Unit 6: Day 10 & 11: Let's Build a Gazebo!

Minds On: 10 Action: 100 Consolidate:40 Total=150 min	**Learning Goal**: • Measure the area of a polygon shaped figure requiring use of trigonometry to determine missing sides.	**Materials** • Ruler • Protractor • several class computers with GSP installed (optional) • BLM 6.10.1

Assessment Opportunities

Minds On…	**Whole Class → Discussion & Brainstorming** Present the problem to the class as outlined on BLM 6.10.1. Students engage in a discussion of photos/brochures of different gazebo designs. Brainstorm to identify the information required to calculate the area of the gazebo floor and the amount of material needed to cover the floor. Consider costs of flooring using available flyers or information from the internet. Students discuss strategies that may be used to solve this problem. Although suggestions may include methods other than trigonometry, remind them that the problem can also be solved by applying what they have learned about trigonometry. **Mathematical Process Focus: Selecting Tools and Computational Strategies:** Students are required to select appropriate tools (sine law or cosine law) and other strategies to solve the problem	Literacy strategies used: Brainstorming Electronic resources: GSP file: U6L10GSP1.gsp U6L10GSP2.gsp U6L10GSP3.gsp U6L10GSP4.gsp Gazebo images: Many gazebo images can also be found using Google image search and in brochures
Action!	**Groups → Investigation – Part 1** Divide the class into 3, 6, or 9 groups. Assign numbers of sides to each group as follows: (group 1: 4 sides and 9 sides; group 2: 5 sides and 8 sides; group 3: 6 sides and 7 sides and then start back at group 1 again for 6 or 9 groups) Instruct students to complete Part 1 of the investigation in their groups. Circulate to check on the progress of the students This first part should be completed by the end of the first day. **Large Groups Investigation – Part 2 and 3** Have students form a larger group by joining with another group. Large groups complete parts 2 and 3 on the second day. If GSP is available, groups can be instructed to verify their findings/conclusions. **Learning Skills(Teamwork)/Observation/Mental Note:** Observe students for amount of time on task and teamwork with their partners. **Mathematical Process Focus: Selecting Tools and Computational Strategies:** Students are required to select appropriate tools and strategies to solve the problem	Hand out necessary equipment so students can draw an accurate scale diagram of the gazebo floor.
Consolidate Debrief	**Large Groups → Presentation** Large groups present their proposal to the class. **Expectations/Presentation/Anecdotal Comments:** Give students anecdotal feedback on their presentations.	
Exploration *Application*	**Home Activity or Further Classroom Consolidation** Complete work on gazebo design and add any additional information or details after presentations.	

6.10.1: Let’s Build a Gazebo

The Wildflower Conservation Area (WCA) is planning to build a gazebo at a nature centre. The gazebo will have a *diagonal width* of 24 feet. The floor must be in the shape of a regular polygon.

Mr. Build-It Wood Store has generously offered to supply all the railings and support beams needed and Ruff-Ruff Roofers has offered to supply the gazebo top; thus, WCA only has to pay for the plywood needed for the floor!

This class has been hired to suggest the number of sides the gazebo should have and determine how much plywood must be purchased for the floor.

Part 1 – Investigating the Possibilities

The first step in choosing a polygon is determining information about a variety of gazebo shapes. Your group will be in charge of looking at 2 possible gazebo designs and determining how much plywood would be needed to cover each of the floors.

Make sure you have your own copy of the solutions for both polygons – you’ll need them later!

Steps:

1. Draw a neat sketch of the polygon. Determine the centre of the polygon. Use this point to help draw identical triangles inside the polygon.
2. Determine the measure of the central vertex angle in each triangle. This angle is at the centre of the polygon.
3. Determine the length of each triangle side that joins the centre of the polygon to the outside edge. This length is the radius of a circle enclosing the polygon.
4. Now focus on one of the triangles and determine the length of the outside edge of one of the triangles. Use Sine Law for your first polygon and Cosine Law for your second.
5. Determine the height (or altitude) of this same triangle. Remember, the height is the perpendicular distance between the centre of the polygon and the outside edge of the triangle.
6. Use the information you have obtained so far to determine the area of this polygon.
7. Each of the plywood sheets is 4’ x 8’ in size. Determine the number of sheets needed to cover the floor assuming that whole sheets must be purchased and that the builders will minimize wasted materials. What assumptions must you make in order to perform this calculation?

Part 2 – Large groups to analyse polygon data

Join with another group. Ensure that you now have information about four possible gazebos.

