# Unit 2: Day 1: Linear and Quadratic Functions

**MCT 4C**

Minds On: 15
Action: 20
Consolidate: 40
Total =75 min

**Learning Goals**

- Activate prior knowledge by reviewing features of linear and quadratic functions such as what the graphs look like, how could the graphs be described, and whether or not the graphs represent functions.
- Consolidate understanding of domain and range
- Learn end behaviour terminology and the definition of the leading coefficient

**Materials**

- PPT 2.1.1
- BLM 2.1.1
- BLM 2.1.2
- BLM 2.1.3
- LCD projector

**Assessment Opportunities**

## Minds On…

**Individuals or Pairs → Activity**

Students complete BLM 2.1.1 using prior knowledge of linear and quadratic functions

**Curriculum Expectations/Observations/Checklist**

Observe students as they complete BLM 2.1.1 and assess their prior knowledge, in particular what they recall about linear and quadratic functions. Use this information to determine the depth in which these functions need to be reviewed as a class.

## Action!

**Whole Class → Discussion**

Students share the results of the activity with the class to verify the correct answers.
Students will discuss any concerns that arise when the answers are presented.

Review necessary terminology and introduce the new terminology (end behaviour and leading coefficient) using the PowerPoint presentation PPT 2.1.1

**Mathematical Process Focus:**
Reflecting - **Reflect** on prior knowledge of linear and quadratic functions;
Connecting – Students **connect** prior content to new terminology introduced

## Consolidate Debrief

**Small Group → Activity**

Students will work in small groups (2 to 4) on the assigned function.
Students fill in their information on the BLM 2.1.2 worksheet for the function that they are assigned by the teacher. Students may have to approximate the y-values for the range.

**Whole Class → Presentation**

Students present their results to the class, allowing all groups to fill in BLM 2.1.2 for all of the functions.

*Concept Practice*

**Home Activity or Further Classroom Consolidation**

Complete BLM 2.1.3.

If necessary, review how to graph a function using a table of values.

# 2.1.1: Match It!

**Match each given function with the graph on the right-hand side.**

1. ______ $y = 2x + 1$

2. ______ $y = x^2 - x - 6$

3. ______ $y = -x^2 + x + 6$

4. ______ $y = 2x - 1$

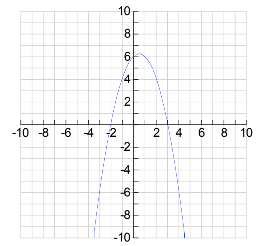
5. ______ $y = x^2 + x - 6$

6. ______ $y = -2x - 1$

7. ______ $y = -x^2 - 5x - 6$

8. ______ $y = -2x + 1$

A



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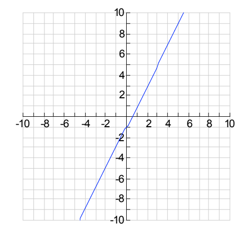

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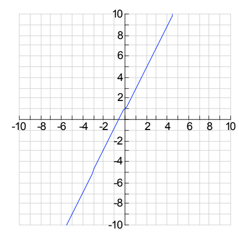

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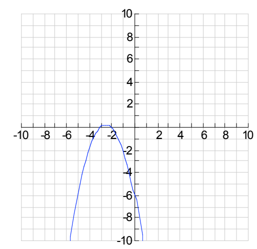

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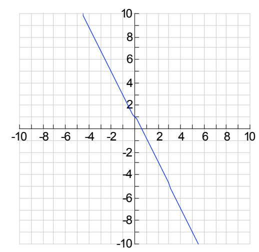

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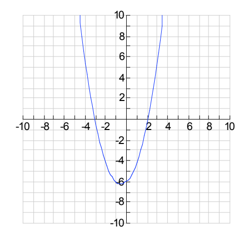

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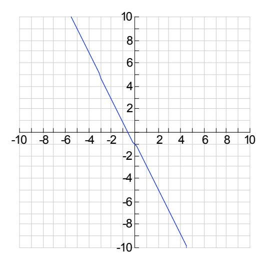

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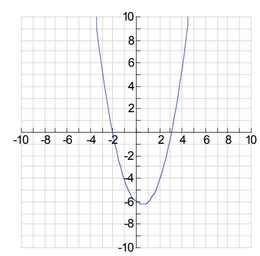


## 2.1.2: Linear and Quadratic Functions

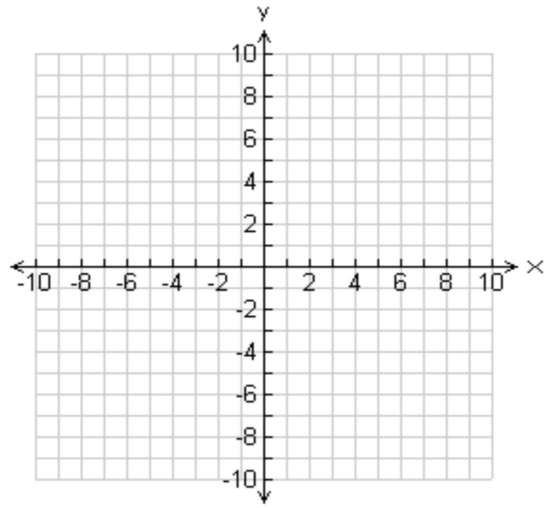
| Function | Domain and Range | Degree | Leading Coefficient | End Behaviour |
|---|---|---|---|---|
| 1. $y = 2x + 1$ | | | | |
| 2. $y = x^2 - x - 6$ | | | | |
| 3. $y = -x^2 + x + 6$ | | | | |
| 4. $y = 2x - 1$ | | | | |
| 5. $y = x^2 + x - 6$ | | | | |
| 6. $y = -2x - 1$ | | | | |
| 7. $y = -x^2 - 5x - 6$ | | | | |
| 8. $y = -2x + 1$ | | | | |

# 2.1.3: Linear and Quadratic Functions – Practice

For each of the given functions, sketch the graph of the relation, creating a table of values if necessary. Use the graph and the equation to fill in the table relating to each graph.

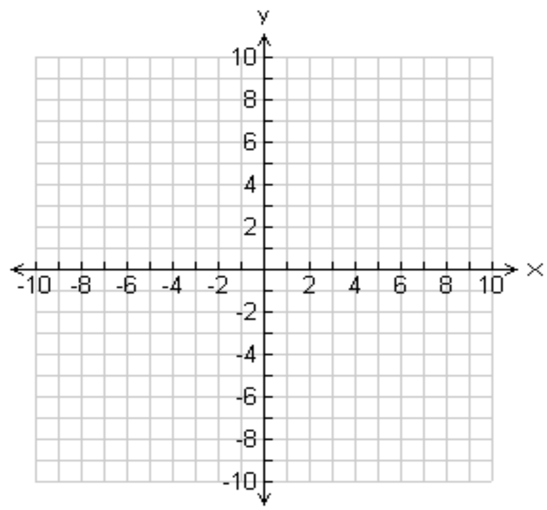
1. $y = -3x + 2$

| | |
|---|---|
| Domain | |
| Range | |
| Degree | |
| Sign of Leading Coefficient | |
| End Behaviour | |
| Is the relation a function? | |




2. $y = -x^2 - 5$

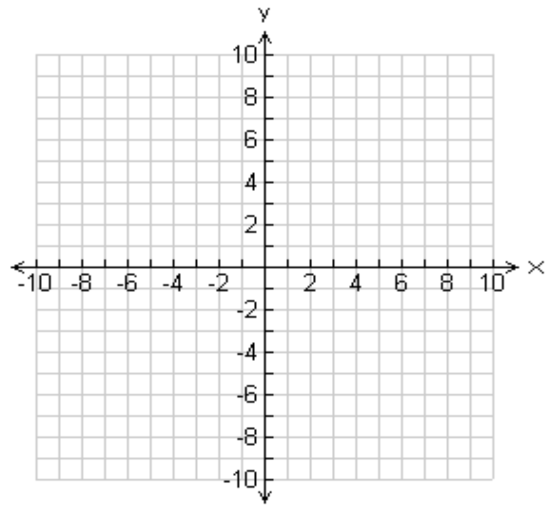
| | |
|---|---|
| Domain | |
| Range | |
| Degree | |
| Sign of Leading Coefficient | |
| End Behaviour | |
| Is the relation a function? | |



# 2.1.3: Linear and Quadratic Functions – Practice (continued)

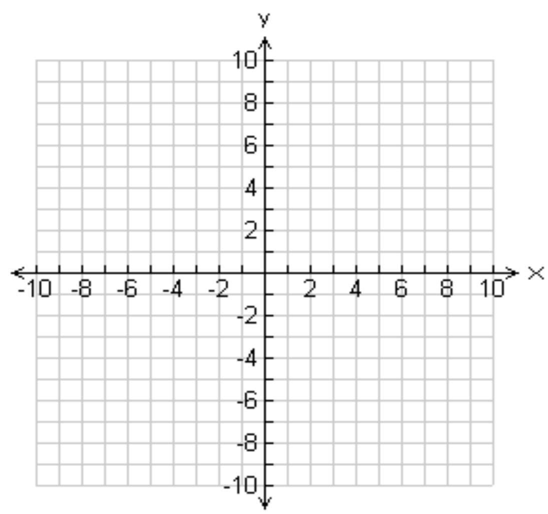
3. $y = \frac{1}{2}x + 4$

| | |
|---|---|
| Domain | |
| Range | |
| Degree | |
| Sign of Leading Coefficient | |
| End Behaviour | |
| Is the relation a function? | |




4. $y = x^2 - 6x + 9$

| | |
|---|---|
| Domain | |
| Range | |
| Degree | |
| Sign of Leading Coefficient | |
| End Behaviour | |
| Is the relation a function? | |




5. Is it possible to graph a line of the form $y = mx + b$ that will not result in a function? Explain your reasoning.

6. Is it possible to graph a quadratic relation of the form $y = ax^2 + bx + c$ that will not result in a function? Explain your reasoning.

