Unit 1: Day 2: Can You Solve These Mysteries?

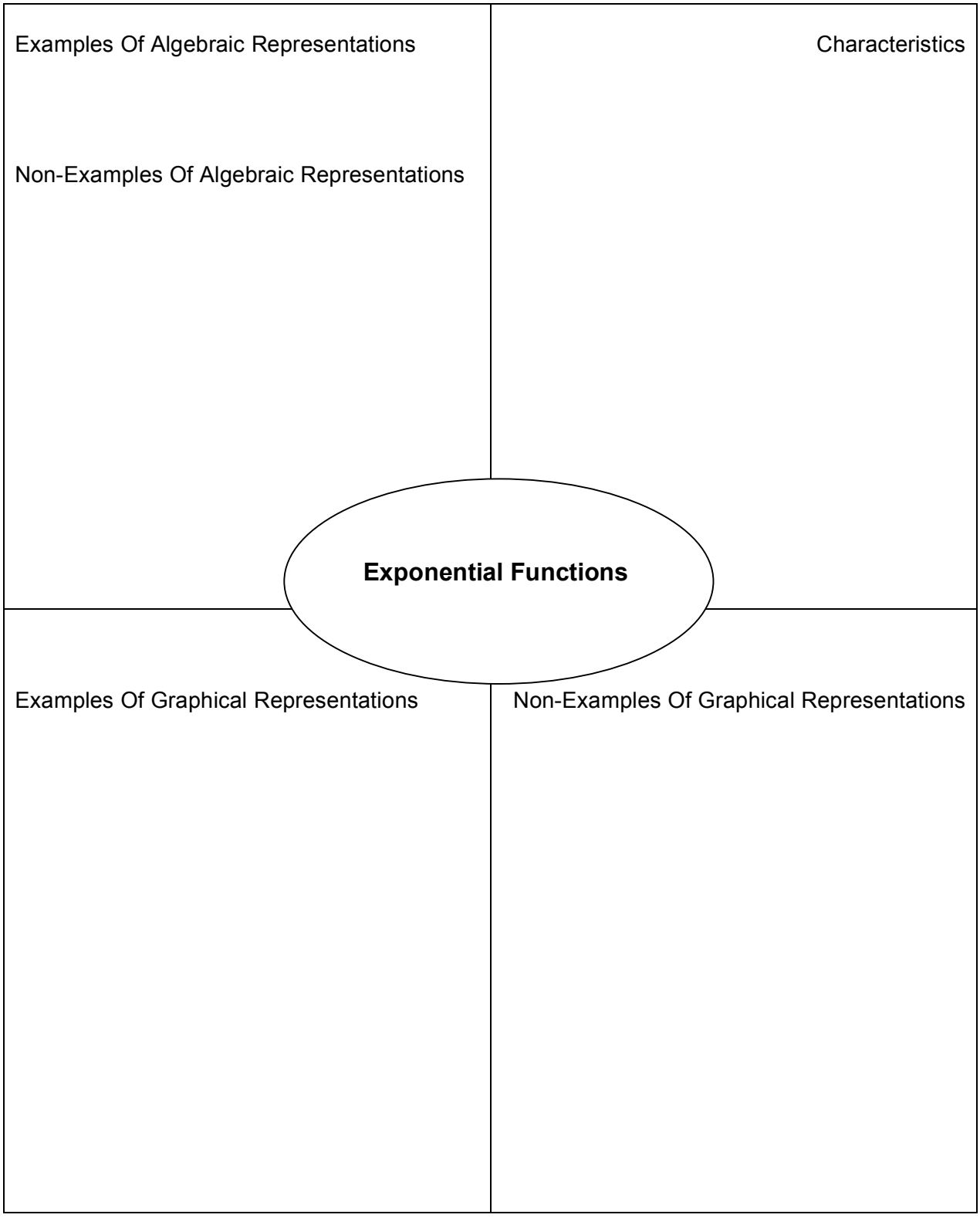
MCT 4C

Minds On: 5 Action: 50 Consolidate:20 Total=75 min	**Math Learning Goals:** • Solve simple exponential equations numerically and graphically, with technology • Connecting algebraic and graphical representations of exponential equations