Steps:

1. Create a chart to summarize the information you and your group members have brought into this group.
2. Create a scatter plot showing area vs. the number of sides in a regular polygon. You may wish to calculate the area of a 3-sided polygon first.
3. Does your scatter plot follow a trend? If so describe the trend.
4. What restrictions would you put on the graph? I.e. can you have any number of sides for your polygon? Will your graph continue to climb infinitely upwards?
5. What does your graph tell you about the shape or number of sides needed to create a maximum flooring area? Does this answer seem reasonable to you? Would a gazebo made this way be easy or difficult to build? Explain.
6. Most gazebos are either 6 sided or 8 sided with 8 sided being the most popular. Why do you think this is the case?

Part 3 – Making a Decision

Now that you have examined all the data, it's time for you to make a decision about the number of sides the gazebo should have.

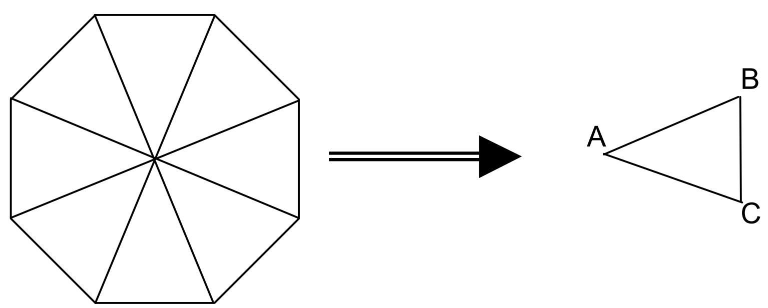
Create a short proposal to The Wildflower Conservation Area. Include the following in your proposal:

1. The number of sides the gazebo should have
2. The area of the gazebo floor and the number of pieces of plywood that will need to be purchased.
3. A paragraph justifying why you think the Gazebo should have the number of sides you suggest. Hint: Use the information & conclusions found in part 2 in your paragraph.

6.10.1: Let’s Build a Gazebo (Part 1 Solutions)

Octagon

1. Draw a neat sketch of a regular octagon. Determine the centre of the octagon. Use this point to help draw eight identical triangles inside the octagon.

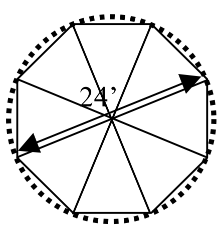


2. Determine the measure of the central vertex angle in each triangle. This angle is at the centre of the octagon.

There are 8 central vertex angles. The measure of one angle is: $A = \dfrac{360°}{8} = 45°$

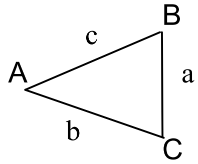
3. Determine the length of each triangle side that joins the centre of the octagon to the outside edge.

The diameter of the octagon is 24’. The lengths AB and AC are both 12’ because both B and C are on the radius of the circle passing through all vertices of the pentagon. Each of these vertices is a radius distance from the center.



4. Now focus on one of the triangles. Use the Sine Law or the Cosine Law to determine the length of the outside edge of the triangle.

For △ABC:



$\angle A = 45°$

$AB = c = 12'$

$BC = a = 12'$

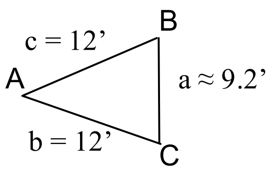
Using the Cosine Law:

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$a^2 = (12)^2 + (12)^2 - 2(12)(12)\cos 45°$$
$$a^2 \approx 84.353$$
$$a \approx 9.2$$

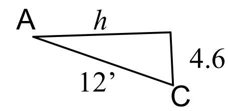
Therefore, each outside edge is approximately 9.2’.

6.10.1: Let's Build a Gazebo (Part 1 Solutions Cont)

5. Determine the height (or altitude) of this same triangle using the Pythagorean Theorem. The height is the perpendicular distance between the centre of the octagon and the outside edge of the triangle.



ΔABC is isosceles so the height is measured from A to the midpoint



$$h^2 + (4.6)^2 = (12)^2$$
$$h^2 = 144 - 21.16$$
$$h^2 = 122.84$$
$$h \approx 11.1$$

Therefore the height is approximately 11.1'.

6. Determine the area of this triangle and then use that value to determine the total area of the regular octagonal floor.