## Unit 2: Day 2: A Higher Degree

**MCT 4C**

Minds On: 10

Action: 60

Consolidate: 5

Total =75 min

**Learning Goals**:

- Investigate cubic and quartic functions and explain why they are functions.
- Graph the equations of cubic and quartic functions and investigate end behaviours, domain and range.
- Describe end behaviours and the impact of the leading coefficient (positive and negative values).

**Materials**

- BLM 2.2.1
- BLM 2.2.2
- BLM 2.2.3
- Graphing calculators

**Assessment Opportunities**

### Minds On…

**Pairs → Activity**

Pairs are given a section of BLM 2.2.1 and take a few minutes to consider the relations. Each pair will have two functions from $y = x$, $y = x^2$, $y = x^3$, and $y = x^4$ and must hypothesize about how the relations will be the same and how they will be different, with reference to the following items:

effect of leading coefficient, end behaviour, degree, maximum number of x-intercepts, domain, range, whether or not the relations are functions

Students will fill in the Venn Diagram in BLM 2.2.1 with their results.

Teacher should copy BLM 2.2.1 and cut the page in half. There should be one Venn diagram for each pair of students in the class.

### Action!

**Small Groups → Jigsaw (Home Groups)**

In groups of 4 , students should each select a different chart to complete on BLM 2.2.2.

**Small Groups → Jigsaw (Expert Groups)**

Students form expert groups according to the chart that selected. Each group creates graphs for each function in BLM 2.2.1 using a graphing calculator. Students make conclusions about the behaviour of cubic and quartic functions based on the investigations.

**Curriculum Expectations/Oral Questions/Rubric**

Assess students as they complete BLM 2.2.1 in their expert groups on their understanding of the key components of functions.

**Small Groups → Jigsaw (Home Groups)**

Students regroup too share their results. Students use the information shared to complete all charts and questions.

**Mathematical Process Focus:**

Reasoning and Proving – Students use their **reasoning** skills to determine patterns related to properties of polynomials.

### Consolidate Debrief

**Pairs→ Activity**

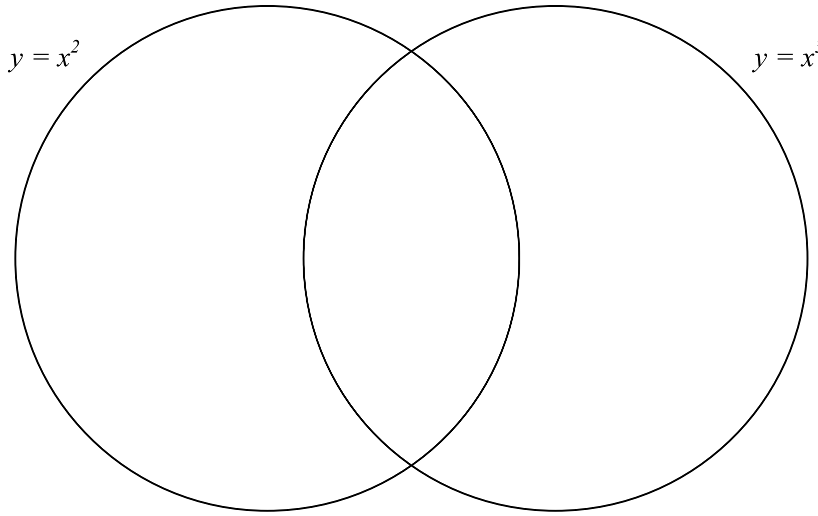
Students work in the same pairs from the Minds On activity and compare their hypotheses with the actual results. They should make any necessary changes to their Venn Diagram.

*Reflection*

**Home Activity**

Complete a Frayer Model (BLM 2.2.3) for cubic and quartic functions.

## 2.2.1: Comparing Functions by Degree

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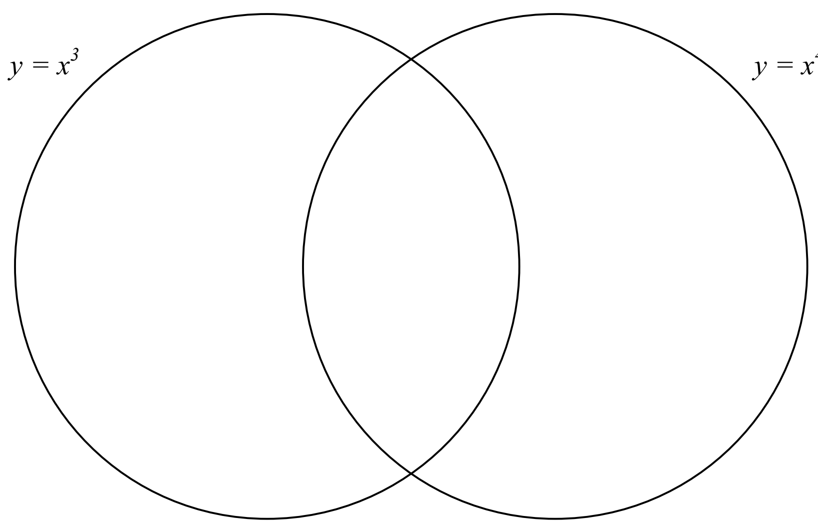
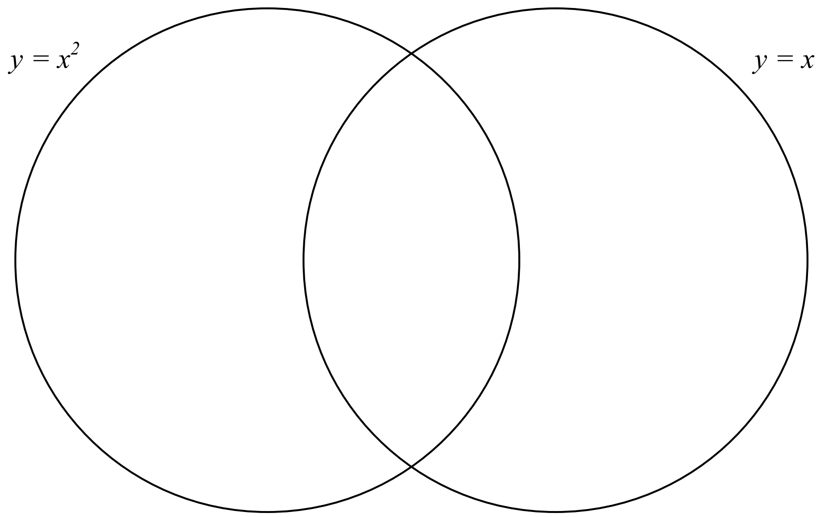
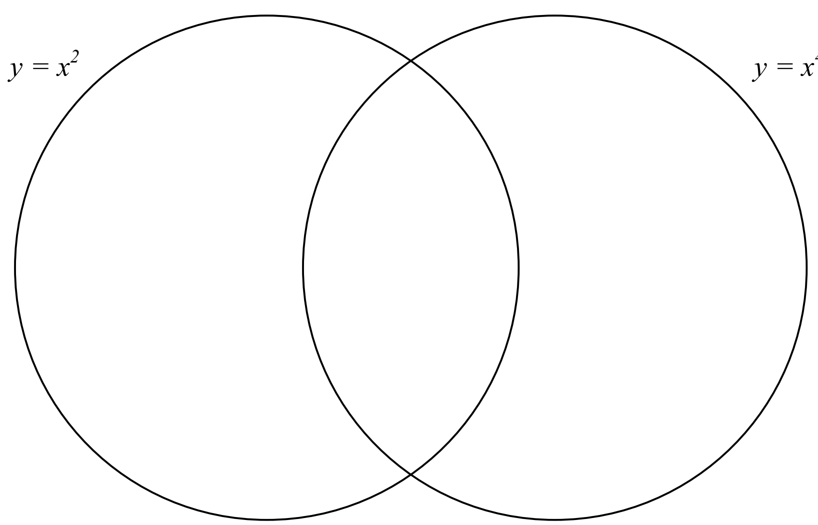
## 2.2.1: Comparing Functions by Degree



