

Unit 8

Proportional Reasoning

Grade 8

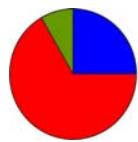
Lesson Outline

BIG PICTURE

Students will:

- develop an understanding that proportions are multiplicative relationships;
- compare and determine equivalence of ratios;
- solve proportions in a variety of contexts;
- solve problems involving rates.

Day	Lesson Title	Math Learning Goals	Expectations
1	Size It Up	<ul style="list-style-type: none">• Investigate proportional situations using everyday examples.• Identify proportional and non-proportional situations.	8m26, 8m27, 8m33, 8m68, 8m70 CGE 4b, 5a, 5e
2	Interpreting Proportional Relationships	<ul style="list-style-type: none">• Use multiple representations to determine proportions.• Through exploration and inductive reasoning, determine what makes a situation proportional.	8m26, 8m27 CGE 3b, 3g
3	Around the World in Eight Days	<ul style="list-style-type: none">• Solve problems involving proportions using concrete materials.	8m27 CGE 5a, 5b
4	Go Fish	<ul style="list-style-type: none">• Solve problems involving proportions.• Connect to an everyday sampling problem.	8m26, 8m27, 8m68, 8m73 CGE 5a, 7i
5	Just Graph It	<ul style="list-style-type: none">• Create a table of values and graph the relationship.• Identify characteristics of a proportional relationship that is shown graphically.	8m26, 8m29, 8m71, 8m76, 8m78 CGE 5b, 7b
6	Do You Agree?	<ul style="list-style-type: none">• Assess students' understanding of proportional reasoning and their ability to use a variety of approaches to solve problems.• Reinforce concepts of proportionality.• Reflect on current understanding.	8m27, 8m29 CGE 5e, 7f
7	Just for One	<ul style="list-style-type: none">• Connect rates to proportional relationships.• Solve problems involving unit rates.	8m29, 8m33, 8m78 CGE 7i
8	Best Buy	<ul style="list-style-type: none">• Compare unit rates.• Problem solving with unit rate and unit prices	8m29 CGE 3c, 4f

**Math Learning Goals**

- Investigate proportional situations using everyday examples.
- Identify proportional and non-proportional situations.

Materials

- relational rods
- measuring tapes
- BLM 8.1.1, 8.1.2, 8.1.3, 8.1.4
- assorted cylinders

Assessment Opportunities

See Think Literacy Mathematics: Grades 7–9, Anticipation Guide, p. 10.

Minds On...**Pairs → Anticipation Guide**

Distribute BLM 8.1.1. Students highlight key words in each of the six statements, then complete the Before column of the Anticipation Guide for Proportional Reasoning. Upon completion students explain their reasoning to a partner. Volunteers explaining their reasoning.

Action!**Small Groups → Investigation**

Explain the instructions at each station (BLM 8.1.2 and 8.1.4). Students rotate through three of them (or more if time allows). Students will record data on BLM 8.1.3.

Whole Class → Discussion

Compare the data collected at each station. Discuss data that doesn't fit due to incorrect measurements or calculations. Identify proportional and non-proportional situations (BLM 8.1.4).

Communicating/Observation/Mental Note: Observe as students rotate through the stations. Note any potential misunderstandings. These can be addressed in **Consolidate Debrief**.

See Think Literacy Mathematics: Grades 7–9, p. 38.

Consolidate Debrief**Whole Class → Discussion**

Groups discuss their findings for each station.

Complete and post a class Frayer model for the word Proportion (BLM 8.1.4). Students revisit their original responses on the anticipation guide and complete the After column.

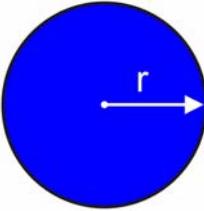
*Concept Practice***Home Activity or Further Classroom Consolidation**

Find some examples of proportional situations at home and add them to the Frayer model.

8.1.1: Anticipation Guide for Proportional Reasoning

Instructions:

- Read each statement below and highlight the key words.
- In pen, check **Agree** or **Disagree** beside each statement below in the **Before** column.
- Compare and discuss your choice with a partner.

Before		Statement	After	
Agree	Disagree		Agree	Disagree
		1. People always get taller as they get older.		
		2. When multiplying the radius of a circle by 2, the result is always the length of the diameter. 		
		3. If you need to make two boxes of macaroni and cheese, you have to triple the amount of milk that you add.		
		4. When converting from cm to m, you always multiply by the same number.		
		5. If Chan buys 6 golf balls for \$4.00, then he can buy 8 of the same golf balls for \$6.00.		
		6. When making lemonade from frozen concentrate, one can of juice to three cans of water is equally proportional to two cans of juice and six cans of water.		

8.1.2: Station Cards

Station 1: Height of Relational Rods to Screen Length

1. Measure the length of each relational rod to the nearest centimetre. Record these values in the table.
2. Place the rod on the overhead projector, and measure the length of the rod on the screen. Record your measurement.
3. Check that you have measurements of all rods.