Area of a triangle $= \dfrac{(base)(height)}{2}$

$= \dfrac{(9.2)(11.1)}{2}$

$= 51.06$

Total gazebo area = 8 triangles x 51.06 sq. ft. = 408.48 sq. ft.

Therefore the area of this triangle is approximately 408.5 sq. ft.

7. Each of the plywood sheets is 4' x 8' in size. Determine the number of sheets needed to cover the floor assuming that Bill and Joanne must buy whole sheets and that they will minimize wasted materials.

Area of one plywood sheet = 4' x 8 ' = 32 sq. ft.

# of sheets needed $= \dfrac{408.5\ sq.ft.}{32\ sq.ft.} \approx 12.8$

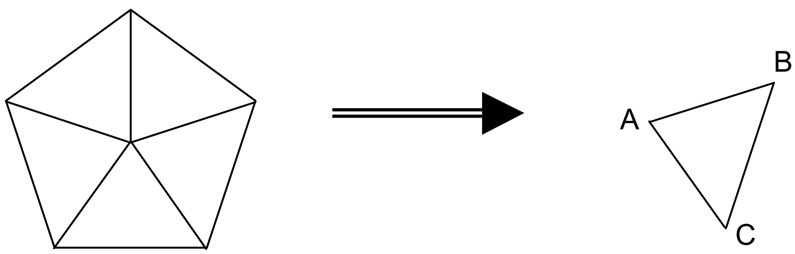
Therefore, Bill and Joanne need to buy 13 full sheets of plywood.

6.10.1: Let's Build a Gazebo (Part 1 Solutions Cont)

Pentagon

1. Draw a neat sketch of a regular pentagon. Determine the centre of the pentagon. Use this point to help draw seven identical triangles inside the pentagon.

   Note that the total number of degrees in a polygon = (# of sides - 2) x 180º
   So, a pentagon has (5 – 2) x 180º = 540º
   Each one of the five outside angles of the pentagon measures approx. 108º.

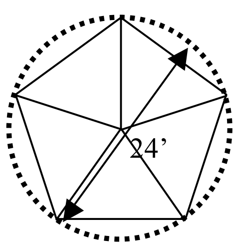


2. Determine the measure of the central vertex angle in each triangle. This angle is at the centre of the pentagon.

   There are 5 central vertex angles. The measure of one angle is: $A = \frac{360°}{5} \approx 72°$

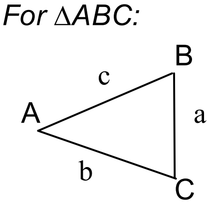
3. Determine the length of each triangle side that joins the centre of the pentagon to the outside edge.

   The diameter of the pentagon is 24'. The lengths AB and AC are both 12' because both B and C are on the radius of the circle passing through all vertices of the pentagon. Each of these vertices is a radius distance from the center.



4. Now focus on one of the triangles. Use the Sine Law or the Cosine Law to determine the length of the outside edge of the triangle.

For ΔABC:



$\angle A = 72°$

$AB = c = 12'$

$BC = a = 12'$

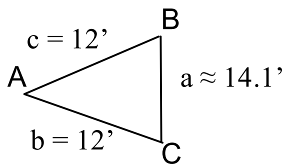
Using the Cosine Law:

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$a^2 = (12)^2 + (12)^2 - 2(12)(12)\cos 72°$$
$$a^2 \approx 199.00$$
$$a \approx 14.1$$

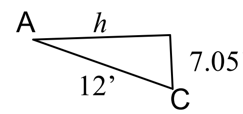
Therefore, each outside edge is approximately 14.1'.

6.10.1: Let’s Build a Gazebo (Part 1 Solutions Cont)

5. Determine the height (or altitude) of this same triangle using the Pythagorean Theorem. The height is the perpendicular distance between the centre of the pentagon and the outside edge of the triangle.



ΔABC is isoceles so the height is measured from A to the midpoint of BC.



$$h^2 + (7.05)^2 = (12)^2$$
$$h^2 = 144 - 49.7025$$
$$h^2 = 94.2975$$
$$h \approx 9.7$$

Therefore the height is approximately 9.7’.

6. Determine the area of this triangle and then use that value to determine the total area of the regular pentagonal floor.

Area of a triangle $= \dfrac{(base)(height)}{2}$

$$= \frac{(14.1)(9.7)}{2}$$
$$= 68.39$$

Total gazebo area ≈ 5 triangles x 68.39 sq. ft. ≈ 341.95 sq. ft.

Therefore the area of the gazebo is approximately 342.0 sq. ft.

