Unit 5 Geometry

MAP 4C

Foundations for College Mathematics

BIG PICTURE

Students will:

- Understand the relationships between imperial and metric units
- Consolidate understanding of perimeter, area, surface area, and volume through real-life problems
- Explore optimization of two-dimensional and three-dimensional figures

Day	Lesson Title	Math Learning Goals	Expectations

1	Exploring Relationships between Metric and Imperial
	
	Lesson Included
	New Jan/08*
		• Explore relationships that exist between inches and centimeters
		(measuring tools: string, both types of rulers, or tapes)
		  • Reading ruler, measuring tape (fraction)
		  • Create a scatter plot from the student's data
		  • Perform a linear regression and get the equation
		  • Connect to the actual conversion (inches <-> centimetres)
			GT1.1

2		• Trundle wheel activity for perimeter • Converting mixed imperial measurements <-> metric Example convert 5 1/8" to cm	GT1.1
3		• Finding the area of rectangles, triangles, and circles, and of related composite shapes, in situations arising from real-world applications • Using imperial, metric and conversions when necessary	GT1.2
4	Maximizing Area for a Given Perimeter *Lesson Included*	• Maximum area for a given perimeter **Problem**: Cagey Problem, Why are copper wires round?	GT2.2,GT2.1
5	Minimizing Perimeter *Lesson Included* ***New Jan/08****	• Minimum perimeter for a given area **Problem**: Fencing	GT2.2,GT2.1
6		• Jazz Day	
7		• Volume problems involving rectangular prisms, triangular prisms, cylinders, and composite figures • Using imperial, metric and conversions when necessary **Example**: Volume of Concrete Pad in cubic meters with initial measurements in feet and inches. Example 8' x 24' x 4"	GT1.3
8		• Surface area problems involving rectangular prisms, triangular prisms, cylinders, and composite figures • Using imperial, metric and conversions when necessary	GT1.3
9	Maximum Volume of a Triangular Prism *Lesson Included*	• Maximum volume for a given surface area • Using imperial, metric and conversions when necessary	GT2.3,GT2.1

10	Minimum Surface Area of a Cylinder *Lesson Included*	• Minimum surface area for a given volume • Using imperial, metric and conversions when necessary	GT2.3,GT2.1
11-13	Keeping the Kandies *Summative Included M.O.**	Summative Task • Packaging Project	

Unit 5: Day 1: Exploring Relationships between Metric and Imperial

Minds On: 10 Action: 35 Consolidate:30 Total=75 min	**Math Learning Goals:** • Explore relationships that exist between metric and imperial measuring systems	**Materials** • BLM 5.1.1 • BLM 5.1.2 • BLM 5.1.3 • Measuring tapes with metric and imperial measures • Graphing Calculators or statistics software

			Assessment Opportunities
	Minds On…	**Whole Class → Discussion** Put the titles "Metric" and "Imperial" on the board. Ask students to list several units of measure (for distance, weight or volume) and discuss whether the suggestions belong to the metric or imperial system. Determine if students are already aware of relationships between metric and imperial systems. (e.g. Some students may already know that there are 2.54 cm in an inch or that there are 12 inches in a foot.) Discuss where each system of measure is currently in use and the need to be able to convert between systems. **Mathematical Process Focus: Reflecting** Students **reflect** on prior knowledge of measurement units and conversions	When adding units to the list, question students whether they are distance, volume, mass, etc. When adding several measures of one type, ask students to order them by size (eg. If students say yards, inches and feet are all imperial distance measures, ask them which is biggest and which is smallest of the three given.
	Action!	**Pairs → Investigation** Explain to students that they will be measuring several objects around the classroom today in both metric and imperial units. They will then be required to perform a regression (as in unit 2 of the course) to determine the relationship between the two units of measure. Distribute BLM 5.1.1 to each pair to perform the investigation. **Learning Skills/Observation/Mental Note:** Assess teamwork skills as you circulate. **Mathematical Process Focus: Connecting –** students **connect** prior knowledge of linear regression with finding a relationship between the lengths of inches and centimetres, yards and metres, etc.	
	Consolidate Debrief	**Whole Class → Sharing Results** For each activity, have students write their equation of the line of best fit on the board as shown in BLM 5.1.3. Using the equations, decide on the appropriate conversion factor for each activity. **Whole Class → Board Note** Using BLM 5.1.3 as a guide, show students how to perform conversions of distances and areas. **Whole Class → Discussion** Discuss other units of measure and their conversion factors. Give the example of the Gimli Glider incident to show the importance of proper/correct conversions (and to peak their interests). In particular, ask the students to determine how much fuel they were short for the flight (the plane needed 22,300kg of fuel, but was loaded with 22,300pounds instead)	Gimli Glider info can be found at: http://en.wikipedia.org/wiki/Gimli_Glider And the news report at: http://archives.cbc.ca/IDC-1-69-240-1155-20/that_was_then/life_society/gimli_glider
Exploration Skill Practice		**Home Activity or Further Classroom Consolidation** Complete BLM 5.1.2	