Station 2: Circumference of a Circle to the Diameter

1. For each circle, measure the circumference to the nearest centimetre and record it in the table.
2. Measure the diameter of each of the circles to the nearest centimetre and record the values.
3. Check that you have measurements of all circles.

Station 3: Length of the Diagonal to the Perimeter of a Rectangle

1. For each of the rectangles, measure the length of the diagonal shown to the nearest centimetre and record it in the table.
2. Measure the perimeter of each of the rectangles to the nearest centimetre and record it in the table.
3. Check that you have measurements of all rectangles.

Station 4: Length of Line Segments Measured in Inches and Millimetres

1. Measure each line segment in inches. Record the values on the table.
2. Measure each line segment in millimetres and record the value.
3. Check that you have measurements for all line segments.

Station 5: Height of a Cylinder to the Circumference of the Base

1. For each of the cylinders provided, measure the height to the nearest centimetre and record it in the table.
2. Measure the circumference of the base of each cylinder and record your value to the nearest centimetre.
3. Check that you have measurements for all cylinders.

8.1.3: Exploring Proportional Relationships

1. Relationship: Length of Relational Rods to Shadow Length

Rod (colour)	Length of Rod	Length on Screen	$\frac{\text{Length of Rod}}{\text{Length of Screen}}$

- What do you notice about the ratio length $\frac{\text{length of rod}}{\text{length on screen}}$?
- How does this help you to determine the length of any rod on the screen?

2. Relationship: Circumference of a Circle to the Diameter (Round to 2 decimal places)

Circle	Circumference	Diameter	$\frac{\text{Circumference}}{\text{Diameter}}$
1			
2			
3			
4			
5			

- What do you notice about the ratio $\frac{\text{circumference}}{\text{diameter}}$?
- How could this help you to find the circumference of any circle?

8.1.3: Exploring Proportional Relationships

3. Relationship: Length of the Diagonal to the Perimeter of a Rectangle

Rectangle	Length of Diagonal	Perimeter	$\frac{\text{Diagonal}}{\text{Perimeter}}$
1			
2			
3			
4			
5			

- a) What do you notice about the ratio $\frac{\text{diagonal}}{\text{perimeter}}$?
- b) Explain why your findings make sense. Could this help you determine any diameter if you know the perimeter? Explain.
4. Relationship: Inches to Millimetres

Line Segment	Inches	Millimetres	$\frac{\text{Inches}}{\text{Millimetres}}$
1			
2			
3			
4			
5			

- a) What do you notice about the ratios $\frac{\text{inches}}{\text{millimetres}}$?
- b) If a line segment were 20 inches in length, how many millimetres would it be? _____
5. Relationship: Height of a Cylinder to the Circumference of the Base

Cylinder	Height	Circumference	$\frac{\text{Height}}{\text{Circumference}}$
1			
2			
3			
4			
5			

- a) What do you notice about the ratio $\frac{\text{height}}{\text{circumference}}$?
- b) Explain why your findings make sense.

8.1.4: Exploring Proportional Relationships (Teacher)

Station 1

Materials: 5 different heights of relational rods.
An overhead projector at a fixed position from the screen.

Answer: This is proportional relationship.

Station 2

Materials: Different size lids or cut-out circles numbered 1 to 5, with the centres marked.

Answer: Circumference of a circle to the diameter of the same circle is a proportional relationship (π).

Station 3

Materials: 5 different rectangles in a variety of shapes numbered 1 to 5, e.g., long and thin, close to square...

Answer: The diagonal of a rectangle does not have a proportional relationship with the perimeter of the rectangle.

Station 4

Materials: 5 different line segments measured in inches (2, 4, 5, 7, 9) numbered 1 to 5.

Answer: This is a proportional relationship in which 1 inch = 25.4 mm.