7. Each of the plywood sheets is 4’ x 8’ in size. Determine the number of sheets needed to cover the floor assuming that Bill and Joanne must buy whole sheets and that they will minimize wasted materials.

Area of one plywood sheet = 4’ x 8 ‘ = 32 sq. ft.

# of sheets needed $= \dfrac{342.0\ sq.ft.}{32\ sq.ft.} \approx 10.7$

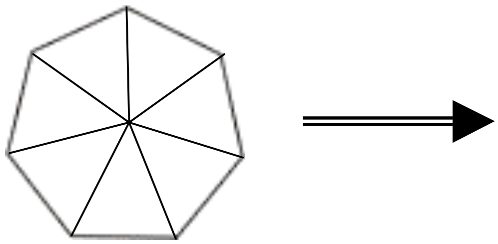
Therefore, Bill and Joanne need to buy 11 full sheets of plywood.

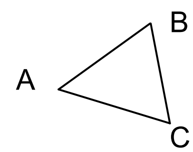
6.10.1: Let's Build a Gazebo (Part 1 Solutions Cont)

Heptagon

8. Draw a neat sketch of a regular heptagon. Determine the centre of the heptagon. Use this point to help draw seven identical triangles inside the heptagon.

   Note that the total number of degrees in a polygon = (# of sides - 2) x 180º
   So, a heptagon has (7 – 2) x 180º = 900º
   Each one of the seven outside angles of the heptagon measures approx. 129.6º.



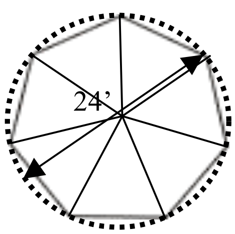


9. Determine the measure of the central vertex angle in each triangle. This angle is at the centre of the heptagon.

   There are 7 central vertex angles. The measure of one angle is: $A = \dfrac{360°}{7} \approx 51.4°$

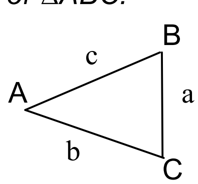
10. Determine the length of each triangle side that joins the centre of the heptagon to the outside edge.

    The diameter of the heptagon is 24'. The lengths AB and AC are both 12' because both B and C are on the radius of the circle passing through all vertices of the heptagon. Each of these vertices is a radius distance from the center.



11. Now focus on one of the triangles. Use the Sine Law or the Cosine Law to determine the length of the outside edge of the triangle.

For ΔABC:



$\angle A = 51.4°$

$AB = c = 12'$

$BC = a = 12'$

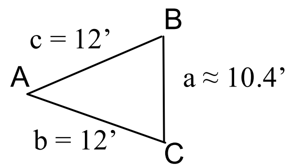
Using the Cosine Law:

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$a^2 = (12)^2 + (12)^2 - 2(12)(12)\cos 51.4°$$
$$a^2 \approx 108.32$$
$$a \approx 10.4$$

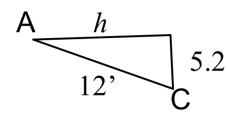
Therefore, each outside edge is approximately 10.4'.

6.10.1: Let's Build a Gazebo (Part 1 Solutions Cont)

12. Determine the height (or altitude) of this same triangle using the Pythagorean Theorem. The height is the perpendicular distance between the centre of the heptagon and the outside edge of the triangle.



ΔABC is isoceles so the height is measured from A to the midpoint of BC.



$$h^2 + (5.2)^2 = (12)^2$$
$$h^2 = 144 - 27.04$$
$$h^2 = 116.96$$
$$h \approx 10.8$$

Therefore the height is approximately 10.8'.

13. Determine the area of this triangle and then use that value to determine the total area of the regular heptagonal floor.

Area of a triangle $= \dfrac{(base)(height)}{2}$

$= \dfrac{(10.4)(10.8)}{2}$

$= 56.16$

Total gazebo area = 7 triangles x 56.16 sq. ft. = 393.12 sq. ft.

Therefore the area of this triangle is approximately 393.1 sq. ft.

14. Each of the plywood sheets is 4' x 8' in size. Determine the number of sheets needed to cover the floor assuming that Bill and Joanne must buy whole sheets and that they will minimize wasted materials.

Area of one plywood sheet = 4' x 8 ' = 32 sq. ft.

# of sheets needed $= \dfrac{393.1\, sq.ft.}{32\, sq.ft.} \approx 12.3$

Therefore, Bill and Joanne need to buy 13 full sheets of plywood.