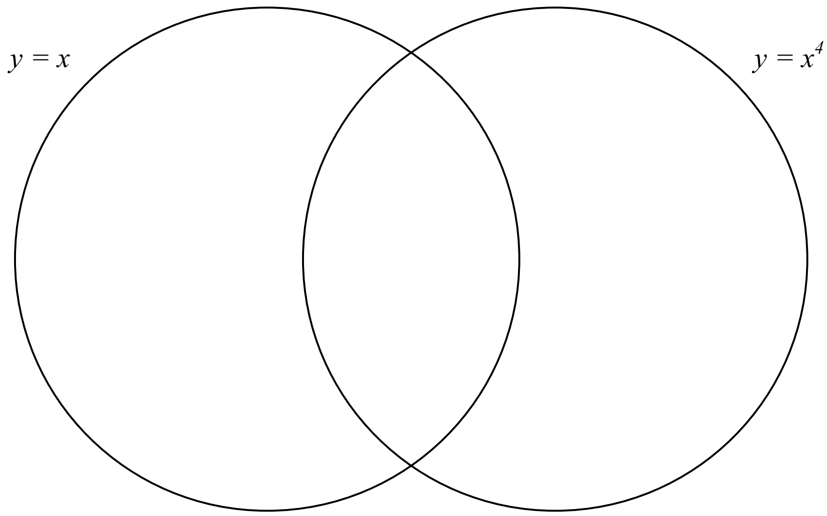
## 2.2.1: Comparing Functions by Degree




## 2.2.1: Comparing Functions by Degree



## 2.2.1: Comparing Functions by Degree

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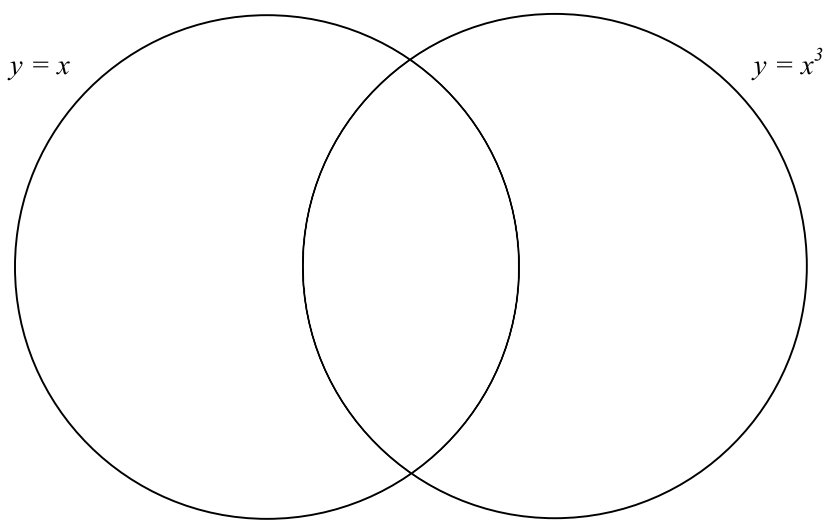
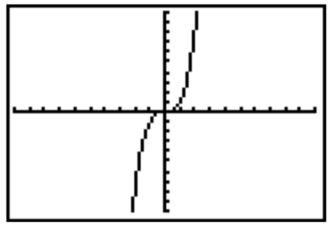
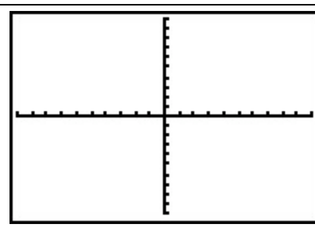
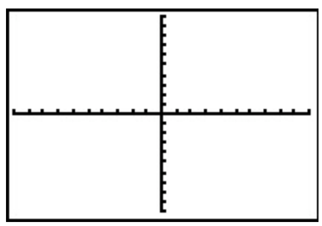
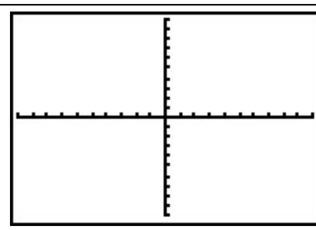
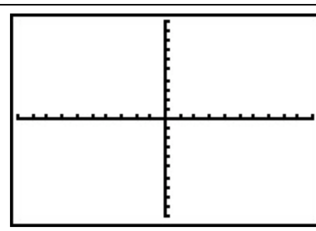
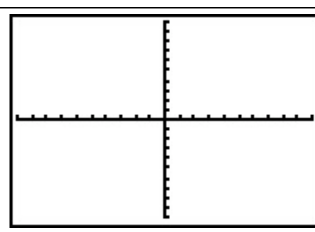
## 2.2.1: Comparing Functions by Degree



# 2.2.2: Investigation - Cubic and Quartic Functions Part 1

1. Look at each equation and state the value of the leading coefficient. Fill in the information in the column specified below. The first one is done for you.
2. Graph each of the following functions on a graphing calculator and sketch a copy of what you see on the given grids. Use the sketch to fill in the other columns in the table. Again, the first one is done for you.

| Equation | Leading coefficient | Graph | Number of x-intercepts | End behaviour |
|---|---|---|---|---|
| $y = x^3$ | 1 |  | 1 | As $x \to \infty$, $y \to \infty$ and as $x \to -\infty$, $y \to -\infty$ |
| $y = x^3 - 2x^2$ | |  | | |
| $y = 2x^3 - 3$ | |  | | |
| $y = x^3 + 3x^2 - x - 3$ | |  | | |
| $y = 3x^3 - 9x$ | |  | | |
| $y = 3x^3 + x$ | |  | | |

## 2.2.2: Investigation - Cubic and Quartic Functions Part 1 (continued)

Refer to the chart that you just completed on the previous page to answer questions 3 – 7.

3. What is true about the leading coefficient of all of the polynomials?

4. What is true about the degree of all of the polynomials?

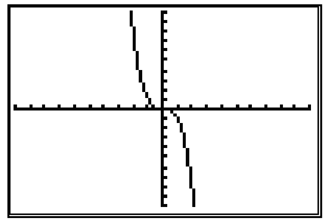
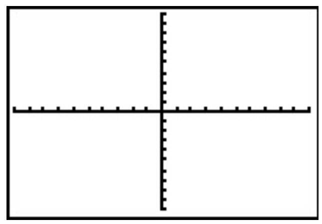
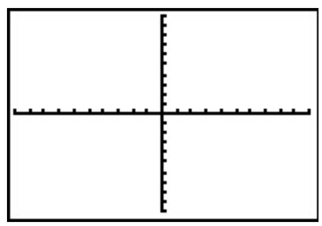
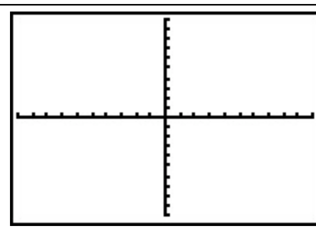
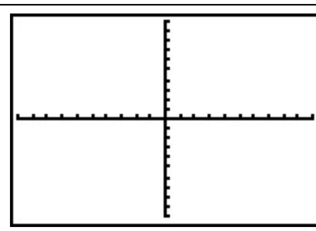
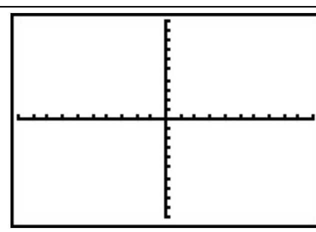
5. What is true about the end behaviour of all of the polynomials?

6. What is the maximum number of x-intercepts for all of the polynomials?

7. Do the graphs of the relations represent functions? Explain.

# 2.2.2: Investigation - Cubic and Quartic Functions Part 2

1. Look at each equation and state the value of the leading coefficient. Fill in the information in the column specified below. The first one is done for you.
2. Graph each of the following functions on a graphing calculator and sketch a copy of what you see on the given grids. Use the sketch to fill in the other columns in the table. Again, the first one is done for you.

| Equation | Leading coefficient | Graph | Number of x-intercepts | End behaviour |
|---|---|---|---|---|
| $y = -x^3$ | -1 |  | 1 | As $x\to\infty$, $y\to-\infty$ and as $x\to-\infty$, $y\to\infty$ |
| $y = -x^3 - 2x^2$ | |  | | |
| $y = -2x^3 - 1$ | |  | | |
| $y = -2x^3 + x^2 + 2x - 1$ | |  | | |
| $y = -2x^3 - 6x$ | |  | | |
| $y = -2x^3 + 4x$ | |  | | |

## 2.2.2: Investigation - Cubic and Quartic Functions Part 2 (continued)

Refer to the chart that you just completed on the previous page to answer questions 3 – 8.