Note: $20 \text{ inches} \times 25.4 = 508 \text{ mm}$

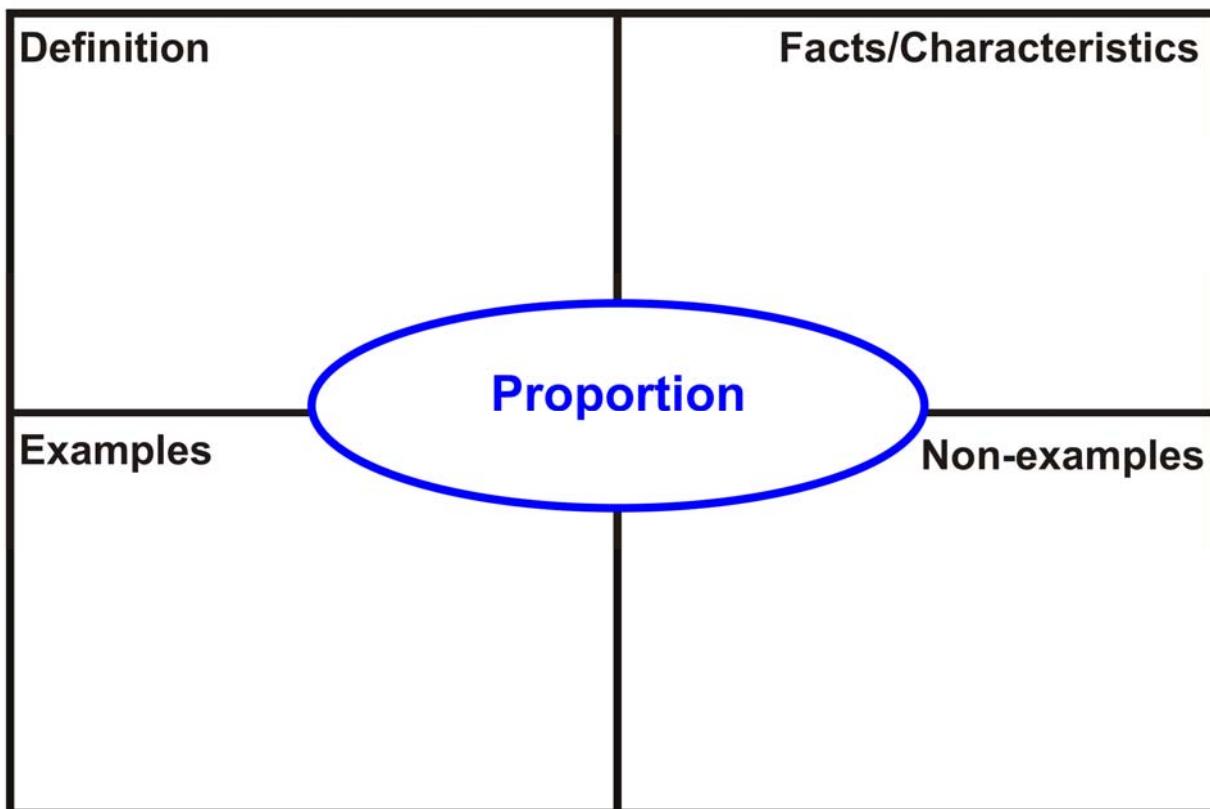
Station 5

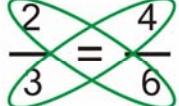
Materials: 5 different cylinders, e.g. coffee can, potato chips, orange juice can, numbered 1 to 5.

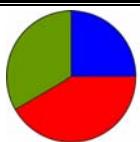
Answer: This is not a proportional relationship.

8.1.4: Exploring Proportional Relationships (Teacher) (continued)

The Frayer Model: Sample Answer



Definition	Facts/Characteristics
A proportion is a statement of two equal ratios or $2 : 3 = 4 : 6$	Two equal fractions $\frac{2}{3} = \frac{4}{6}$  $2 \times 6 = 3 \times 4$
Proportion Examples $\frac{c}{d} = \pi$ - If $6:15 = n:25$ then $n = 10$ $\frac{6}{15} = \frac{10}{25} = \frac{2}{5}$ - # boys to girls = 13:27 - ammonia is 1:3 ratio of nitrogen to hydrogen - mixing frozen orange juice - circumference of cylinder to diameter	Non-examples  because $2 \times 9 \neq 3 \times 4$ or because $\frac{2}{3} > \frac{1}{2}$ and $\frac{4}{9} < \frac{1}{2}$ - diagonals of rectangles to perimeter - circumference of cylinders to height

**Math Learning Goals**

- Use multiple representations to determine proportions.
- Through exploration and inductive reasoning, determine what makes a situation proportional.

Materials

- manipulatives
- BLM 8.2.1
- chart paper
- markers

Assessment Opportunities

Unit-rate strategy:
how many for one?

Factor-of-change
strategy: “times as
many” method

Fraction strategy: use
unit rates as fractions
and create equivalent
fractions

Cross product
algorithm: set up a
proportion, form a
cross product, and
solve the equation by
dividing

“Connecting
Research to Teaching
Proportional
Reasoning” by
Kathleen Cramer and
Thomas Post

(http://education.umn.edu/rationalnumberproject/93_2.html)

Minds On...**Whole Class → Discussion**

Add student examples to the Frayer model from the Home Activity in Day 1.
Discuss why the student examples are proportional or non-proportional.

Pairs → Problem Solving

Students solve the problem and share how they came to their solution:

Jack and Jill were driving the same speed along a highway. It took Jack 25 minutes to drive 50 kilometres. How long did it take Jill to drive 125 kilometres? Explain different methods of arriving at the same solution.

Highlight methods for problem solving:

- unit rate strategy: unit rate (25 minutes for 50 km, $\frac{1}{2}$ min for 1 km,

$$\frac{1}{2} \times 125 = 62.5 \text{ mins for 125 km}$$

- factor-of-change strategy: 125 is 2.5 times as far.

Therefore, $25 \times 2.5 = 62.5$ mins

- fraction strategy: $\frac{25}{50} = \frac{1}{2}, \frac{1}{2} = \frac{62.5}{125}$

- cross-product algorithm: $\frac{25}{50} = \frac{x}{125}, x = \frac{125 \times 25}{50}$.

Action!**Small Groups → Investigation**

Groups solve the given problems (BLM 8.2.1) using two methods. One person from the group explains the methods they used. Post the solutions.

Circulate to monitor progress, offer suggestions, and note the variety of strategies used. Distribute chart paper and markers to groups as they are ready.