Materials
- BLM 1.2.1
- BLM 1.2.2
- BLM 1.2.3
- BLM 1.2.4
- Graphing calculators

			Assessment Opportunities
Minds On…	**Groups of 3 or 4 → Brainstorm** Have students complete BLM 1.2.1 to connect to previous lesson on exponential functions. A large Frayer model on chart paper or overhead transparency can be used to capture the responses from the class.		**Optional:** Chart copy or overhead transparency copy of BLM 1.2.1 and BLM 1.2.3
Action!	**Individual → Guided Exploration** Introduce BLM 1.2.2 and recall the meaning of "solving an equation". Students will have to "solve" each of the four mysteries. As they work through each mystery they will be introduced to different strategies for solving exponential equations (i.e. inspection; trial and error; graphically with technology). **Pairs → Practise** For question #15, students may exchange the equations they created and see if their partner can solve their equations using whatever strategy they choose. **Learning Skills/Teamwork/Checkbric:** Observe work habits and initiative as students work through the investigation individually. **Mathematical Process Focus: Connecting – Connect** graphical and algebraic representations for solving equations.		Provide examples and further practice for solving simple equations algebraically. BLM 1.2.2 question 5 could use TRACE or table of values with the calculator. A review for finding the intersection of two curves using graphing technology may be needed here. .
Consolidate Debrief	**Groups of 3 or 4 → Brainstorm** Students complete a Frayer model (BLM 1.2.3) for Exponential Equations. Highlight the differences and similarities between functions and equations. **Whole Class → Discussion** Share group responses to the Frayer model. Ensure students add anything that is missing to their own. A large Frayer model on chart paper or overhead transparency can be used to capture the responses from the class.		
Differentiated Exploration Application Reflection	**Home Activity or Further Classroom Consolidation** Students complete BLM 1.2.4 for home activity for further consolidation. **Curriculum Expectation/Reasoning /Rubric: Next day collect BLM 1.2.4 and assess students' understanding of exponential equations.**		

1.2.1: Frayer Model – Exponential Functions



1.2.2: Can You Solve This Mystery?

Recall:"To solve an equation" means to determine the value of the variable that makes the equation true.

MYSTERY #1 – How can you solve exponential equations?

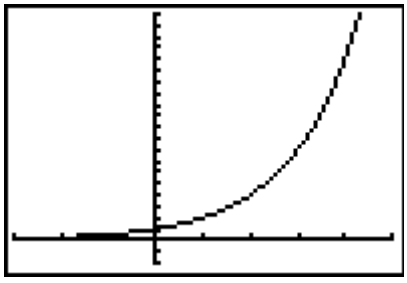
1. Solve these exponential equations. Match the solution with the equation.

Equation		Solution
_____	1. $2^x = 8$	A. x = -2
_____	2. $2^x = 16$	B. x = 0
_____	3. $2^x = \frac{1}{4}$	C. x = 3
_____	4. $2^x = 1$	D. x = 4

2. Describe the process that you used to solve the equations above.

3. Solve $2^x = 5$. Round your answer to the nearest hundredth.

4. Use a graphing calculator and window settings to graph the function $y = 2^x$.



5. Complete the following statements.

a) The value of the function is 16 when x = _____

b) The value of the function is $\frac{1}{4}$ when x = _____

c) The value of the function is 8 when x = _____

d) The value of the function is 5 when x = _____ (round your answer to two decimal places)

1.2.2: Can You Solve This Mystery? (continued)

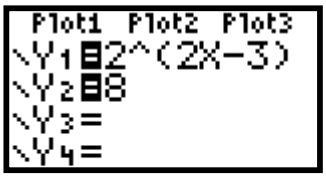
6. Explain how to use the graph of the function $y = 2^x$ to solve the equations in #1.

7. Explain why you **<u>can not</u>** use the graph of the function $y = 2^x$ to solve the equation $2^{2x-3} = 8$ but you **<u>can</u>** use $y = 2^x$ to solve the equation $2^x = 8$.