3. What is true about the leading coefficient of all of the polynomials?

4. What is true about the degree of all of the polynomials?

5. What is true about the end behaviour of all of the polynomials?

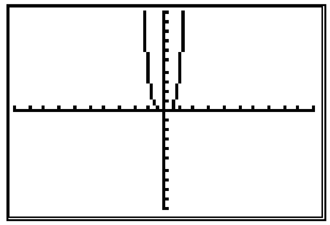
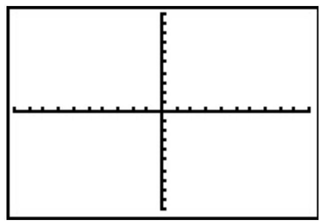
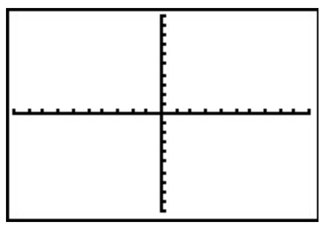
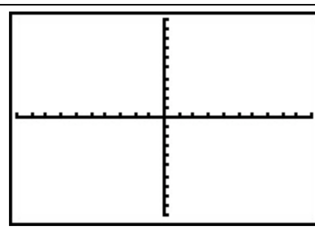
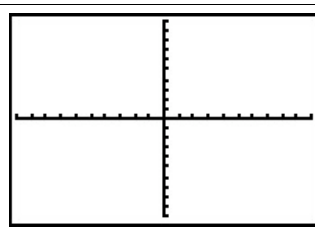
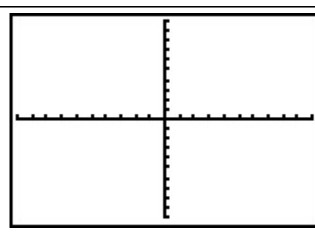
6. What is the maximum number of x-intercepts for all of the polynomials?

7. Do the graphs of the relations represent functions? Explain.

8. What impact do the signs of the leading coefficients seem to have on the graphs from the first chart and the second chart?

# 2.2.2: Investigation - Cubic and Quartic Functions Part 3

1. Look at each equation and state the value of the leading coefficient. Fill in the information in the column specified below. The first one is done for you.
2. Graph each of the following functions on a graphing calculator and sketch a copy of what you see on the given grids. Use the sketch to fill in the other columns in the table. Again, the first one is done for you.

| Equation | Leading coefficient | Graph | Number of x-intercepts | End behaviour |
|---|---|---|---|---|
| $y = 3x^4$ | 3 |  | 1 | As $x \to \infty$, $y \to \infty$ and as $x \to -\infty$, $y \to \infty$ |
| $y = x^4 - 3x^3$ | |  | | |
| $y = 2x^4 + 4$ | |  | | |
| $y = x^4 - 5x^2 + 4$ | |  | | |
| $y = x^4 - x^3 - 3x^2 + 3x$ | |  | | |
| $y = x^4 - 4x$ | |  | | |

## 2.2.2: Investigation - Cubic and Quartic Functions Part 3 (continued)

Refer to the chart that you just completed on the previous page to answer questions 3 – 7.

3. What is true about the leading coefficient of all of the polynomials?

4. What is true about the degree of all of the polynomials?

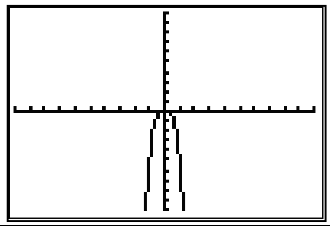
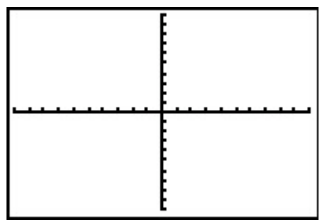
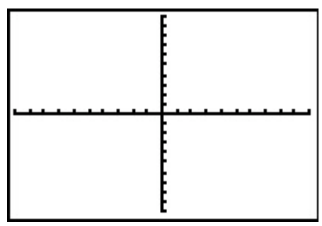
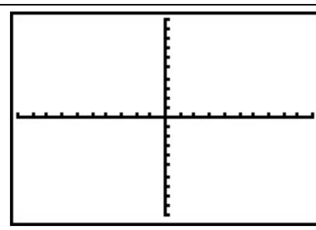
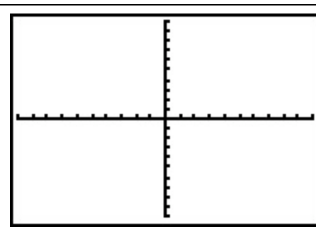
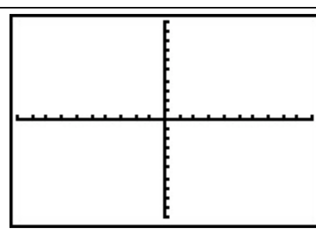
5. What is true about the end behaviour of all of the polynomials?

6. What is the maximum number of x-intercepts for all of the polynomials?

7. Do the graphs of the relations represent functions? Explain.

# 2.2.2: Investigation - Cubic and Quartic Functions Part 4

1. Look at each equation and state the value of the leading coefficient. Fill in the information in the column specified below. The first one is done for you.
2. Graph each of the following functions on a graphing calculator and sketch a copy of what you see on the given grids. Use the sketch to fill in the other columns in the table. Again, the first one is done for you.

| Equation | Leading coefficient | Graph | Number of x-intercepts | End behaviour |
|---|---|---|---|---|
| $y = -5x^4$ | -5 |  | 1 | As $x \to \infty$, $y \to -\infty$ and as $x \to -\infty$, $y \to -\infty$ |
| $y = -x^4 - 3x^3$ | |  | | |
| $y = -2x^4 + 4$ | |  | | |
| $y = -x^4 + x^3 + 4x^2 - 4x$ | |  | | |
| $y = -x^4 + 4x^2$ | |  | | |
| $y = -x^4 + 5x^2 - 4$ | |  | | |

# 2.2.2: Investigation - Cubic and Quartic Functions Part 4
## (continued)

Refer to the chart that you just completed on the previous page to answer questions 3 – 9.

3. What is true about the degree of all of the polynomials?

4. What is true about the leading coefficient of all of the polynomials?

5. What is true about the end behaviour of all of the polynomials?

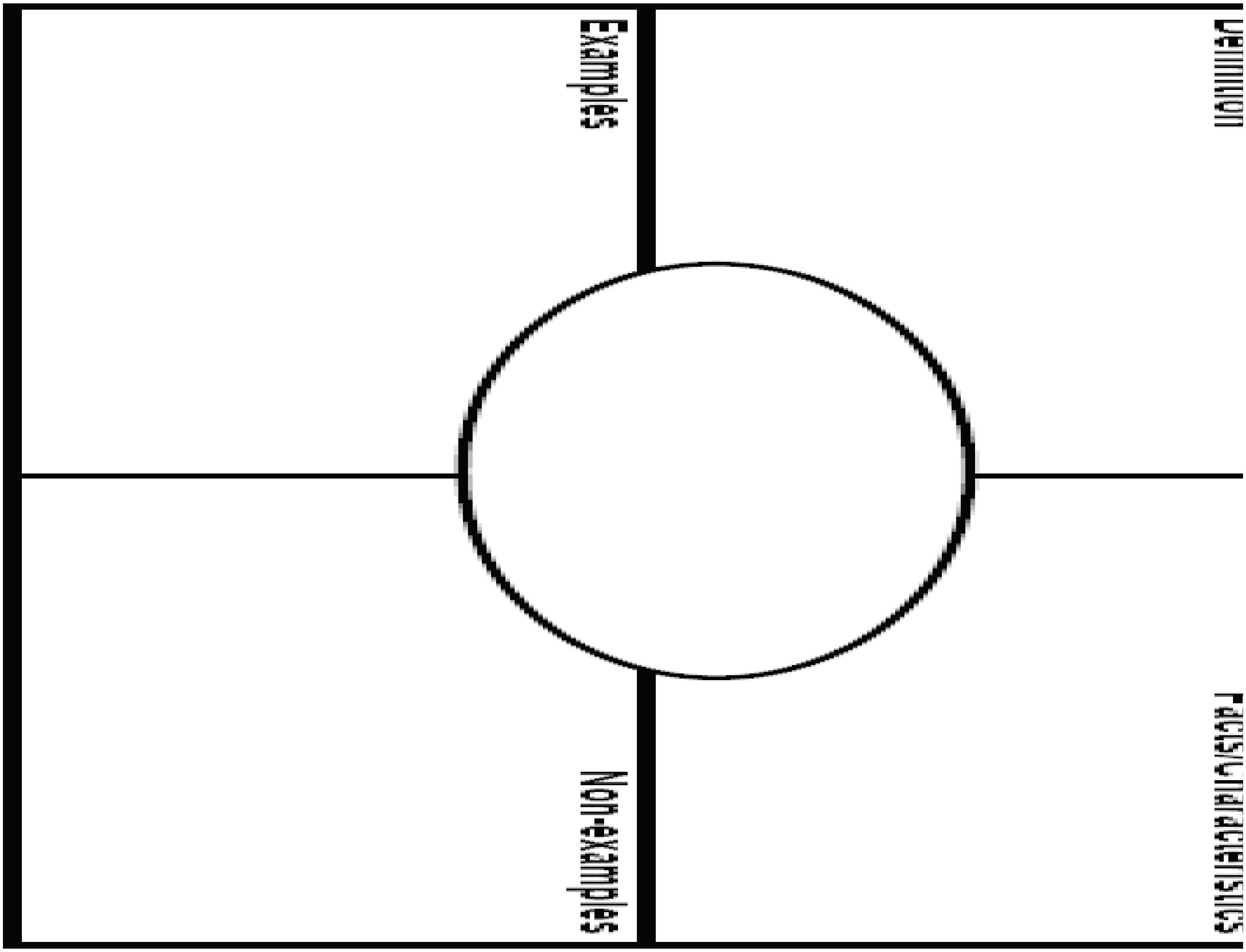
6. What is the maximum number of x-intercepts for all of the polynomials?