Reasoning & Proving/Demonstration/Anecdotal: Observe reasoning skills during the investigation and select groups to present so that all methods are shared.

**Consolidate
Debrief****Whole Class → Discussion**

Revisit the posted solutions to reinforce the strategies used. All methods use multiplicative reasoning (unit rate strategy, factor of change strategy, fraction strategy, and cross-product algorithm) and it is this multiplicative property that makes a proportion.

Home Activity or Further Classroom Consolidation

Solve the problem and validate your solution using a second strategy: If you can type 45 words per minute, how long will it take to type a 900-word essay?
Show your work.

8.2.1 Sample Problems (Teacher)

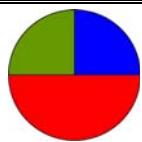
Some of these problems are more difficult than others. Decide in advance which groups should solve each problem and how you want to set up the groups. Each group should show two different methods for determining the solution. Advise students that solutions will be posted on chart paper.

Sample Problems

1. If three apples cost \$1.97, how much would six apples cost? How much would seven apples cost?
2. If two shirts cost \$29.95, how much would one cost? How much would you pay for four shirts or five shirts?
3. If five graduation tickets cost \$35.00, how many could you buy with \$14.00? How much would it cost you to buy eight tickets?
4. You get six pieces of gum in a package that costs \$0.87. If you could buy a package with three pieces of gum, how much would it cost? How much would you pay for a package of eight?
5. If Sam made \$2,550 selling 200 CDs, how much money would he make if he sold 50? What would he make if he sold 900? or 273?
6. The standard sizes for photographs are: 4×6 , 5×7 , and 8×10 . Can you use a photocopier to enlarge a 4×6 photo to one of the other standard sizes? Explain.
If you reduce an 8×10 photograph, what sizes could you make?
7. You want to buy four coloured markers. The smallest package available has six markers in it and costs \$9.00. One package has been opened and contains only five markers. You ask the sales clerk how much it would cost. What would the cost of this damaged package be? Show the answer at least two different ways.

Answers to Sample Problems

1. Six apples: \$3.94 Seven apples: \$4.60	5. 50 CDs: \$637.50 900 CDs: \$11,475.00 273 CDs: \$3,480.75
2. One shirt: \$14.98 (14.975) Four shirts: \$59.90 Five shirts: \$74.90 (\$74.875 or \$74.88 depending on rounding)	6. This is not a proportional relationship. You cannot enlarge your pictures appropriately using a photocopier.
3. \$14.00: 2 tickets Eight tickets: \$56.00	7. Package of 5 should be \$7.50 Unit rate $\$9.00 \div 6 = \1.50 $\frac{9.00}{6} = \frac{\text{cost}}{5}$
4. Three pieces of gum: \$0.44 Eight pieces of gum: \$1.16	

**Math Learning Goals**

- Solve problems involving proportions using concrete materials.

Materials

- linking cubes
- pattern blocks
- grid paper
- BLM 8.3.1

Assessment Opportunities**Minds On...****Whole Class → Discussion**

Pose the following problem:

Two players on the school basketball team scored all the points in the last game. The ratio of points scored was 2:5. The team scored 35 points in total. How many points did each player score?

Use manipulatives to model the problem (linking cubes, pattern blocks, grid paper). Students share a variety of strategies and their reasoning.

Students requiring additional practice can complete BLM 8.3.1.

Action!**Whole Class → Instruction**

Demonstrate connections between ratio, proportion, and fractions using a graphic organizer.

Pairs → Investigation

Provide a number of packages with two items such as linking cubes, pattern blocks, coloured tiles in specific proportions that can be reduced to simplest form. Include a problem to be solved. Students use the contents to solve the problem and determine the ratio of the items in it. They reduce the ratio to simplest form.

They repeat the investigation with a different package.

Students present the problems they solved and their ratio. Classmates ask presenters questions so that they understand.

Communicating/Presentation/Anecdotal: Observe students' use of appropriate terminology and clarity of explanation.

**Consolidate
Debrief****Pairs → Connecting**

Students create a mind map connecting the ideas and key information of proportion and share and compare with a partner.

*Practice***Home Activity or Further Classroom Consolidation**

Complete the problem:

Kerry said that the Japanese Bullet Train takes about 6 minutes to travel 22.2 km. Jerry said that at this rate, he could travel around the world at the equator in less than 8 days. Kerry disagrees – she thinks it will take longer.

Who is correct? Justify your response.

The diameter of Earth is approximately 38 250 km.

8.3.1 Ratios and Proportions

Name:

1. Circle the letter if the pairs of ratios are equivalent.

a) 3:4 and 6:8

b) 2:5 and 22:25

c) 3 to 5 and 3:5

d) 4:15 and 2:10

2. Make an equivalent ratio by finding the missing value.

a) 2 to 7 = 4 to y

b) $x:10 = 6:30$

c) $\frac{3}{8} = \underline{\hspace{2cm}} \text{ to } 72$

d) $\frac{9}{x} = \frac{3}{4}$

3. Write the ratio represented by each situation:

a) You make orange juice using one can of frozen concentrate and three cans of water.