5.1.1: Metric and Imperial: What's the Relationship?

Exploring Centimetres and Inches

Data Collection

1. Measure 5 objects of varying sizes around the classroom.
2. Measure each object two times: once in centimetres and once in inches.
3. Record your results in the table.

Object	Length in Inches	Length in Centimetres

Analyze the data

4. Enter your data into a spreadsheet or graphing calculator.
5. Perform a linear regression to obtain the equation of the line of best fit.

Line of Best Fit: _____

Interpret your findings

6. Measure another object in the classroom in inches.

Object: _____ Length in inches: _____

7. Use your line of best fit to calculate the length in centimetres.

8. Now measure the item in centimetres. How close was your calculation to your measurement?

9. How do you convert lengths given in inches to centimetres?

10. How do you convert lengths given in centimetres to inches?

5.1.1: Metric and Imperial: What's the Relationship?
(Continued)

Exploring Centimetres^2 and Inches^2

You have determined a relationship between centimetres and inches, which can be used to convert lengths or distances. Now, you will determine a relationship to convert **areas** in inches^2 and centimetres^2.

Hypothesis

1. Do you think the relationship between inches^2 and centimetres^2 is the same as the relationship you found between centimetres and inches? Explain.
   (example: how would you convert 12 cm^2 to inches^2?)

Data Collection

2. Measure **the area** of 5 objects of varying sizes around the classroom.
3. Measure the area of each object two times: once in inches^2 and once in centimetres^2.
4. Record your results in the table.

Object	Area in Inches^2	Area in Centimetres^2

Analyze the data

5. Enter your data into a spreadsheet or graphing calculator.
6. Perform a linear regression to obtain the equation of the line of best fit.

Line of Best Fit: _____

Interpret your findings

7. Do your findings agree or disagree with your hypothesis? Explain.

8. How do you convert areas given in inches^2 to centimetres^2?

10. How do you convert areas given in centimetres^2 to inches^2?

5.1.1: Metric and Imperial: What's the Relationship?
(Continued)

Exploring Metres and Yards

Data Collection
1. Measure 5 objects of varying sizes around the classroom.
2. Measure each object two times: once in metres and once in yards.
3. Record your results in the table.

Object	Length in Metres	Length in Yards

Analyze the data
4. Enter your data into a spreadsheet or graphing calculator.
5. Perform a linear regression to obtain the equation of the line of best fit.

Line of Best Fit: _____

Interpret your findings
6. Measure another object in the classroom in metres.

Object: _____ Length in metres: _____

7. Use your line of best fit to calculate the length in yards.

8. Now measure the item in yards. How close was your calculation to your measurement?

9. How do you convert lengths given in metres to yards?

10. How do you convert lengths given in yards to metres?

5.1.2: Metric and Imperial Conversions

1. Perform the following conversions using the relationships determined in class today.

   a) 15 cm = _____ inches
   b) 20 inches = _____ cm
   c) 7 3/8 inches = _____ cm
   d) 10.4 cm = _____ inches
   e) 6 yards = _____ m
   f) 18 m = _____ yards
   g) 4 yards 2 feet = _____ m
   h) 16 m 52 cm = _____ yards

2. Another set of distance measurements commonly used are miles and kilometres.

   a) What unit of distance do you *think* is longer, a mile or a kilometre?
   b) There are 1760 yards in a mile. Convert this number to metres.
   c) How many metres are in a kilometre?
   d) Which unit of distance is longer, the mile or the kilometre?
   e) Determine the relationship between miles and kilometres based on your above calculations.

3. Converting areas is not the same as converting distances. Below is a different method you can use to convert areas in different units of measurement if you already know the appropriate conversions for lengths.