7. Do the graphs of the relations represent functions? Explain.

8. What impact do the signs of the leading coefficients seem to have on the graphs in the third and fourth chart?

9. In general, what is the relationship between the degree of a polynomial and the maximum number of x-intercepts for the polynomial?

# 2.2.3: Frayer Model - Cubic and Quartic Functions



## Unit 2: Day 3: Cubic and Quartic Functions

MCT 4C

| Time | Learning Goals | Materials |
|---|---|---|
| Minds On: 20<br>Action: 30<br>Consolidate:25<br>Total=75 min | **Learning Goals:**<br>• Investigate cubic and quartic functions.<br>• Consolidate results from the previous activity to reinforce end behaviours, domain, range, sign of the leading coefficient, and maximum number of zeros for cubic and quartic functions. | **Materials**<br>• PPT 2.3.1<br>• LCD Projector<br>• BLM 2.3.1<br>• BLM 2.3.2<br>• BLM 2.3.3 |

| | | Assessment Opportunities |
|---|---|---|
| **Minds On…** | **Pairs → Activity**<br>Students complete BLM 2.3.1.<br><br>**Whole Class → Discussion**<br>Discuss student responses to BLM 2.3.1. Do not ask for support for answers as students will have an opportunity to revisit their answers after the Action portion of the lesson. | This activity will give the students an opportunity to share their findings from the work they completed the day before. You will be able to assess what is to be focused on today, based on students' demonstrated understanding of a concept. |
| **Action!** | **Whole Class → Demonstration**<br>The teacher should reiterate the topics mentioned in the previous lesson using the PowerPoint presentation PPT 2.3.1<br><br>**Individual → Note Making**<br>During the PowerPoint presentation, PPT 2.3.1, students should make notes on their copy of BLM 2.3.2<br><br>**Mathematical Process Focus:**<br>Reflecting – **Reflect** on findings from the previous lesson to the PowerPoint shown. | You may want to have students fill in portions of the BLM 2.3.2 before the slide appears in the presentation.<br><br>You may want to ask "Who answered 'yes' to question one?" and then ask "Why?". Repeat the process for the rest of the questions. |
| **Consolidate Debrief** | **Pairs → Activity**<br>Each pair from the Minds On portion of this lesson will revisit BLM 2.3.1 and make any necessary changes and/or additions.<br><br>**Whole Class → Assessment/Discussion**<br>Discuss BLM 2.3.1 by asking for examples that support a true statement and non-examples that support a false statement.<br><br>**Mathematical Process/Discussion/Mental Note**<br>As students share their answers and provide examples to support their thinking, assess reasoning skills. | |
| *Concept Pratice Reflection* | **Home Activity or Further Classroom Consolidation**<br>In your own words, describe/define the following:<br>• Effect of leading coefficient<br>• End behaviour<br>• Degree<br>• Maximum number of x-intercepts<br>• Domain<br>• Range<br>• Are the relations functions?<br>Complete BLM 2.3.3 | |

# 2.3.1: True or False?

**Read each of the following statements and circle True or False below.**

1. A degree of four is the highest degree that a cubic function can have. **True** **False**
2. A cubic function has to have at least one x-intercept. **True** **False**
3. The leading coefficient for the function $2x^3 – 5x^2 + 10x + 3$ is $2x^3$. **True** **False**
4. The domain and range for all quartic functions will never be restricted. **True** **False**
5. The domain and range for all cubic functions will never be restricted. **True** **False**
6. A quartic function can have three x-intercepts. **True** **False**
7. A cubic function can have four x-intercepts. **True** **False**
8. A quartic function can resemble a quadratic function when graphed. **True** **False**
9. Sometimes a quartic relation is not a function. **True** **False**
10. As $x \rightarrow \infty$, $y \rightarrow -\infty$ and $x \rightarrow -\infty$, $y \rightarrow -\infty$ means the graph is starting on the left in quadrant 3 and ending on the right in quadrant 4. **True** **False**
11. The leading coefficient does not influence the graph of a quartic function. **True** **False**
12. The x-intercepts do not change when the graph is reflected on the x-axis. **True** **False**
13. The function $y = (x – 3)^2 (x + 1)$ would have two x-intercepts. **True** **False**
14. The function $y = (x – 2)(x – 2)(x – 2)$ would have three x-intercepts. **True** **False**
15. The function $y = (x + 4)^2(x – 4)^2$ would create a “W” shape. **True** **False**
16. The end behaviour for the function $y = -x^4 + 2x^3 – x^2 + 3x – 10$ would be as $x \rightarrow \infty$, $y \rightarrow \infty$ and $x \rightarrow -\infty$, $y \rightarrow \infty$. **True** **False**

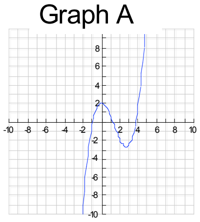
## 2.3.2: PowerPoint Presentation

SLIDE 5:

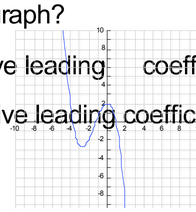
# Cubic Polynomials

Look at the two graphs and discuss the questions given below.

Graph A

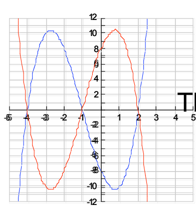


Graph B



1. How can you check to see if both graphs are functions?
2. How many x -intercepts do graphs A & B have?
3. What is the end behaviour for each graph?
4. Which graph do you think has a positive leading coefficient? Why?
5. Which graph do you think has a negative leading coefficient? Why?

SLIDE 6:



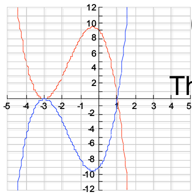
# Cubic Polynomials

The following chart shows the properties of the graphs on the left.

| Equation Factored form & Standard form | X-Intercepts | Sign of Leading Coefficient | End Behaviour | Domain and Range |
|---|---|---|---|---|
| Factored $y=(x+1)(x+4)(x-2)$ Standard $y=x^3+3x^2-6x-8$ | | | | |
| Factored $y=-(x+1)(x+4)(x-2)$ Standard $y=-x^3-3x^2+6x+8$ | | | | |

## 2.3.2: PowerPoint Presentation (continued)

SLIDE 7:

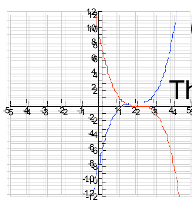


# Cubic Polynomials

The following chart shows the properties of the graphs on the le ft.

| Equation<br>Factored form &<br>Standard form | X-Intercepts | Sign of Leading Coefficient | End Behaviour | Domain and Range |
|---|---|---|---|---|
| Factored<br>$y=(x+3)^2(x-1)$<br>Standard<br>$y=x^3+5x^2+3x-9$ | | | | |
| Factored<br>$y=-(x+3)^2(x-1)$<br>Standard<br>$y=-x^3-5x^2-3x+9$ | | | | |

SLIDE 8:



# Cubic Polynomials

The following chart shows the properties of the graphs on the le ft.

| Equation<br>Factored form &<br>Standard form | X-Intercepts | Sign of Leading Coefficient | End Behaviour | Domain and Range |
|---|---|---|---|---|
| Factored<br>$y=(x-2)^3$<br>Standard<br>$y=x^3-6x^2+12x-8$ | | | | |
| Factored<br>$y=-(x-2)^3$<br>Standard<br>$y=-x^3+6x^2-12x+8$ | | | | |

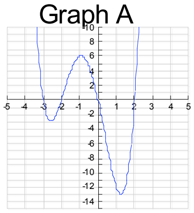
## 2.3.2: PowerPoint Presentation (continued)

SLIDE 9:

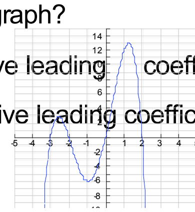
# Quartic Polynomials

Look at the two graphs and discuss the questions given below.

Graph A

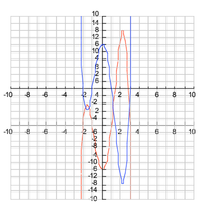


Graph B



1. How can you check to see if both graphs are functions?
2. How many x -intercepts do graphs A & B have?
3. What is the end behaviour for each graph?
4. Which graph do you think has a positive leading coefficient? Why?
5. Which graph do you think has a negative leading coefficient? Why?

