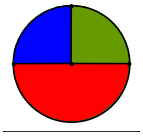
Day 13: Exploring the Pythagorean Relationship using *The Geometer's Sketchpad®*

Grade 8



Description

- Use *The Geometer's Sketchpad* ® to investigate the Pythagorean relationship.
- Apply the Pythagorean relationship.

Materials

- carpenter's triangle
- *The Geometer's Sketchpad 4* ®
- BLM 13.1
- GSP file: Pythagorean Relationship

Assessment Opportunities

Minds On…

Whole Class → Connections to Careers and Posing a Question

Curriculum Expectation and Learning Skills/Portfolio/Marking Scheme and Checklist: Collect student work on BLM 12.3 for assessment.

Use a carpenter's triangle as an example of a tool. Explain that it is based on the 3:4:5 Pythagorean Triple (a set of 3 whole numbers which are the lengths of the sides of a right-angled triangle) used by carpenters to ensure that their walls are "square."

The 3:4:5 Pythagorean Triple was also used by the Egyptians in the building of pyramids.

Using students' entries on BLM 12.2, identify Pythagorean Triples and start a cumulative class list to be augmented as the lessons continue.

Draw student attention to the 3:4:5 and 6:8:10 triples. Might there be a relationship between these two? (multiples) Can we use this to generate other triples?

Extension: Find another Pythagorean Triple besides the 3:4:5, 6:8:10, and 5:12:13 encountered on BLM 12.2, e.g., 8:15:17

Hypothesize whether or not the Pythagorean relationship is true for all types of triangles. How could we confirm or refute your hypothesis? What tool could we use to test the hypothesis?

Action!

Pairs → Guided Exploration

Mixed-ability pairs use *The Geometer's Sketchpad*® and follow instructions on BLM 13.1 to discover that the Pythagorean relationship is unique to right-angled triangles.

Learning Skills (Co-operation with others)/Question & Answer/Checklist: Observe students as they work through the activity.

Students who finish early can develop and explore "what if" questions they can pose in relation to the Pythagorean relationship.

Consolidate Debrief

Whole Class → Demonstrating Understanding

Use GSP file: The Pythagorean Relationship (p. 47) to consolidate student understanding and to introduce them to different visual proofs.

Say, "Now that we understand what the Pythagorean relationship is and know that it is unique to right-angled triangles, we want to see where it applies to real life situations." Return to the original problem from Lesson 11 and assign it.

Application
Concept Practice

Home Activity or Further Classroom Consolidation

Solve the window and plywood problem from Day 11.

13.1: The Pythagorean Relationship – Investigation

Construct a Right Triangle using the following steps

- Select GRID from the overhead menu and click on "snap to grid."
- Select the POINT TOOL from the side menu to construct a point where the x-axis and y-axis intersect on the grid.
- Create another point at (0, 4) which is four steps up the vertical or y-axis.
- Create a third point at (3, 0), three steps along the horizontal or x-axis.
- Highlight the three points you have just constructed by holding down the shift button while clicking on each with the SELECT TOOL.
- Click on CONSTRUCT and then select "Segment."
- You should now have a right-angled triangle. Use the LABELLING TOOL to name each vertex A, B and C respectively.

Construct a Square on each of the three sides of the triangle

- Double click on a vertex (look for a "bulls eye"- this marks that point as the centre of rotation).
- Select an adjacent side and the point at the end of that segment
- Click on TRANSFORM, select "Rotate."
- Choose 90 degrees (or –90 degrees, depending on which side was selected).
- If there is no point at the end of this line segment – select the segment, click on CONSTRUCT, select "Point on Object". Then, drag this point completely to the end of the segment.
- Double click on this newly created point (to mark it as the centre of rotation).
- Select the segment.
- Click on TRANSFORM, select "Rotate."
- Choose 90 degrees (or –90).
- Repeat until a square is constructed on each of the 3 sides.

Measure the area of each square

- Holding the SHIFT key down, point and click on each of the 4 vertices in clockwise or counter clockwise order.
- Click on CONSTRUCT.
- Select "Polygon Interior". Colour each square's interior differently if you choose.
- Click on "Measure."
- Select "Area."
- Repeat for each of the 3 squares.

Use the Geometer's Sketchpad Calculator

- Click on "Measure."
- Select "Calculate"
- Highlight the area of the smallest square.
- Click the "Add" button on the calculator.
- Highlight the area of the next smallest square.
- Click on "OK" (there is no = sign).

Look for a relationship between the values.

13.1: The Pythagorean Relationship – Investigation (continued)

Experiment

- Click and drag point any vertex other than the 90 degree one.
- Examine the area values as they change. What changes? What stays the same?
- Ask some "what if" questions here and experiment.
- Talk with your partner to further clarify any relationships you notice. Consider how these relationships might be important in mathematics. Each student writes a journal entry as a personal interpretation of the relationship.

Journal

Students use the following prompts to write and reflect upon their learning:

"After investigating the squares on the sides of right angle triangles using Geometer's Sketchpad, my partner _____ and I discovered that…
We experimented with … and found that …
We also developed the following "what if" questions.

What if?

Encourage students to develop and explore "what if" questions:
"What if the triangle is not a right angled triangle? Will the relationship still hold true?"
Students can explore this question quickly and easily with GSP.

"What if a semi-circle or some other geometric figure is built on each side of the right triangle, will the relationship still exist?
Encourage students to investigate this on GSP.

The Pythagorean Relationship (GSP file)

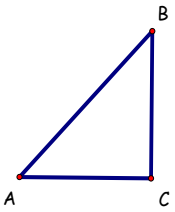
Download this file at www.curriculum.org/occ/tips/downloads.shtml

The Pythagorean Relationship

Given: Right Triangle ABC, ∠C = 90°.

Squares will be drawn on the three sides.
Click on the link below to show the squares.

Show Squares of the Sid

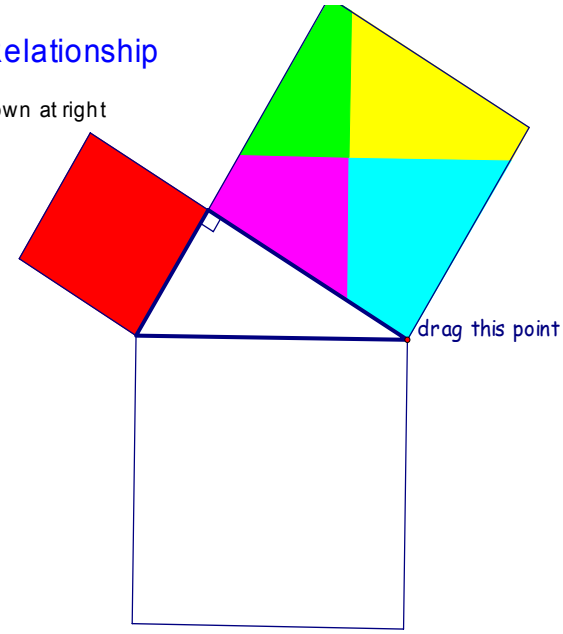


The Pythagorean Relationship

A right angled triangle is shown at right
with a right angle at C.

Show Pythagorean Theor

Reset



The Pythagorean Relationship

A right angled triangle is shown at right
with a right angle at A.

Follow the steps below.

1) Show Squares of Side

2) Show Altitude

3) Show Quadrilatera

4) Show Area Measuremer