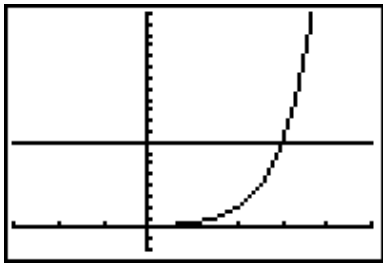
<u>MYSTERY #2 – How can you solve more difficult exponential equations?</u>

8. Fear not! There is a way to solve the equation $2^{2x-3} = 8$.

   Enter the left side of the equation as one function and the right side of the equation as another function using the ! editor of your calculator as shown.



9. Label the functions as $y = 2^{2x-3}$ and $y = 8$ on the screen shot below.



y = _____ (equation of curve)

y = _____ (equation of line)

10. Use the intersect operation to find the point of intersection of the two functions and complete the following statements.

    The point of intersection occurs when x = _____ and y = _____.

    The solution to the equation is x = _____.

    When x = _____, both functions have a value of _____.

<u>MYSTERY #3</u>

11. Use your graphing calculator and the Intersection Method to solve the following equations. Record the solution.

Equation	Y_1	Y_2	Solution to Equation
a) $2^{2x-3} = 8$	$Y_1 = 2^{2x-3}$	$Y_2 = 8$	x =
b) $2^{2x-3} - 6 = 2$			
c) $2^{2x-3} + 5 = 13$			

12. Solve the mystery…Why are all of the solutions to these equations the same?

1.2.2: Can You Solve This Mystery? (continued)

MYSTERY #4

13. True or false (check one)? "The solution to any exponential equation is always an exact value." ❒ true or ❒ false Justify your choice.

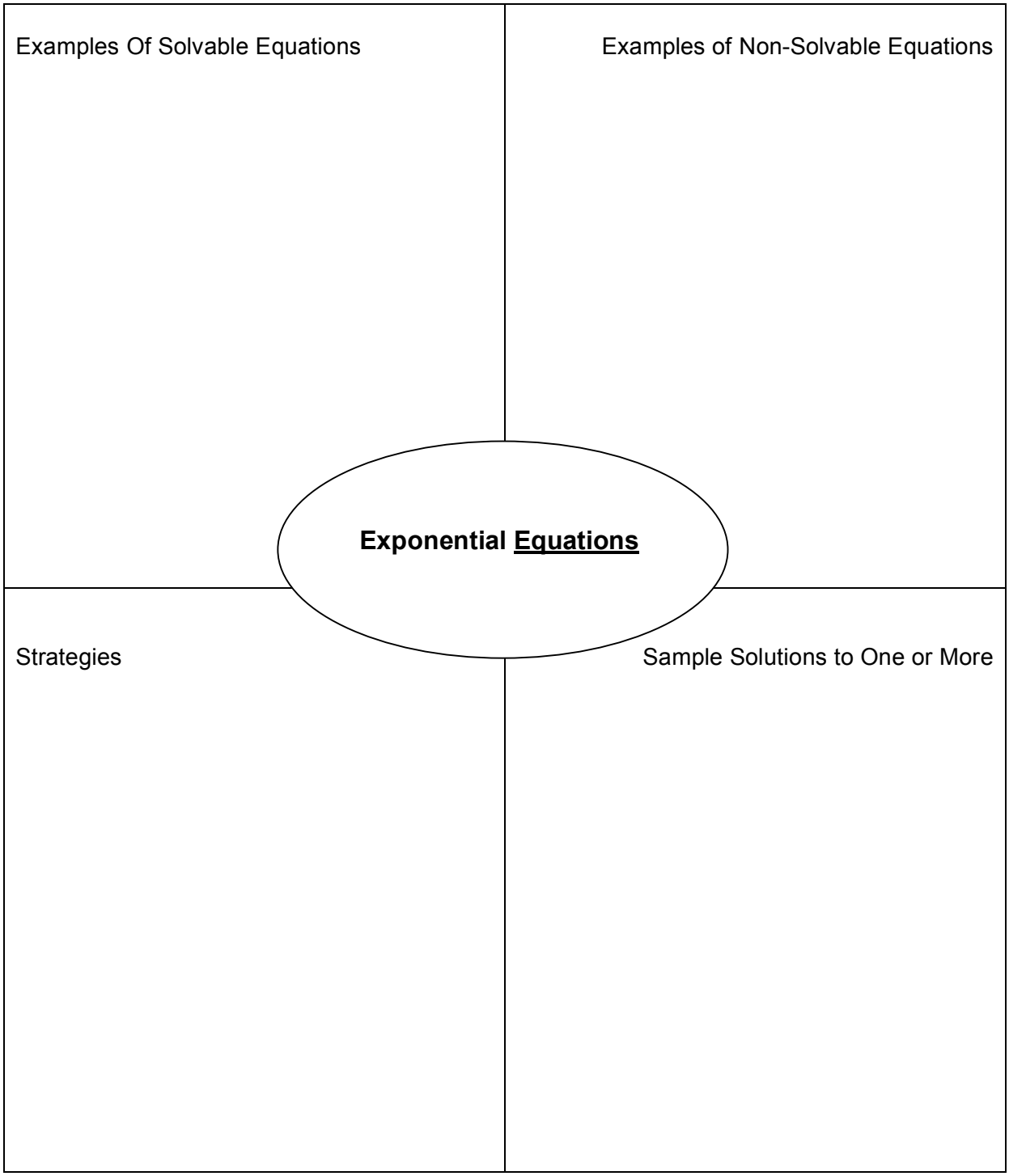
14. Use trial and error on your calculator to determine the solution to the following equations. Compare your solution using the Intersection Method.