Ratio = _____ juice: _____ water

b) You make hot chocolate using two scoops of powder in one cup of hot water.

Ratio = _____ powder: _____ water

c) The days of the workweek compared to the weekend

Ratio = _____ workdays: _____ weekend

d) The number of boys compared to the number of girls in the class.

Ratio = _____ boys: _____ girls

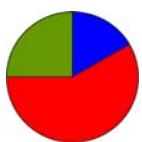
4. Write a situation that represents each ratio:

a) 8 to 5

b) $\frac{7}{10}$

c) 3:7

d) 7:3

**Math Learning Goals**

- Solve problems involving proportions.
- Connect to a everyday sampling problem.

Materials

- paper bags
- linking cubes
- masking tape

Assessment Opportunities**Minds On...****Whole Class → Investigation**

Students create ratios by moving to different areas within the classroom, based on an attribute chosen by the teacher. Record the appropriate ratios on the board that reflect the class population. Possible ratios: 1) boys: girls: adults; 2) shirt colour – light: dark: medium.

Discuss the ratios and demonstrate when they can be reduced to simplest form.

Use the term *simplest form*, as it will be required in Lesson 6.

All parts of the ratio together represent the whole class.

Action!**Small Groups → Exploration**

Each group receives a paper bag filled with 30 linking cubes of one colour. One student removes six cubes, puts a piece of masking tape on each cube, and returns them to the bag. Another group member shakes the bag, takes out five cubes, records how many of these cubes are taped and how many are not, and returns the cubes to the bag. Each group member repeats this process of taking out five cubes, recording, and returning cubes to the bag. Compare results and estimate how many cubes are in the bag.

Lead a discussion on how this experiment can be used to determine the total number of cubes in the bag (equivalent ratio – 6 out of 30 equivalent to 1 out of 5).

Repeat with 20 cubes, 5 of which are taped. Students take out 4 cubes at a time, determine the ratio of taped cubes to those that are not taped, and make predictions using the ratios of taped cubes to total cubes to estimate the number of cubes in the bag.

Reasoning & Proving/Observation/Anecdotal: Observe groups as they work through their exploration and listen to their reasoning.



Students do not know how many cubes are in the bag.

Consolidate Debrief**Whole Class → Connecting**

Groups share their estimates and explain their thinking. Work through the estimation for the problem: Scientists often use the catch, band, and release method to estimate the size of wildlife populations. For example, 250 trout were caught, banded, and released into a small lake in Northern Ontario. One month later, another 250 trout were caught in the lake, 30 of them had bands. From this information scientists could estimate the size of the trout population of the lake. (Approximately 1708 trout were in the lake.)

Students explain why they wait for a month to catch fish.

Provide the population for your school and community.

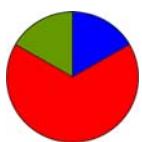
Remind students that all parts of whole ratios represent the total population.

Concept Practice**Home Activity or Further Classroom Consolidation**

Assuming that the ratio of eye colour of the class is the same within the wider community, estimate how many people have eye colour that is blue, brown, or other in the whole school, the community, the province, and the country.

Students record any assumptions that they make.

Population: Ontario – approximately 11.5 million; Canada – approximately 33 million (July 2005)

**Math Learning Goals**

- Create a table of values and graph the relationship.
- Identify characteristics of a proportional relationship that is shown graphically.

Materials

- graphing tools
- BLM 8.5.1, 8.5.2

Assessment Opportunities**Minds On...****Whole Class → Discussion**

Demonstrate how to set up the data in a table and graph it.

Your group is going to make an orange drink from a mix. The recipe says to use 3 scoops of mix to make 2 cups of orange drink. You want to make 4 cups of orange drink. How much mix do you need to use? How much for 8?

Note: Situations 1 and 2 are proportional, but 3 and 4 are not.

Action!**Pairs → Investigation**

Using either a technology tool (Fathom™, GSP®4, TinkerPlots™, graphing calculators) or pencil and paper, students complete and graph the situations in questions 1–4 (BLM 8.5.1).

Whole Class → Discussion

Discuss that the resulting graph forms a straight line that goes through the origin if the relationship is proportional.

Learning Skills/Rating Scale: Observe students as they work on the pairs investigation. Ask questions so that they can explain their thinking. Discuss any misunderstanding during **Consolidate and Debrief**.

**Consolidate
Debrief****Whole Class → Discussion**

Discuss and complete responses for questions 5 and 6 (BLM 8.5.1), paying particular attention to the characteristics of a graph that shows a proportional relationship.

*Concept Practice***Home Activity or Further Classroom Consolidation**

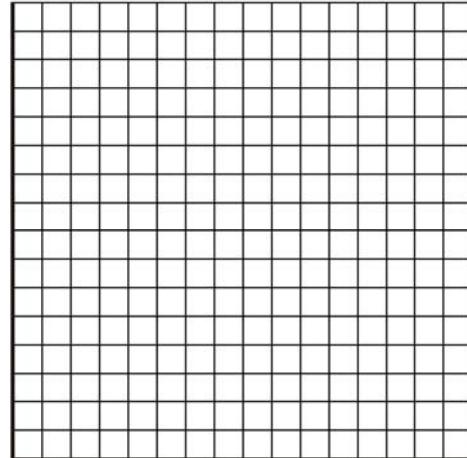
Complete worksheet 8.5.2.