SLIDE 10:



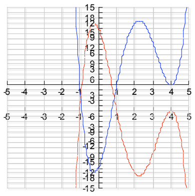
# Quartic Polynomials

The following chart shows the properties of the graphs on the left.

| Equation Factored form & Standard form | X-Intercepts | Sign of Leading Coefficient | End Behaviour | Domain and Range |
|---|---|---|---|---|
| Factored $y=(x-3)(x-1)(x+1)(x+2)$ Standard $y=x^4-x^3-7x^2+x+6$ | | | | |
| Factored $y=-(x-3)(x-1)(x+1)(x+2)$ Standard $y=-x^4+x^3+7x^2-x-6$ | | | | |

## 2.3.2: PowerPoint Presentation (continued)

SLIDE 11:

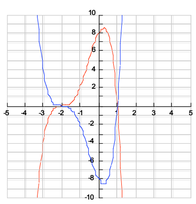


# Quartic Polynomials

The following chart shows the properties of the graphs on the left.

| Equation<br>Factored form & Standard form | X-Intercepts | Sign of Leading Coefficient | End Behaviour | Domain and Range |
|---|---|---|---|---|
| Factored<br>$y=(x-4)^2(x-1)(x+1)$<br>Standard<br>$y=x^4-8x^3+15x^2+8x-16$ | | | | |
| Factored<br>$y=-(x-4)^2(x-1)(x+1)$<br>Standard<br>$y=-x^4+8x^3-15x^2-8x+16$ | | | | |

SLIDE 12:



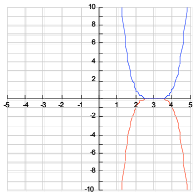
# Quartic Polynomials

The following chart shows the properties of the graphs on the left.

| Equation<br>Factored form & Standard form | X-Intercepts | Sign of Leading Coefficient | End Behaviour | Domain and Range |
|---|---|---|---|---|
| Factored<br>$y=(x+2)^3(x-1)$<br>Standard<br>$y=x^4+5x^3+6x^2-4x-8$ | | | | |
| Factored<br>$y=-(x+2)^3(x-1)$<br>Standard<br>$y=-x^4-5x^3-6x^2+4x+8$ | | | | |

# 2.3.2: PowerPoint Presentation (continued)

SLIDE 13:



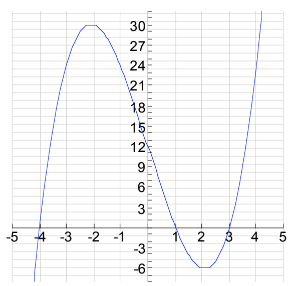
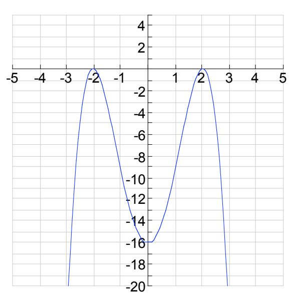
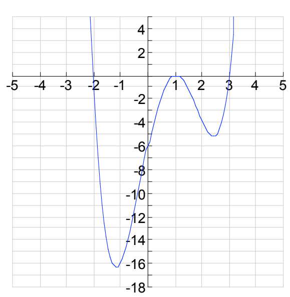
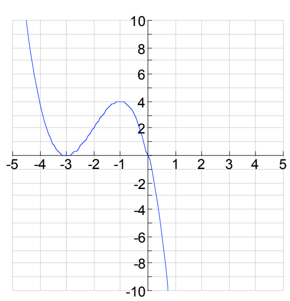
## Quartic Polynomials

The following chart shows the properties of the graphs on the le ft.

| Equation<br>Factored form & Standard form | X-Intercepts | Sign of Leading Coefficient | End Behaviour | Domain and Range |
|---|---|---|---|---|
| Factored<br>$y=(x-3)^4$<br>Standard<br>$y=x^4-12x^3+54x^2-108x+81$ | | | | |
| Factored<br>$y=-(x-3)^4$<br>Standard<br>$y=-x^4+12x^3-54x^2+108x-81$ | | | | |

# 2.3.3: Properties of Cubic and Quartic Functions

1. Based on the graphs given, complete the chart.

| | Sign of Leading Coefficient | Number of x-intercepts | End Behaviour | Domain | Range | Type of Function Cubic or Quartic? |
|---|---|---|---|---|---|---|
|  | | | | | | |
|  | | | | | | |
|  | | | | | | |
|  | | | | | | |

# Unit 2: Day 10: Using Polynomial Functions to Model Real Life Data

MCT 4C

| | | |
|---|---|---|
| Minds On: 10<br>Action: 60<br>Consolidate:5<br>Total =75 min | **Learning Goals**<br>• Activate prior knowledge by reviewing features of the graphing calculator such as regression analysis, if needed.<br>• Apply knowledge of polynomial functions to a set of data to determine an appropriate model for the data. | **Materials**<br>• BLM 2.10.1<br>• BLM 2.10.2<br>• Graphing calculators<br>• Overhead panel or TI-Smartview, if available |

**Assessment Opportunities**

| | | |
|---|---|---|
| Minds On… | **Pairs → Activity**<br>Students complete BLM 2.10.1 to review how to plot data and use the graphing calculator for regression analysis. | This activity is only needed if students have not recently used the graphing calculators for regression analysis. |
| Action! | **Individuals or Pairs→ Activity**<br>Students complete BLM 2.10.2 using the graphing calculators.<br><br>Note: It is very difficult to find data that fits a cubic or quartic function. The given data does not really fit a quartic model outside of the given data. The website calculator.maconstate.edu/cubic_modeling_lesson/index.html has a set of data that fits a cubic model nicely. However, the statistics are US based. Teachers may decide to use this data instead of the given data.<br><br>Mathematical Process/Observation/Checklist<br>Assess students' ability to reflect and connect as they complete the task.<br><br>**Mathematical Process Focus:**<br>Reflecting **- Reflect** on understanding of regression to complete the journal for the home activity<br>Connecting – Students **connect** prior content to new terminology introduced | If time is a concern, have students complete all of the parts that require the use of the graphing calculator during class and the conclusions (#7) can be done for homework or in class the following day. |
| Consolidate Debrief | **Whole Class→ Discussion**<br>Discuss the assignment, once it has been handed in (if it is being assessed), to determine which model is the better fit. Also discuss the limitations that students found in the model and their confidence in using the model to predict values that are outside of the data range. Explain that when given a set of real-life data, there are two major considerations when determining a best-fit model for the data. The first consideration is whether or not the model is a good mathematical fit. This entails looking at the graph to see if the model is close to the data points and checking the value of $R^2$. The second consideration should be whether the fit seems reasonable based on what the data is representing. This entails looking at what happens to the function and determining if it makes sense that the data would continue to increase where the function is increasing and to decrease where the function is decreasing. | |
| *Journal* | **Home Activity**<br>Reflect on the activity and complete the following statements:<br>1. The hardest part of the assignment was…<br>2. I think I could do better if … | |

# 2.10.1: Modelling Data Using a Graphing Calculator

**Objective:** To plot the following data and model the data using an appropriate polynomial function.

| x | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|
| y | -43 | -10 | -1 | 2 | 1 | 15 |

**Clearing Lists**

When drawing a scatter plot, it is advisable to first clear all the lists – this will delete any data that is already in the lists.

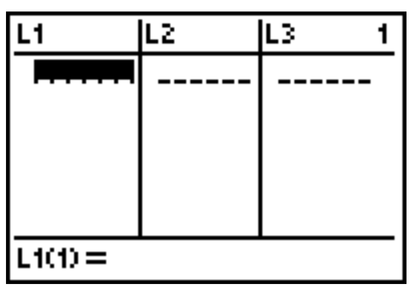
To do this, press □, □ and choose option **4: ClrAllLists** and press Enter.

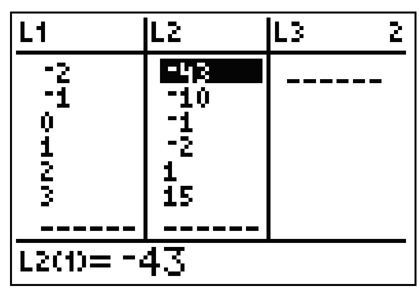


**Entering Data**

The data must be entered in lists before it can be graphed. To do this, enter the *x*-values in $L_1$ and the *y*-values in $L_2$.

Press **STAT** and then select **1:Edit.** Enter the ***x*-values** in $L_1$. Now press □ so the cursor is in $L_2$ and enter the ***y*-values**.

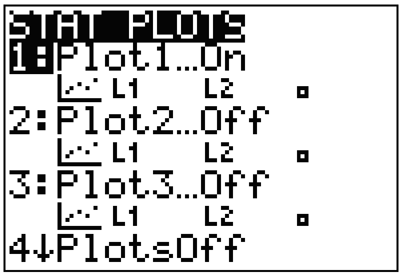



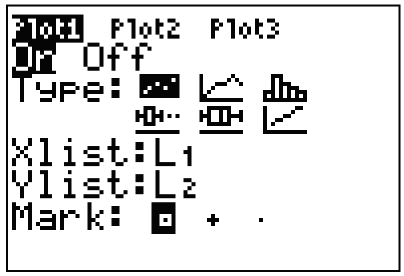
# 2.10.1: Modelling Data Using a Graphing Calculator (continued)

**Plotting Data**

To plot the data, it is necessary to turn the plotting feature of the calculator on so that it will graph the data in the lists. The lists to be used must also be specified. Remember that the x-values have been entered in $L_1$ and the y-values have been entered in $L_2$.

Press □, □ and choose **1: Plot1.** Move the cursor to On and press □ so that Plot1 is on. Note that the Xlist is $L_1$ and the Ylist is $L_2$.