Equation	Solution Using Trial And Error On Your Calculator (2 decimal places)	Solution Using The Intersection Method (2 decimal places)
a) $3.05^x = 15$	x =	x =
b) $2.3^x = 6$	x =	x =
c) $4.2^{-x} = 7$	x =	x =
d) $3.05^x = 15$	x =	x =

15. Make an exponential equation, with base 5, where the solution is

a) an exact value	b) 0.75
c) a negative integer	d) an irrational number (not exact)

1.2.3: Frayer Model – Exponential Equations



1.2.4: Further Mysteries to Ponder

1. Explain why $5^x = -2$ has **no** solution but $2^x - 5 = -2$ has a solution.
   Provide graphical and algebraic support.

2. Is it possible to create an exponential equation that has more than one solution?
   Provide graphical and algebraic support.

3. Compare and contrast the solutions in group A with those in group B.

Group A	Group B
$2^{-x} = 8$	$2^x = 8$
$2^{-x} = 1$	$2^x = 1$
$2^{-x} = \frac{1}{4}$	$2^x = \frac{1}{4}$
$2^{-x} = 16$	$2^x = 16$

Unit 1: Day 11: Summative Assessment: Expo-Experts

MCT 4C

Minds On: 20 Action: 50 Consolidate:5 Total=75 min	**Math Learning Goals:** • Representing an exponential function with an equation, given its graph or properties • Connect algebraic and graphical models of exponential functions for collected data • Identifying properties of exponential functions, including those that arise from real-world applications and explain any restrictions that the context places on the domain and range • Compare and contrast properties of exponential functions that arise from real-world applications	**Materials** • 35 baggies of M&M's® • Graphing calculators • 35 paper plates • 35 sheets of letter sized paper • Scissors • Glue sticks • Rulers • BLM 1.11.1-1.11.5

