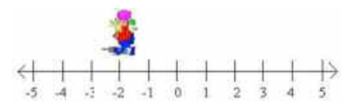
2 (The gnome takes two steps and turns back to the positive direction.)



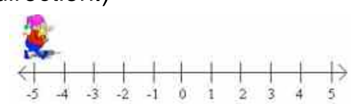
\+ (The gnome stays put.)



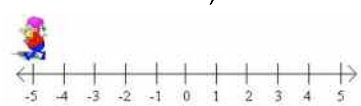
– (The gnome turns around.)



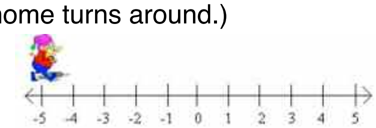
3 (The gnome takes three steps and turns back to the positive direction.)



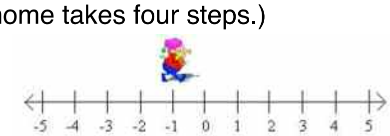
– (The gnome turns around.)



– (The gnome turns around.)



4 (The gnome takes four steps.)



He is now at –1! Try "to give" the gnome a few addition/subtraction questions (in standard or non-standard form) to calculate, and you'll see that he never, ever makes an error.

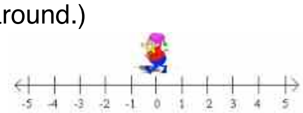
When my students "played" enough with addition/subtraction questions (and they actually played, acting as Computing Gnomes on a number line that I had drawn on the floor), they were ready to investigate how the gnome performed multiplication/division questions. Fortunately for our gnome, multiplication/division is a short form for repeated addition/subtraction, so he is able to perform multiplication or division following the same rules. The only difficulty is that he cannot perform the calculations step by step. He has to “unpack” the addition/subtraction question from its multiplication/division form. “Unpacking” means defining in what direction the gnome has to move and how many steps he has to take. Although the following rule may seem a bit arbitrary to some, students seem to be comfortable with the idea.

First, the gnome performs all the sign commands and thus defines the direction for his movement. Then, after having dealt with the direction, he can take as many steps as necessary (e.g., two times four steps, three times eleven steps).

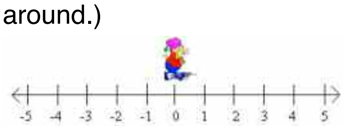
For instance, let him calculate (+2) x (–2). First, the gnome “reads” the signs: + (He stays put.); – (He turns around.). Now when the direction is chosen, he can deal with magnitude. Computing Gnome takes two times 2 steps and ends up at negative 4.

And finally, we can solve the mystery of *Minus times minus equals plus*. Let our gnome calculate (–2) x (–2). First, the gnome “reads” the signs:

– (He turns around.)



– (He turns around.)



Then he takes 4 steps and ends up at positive 4!

Division questions are performed using the same algorithm: the gnome performs all the signs commands and then takes as many steps as necessary. Also, my examples use integers, but the gnome’s way to perform calculations can be easily applied to questions with fractions or decimals.

Note that Computing Gnome does not make distinctions between the plus and minus signs as symbols representing positive and negative and symbols representing addition and subtraction. It is quite common to suggest that “minus” is an operation performed on two numbers, while “negative” is an operation performed on a single number. But if negative numbers are defined as opposites to positive numbers on the number line, then –6 (negative 6) is a short form for 0 – 6 (0 subtract 6). Minus is just a sign for “opposite” direction, whether we use it as a sign for subtraction or a negative sign.

Conclusion

The Computing Gnome approach is based on the recognition that signed numbers are defined by their direction and their magnitude (absolute value). Accordingly, our gnome first breaks every arithmetic question into a set of two types of commands: commands for the direction of his movements and commands for the length (magnitude) of his movements. Secondly, negative numbers are defined through positive numbers as their opposites. Positive (counting) numbers are conceptually (and historically) primary numbers. That is why the Computing Gnome always starts a new move facing right on a number line (i.e., in positive direction).

It is also interesting to note that some cognitive linguists describe the metaphor for arithmetic as “a motion along a path” (Lakoff & Núñez, 2000, p. 72). The Computing Gnome approach is an extension of this metaphor.

The Computing Gnome has persuaded my students that “minus times minus equals plus” and that “the reason for that” we definitely need to discuss. Try the Computing Gnome in your classroom and see if he does the trick for your students.

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