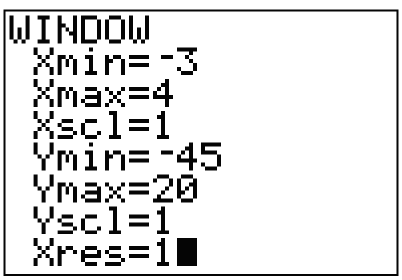


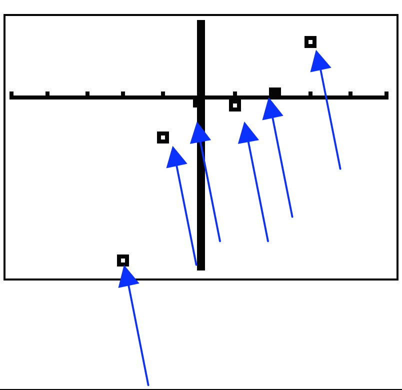

**Adjusting Viewing Window**

It is important to let the calculator know what values for x and y need to be plotted. For our data, the *x*-values range from -2 to 3 and the *y*-values range from -43 to 17.

Press □ and set the *x*- and *y*-values to include these ranges.



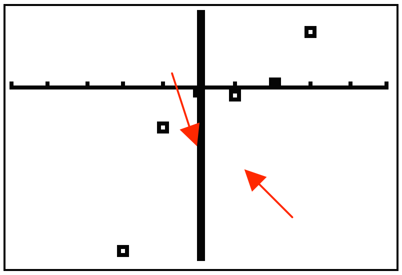

Now press □ and the scatter plot should appear. There were six points to be plotted so make sure that 6 points are visible in the graph. If this is not the case, adjust the viewing window. (The arrows on the graph indicate the six points.)



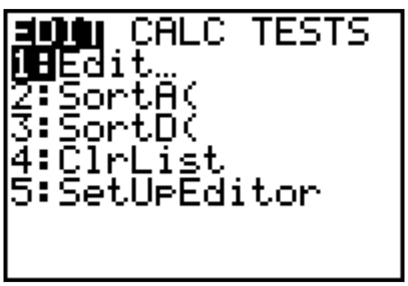
# 2.10.1: Modelling Data Using a Graphing Calculator (continued)

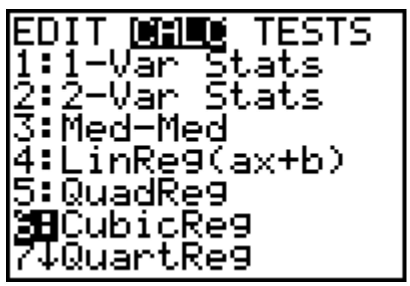
**Regression Analysis**

Now it is necessary to determine what polynomial function would be a good model for the data. Since the data goes from Q3 to Q1 and has 2 ‘turnaround’ points, it would be logical to say that a cubic function would be a good model for the data.

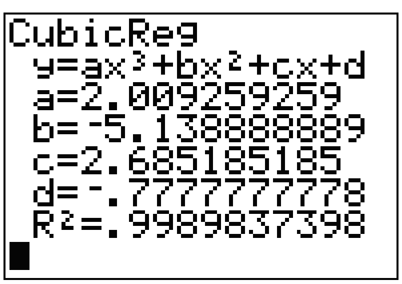


Now, to have the calculator create a cubic function that fits the data.
Press □, then □ to select the **CALC** menu, and then select **6:CubicReg** for cubic regression. Press □, □, □, □, □, □, □ and □. Your screen should look like the screen below on the far right. This will perform the regression on the data in $L_1$ and $L_2$ and store the equation of the cubic function in $Y_1$.



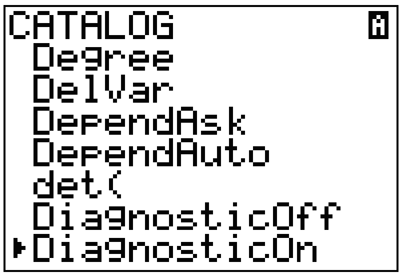





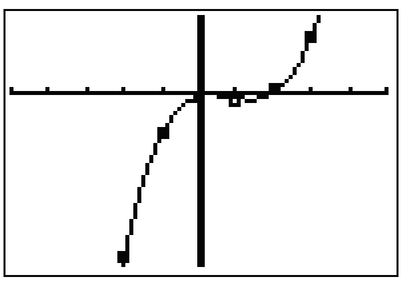



The screen to the left shows the general cubic function and the values for a, b, c, and d. It also shows a value for $R^2$. This is called the coefficient of determination. This value is useful to determine if the function is a good fit to the data – the closer this value is to 1, the better the function fits the data! Since $R^2$ is about 0.99898, this shows that the function is a great fit. If your calculator doesn’t show $R^2$, refer to the next page.

If your calculator doesn't show $R^2$, perform the following sequence of commands. Press □, □ and choose **DiagnosticOn** from the list and press □. You will have to do the regression analysis over again but this time the $R^2$ should show up. This is very helpful to assist you in determining whether a model is a good fit for data. Remember, the closer the $R^2$ value is to one, the stronger the fit. The further the $R^2$ value is from one, the weaker the fit. For example, $R^2 = 0.8$ is a reasonably good fit while $R^2 = 0.5$ is not a good fit.




Press □ to see the data points and the cubic regression function together.



As shown in the graph, the cubic function is almost a perfect fit for the data. This was also apparent from the $R^2$ value of 0.99, showing a value very close to one!

# 2.10.2: Regression, Regression, Regression

**The data in the following table lists conventional oil consumption (in thousands of barrels per day) in Ontario.**

**Conventional Oil Consumption**

| **Year** | **1998** | **1999** | **2000** | **2001** | **2002** | **2003** | **2004** | **2005** |
|---|---|---|---|---|---|---|---|---|
| **Consumption** | **3.8** | **4.2** | **4.0** | **4.3** | **3.8** | **3.4** | **2.6** | **2.4** |

**Source: www.capp.ca**

1. Enter the data into a graphing calculator and create a scatter plot.

2. Based on the scatter plot, what type of polynomial function appears to be the best fit for the data? Explain why you chose this polynomial function.

3. Find a polynomial function of degree 3 that models the data. Is this function a good fit for the data? Explain.

4. Find a polynomial function of degree 4 that models the data. Is this function a good fit for the data? Explain.

5. Choose the polynomial function that is the best fit (from #3 and 4) and give reasons for your choice. State the equation of this polynomial function.

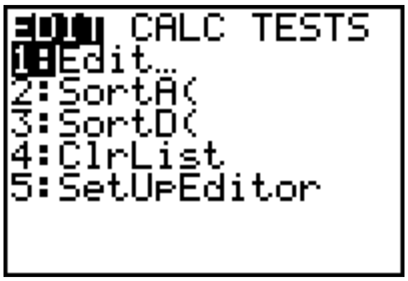
6. Use your model to estimate the conventional oil consumption in Ontario in 2007.

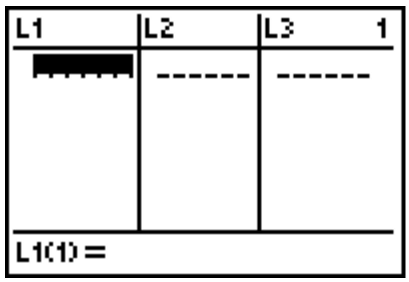
7. What do you think are limitations to the model that you created? Explain any concerns that you might have using this data to make predictions about oil consumption in other years.

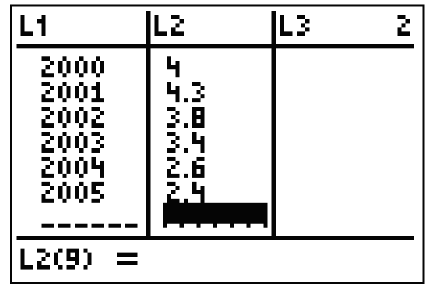
# 2.10.2: Regression, Regression, Regression (Solutions)

1. Plot the data using a graphing calculator.

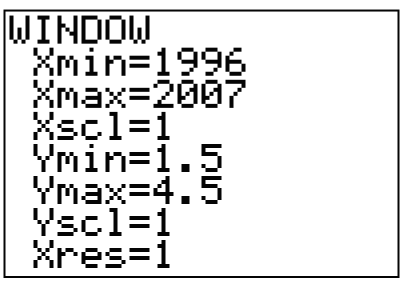
First clear all lists in the calculator. Press □ and then select **1:Edit.** Enter the ***x*-values** (the years) in $L_1$. Now press the □ so the cursor is in $L_2$ and enter the ***y*-values** (the consumption).