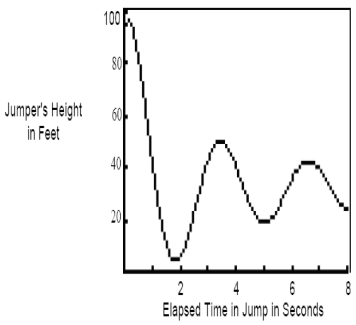
			Assessment Opportunities
Minds On…	**Pairs → Activity** Distribute copies of BLM 1.11.1 per pair. Students will need scissors and glue sticks for the matching sheet. Collect their matching sheet and move onto individual investigations. Make sure students are aware of time constraints. **Learning Skills/Teamwork/Checkbric:** Observe teamwork as pairs work through investigation.		. The paired investigation may be completed individually and assessed as a part of the summative. Adjustment in timing can be made vby using only one of the investigations or completing the second investigation on the next day using one of the two jazz days in Unit 2.
Action!	**Individual → Investigation** Students use graphing calculators as they work through the summative assessment on BLM 1.11.2 and BLM 1.11.3. **Mathematical Process Focus: Connecting** – Students **connect** different models of exponential data. **Mathematical Process Focus: Representing** – Students **represent** real-world applications that can be modelled by exponential functions.		For those students who need assistance with calculator skills, a brief review can be provided the day prior or a tip sheet (BLM 1.11.4) can be distributed to students.
Consolidate Debrief	**Whole Class → Debrief** Collect all materials from each student. Discuss the nature of the two investigations and which ones were examples of exponential growth and decay. **Curriculum Expectations /Rubric: Use BLM 1.11.5 to evaluate student work from the investigations.**		
Concept Practice Application	**Home Activity or Further Classroom Consolidation** Prepare for pencil and paper test that addresses the expectations that were not covered in the final summative task.		

1.11.1: Sort this out

Instructions:

1. Cut along each of the solid lines and sort each function as either an exponential or non-exponential function.
2. With the group of exponential functions, sort them as either growth or decay functions. When you are satisfied with your sorted functions glue them to the recording sheet.
3. Based on your thinking, write a justification next to each function explaining why you have categorized the function in that way.

A



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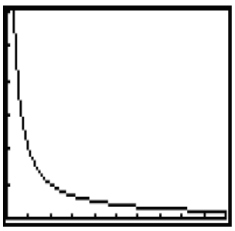

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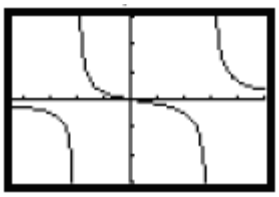

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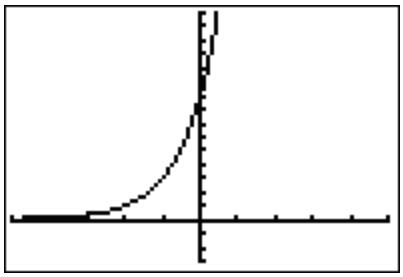

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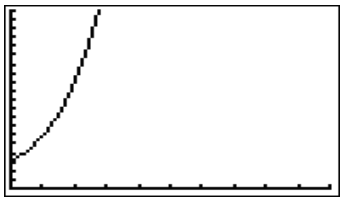

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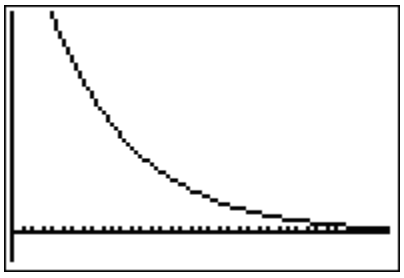

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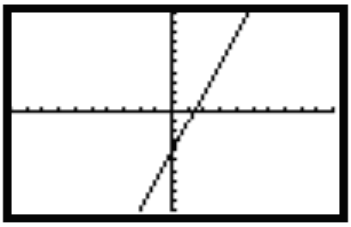

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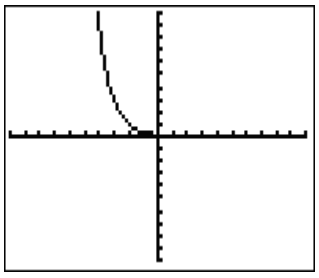


1.11.1: Sort this out (continued)

Instructions: Glue all of the functions onto this sorting sheet. Make sure to write a justification for each choice next to each function.