8.5.1: What Does the Graph Tell Us?

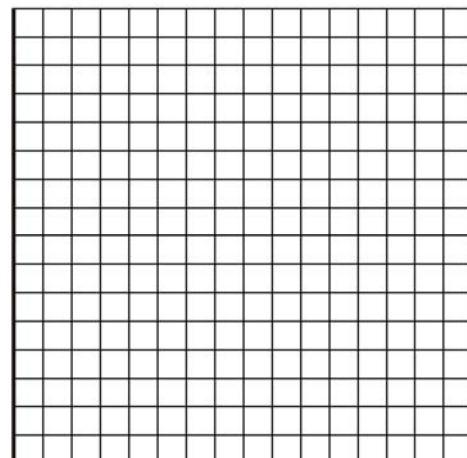
1. Dakota earns \$15.00 caring for a child for 3 hours.
 - a) At this rate, how much would she earn for 1 hour?
For 5 hours?
 - b) Fill in the given table of values.

c)

Hours	Dollars Earned	\$ Per Hour



2. Jordan starts at his home and bikes along a country road. He bikes at a constant speed. Twenty minutes later he passes Stephanie's house; he knows he is 2 kilometres away from his home. Another one of his friends, Jacob, lives 4 kilometres further down the same road.
 - a) How much longer would Jordan have to ride his bike to reach Jacob's house?
 - b) How long will Jordan take to ride his bike 10km? Fill in the table of values below.
- c)
- | Time Taken | Total Distance Travelled | $\frac{\text{Distance}}{\text{Time}}$ |
|------------|--------------------------|---------------------------------------|
| | | |
| | | |
| | | |

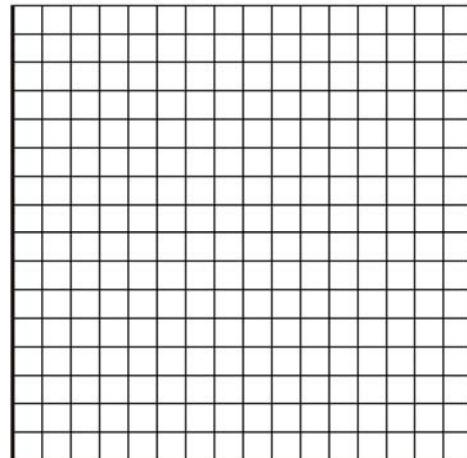


8.5.1: What Does the Graph Tell Us? (continued)

3. Main Street Market purchases a new freezer for its store. When the freezer is first plugged in, it has an inside temperature of 18° C . The temperature drops 3° C during the first 15 minutes, 6° C during the next 15 minutes, and 9° C during the next 15 minutes, until it reaches -3° C .

- a) Generate a table of values showing the drop in temperature for the first hour.

Time	Temperature (C°)
0 minutes	18° C
15 minutes	15° C
30 minutes	9° C

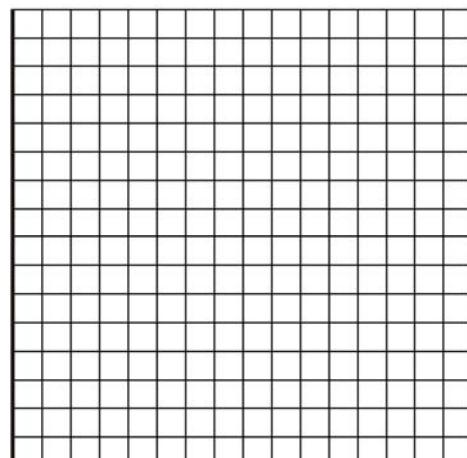


- b) Graph this relationship.
c) Describe the graph. What do you notice?
d) Does the temperature drop at a constant rate?
Justify your answer.

4. Jenna is visiting a relative in Ottawa and wants to rent a bike. She calls Past Premium Bikes and finds out that the cost of renting a bike is \$5.00 plus \$2.50 per hour.

- a) How much will it cost her to rent a bike for 3 hours?

Number of Hours	Cost	Cost Per Hour
1	\$7.50	\$7.50
2	\$10.00	\$5.00
3		



- b) How much will it cost her to rent a bike for 6 hours?
c) Graph this relationship.
d) Describe the graph. What do you notice?

5. Which of the above situations represent a proportional relationship? How do you know?

6. Describe two characteristics of the graphs of the proportional relationships.

1.

2.