   Convert 16 miles^2 to km^2 by following the steps outlined below.

   a) Consider 1 mile^2. Convert the side lengths to kilometres.

   b) Calculate the area in km^2.
   c) Use the information obtained in parts a) and b) to convert 16 miles^2 to km^2.

4. Use any method to convert 30 inches^3 to cm^3.

5.1.3: Metric and Imperial Conversions (Teacher Notes)

Board Note (for Consolidation):

Line of Best Fit (Inches – Centimetres)	**Line of Best Fit (Inches^2 – Centimetres^2)**	**Line of Best Fit (Metres – Yards)**

(*Have students write their lines of best fit for each of these scenarios on the board.)*

There are approximately 2.54 cm in an inch.
There are approximately 6.45 cm^2 in an inch^2.
There are approximately 1.09 yards in a metre.

<u>Performing Conversions</u>

1. Perform the following conversions using the relationships determined in class today.

a) 5 7/8 inches = _____ cm

$$\frac{inch}{cm}\frac{1}{2.54} = \frac{5\frac{7}{8}}{x}$$

$$\frac{1}{2.54} = \frac{5.875}{x}$$

$$1x = (2.54)(5.875)$$

$$x \approx 14.9cm$$

b) 21.2 cm = _____ inches

$$\frac{inch}{cm}\frac{1}{2.54} = \frac{y}{21.2}$$

$$(21.2)(1) = 2.54y$$

$$\frac{21.2}{2.54} = y$$

$$y \approx 8.3inches$$

c) 3 yards 1 foot = _____ m

d) 14 m 33 cm = _____ yards

e) 42 cm^2 = _____ inches^2

f) 15 inches^2 = _____ cm^2

$$\frac{inch^2}{cm^2}\frac{1^2}{2.54^2} = \frac{z}{42}$$

$$(42)(1) = (2.54)^2 y$$

$$\frac{42}{(2.54)^2} = y$$

$$y \approx 6.5inches^2$$

2. A Canadian Football League playing field is 110 yards by 65 yards. Determine the area of the football field in m^2.

   ** Allow students to try this question as a think-pair-share. Hopefully you will see at least 2 different solutions (determine the area, then convert to metric; convert the dimensions to metric then determine the area)*

Instead of 22,300 kg of fuel, they had 22,300 *pounds* on board

Unit 5: Day 5: Minimizing Perimeter

Minds On: 10 Action: 45 Consolidate:20 Total=75 min	**Math Learning Goals:** • Determine minimum perimeter for a given area. • Compare minimizing perimeter for a set area with maximizing area for a set perimeter	**Materials** • Geometer's Sketchpad • U5L5_GSP1 • BLM 5.5.1 • BLM 5.5.2

			Assessment Opportunities
Minds On…	**Think, Pair, Share → Discussion** Put two questions on the board: 1. *Determine the dimensions of the right triangle with a maximum area where the perimeter must be 32 inches.* 2. *Determine the dimensions of the right triangle with a minimum perimeter where the area must be 40 inches^2.* Have students discuss with a partner the similarities and differences between these two questions. During the discussion, prompt students to focus on what information is given, what is required to be found, and what process might you go through to solve the problems. Point out to students that the first question was from the previous day's lesson and the second question will be the focus of today's lesson.		
Action!	**Individual → Investigation** Distribute BLM 5.5.1 to students and direct them to the GSP file that they should use as an aid in completing the investigation. **Reasoning and Proving/Observation/Mental Note:** Circulate and give students feedback on their work. **Mathematical Process Focus: Reasoning and Proving –** students are required to investigate several different options and must argue why the option they chose is the best one.	⇒	Students should have access to U5L5_GSP1
Consolidate Debrief	**Student Grouping → Instructional Strategy** Distribute BLM 5.5.2 and instruct students on how to manually find the dimensions now that we have determined some special relationships (Refer to BLM 5.5.2 (Teacher notes). (i.e. That the optimal dimensions for a rectangle must be a square and that the optimal dimensions for a right triangle is a right isosceles triangle.)		
Concept Practice	**Home Activity or Further Classroom Consolidation** Complete questions 1 and 2 at the bottom of BLM 5.5.2		

5.5.1: Park Beautification

A local park's officials have been given a grant to beautify its facilities. Along with new playground equipment and repainting of all the buildings, a garden is to be planted somewhere on the grounds.