Your name: _____ Your partner: _____

Exponential Functions *Group exponential functions here and sort them into either **growth or decay** below.*		**Non-exponential functions**
Decay	**Growth**	

1.11.2: Melts in your mouth, not in your hands! ®

Your name: _____

Instructions:

1. Empty your bag of M&M's® onto your paper plate and count the total. Record this initial value as trial number 0 in the table below.
2. Replace M&M's® into the bag and mix them well.
3. Pour the M&M's® back out onto the desk and count the number of M&M's® which have an "m" showing and place them back into the bag. Record this value in the table below. The others can be eaten or removed.
4. Continue to pour M&M's® onto the paper plate, count and record the number of "m's" removing the others until the number of M&M's® remaining is less than 5.

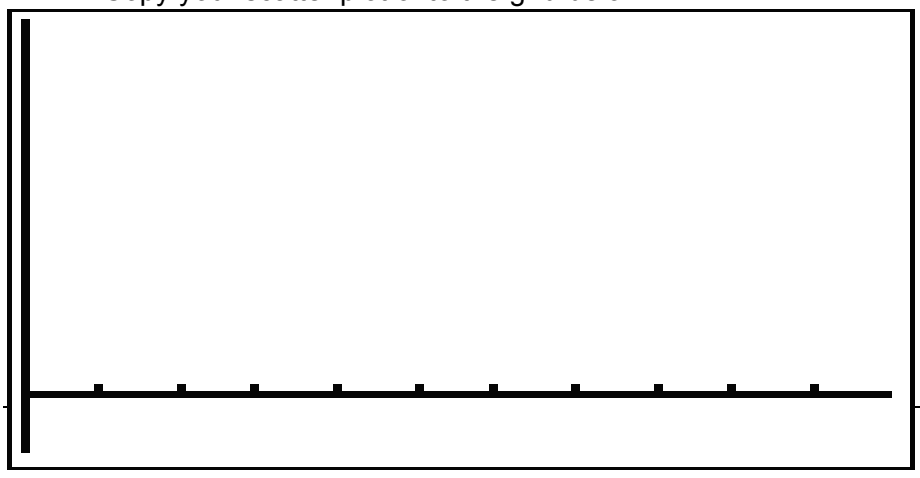
Part 1: Collecting Data

Trial number	Number of M&M's® remaining
0	
1	
2	
3	
4	
5	
6	

Add rows if needed.

Part 2: Graphing

1. Using a graphing calculator, make a scatter plot of the data.
2. Copy your scatter plot onto the grid below.



1.11.2: Melts in your mouth, not in your hands! ® (continued)

Your name: _____

1. The scatter plot of the data is (check one): ❒ exponential OR ❒ quadratic
   Provide two reasons for your choice.

2. Determine the percentage decrease from one trial to the next. Record your answers in the chart.

Trial numbers	Percentage decrease in M&M's®
Trial 0 to Trial 1	
Trial 1 to Trial 2	
Trial 2 to Trial 3	
Trial 3 to Trial 4	
Trial 4 to Trial 5	
Trial 5 to Trial 6	
Trial 6 to Trial 7	

Add rows if needed.

3. Find the average of the percentage decreases from one trial to the next.

4. Using your average, create a model that will predict the number of M&M's® after any trial.

5. Using the graphing calculator, find an equation that will model the data: _____

6. Compare your model with the one given by the graphing calculator. Explain any similarities or differences between the two.

1.11.2: Melts in your mouth, not in your hands! ® (continued)

Part 4: Interpreting the data

1. Using your mathematical model, when x = 3, what is the value of the function? Show your work below. Compare this to the value in the data table. Explain why there is a difference between the value from the model and value from the data.

2. Mr. Johnston's class decided to combine all their M&Ms® and came up with the following model for their data: $y = 2100(0.65)^x$, where x is the trial number and y is the number of M&Ms® remaining. Explain the significance of the values in the model by referring to the number of M&Ms® and the rate of change.

3. At some point during Mr. Johnston's investigation, the class had 100 M&Ms® left. Determine how many trials had taken place. Show your work.

1.11.3: Origami, it's not!

Instructions:

1. Fold a sheet of white paper in half.
2. Unfold the sheet of paper and count the number of sections.
3. Refold and add a new fold.
4. Continue to fold and count the sections after you make each new fold.
5. Record the data in the table below.