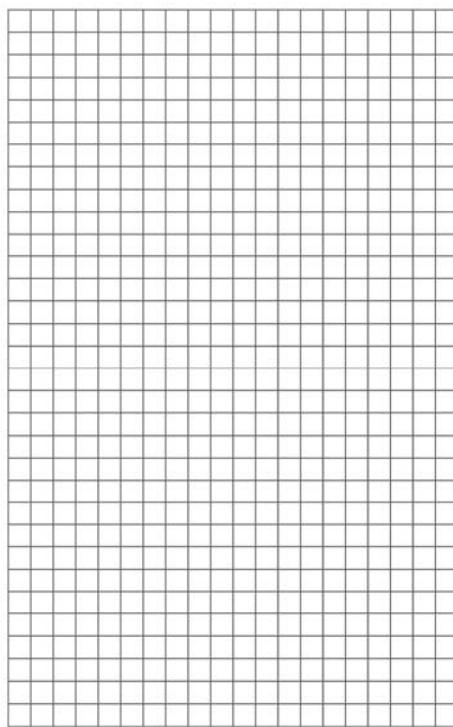
8.5.2 Tea Time

Pat's recipe for making iced tea calls for 2 tablespoons of mix to 3 cups of water. Emmi's recipe uses 3 tablespoons of mix to 5 cups of water.

Complete the table below to calculate the total amount of each ingredient for the given number of containers.

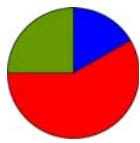
	Pat's Recipe		Emmi's Recipe	
Containers	Iced tea mix (tbsp.)	Water (cups)	Iced tea mix (tbsp.)	Water (cups)
1	2	3	3	5
2	4		6	
3				
4				
5				
6				

1. Which iced tea recipe would have the strongest taste? *Justify your answer.*
2. Draw a line graph to represent the recipes. Use a different colour for each recipe and include a title for your graph.



8.5.2 Tea Time (continued)

3. How are the graphs the same?
 4. How are the graphs different?
 5. Use the graph to support your answer to question 1.

**Math Learning Goals**

- Assess students' understanding of proportional reasoning and their ability to use a variety of approaches to solve problems.
- Reinforce concepts of proportionality.
- Reflect on current understanding.

Materials

- BLM 8.6.1

Minds On...**Whole Group → Opposite Sides**

Designate one side of the room as the agreement side and the opposite side disagreement side.

Students move to the side of the room that represents their position on the question and explain their reasoning. Look for more than one explanation.

- Three T-shirts cost \$18, so two T-shirts will cost \$9. [\[Disagree\]](#)
- Fifteen bowling balls weigh 75 kilograms, so 12 bowling balls weigh 60 kilograms. [\[Agree\]](#)
- A dozen eggs cost \$2.20, so 18 eggs will cost \$3.30. [\[Agree\]](#)
- John gets 10 out of 15 free throws. Jamal gets 8 out of 10 free throws. John has a better free-throw average. [\[Disagree\]](#)
- Swimming $\frac{1}{4}$ of a kilometre burns about the same number of calories as running a kilometre. Rahim runs 10 kilometres. He would have to swim 40 kilometres to burn the same number of calories. [\[Disagree\]](#)

Assessment Opportunities

Opposite Sides
Activity: See *Think Literacy Math: Grades 7–9*, p. 109

Action!**Individual → Practice**

Students complete BLM 8.6.1.

Content Expectations/Assessment/Scoring Guide: Students submit BLM 8.6.1 for assessment.

**Consolidate Debrief****Whole Class → Discussion**

Students pose questions that they still have about proportional reasoning, and other students respond.

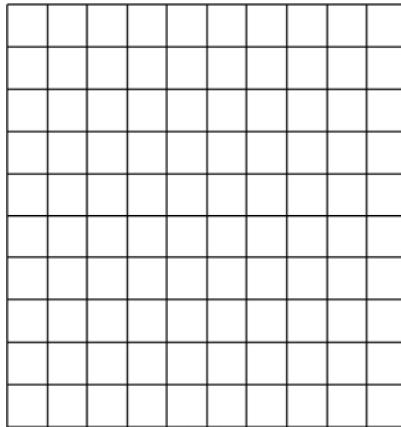
*Reflection***Home Activity or Further Classroom Consolidation**

Complete a journal entry: Learning about proportional reasoning is important because...

8.6.1: Proportional Progress

Name:

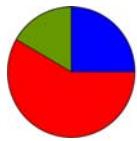
1. This week, Samir purchased 3 apples for \$2.00 and 6 apples for \$4.00. His father is pleased to see him eating healthy snacks, but wants to know the cost of each apple. Samir wants to show the answer to his father using two different methods. He graphed the cost of the apples. Explain how Samir can use the graph and one other method to answer his father's question about the cost of just one apple.
2. Alfreda believes that oranges cost less than the apples that Samir bought. She bought 4 oranges for \$2.50. Is it true that the oranges she bought are less expensive than Samir's apples? Show your work and explain your answer in writing.



3. Floyd and Rebecca ran equally fast along the track. It took Floyd 25 seconds to run 200 metres. How long did it take Rebecca to run 150 metres? Show your work.

4. Complete the following table showing different equivalent proportions:

Fraction	Ratio	Simplest Form
$\frac{14}{20}$		
	8:18	
		4:5
$\frac{21}{77}$		
	2:2	

**Math Learning Goals**

- Connect rates to proportional relationships.
- Solve problems involving unit rates.

Materials

- supermarket flyers

Assessment Opportunities

Provide visuals of two dart boards of different sizes to refer to during the discussion.