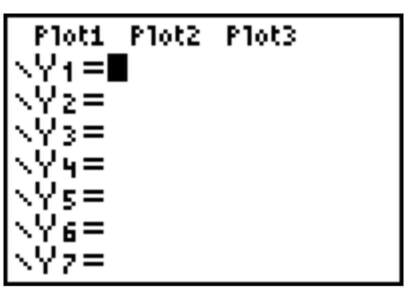


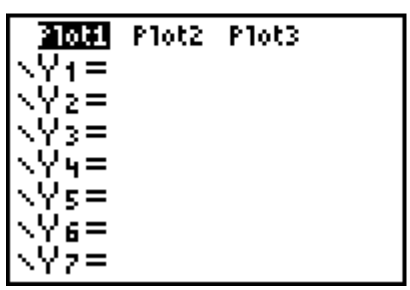


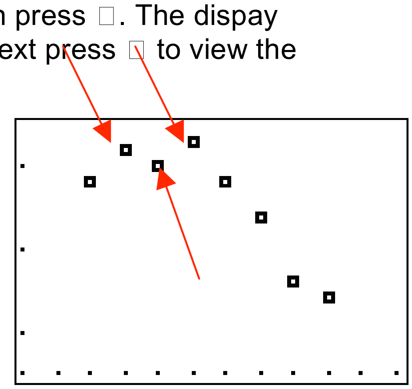

Press □ and enter the values to match the window below.




Next, to graph the **<u>scatterplot</u>,** Press □. Notice if **PLOT1** is highlighted. If it is not as in the first image below, press the □ to highlight **PLOT1,** and then press □. The dispay should now look like the second image below. In either case, next press □ to view the scatterplot.



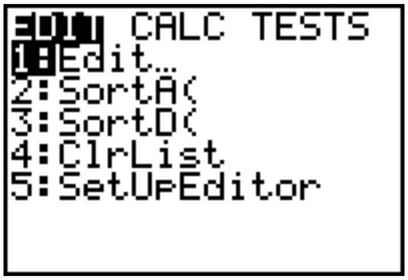


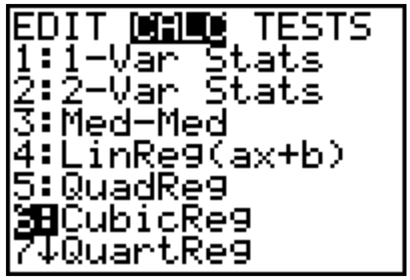


2. Answers may vary here. The data looks like it could be an ‘M’ shape with three turnaround points (see diagram) so the best fit would probably be a quartic function.

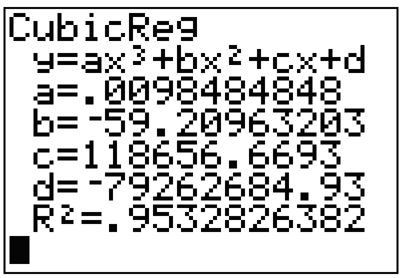
## 2.10.2: Regression, Regression, Regression (Solutions - Continued)

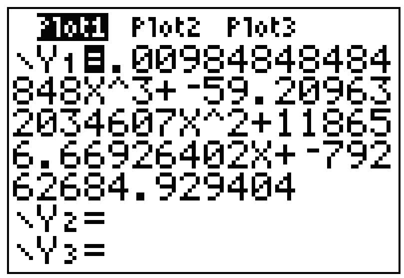
3. Press □, then the □ to select the **CALC** menu, and then select **6:CubicReg** for cubic regression. Press □, □, □, □, □, □, □ and □. This will perform the regression and store the equation of the cubic function in $Y_1$. Press □ to see the data points and the cubic function together.

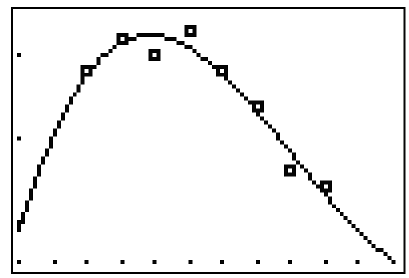






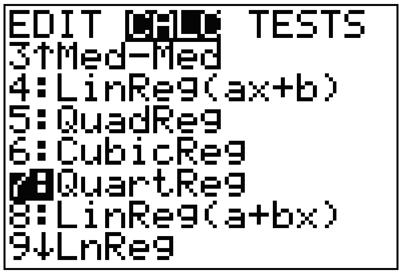



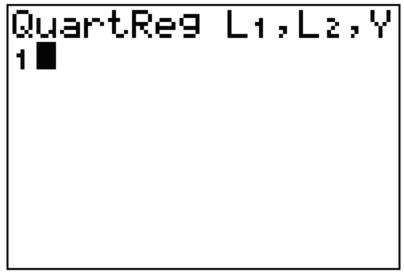



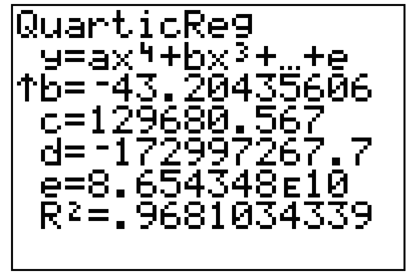
The data is a pretty good fit as shown in the graph, the points are mostly on the curve and by the value of $R^2$, it is about 0.95 which is quite close to 1. However, the graph does not match the data for the year 2000, where the production of oil decreased. The graph shows an increase in the year 2000.

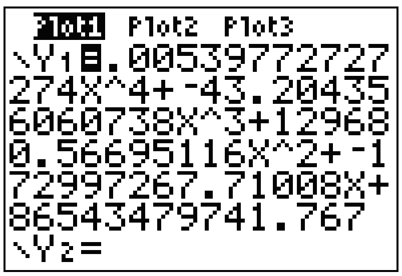
## 2.10.2: Regression, Regression, Regression (Solutions - Continued)

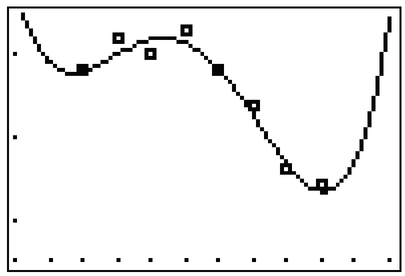
4. The steps are quite similar to those in #3 but quartic regression must be chosen instead of cubic regression.



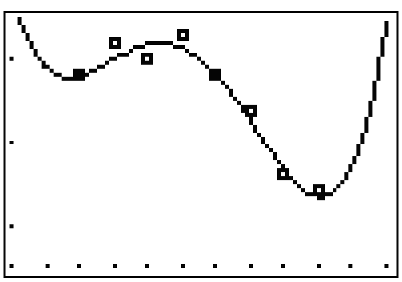



Although the quartic function does not fit the data perfectly, it is closer to the points than the cubic function. Also, the $R^2$ value is about 0.968 and this value is closer to 1 than the value obtained for the cubic function.

5. The quartic model is a better fit.

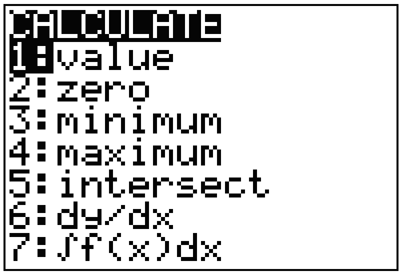


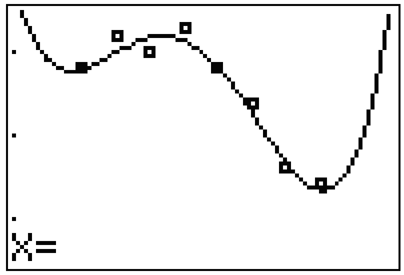
The equation for the quartic model is:

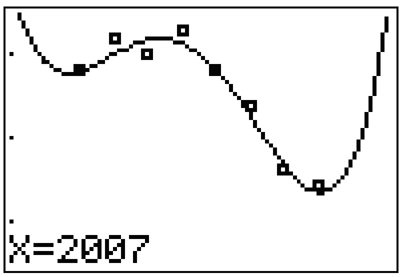
$$y = 0.00539772727274x^4 - 43.204356060738x^3 + 129680.56695116x^2 - 172997267.71008x + 86543479741.767$$

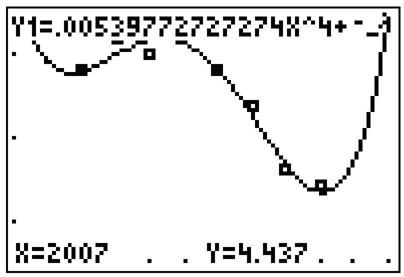
# 2.10.2: Regression, Regression, Regression (Solutions - Continued)

6. There are many ways to do this on a graphing calculator. One can use the □ button or use the calculator to determine the *y*-value when *x* = 2007. The latter method will be used here. To do this, press □, □, □, 2007, □.




Therefore, the oil consumption for 2007 is about 4.437 thousand barrels per day.

7. The model is a fairly good fit for the data that is given but it is not perfect. The quartic function also starts increasing drastically after 2005 and, without actual data for this time, it would not be reasonable to use this data for predictions after 2005. It also does not seem realistic that we will be able to increase the oil production indefinitely as illustrated in the quartic model because natural resources are limited.

Again, it is not known if the oil production level before 1998 was larger and has decreased to the value in 1998 or if the oil production level was smaller and has increased to the 1998 value. The quartic model shows that the production levels should be decreasing before 1998 and this does not seem likely unless there were huge supplies of oil that were to be extracted and these supplies were steadily depleted until a new site was found for oil around 2005, again increasing the production for a few years.

Overall, the model fits the data given but it is not reasonable to use it to extrapolate to values outside the given data with any confidence.