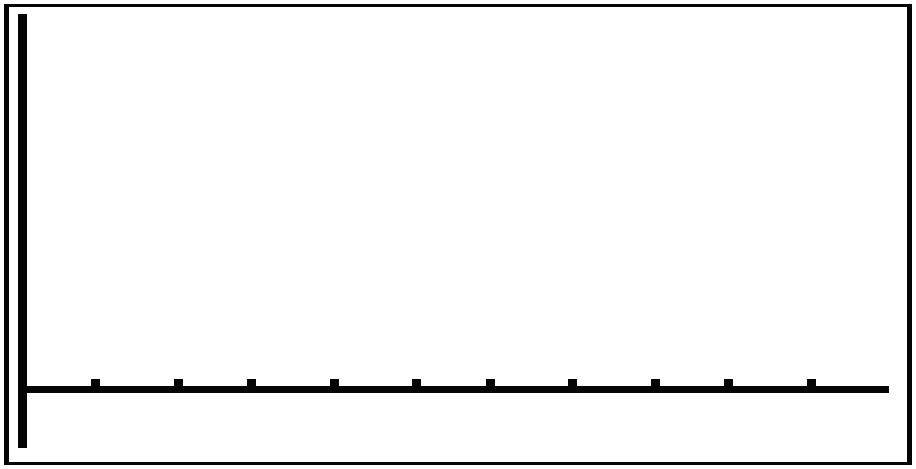
Part 1: Collecting Data

Number of folds	Number of sections
0	
1	
2	
3	
4	
5	
6	

Add rows if needed.

Part 2: Graphing

1. Using a graphing calculator, make a scatter plot of the data.
2. Copy your scatter plot onto the grid below.



1.11.3: Origami it's not! (continued)

Your name: _____

Part 3: The model

Using the graphing calculator, find an equation that will model the data: _____

Part 4: Interpreting the data

1. Using your algebraic model, what is the value of the function when x = 6? Show your work below. Compare this to the value on your graphical model. Explain why there is or is not a difference between the values from the two models (algebraic and graphical).

2. Based on the general exponential function, $y = a(b)^x$, what is the value for b in your algebraic model? What does this value represent in the context of the problem?

3. Compare and contrast the M&M® mathematical models to the paper folding mathematical models.

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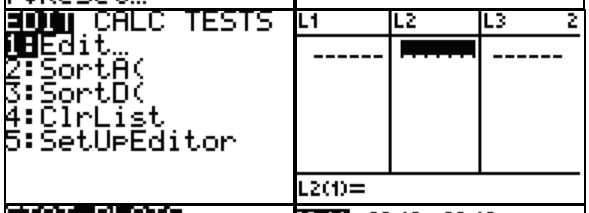
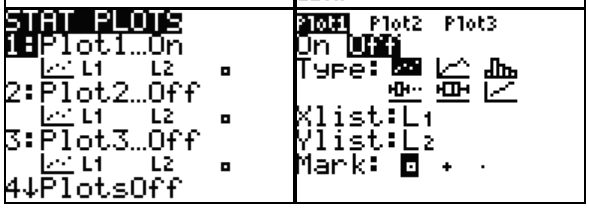
1.11.4: Graphing Calculator Tip Sheet

Clear lists of any data that may be in the calculator by pressing ` > + and selecting 4: ClrAllLists.
The command will be pasted into the home screen.
Press e to execute the command.

To enter data, press S and select the first menu option, 1: Edit.
You will now have an empty list editor. Enter your data into the lists. Make sure that the lists are the same length

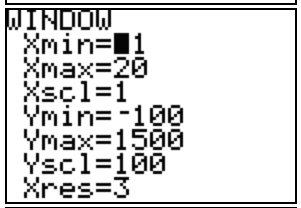


To create a scatter plot, press ` > ! . To create the scatter plot in the first plot, press e.
Turn on the plot by placing the cursor over the word "On" and pressing e.
The scatter plot is the first type and should be highlighted.
If your data is in Ω and æ, then the Xlist and Ylist should be set to Ω and æ respectively.
Select the mark you prefer.
The final window should look like the final screen capture at right.