Minds On...**Whole Class → Discussion**

Pose the problem:

Trevor threw 5 darts at a large dartboard and Saleem threw 5 darts at a much smaller dartboard. All darts hit the dartboards.

In comparison to Saleem, are Trevor's darts closer together, farther apart, or is it impossible to tell? [impossible to tell]

How would your answer change if you were also told that the darts were evenly spaced across Trevor's and Saleem's respective dartboards?

Action!**Individual → Exploration**

Students find their pulse by starting to count when they get the signal. After 30 seconds, students stop counting and record number of beats they counted. This is the resting heart rate. Students engage in physical exercise for 1 minute and count their pulse rate again, this time for 20 seconds.

To make the count standard, change both heart rates to beats per minute.

Ask: What is the advantage of beats per minute?

Students explain why their heart rates are not the same.

Use this example to define unit rate. Have students identify other unit rates that they are familiar with, e.g., kilometres per hour, metres per second, cost per hour.

Pairs → Investigation

Students look through supermarket flyers to find examples of unit prices e.g., fruit in \$/kg.

Students choose 10 items and find the unit cost for each, e.g., price/mL, price/gram.

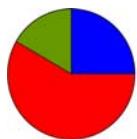
**Consolidate
Debrief****Individual → Presentation**

Volunteers present one of the items for which they found the unit price. Students explain why knowing the unit rate is useful for shopping. Students explain their understanding of unit rates and unit prices. **Note:** Unit prices are always \$ per _____.

Communicating/Presentation/Anecdotal: Observe students as they share their examples and thinking. Encourage the class to question each other until they understand.

Exploration**Home Activity or Further Classroom Consolidation**

Find everyday examples of unit rate on your way home and at home, e.g., speed limit sign, gas prices, and record them in your journal.

**Math Learning Goals**

- Compare unit rates.
- Problem solving with unit rate and unit prices.

Materials

- BLM 8.8.1, 8.8.2

Assessment Opportunities**Minds On...****Pairs → Problem Solving**

Students complete math analogies and share their reasoning.

- Seventy is to seven as dime is to _____.
- Metre is to kilometre is millilitre is to _____.
- Eight is to two as twelve is to _____.
- Cent is to dollar as centimetre is to _____.
- Half is to one as two is to _____.
- Nickel is to quarter as dime is to _____.
- Penny is to dime as dime is to _____.

Discuss any analogies that students don't agree on.

Action!**Whole Class → Problem Solving**

Set a context for problem solving:

The local food bank has started its semi-annual food drive. In support, local grocery stores have advertised a sale on canned soup. Three different brands of soup are available in large quantities. Yummy in Your Tummy soup is being sold at \$18.89 for 12 cans of 284 ml. Canned Creations soup is being sold at \$30.69 for 24 cans of 284 ml.

Ask: Which is the better deal between these two brands? Justify your answer using two different solutions. [Canned Creations soup is the better deal since you get double the quantity for less than double the price.]

In pairs, students solve a further problem and explain their reasoning:

- Super Soup is being sold at \$60.99 for 24 larger cans of 568 ml. Which is the better deal now?
- What was the most economical way to purchase the soup?
- If the school raises \$200 to buy for the food bank, what is the most soup they could buy?

Individual → Practice

Students complete BLM 8.8.1.

Curriculum Expectations/Observation/Checklist: Ask questions to determine that they understand unit rates and observe their reasoning.

**Consolidate
Debrief****Whole Class → Discussion**

Students discuss quantity purchased and determine the strategies for purchasing the most items for the least amount.

Exploration

Home Activity or Further Classroom Consolidation

Complete worksheet 8.8.2.

8.8.1: Best Buy

You're staying with your aunt this weekend and have been sent out to buy some things at the grocery store. She expects you to get the best value for the \$20.00 she has given you. You must buy the following items from the shopping list. There is no tax on food items.

Shopping List

skim milk

yogurt

granola bars

hamburger buns

veggie burgers

When you get to the store, you see the following options:

skim milk

1 litre for \$1.69

4 litres for \$5.29

yogurt

4 containers of 85 ml for \$2.69

2 containers of 150 ml for \$2.59

granola bars

package of 6 for \$3.19

package of 8 for \$3.79

hamburger buns

package of 12 for \$2.29

package of 8 for \$1.99

veggie burgers

package of 2 for \$1.85

package of 4 for \$3.40

Calculate the unit rate for each item to decide which is the better buy.

Show your calculations on the back of this page, then record which option you would purchase.

Calculate how much money you spent and how much is left over from the \$20.00.

8.8.2: Best Bite

Using the nutritional information on the table, calculate which is the healthier cookie on the basis of calories and total fat. Calories and fat are measured using different units and therefore require separate calculations. Consider all the information in the table.

Nutritional Information

Cookie and serving size (g)	Oatmeal 100	Shortbread 75	Vanilla 24	Raisin 25
Calories (kcal)	480	360	117	120
Total Fat (g)	24.3	19.8	5.7	5.9

Show your work for each criteria, then clearly identify which is the healthier cookie. Explain your reasoning.