Three different locations have been chosen for the garden. However, the brick used to line the perimeter of the garden is quite expensive. You will examine the three different locations for a garden area of 20 m^2 and decide which option will be cheapest to border with the chosen brick.

Using Geometer's Sketchpad, open the provided file and examine the three different options.

Option 1: A Circular Garden with Fountain

The beautification committee has chosen the centre of the park as its first option. They would like to build a circular garden and place a fountain in the middle.

In this case, there will only be one possible set of dimensions. Determine the radius and circumference of a circle with an area of $20m^2$.

Area = $20m^2$ Radius = _____ Circumference = _____

Option 2: A Rectangular Garden Surrounding the Park Sign

The committee's second option is near the entrance of the park. Here, the garden would surround the welcome sign to the park.

For this case, you must consider at least 8 different dimensions for the length and width of the garden. Remember, you would like an area of $20m^2$ and you are trying to *minimize* the perimeter of the garden.

Length	Width	Area	Perimeter

Hypothesis: Based on your trials, what do you think are the *best* dimensions of the garden for this option?

5.5.1: Park Beautification (Continued)

Option 3: A Right Triangular Garden Near the Playground

The committee's final option is in a corner of the park near the playground.

For this case, you must consider at least 8 different dimensions for the height and base of the right triangle. Remember, you would like an area of $20m^2$ and you are trying to *minimize* the perimeter of the garden.

Height	Base	Area	Perimeter

Hypothesis: Based on your trials, what do you think are the *best* dimensions of the garden for this option?

Selecting the Best Option

You have now had a chance to investigate several different options for the placement of a garden in the park. What recommendation would you make to the park beautification committee based on your research? Be sure to justify your recommendation.

5.5.2: Calculating Dimensions: Maximum Area / Minimum Perimeter

1. Determine the perimeter of a circle with an area of 144 inches^2.

2. Determine the minimum perimeter of a rectangle with an area of 18 mm^2.

3. Determine the minimum perimeter of a right-angled triangle with an area of 52 ft^2.

Maximizing Area and Minimizing Perimeter: Practise

Complete the following questions on lined paper.

1. Determine the maximum area for each of the following shapes.
   a) a circle with a circumference of 15 cm.
   b) a rectangle with a perimeter of 34 m.
   c) a right-angled triangle with a perimeter of 18 mm.

2. Determine the minimum perimeter for each of the following shapes.
   a) a circle with an area of 256 yards^2.
   b) a rectangle with an area of 92 inches^2.
   c) a right-angled triangle with an area of 13 dam^2.

3. Determine the minimum amount of fencing needed for each of the following: (Note: you will have to create a table for these ones)
   a) A rectangle garden to be fenced on all 4 sides, with an area of 9 yards^2.
   b) A rectangle garden only to be fenced on **3** sides, with an area of 18 yards^2.
   c) A rectangle garden only to be fenced on **2** sides, with an area of 36 yards^2.
   d) What do you notice about your answers from a – c? Why do you think this is the case?

5.5.2: Calculating Dimensions: Maximum Area / Minimum Perimeter (Teacher Notes)

1. Determine the perimeter of a circle with an area of 144 inches^2.

$A = \pi r^2$
$144 = \pi r^2$
$\frac{144}{\pi} = \frac{\pi r^2}{\pi}$

$\frac{144}{\pi} = r^2$
$\sqrt{\frac{144}{\pi}} = r$
$r = 6.8 inches$

$C = 2\pi r$
$C = 2\pi(6.8)$
$C = 42.7 inches$

2. Determine the minimum perimeter of a rectangle with an area of 18 mm^2.

To get optimal dimensions, we must have a square, so l=w

$A = l^2$
$18 = l^2$
$\sqrt{18} = l$
$l = 4.2 mm$

$P = 4l$
$P = 4(4.2)$
$P = 16.8 mm$

3. Determine the minimum perimeter of a right-angled triangle with an area of 52 ft^2.

$A = \frac{bh}{2}$
$52 = \frac{bh}{2}$
$104 = bh$

but b=h since we have an isosceles triangle

$104 = b^2$
$\sqrt{104} = b$
$b = 10.2 feet$

We know from yesterday's work:

$P = 3.414b$
$P = 3.414(10.2)$
$P = 34.8 feet$