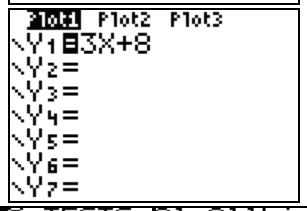


Final scatter plot window!

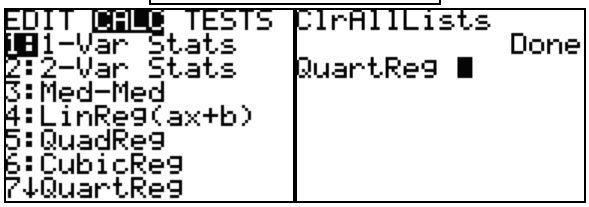
To change the axes and scale, press @.
Set the values on the screen here to capture a good view of all the data.



To see the scatter plot, press %. Adjust the window if needed.
If there is a function that is being graphed that you don't want, press ! and place the cursor on the line with the unwanted function and press C.



To perform a regression, press S and move over to the CALC menu. Select the regression you want and press e. The regression will be preformed on Ω and æ by default. Press e to execute the command.
If the regression doesn't fit your data well, try another one.



1.11.5: Expo-Experts Assessment Rubric

Name:_____

Reasoning and Proving				
Criteria	**Level 1**	**Level 2**	**Level 3**	**Level 4**
Degree of clarity in explanations and justifications in reporting **11.1** **11.2 Part 3 #1 & 6** **11.3 Part 4 #3**	Explanations and justifications are partially understandable	Explanations and justifications are understandable by me (for a teacher), but would likely be unclear to others who have not studied exponential functions	Explanations and justifications are clear for a range of audiences	Explanations and justifications are particularly clear and detailed
Exploring and Reflecting				
Ability to apply the processes of inquiry and problem solving i.e., reflecting, revisiting and revising **11.2 Part 4 #1** **11.3 Part 4 #1**	Applies the processes to the assigned task with significant prompts	Applies the processes to the assigned task with minor prompts	Applies the processes to the assigned task without prompts	Applies the processes to the assigned task with a broader view of the task without prompts
Selecting Tools and Computational Strategies				
Select and use tools and strategies to solve a problem **11.2 Part 3 #2,3; Part 4 #3**	Selects and applies appropriate tools, with major errors, omissions, or mis-sequencing	Selects and applies appropriate tools, with minor errors, omissions or mis-sequencing	Selects and applies appropriate tools, accurately, and logically sequenced	Selects and applies the most appropriate or variety of appropriate tools, accurately and logically sequenced
Connecting				
Relate mathematical ideas to situations drawn from other contexts **11.2 Part 4 #2** **11.3 Part 4 #2**	Makes weak connections	Makes simple connections	Makes appropriate connections	Makes strong connections

1.11.5: Expo-Experts Assessment Rubric (continued)

Representing				
Criteria	**Level 1**	**Level 2**	**Level 3**	**Level 4**
Creation of a model to represent the data [e.g., numerical, algebraic, graphical, physical, or scale model, by hand or using technology] **11.2 Part 1 & 2; Part 3 #4 & 5** **11.3 Part 1 & 2; Part 3**	Creates a model that represents little of the range of data	Creates a model that represents some of the range of data	Creates a model that represents most of the range of data	Creates a model that represents the full range of data
Communicating				
Correct use of mathematical symbols, labels, units and conventions **11.2 Part 2; Part 3 #3 & 4; Part 4 #3** **11.3 Part 2;**	Sometimes uses mathematical symbols, labels and conventions correctly	Usually uses mathematical symbols, labels and conventions correctly	Consistently uses mathematical symbols, labels and conventions correctly	Consistently and meticulously uses mathematical symbols, labels and conventions, recognizing novel opportunities for their use
Appropriate use of mathematical vocabulary **11.1** **11.2 Part 3 #1 & 6; Part 4 #1 & 2** **11.3 Part #1, 2 & 3**	Sometimes uses mathematical vocabulary correctly when expected	Usually uses mathematical vocabulary correctly when expected	Consistently uses mathematical vocabulary correctly when expected	Consistently uses mathematical vocabulary correctly, recognizing novel opportunities for its use